A meaning and interpretation of minus areas of figures by means of division by zero

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February 12, 2019

Abstract: In this paper, we give a simple and surprising interpretation of minus areas of figures by means of the division by zero calculus $1/0 = 0$.

Key Words: Division by zero, division by zero calculus, $0/0 = 1/0 = z/0 = 0$, infinity, discontinuous, minus area.

2010 Mathematics Subject Classification: 00A05, 00A09

1 Introduction

We first recall the typical example for the area of a triangle.

The area S of the triangle $P_1P_2P_3$ with $P_j(x_j, y_j), j = 1, 2, 3$ is given by

$$S = \pm\frac{1}{2} \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

For the sigh, when we consider P_1, P_2, P_3 for the direction of the triangle, $+$ and in the converse (reverse) direction, $-$. This property shows a beautiful relation of geometry and algebra. We can see many and many examples as a beautiful property.

In this paper, we will give a reason why such a relation exists. In this concrete case, we can say surprisingly that the minus area shows the area of the outside of the triangle **in a new sense** that is derived naturally by the division by zero.

2 Background - division by zero

For our purpose, we will give simply the background on the division by zero.

For the long history of division by zero, see [2, 17]. The division by zero with the mysterious and long history was indeed trivial and clear as in the followings:

By the concept of the Moore-Penrose generalized solution of the fundamental equation $ax = b$, the division by zero was trivial and clear as $a/0 = 0$ in the **generalized fraction** that is defined by the generalized solution of the equation $ax = b$. Here, the generalized solution is always uniquely determined and the theory is very classical. See [3] for example.

Division by zero is trivial and clear from the concept of repeated subtraction - H. Michiwaki.

Recall the uniqueness theorem by S. Takahasi on the division by zero. See [3, 23].

The simple field structure containing division by zero was established by M. Yamada ([6]). See also Okumura [15] for a simple introduction.

Many applications of the division by zero to Wasan geometry were given by H. Okumura. See [9, 10, 11, 12, 13, 14] for example.

The division by zero opens a new world since Aristotele-Euclid. See the references for recent related results.

For the fundamental function $W = 1/z$ we did not consider any value at the origin $z = 0$, because we did not consider the division by zero $1/0$ in a good way. Many and many people consider its value by the limiting like $+\infty$ and $-\infty$ or the point at infinity as ∞. However, their basic idea comes from **continuity** with the common sense or based on the basic idea of Aristotle. – For the related Greece philosophy, see [24, 25, 26]. However, as the division by zero we will consider its value of the function $W = 1/z$ as zero at $z = 0$. We will see that this new definition is valid widely in mathematics and mathematical sciences, see ([7, 8]) for example. Therefore, the division by zero will give great impacts to calculus, Euclidian geometry, analytic geometry, complex analysis and the theory of differential equations at an undergraduate level and furthermore to our basic ideas for the space and universe.

The behavior of the space around the point at infinity may be considered by that of the origin by the linear transform $W = 1/z$ (see [1]). We thus see that

$$\lim_{z\to\infty} z = \infty, \tag{2.1}$$

however,

$$[z]_{z=\infty} = 0, \tag{2.2}$$

by the division by zero. Here, $[z]_{z=\infty}$ denotes the value of the function $W = z$ at the topological point at the infinity in one point compactification by Aleksandrov. The difference of (2.1) and (2.2) is very important as we see clearly by the function $W = 1/z$ and the behavior at the origin. The limiting value to the origin and the value at the origin are different. For surprising results, we will state the property in the real space as follows:

$$\lim_{x\to+\infty} x = +\infty, \quad \lim_{x\to-\infty} x = -\infty,$$

however,

$$[x]_{+\infty} = 0, \quad [x]_{-\infty} = 0.$$

Of course, two points $+\infty$ and $-\infty$ are the same point as the point at infinity. However, $\pm$ will be convenient in order to show the approach directions. In [7], we gave many examples for this property.

In particular, in $z \to \infty$ in (2.1), ∞ represents the topological point on the Riemann sphere, meanwhile ∞ in the left hand side in (2.1) represents the limit by means of the ϵ - δ logic. That is, for any large number M, when we take for some large number N, we have, for $|z| > N$, $|z| > M$.

3 Interpretation and conclusion

Since a general result and a special case are in the same situation, we shall state our conclusion in the special case in the introduction.

We shall consider a large disc containing the triangle $P_1P_2P_3$ with radius R with center at the origin. Then, the area $S(R)$ of $\{ \{x^2+y^2 < R^2\} \setminus \triangle P_1P_2P_3\}$ is given by

$$S(R) = \pi R^2 - S.$$

Of course,

$$\lim_{R\to\infty} S(R) = +\infty.$$

However, by the division by zero, for $R = \infty$, we obtain

$$S(\infty) = -S,$$

that means the area of the outside of the triangle.

This is the conclusion of this paper.

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