Quantization of Poisson brackets

A.Balan

February 9, 2019

Abstract

We show a quantization of the Poisson brackets. A non-commutativity is introduced and generalizes the classical brackets

1 The classical Poisson brackets

We take a classical symplectic manifold (M, w), the Poisson brackets are defined:

$$\{f, g\} = w(df, dg)$$

We have:

$$\{f, g\} = \sum_i (e_i f)(e'_i g) - (e'_i f)(e_i g)$$

2 The quantization

We consider a fiber bundle with flat connection (E, ∇). Then, the quantization is given by a symplectic form W,

$$W \in \Lambda^2(TM) \otimes End(E)^3$$

We have the non-commutativ Poisson brackets:

$$\{f, g\} = W(\nabla f, \nabla g)$$

with f, g two endomorphisms.

$$[f, g] = \sum_i (\nabla_{e_i} f) \circ (\nabla_{e'_i} g) - (\nabla_{e'_i} f) \circ (\nabla_{e_i} g)$$

$$\{f, g\} = [f, g] - [g, f]$$

Due to the fact that W is closed and that the connection is flat, we have a non-commutativ Jacobi identity.

3 Bibliography

W.Greiner J.Reinhardt, "Field Quantization", Springer, Berlin, 1996.