Quantum mechanics as a reactive probabilistic system

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Abstract

In this paper, we present the quantum mechanics as a reactive system. To show this we introduce the holography as a reactive system. Illumination of the object and reference beam create sources (hologram) that activated by the same reference beam react in a way to produce the image   Controlled chemical system react to the external or internal change generate new sources (chemical product) that diffuses in the body environment establish a new equilibrium. The droplets bouncing on a vertical vibrating fluid bath that simulate the main quantum phenomena is a reactive system between droplets and vibrating fluid. We show that quantum mechanics is a probabilistic reactive system between quantum potential and pseudo kinetic energy in which an integral is the Fisher information. We show that quantum mechanics can be built by only the probability normalization properties without assuming any other hypothesis. With join probability and Euler Lagrange equation we can found the quantum potential and with the continuity equation we show that with only probability approach it is possible to give a meaning to the Shrodinger equation without any thermo-dynamical model of quantum mechanics. The reactive system can be denoted as a morphogenetic system where the form is generated by some designed rules or properties.

1. Introduction

In this paper, we introduce the meaning of the reactive system (morphogenesis) with field and sources that react in a control way to the field in a way to generate needed constraint in the system sources field. First example of reaction system is the 'holography'[2]. Another example is 'Turing morphogenesis' by chemical system and 'diffusion'. Designed constrains of the field structure can be generated by synchronic action of field control sources by designed rules (morphogenetic software) . Sources and field interact in a control way to generate the wanted pattern or constraint. So field activate sources which action generate the wanted structured field in a reactive system. Quantum mechanics simulation (by droplets bouncing experiments on a vertical vibrating fluid path) is presented as a reactive system. Vibrating fluid is an active environment that react with the droplets and the same droplets react again with the vibrating fluid. The morphogenetic process is created by the interaction between fluid and drop object. The droplets and vibrated fluid generate behaviors those are similar to quantum mechanics. In this paper, we'd like to go beyond the case of droplets and vibrating fluid. Rather with the concept of probability only, we'll show that there exist a pseudo kinetic energy in which the integral is the Fisher information of which some minimum condition gives us the required quantum potential. Quantum potential and equation of the continuity for the probability shows the possibility to justify the meaning of the Schrodinger equation and also all the quantum mechanics structure. So the join probability in many-particles system is the fundamental and unique concept to justify the entanglement and many other phenomena in quantum mechanics.

2. Morphogenetic systems (reactive system)

Given this wave model

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 \rho}{\partial t^2} = 0 \qquad (1)$$

The solution of it is a function of the space time variables and of the initial value of the density ρ .
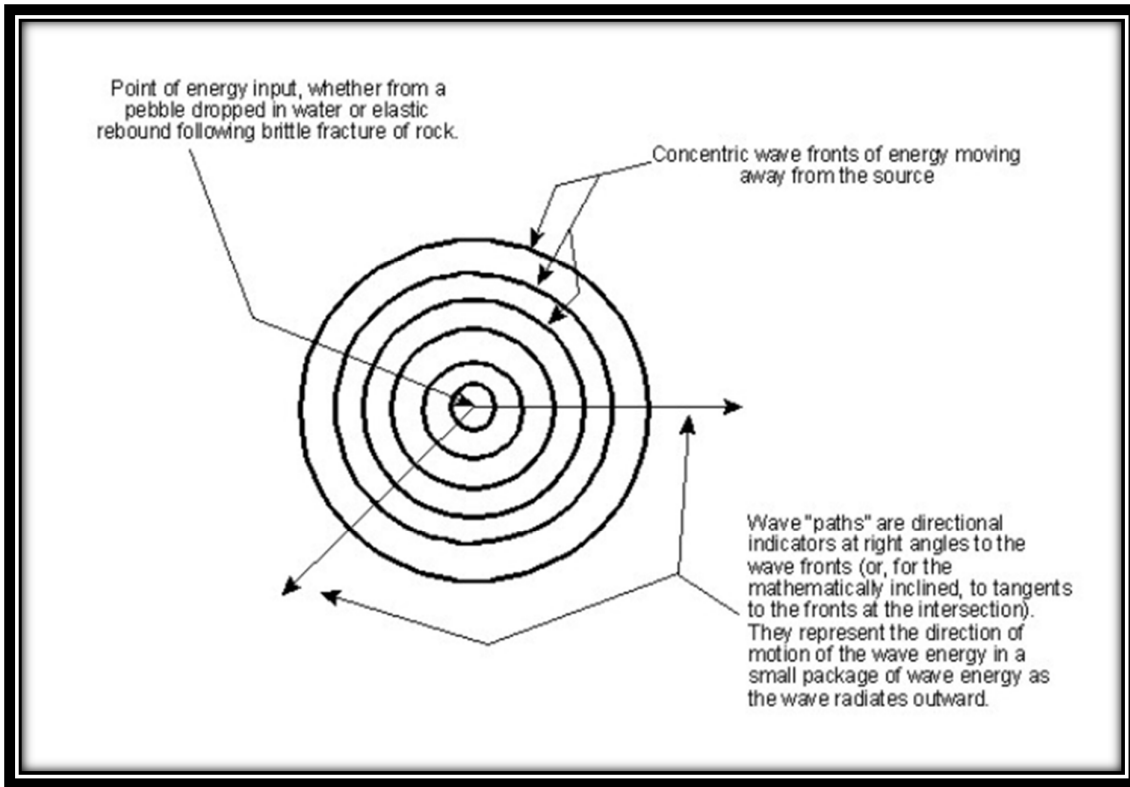
[**Figure 1**: wave generate by a drop into the water. The drop generate the initial condition and after the wave move independently from the initial drop, we have no real sources.]

The homogeneous equation (1) can substituted with a non-homogeneous equation

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 \rho}{\partial t^2} = S(x,y,t) \qquad (2)$$

Where S is an ordinary sources that changes in the space and time. The source S superpose its ρ density fields on the homogeneous solution of the wave equation (1). The equation (2) is a non-reactive system because S is completely independent from the density ρ.
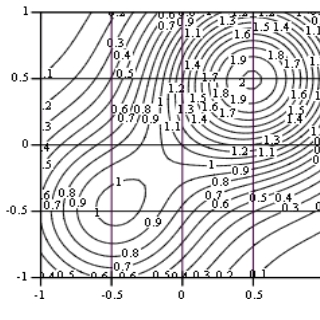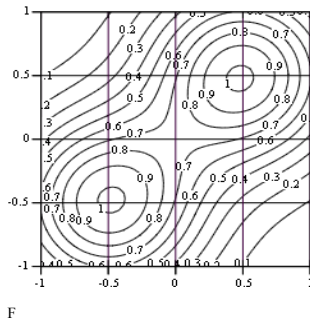
[Figure 2: wave superposition by different non-reactive sources.]

All the previous equation describes the non-reactive sources.

Now we make a transformation of the density by the transformation U. So we have

$$\frac{\partial^2 U\rho}{\partial x^2} + \frac{\partial^2 U\rho}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 U\rho}{\partial t^2}$$

$$= US(x,y,t) + (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})U\rho - U(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\rho \qquad (3)$$

$$= US(x,y,t) + [(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}),U]\rho$$

$$= US + [A,U]\rho$$

Now this becomes a reactive system which is generated by the transformation U of the density given by the equation (2). The reactive sources are defined in this way

$$S_R = [A,U]\,\rho \qquad (4)$$

The equation (3) is the 'prototype of the morphogenetic system (reactive system)' which organizes itself by the detection of the local field in (2). With the information of the local field, it generates new waves actively, which superpose with the previous field, to create the designed transformation U.

Now, if the reactive sources are equal to zero, then in this case

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})U\rho - U(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\rho = 0$$

We have

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})U\rho = U(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\rho = US$$

For which when the source S change by U the density ρ change in the same way. Without the reactive sources the density can change only in the same way of the sources. Only reactive sources can give to the field of waves any type of constrain or form.

When the source **depend** on the field the commutator $S_R = [A,U]\rho$ is different from zero. We remark that to generate the reactive source is necessary a **non-local connection** among field and sources as in the holographic process. All the sources must be active in that same time (synchronic condition)
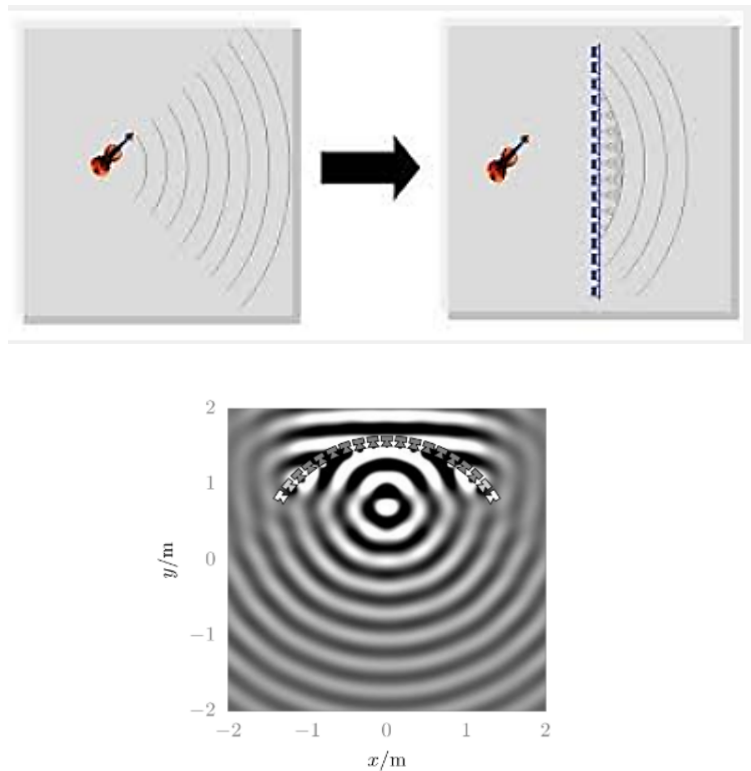
Example of reactive sources process

Figure 4 . Reactive sources. [3]

The first image shows a contradictory or inconsistent state. In fact in the first figure we have a violin and we listen the real violin. In the second we create reactive sources that produce the same sound but without the violin. Now, we have a complex wave pattern that we have designed by the reactive sources. The equation 3 is nothing but the equation for the morphogenetic of designed pattern by reactive sources.

3. Holographic process as a reactive system

'Holographic process' is again a morphogenetic process for which a coherent light interfere with the object light. The superposition of the two optical field is memorized in a sensitive paper (hologram). So we print the hologram of the light in the paper. This hologram [2] is a passive element. Now when we illuminate with the coherence light in the paper, the passive hologram became a set of active sources of light that generate back the image of the object with its three dimensional form. So the image is created by a reaction of the sensitive paper to the coherent light. The active sources depends on the original field superpose to the coherent light. The holographic process creates reactive sources by means of which we can read or rebuilt the three dimension image of the object. First part of figure-3, is WRITE or source building, second part is an activation of the sources and generation of the object image that denoted as READ.
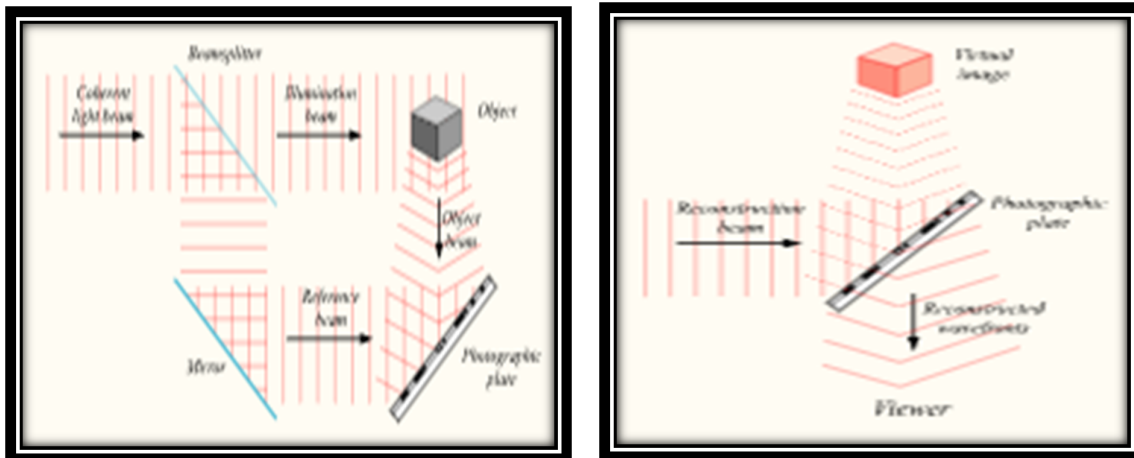

Figure-3: First part is WRITE that generate hologram memory, the second is the activation of the source (reaction) by coherent beam or READ process that generate the three dimensional image of the object.

4. Turing Morphogenetic chemical diffusion reactive system

We know that 'diffusion processes' destroy in time any structure or order to obtain a homogeneous density. 'Turing' discovers that when we include in the diffusion process a chemical system with a network of chemical reactions, we generate in particular cases structures far from homogeneous states. When we want generate structures or forms we can design special chemical reactions as active sources of molecule. The chemical reaction with diffusion generate different wanted density of molecule in the body. This sources are created by chemical reactions so that they are dependent from the initial density of all the molecule in the system (building of the sources) after the generate molecules gets diffused in all the system to generate a pattern or a form in all the system. The wanted structure or conceptual object is the engine that control the sources that in the diffusion process generate the image of the

conceptual object. The Turing process is a morphogenetic system (reactive system) between the chemical reaction system (sources) and the field of the diffused molecule. In a mathematical way, we have the diffusion process by sources independent from the initial density [7]

$$\frac{\partial^2 \rho_j}{\partial x^2} + \frac{\partial^2 \rho_j}{\partial y^2} - \frac{1}{D}\frac{\partial \rho_j}{\partial t} = S_j$$

Suppose, for given the chemical reactions system

$$
\begin{aligned}
A &\xrightarrow{k_1} B \\
B &\xrightarrow{k_2} C \\
B+2C &\xrightarrow{k_3} 3C \\
C &\xrightarrow{k_4} D
\end{aligned}
\tag{5}
$$

The muster equation for this chemical system is

$$
\left\{
\begin{aligned}
\frac{\partial \rho_A}{\partial t} &= -k_1 \rho_A \\
\frac{\partial \rho_B}{\partial t} &= k_1 \rho_A - k_2 \rho_B - k_3 \rho_B \rho_C^2 \\
\frac{\partial \rho_C}{\partial t} &= k_2 \rho_B + k_3 \rho_B \rho_C^2 - k_1 \rho_C \\
\frac{\partial \rho_D}{\partial t} &= k_4 \rho_C
\end{aligned}
\right.
\tag{5(a)}
$$

So the diffusion equation can be written as

$$\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} - \frac{1}{D}\frac{\partial \rho_A}{\partial t} = 0$$

$$\frac{\partial^2 \rho_B}{\partial x^2} + \frac{\partial^2 \rho_B}{\partial y^2} - \frac{1}{D}\frac{\partial \rho_B}{\partial t} = 0$$

5(b)

$$\frac{\partial^2 \rho_C}{\partial x^2} + \frac{\partial^2 \rho_C}{\partial y^2} - \frac{1}{D}\frac{\partial \rho_C}{\partial t} = 0$$

$$\frac{\partial^2 \rho_D}{\partial x^2} + \frac{\partial^2 \rho_D}{\partial y^2} - \frac{1}{D}\frac{\partial \rho_D}{\partial t} = 0$$

Now, we have the reactive system of equations, by applying 5(a) in 5(b) as

$$\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} - \frac{1}{D}(-k_1 \rho_A) = 0$$

$$\frac{\partial^2 \rho_B}{\partial x^2} + \frac{\partial^2 \rho_B}{\partial y^2} - \frac{1}{D}(k_1 \rho_A - k_2 \rho_B - k_3 \rho_B \rho_C^2) = 0$$

(6)

$$\frac{\partial^2 \rho_C}{\partial x^2} + \frac{\partial^2 \rho_C}{\partial y^2} - \frac{1}{D}(k_2 \rho_B + k_3 \rho_B \rho_C^2 - k_1 \rho_C) = 0$$

$$\frac{\partial^2 \rho_D}{\partial x^2} + \frac{\partial^2 \rho_D}{\partial y^2} - \frac{1}{D}(k_4 \rho_C) = 0$$

So we have two systems, the first one tells us the 'behavior of the metabolite A, B, C, D when we change the time for the same position. The second one is a 'reactive diffusion process' for which we can compute the metabolites for the same time as parameter with different positions. For the previous chemical system, we have that 'A' is eliminated from the system so we have a negative source for 'A'. For 'B', we have a positive source that is originated from A, and also a negative source that eliminate B and generate C and 3C. For 'C' this is generated from internal system of B and in the same time is eliminated to produce 'D' which is included in the system. In conclusion, A is destroyed, D is generated by B and C and are oscillated elements that one time they are generated and in a second time they are destroyed. Because we have a diffusion process the oscillation process alimented by A and D that is eliminated , move in the system and generate waves. For the oscillation reaction we have the space time behavior.

[**Figure 5**: Image of the Turing morphogenesis by diffusion and chemical reaction]

5. Simulation of the quantum mechanics by droplets bouncing on a vertical vibrating fluid bath

For the simulation of the quantum mechanics by droplets bouncing on a vertical vibrating fluid bath as we can see in this image
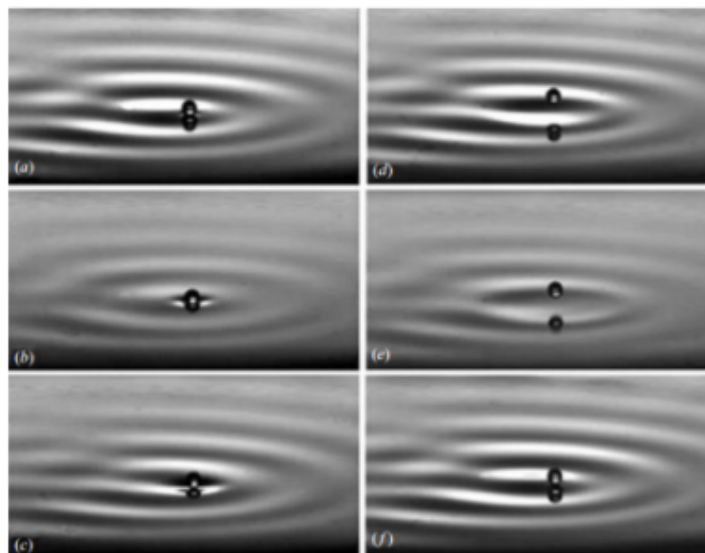
[**Figure 6:** image of droplets bouncing on the vertical vibrating fluid by which we can simulate a lot of properties in quantum mechanics.]

The simple equation of the droplets bouncing on the vertical vibrating fluid is [1]

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} = \rho\sin(w\frac{dx}{dt}) \qquad (7)$$

Where

$$S_R = \rho\sin(w\frac{dx}{dt}) \text{ is a reactive source.}$$

In the polar coordinates we have

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} = \rho(i\sin(w\frac{dx}{dt}) + \rho\cos(w\frac{dx}{dt}))$$

We remark that

$$m\frac{d^2x}{dt^2} + a\frac{dx}{dt} = 0 \text{ is the model for the 'dumping process'.}$$

The reactive source gives the energy to the droplet to move under the action of the vibrating fluid. So we have a couple between the vibrating and the droplet bouncing. The energy move from the droplet to the fluid and the other way around. For the transformation U we have

$$m\frac{d^2Ux}{dt^2} + a\frac{dUx}{dt} = m\frac{d}{dt}(\frac{dUx}{dt}) + aU\frac{dx}{dt} + ax\frac{dU}{dt}$$

$$= m\frac{d}{dt}(U\frac{dx}{dt} + x\frac{dU}{dt}) + a(U\frac{dx}{dt} + x\frac{dU}{dt})$$

$$= m(U\frac{d^2x}{dt^2} + 2\frac{dU}{dt}\frac{dx}{dt} + x\frac{d^2U}{dt^2}) + a(U\frac{dx}{dt} + x\frac{dU}{dt}) \qquad (8)$$

$$= U(m\frac{d^2x}{dt^2} + a\frac{dx}{dt}) + 2m\frac{dU}{dt}\frac{dx}{dt} + mx\frac{d^2U}{dt^2} + ax\frac{dU}{dt}$$

$$= 2m\frac{dU}{dt}(\frac{dx}{dt} + ax) + m\frac{d^2U}{dt^2}$$

$$= S_R$$

Given the dumping solution x, we can found the solution for U. So we can compute the transformation by which we can know the transformation of x. We can also design the transformation U and with 'x' we can compute the reactive sources to generate the transformation U. This is the process by which we can compute the

sources to obtain the transformation of x. Many experiments shows the analogy between quantum mechanics and the reactive previous phenomena.

6. Fisher information and join probability

The probability to have joined set of data and states is given by the expression

$$p(v) = p(s_1,...,s_n, q_1,....,q_m) = p(s_j, q_k) \qquad (9)$$

Where we have two type of variables. The variables q_k are the internal or macro variable to the particle, and s_j are external variables that generate noise to the states of the particle as in the Brownian movement. Any particle in quantum mechanics is in correlation with all the other particles in the universe that externally change the state of any particle. In the real world we must consider the state of all the universe, but because this is impossible , we perceive the other part of universe as noise. In agreement with the previous chapter we have the same separation of sources and variables (states). States and sources are joined by the probability in the multidimensional space. We know that for all possible states q_k we have the fundamental properties

$$\int p(s_j, q_k) dq_1 dq_2 ... dq_n = 1$$

So we have

$$\frac{\partial}{\partial q_k} \int p(s_j, q_k) dq_j = \int \frac{\partial p(s_j, q_k)}{\partial q_k} dq_j =$$

$$= \int \frac{\partial p(s_j, q_k)}{\partial q_k} \frac{1}{p(s_j, q_k)} p(s_j, q_k) dq_j == \int \frac{\partial \log p}{\partial q_k} p dq_j = 0$$

With another derivatives we have

$$\frac{\partial}{\partial q_h} \int \frac{\partial \log p}{\partial q_k} p dq_j = \int \frac{\partial}{\partial q_h} (\frac{\partial \log p}{\partial q_k} p) dq_j = \int (\frac{\partial^2 \log p}{\partial q_h \partial q_k} p + \frac{\partial \log p}{\partial q_k} \frac{\partial p}{\partial q_h}) dq_j =$$

$$\int (\frac{\partial^2 \log p}{\partial q_h \partial q_k} p + \frac{\partial \log p}{\partial q_k} \frac{\partial p}{\partial q_h} \frac{1}{p} p) dq_j = \int (\frac{\partial^2 \log p}{\partial q_h \partial q_k} p + \frac{\partial \log p}{\partial q_k} \frac{\partial \log p}{\partial q_h} p) dq_j = \qquad (10)$$

$$\int (\frac{\partial^2 \log p}{\partial q_h \partial q_k} p dq_j + \int \frac{\partial \log p}{\partial q_k} \frac{\partial \log p}{\partial q_h} p) dq_j = 0$$

Where

$$I = \int \frac{\partial \log p}{\partial q_k} \frac{\partial \log p}{\partial q_h} p) dq_j = E\left(\frac{\partial \log p(s,q)}{\partial q_k} \frac{\partial \log p(s,q)}{\partial q_h}\right) \qquad (10)$$

is the required Fisher information.

For instance, given the like-hood distributions $F(x,\sigma) = e^{-\frac{x^2}{\sigma^2}}$ with $\sigma = 1$
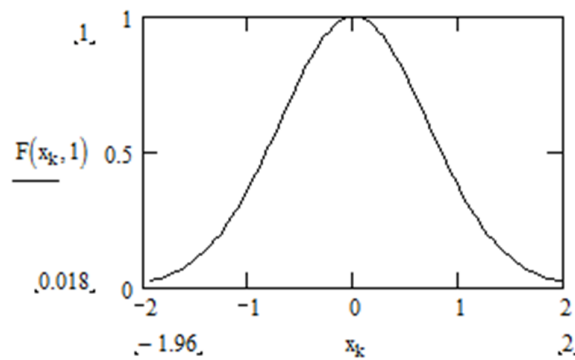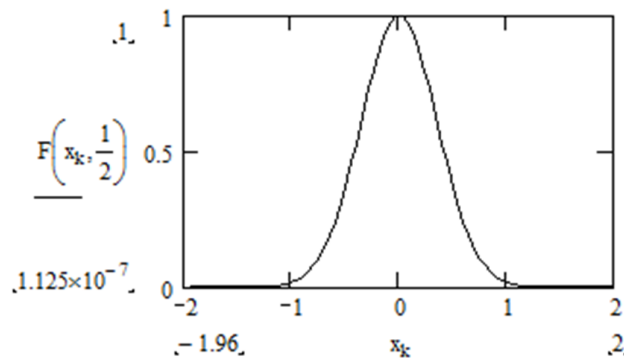we have

Figure 7 Like-hood distribution with low Fisher information

[**Figure 8** Like-hood distribution with higher Fisher information.]

It is easy to show that the Fisher information in the first case, i.e. fig.7 is less than that of the Fisher information in the second case i.e. figure 8. So it can be concluded that less is the variation of the data higher is the Fisher Information. And also, more we are near to the deterministic process more high is the Fisher information.

7. Quantum mechanics as reactive (morphogenetic) information system

From the Fisher information we have

$$S(s_1, s_2, \ldots, s_n) = \int \rho \frac{\partial \log \rho}{\partial q_i} \frac{\partial \log \rho}{\partial q_j}) dq_k \qquad (11)$$

Where we define $\dfrac{\partial \log \rho}{\partial q_i}$ as like the osmotic velocity that we see in Nelson [9]

mechanics and $\dfrac{\partial \log \rho}{\partial q_i} \dfrac{\partial \log \rho}{\partial q_j}$ like a kinetic energy. For the minimum action

principle of the like kinetic energy we have the average value of the like kinetic energy as action S function of the external sources s_j . The minimum variation of the action function of the external variables is given by the expression

$$\delta S(s_1, s_2, \ldots, s_n) = \delta \int \rho \frac{1}{2} \frac{\partial \log \rho}{\partial q_i} \frac{\partial \log \rho}{\partial q_j}) dq_k \qquad (12)$$

Derivation of Euler-Lagrangian interpretation of Schrodinger Equation

The Euler Lagrange equation the minimum variation of the Fisher entropy (12) is given by the expression

$$\delta S = \frac{\partial(\rho \frac{1}{2} \frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j})}{\partial \rho} - \frac{\partial}{\partial x_\mu} \frac{\partial(\rho \frac{1}{2} \frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j})}{\partial \frac{\partial \rho}{\partial x_\eta}}$$

$$= \frac{1}{2}\left(\frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j} - \frac{1}{\rho} \frac{\partial(\frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j})}{\partial \frac{\partial \rho}{\partial x_\mu}} \right)$$

$$= \frac{1}{2}\left(\frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j} - \frac{1}{\rho} \frac{\partial}{\partial x_\mu} \frac{\partial(\frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j})}{\partial \frac{\partial \rho}{\partial x_\eta}} \right)$$

$$= \frac{1}{2} \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j} - \frac{1}{\rho} \frac{\partial^2 \rho}{\partial x_i \partial x_j} = \frac{2m}{(\frac{h}{2\pi})^2} Q = kQ$$

Here, Q is the quantum potential and h is the Plank constant.

In fact we have

$$\frac{1}{2}\frac{1}{p^2}(\frac{\partial p}{\partial q})^2-\frac{1}{p}\frac{\partial^2 p}{\partial q^2}$$

$$=\frac{-1}{p^{\frac{1}{2}}}(-\frac{1}{4p^{\frac{3}{2}}}\frac{\partial p}{\partial q}+\frac{1}{2p^{\frac{1}{2}}}\frac{\partial^2 p}{\partial q^2})$$

$$=\frac{-1}{p^{\frac{1}{2}}}\frac{\partial}{\partial q}(\frac{1}{2\rho^{\frac{1}{2}}}\frac{\partial\rho}{\partial x})$$

$$=\frac{-1}{p^{\frac{1}{2}}}\frac{\partial}{\partial q}(\frac{\partial p^{\frac{1}{2}}}{\partial q})=\frac{-1}{p^{\frac{1}{2}}}(\frac{\partial^2 p^{\frac{1}{2}}}{\partial q^2})=-\frac{\Delta p^{\frac{1}{2}}}{p^{\frac{1}{2}}}=-\frac{\Delta R}{R}=\frac{2mQ}{(\frac{h}{2\pi})^2}$$

For a set of states we have

$$Q=\frac{1}{k}(\frac{\Delta p}{p}-\frac{1}{2}(\frac{\nabla p}{p})^2)$$

$$=2\frac{1}{k}(\frac{\nabla(\nabla p)}{p}-\frac{1}{2}(\frac{\nabla p}{p})^2)$$

$$=\sqrt{\frac{1}{k}}\nabla(\sqrt{\frac{1}{k}}\nabla\log(p))-\frac{1}{2}(\sqrt{\frac{1}{k}}\nabla\log(p))^2$$

$$=\sqrt{\frac{1}{k}}\nabla u-\frac{1}{2}u^2$$

$$\text{i.e. }\boxed{Q=\sqrt{\frac{1}{k}}\nabla u-\frac{1}{2}u^2}$$

Where u is the pseudo osmotic velocity [9]. As we know from quantum potential is possible to generate again the Schrodinger equation in this way

$$\frac{\partial S}{\partial t}+\frac{1}{2m}p_i p_j+V+\frac{(\frac{h}{2\pi})^2}{2m}(\frac{1}{\rho^2}\frac{\partial\rho}{\partial x_i}\frac{\partial\rho}{\partial x_j}-\frac{2}{\rho}\frac{\partial^2\rho}{\partial x_i\partial x_j})$$

$$=\frac{\partial S}{\partial t}+\frac{1}{2m}p_i p_j+V+Q=0$$

Now for

$$R^2=\rho$$

and the Plank constant is equal to 1 we have

$$Q = -\frac{1}{2m}\frac{\nabla^2 R}{R}$$

So for $\nabla S = mv = p$ we have

$$\frac{\partial S}{\partial t} + \frac{1}{2m}p_i p_j + V + Q = \frac{\partial S}{\partial t} + \frac{|\nabla S|^2}{2m} + V + (-\frac{1}{2m}\frac{\nabla^2 R}{R}) = 0$$

Where Q is the Bohm quantum potential that is a consequence for the extreme condition of Fisher information (minimum or maximum condition for the Fisher information).

Now we write the continuity equation for the probability $\rho = R^2$ in this way

$$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$

That can write in this way

$$\frac{\partial R^2}{\partial t} + \nabla(R^2 v) = 2R\frac{\partial R}{\partial t} + \nabla(R^2 v) = 2R\frac{\partial R}{\partial t} + 2R\nabla Rv + R^2\nabla v = 0$$

For $R \neq 0$ we divide the previous expression for 2R so we have

$$2R\frac{\partial R}{\partial t} + 2R\nabla Rv + R^2\nabla v = \frac{\partial R}{\partial t} + \nabla Rv + \frac{1}{2}R\nabla v = 0$$

$$\frac{\partial R}{\partial t} + \nabla Rv + \frac{1}{2}R\nabla v = \frac{\partial R}{\partial t} + \frac{1}{2m}(2\nabla Rmv + R\nabla mv) = 0$$

Because $\nabla S = mv = p$ we have

$$\frac{\partial R}{\partial t} + \frac{1}{2m}(2\nabla R\nabla S + \frac{1}{2}R\nabla\nabla S) = \frac{\partial R}{\partial t} + \frac{1}{2m}(2\nabla R\nabla S + R\nabla^2 S) = 0$$

Now we combine the continuity equation of the probability with the

$$\frac{\partial S}{\partial t} + (\frac{|\nabla S|^2}{2m} - \frac{1}{2m}\frac{\nabla^2 R}{R} + V) + i(\frac{\partial R}{\partial t} + \frac{1}{2m}(R\nabla^2 S + 2\nabla R\nabla S)) = 0$$

Where the real part is consequence of the Fisher information and the imaginary part is due to the continuous equation for the probability.

Now for the equation $\psi = \mathrm{Re}^{iS}$, from the previous equation for S we have the Schrodinger equation.[11.12]

$$i\frac{\partial \psi}{\partial t} = (-\frac{1}{2m}\nabla^2 + V)\psi$$

with

$$\frac{h}{2\pi} = 1$$

In conclusion we can make a reverse process used by Schrodinger we can generate the Schrodinger equation by the information space and the continuity equation of the probability. In this way the Hilbert mechanism can be explained only by probability normalization constraints.

8. Quantum mechanics as a reactive system by like kinetic energy and quantum potential

Given the terms in (10)

$$F_1 = \frac{\partial^2 \log p}{\partial q_h \partial q_k}, F_2 = \frac{\partial \log p}{\partial q_k}\frac{\partial \log p}{\partial q_h} \qquad (13)$$

We compare the classical reaction process between the kinetic energy of a particle with the potential energy with the quantum potential and the pseudo kinetic term in a way to show that also the quantum mechanics is a reactive system like the particle in the field.

The terms (13) can be write in this way

$$F_1 = \frac{\partial^2 \log p}{\partial q_h \partial q_k} = \frac{\partial}{\partial q_h}\frac{\partial \log p}{\partial q_k} = \frac{\partial}{\partial q_h}(\frac{1}{p}\frac{\partial p}{\partial q_k}) = -\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k} + \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k}$$

$$F_2 = \frac{\partial \log p}{\partial q_k}\frac{\partial \log p}{\partial q_h} = \frac{1}{p^2}\frac{\partial p}{\partial q_k}\frac{\partial p}{\partial q_h}$$

The sum of the two functions is

$$F_1 + F_2 = F = -\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k} + \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} + \frac{1}{p^2}\frac{\partial p}{\partial q_k}\frac{\partial p}{\partial q_h} = \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k}$$

$$F = \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - \frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k} + \frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k}$$

Where we have

$$kQ = \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - \frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k}$$

And pseudo kinetic energy is

$$K = \frac{1}{2}\frac{\partial \log p}{\partial q_h}\frac{\partial \log p}{\partial q_k}$$

So we have

$$kQ + K = \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k}$$

The term $\frac{\partial^2 p}{\partial q_h \partial q_k}$ is the Hessian of the probability. We remember that the eigenvalues of the hessian give us the stable condition of the quantum system. If the eigenvalue is positive we have the stable condition when the eigenvalue is negative the state is unstable. Only the stable condition for the energy is possible in quantum system. Given

$$G = K - kQ = K + kQ - 2kQ = \frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - 2kQ$$

When we substitute the values we have

$$G = (\frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - \frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k}) + (\frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k}) - 2(\frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - \frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k}) =$$

$$\frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - 2(\frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} - \frac{1}{2}\frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k}) = -\frac{1}{p}\frac{\partial^2 p}{\partial q_h \partial q_k} + \frac{1}{p^2}\frac{\partial p}{\partial q_h}\frac{\partial p}{\partial q_k} = -\frac{\partial^2 \log p}{\partial q_h \partial q_k}$$

In conclusion we have the reactive system

$$G = L = K - kQ = -\frac{\partial^2 \log p}{\partial q_h \partial q_k}$$

That is comparable with the (12)

$$G = K - kQ = -\frac{\partial^2 \log p}{\partial q_h \partial q_k}$$

and

$$S = \int (K - kQ)\rho dq_k = -\int \frac{\partial^2 \log p}{\partial q_h \partial q_k}\rho dq_k = \int \frac{\partial \log p}{\partial q_k}\frac{\partial \log p}{\partial q_h}\rho dq_k = \int L dq_k$$

So the classical action potential that is the difference between kinetic energy and potential energy can be found also in quantum mechanics by the like kinetic energy and quantum potential.

9. Conclusion

In this paper, we present reactive systems (morphogenetic system) in different contexts. The first the wave reactive system, the second is the Holographic system and the third is the diffusion and chemical Morphogenetic system and the last one is the walking droplets [1, 5]. With the inspiration taken from the concept of 'Bijective methodology' [8, 10], we can present the quantum mechanics as a reactive system. At the end of the presentation of the different reactive systems, we introduce the Fisher information from the joint probability function of the noise sources and physical states. With the minimum condition of the Fisher information **which can be satisfied by Euler Lagrange equation** and the quantum potential, after with the continuity equation of the probability we can obtain the Schrodinger equation that is a direct consequence of the minimum Fisher information. We remark that Schrodinger equation does not use any thermodynamical model as Brownian model. We establish an analogy between classical mechanics where we define the 'Lagrangian' by kinetic energy and the potential energy and quantum mechanics where we define a like kinetic energy and quantum potential, So we can define a Lagrangian and action for

quantum mechanics in the same way as the classical system. In this way the quantum mechanics became a reactive system between like kinetic energy and the field of quantum potential. In this way, Quantum mechanics may became a pure probabilistic phenomena where the wave function and Hilbert space are only a mathematical instrument to compute the joint probability in different particles structures. The entanglement in this quantum reactive system is the most important phenomena that is described by the joint probability

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