

Tres Mega-Integrales

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Resumen. Esta nota muestra tres integrales definidas.

Integral 1.

$$\frac{\pi}{2\sqrt{2}} = \frac{1}{2} + \int_{1/2}^{\infty} \left(1 - \sqrt{\frac{5}{3} - \frac{1}{6x} R(x)^{1/3} - \frac{2x}{3} R(x)^{-1/3}} \right) dx \quad (1)$$

donde

$$R(x) = x(108 + 8x^2 + 12\sqrt{3}\sqrt{4x^2 + 27}) \quad (2)$$

Integral 2.

$$\frac{\pi}{4} = \int_0^{\infty} \left(1 - \sqrt{\frac{5}{3} - \sqrt[3]{f(x)} - \sqrt[3]{g(x)}} \right) dx \quad (3)$$

donde

$$f(x) = \frac{2x^2 + 45}{54x^2} + \sqrt{\frac{4 + 71x^2 + 8x^4}{108x^6}} \quad (4)$$

$$g(x) = \frac{2x^2 + 45}{54x^2} - \sqrt{\frac{4 + 71x^2 + 8x^4}{108x^6}} \quad (5)$$

Integral 3.

$$\frac{3\pi}{4} = \frac{1}{\sqrt{2}} + \int_{1/\sqrt{2}}^{\infty} \left(1 - \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{6} \sqrt{f(x)} - \frac{1}{6} \sqrt{g(x)} \right) dx \quad (6)$$

donde

$$f(x) = \frac{s^2 + 4sx + x^2 + 12}{sx} \quad (7)$$

$$g(x) = -6\sqrt{6}\sqrt{f(x)} + 6\sqrt{h(x)} - j(x) \quad (8)$$

$$h(x) = \frac{s^4 + 2s^3x + (75x^2 - 12)s^2 + (2x^3 + 24x)s + (x^2 + 12)^2}{s^2x^2} \quad (9)$$

$$j(x) = \frac{3s^2 - 24sx + 3x^2 + 36}{sx} \quad (10)$$

$$s = s(x) = \sqrt[3]{x^3 + 72x + 6\sqrt{3}\sqrt{x^4 + 44x^2 - 16}} \quad (11)$$

Referencias

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