Entropy Gradient and Relative Entropy of Physical Systems

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Abstract

Classical thermodynamics is founded on three principles and on the concept of entropy in which heat and temperature are considered physical quantities that are necessary for describing the behaviour of thermodynamic systems. The Principle of Specific Entropy has shown the concept of entropy can be extended to all physical systems with an appropriate choice of energy and of physical quantity that are involved in the physical process. New concepts of entropy gradient and of relative entropy allow to study physical systems from a different viewpoint and to extend their knowledge.

1. Introduction

In classical thermodynamics the following three principles are valid:

1° Principle: when a physical system is supplied with a heat Q, a part is converted into a variation of internal energy ΔU and the remaining part into work L performed by the system on the surrounding ambient, i.e. $Q = \Delta U + L$ (fig.1)

2° Principle: It is impossible in nature a transformation in which the only outcome is the transfer of heat from a colder body to a warmer body without external work (Clausius' definition) or it is impossible in nature a cyclic transformation in which the only outcome is the transformation into work of all heat supplied by a source at constant temperature (Kelvin-Planck's).

3° Principle: It is impossible to reach the absolute zero by a finite number of transformations.

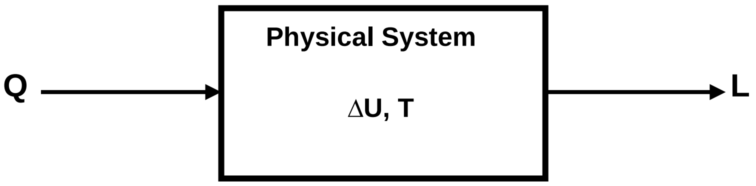

Fig.1 Q represents heat that is supplied to system, L is the produced work, ΔU is the variation of internal energy and T is the temperature of system.

The classical definition of entropy is

$$dS = \frac{dQ}{T} \qquad (1)$$

in which dQ represents the heat exchange between the physical system and the surrounding ambient and dS is the variation of entropy of the system caused by dQ at the temperature T.
In the paper "Specific Entropy of Physical Systems"[1] we have pointed out this definition of entropy is unsatisfactory because it restricts the field of transformations only to exchanges of heat. On this account in that paper we extended the definition to all shapes of energy according to the following formula

$$dS = \frac{dE}{P} \qquad (2)$$

in which S is the "specific entropy" of the system. E and P represent energy, not only heat, and the physical quantity, not only T, that characterize the considered physical process. The physical quantity P coincides with the temperature T for thermodynamic and electrodynamic systems and with the the velocity v for mechanical and gravitational systems. We will see for gravitational systems it is preferable to use the distance r.
As per this new definition we calculated the specific entropy for different types of systems: thermodynamic systems, mechanical systems, gravitational systems, electrodynamic systems (i.e. elementary particles), finding the following equations[1] , with relative graphs:

a. for thermodynamic systems (fig.2)[2], dS=dQ/T e

$$S = S_o \left(1 + \ln \frac{T}{T_o}\right) \qquad (3)$$

in which $S_o = Mc_s$ is entropy for $T = T_o$, M is mass and c_s the specific heat of system.

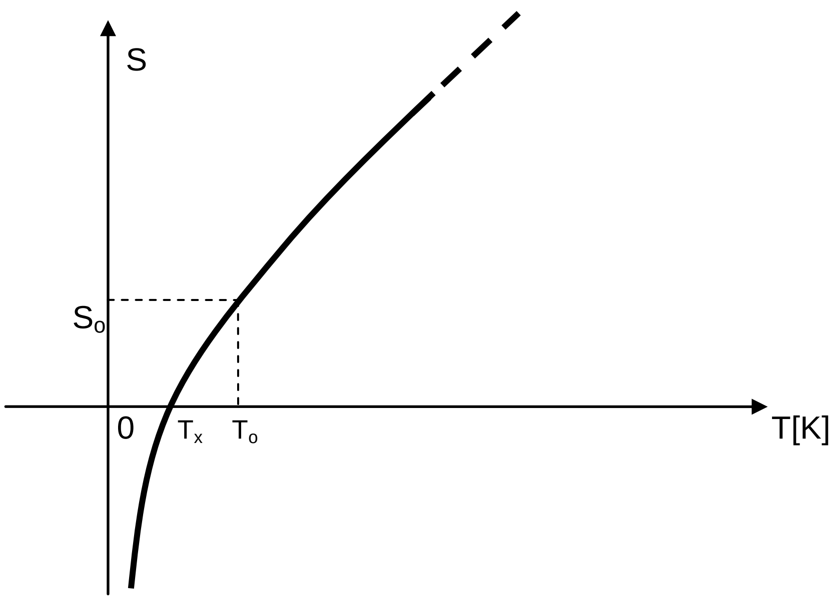

Fig.2 Graph of specific entropy of a thermodynamic system as function of the Kelvin temperature

b. for mechanical systems[1] (fig.3), $dS=dE_c/v$, in which E_c is the kinetic energy and hence

$$S = mv + S_o \qquad (4)$$

in which S_o is the specific entropy initial for velocity v=0.

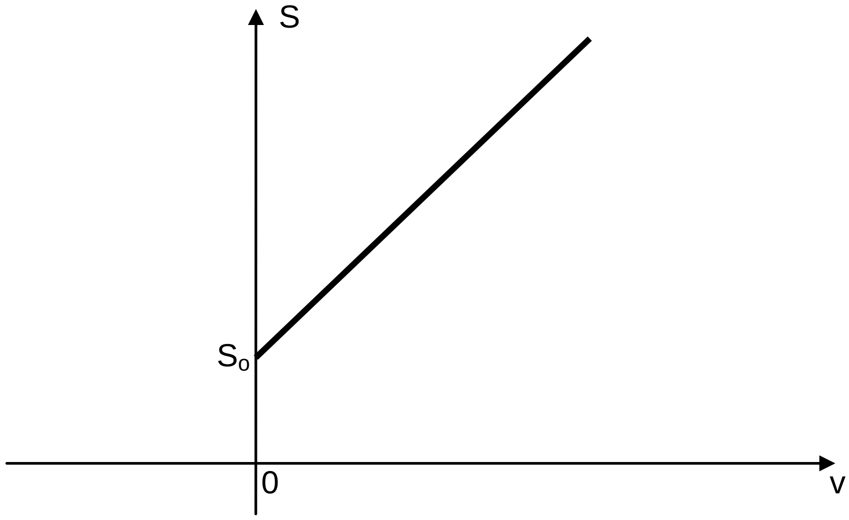

Fig.3 Graph of specific entropy of a mechanical system as function of velocity

c. for gravitational systems in free fall[1][3] , energy is the kinetic energy $E_c=GMm/r$, the characteristic physical quantity for the specific entropy was assumed to be the velocity v and the following formula of the specific entropy was obtained

$$S= S_o + m\sqrt{\frac{GM}{2r}} \qquad (5)$$

A more appropriate analysis suggests to assume directly the distance r like characteristic physical quantity and in that case the specific entropy is given by $dS=dE_c/r$. Integrating we obtain (fig.4)

$$S = S_o + \frac{GMm}{2r^2} \qquad (6)$$

in which S_o is the specific entropy at greatest distance from the centre of gravity ($r=\infty$).

d. for electrodynamic systems (elementary particles) $dS=dE_i/T$ in which E_i is the intrinsic energy of particle. Relations of the specific entropy[1] are

$$S = S_o\left(1 + \ln\left|\frac{T}{T_o}\right|\right) \qquad (7) \quad \text{as function of internal temperature (fig.5)}$$

$$S = S_o\left(1 + \ln\left|1 - \frac{v^2}{2c^2}\right|\right) \qquad (8) \quad \text{as function of velocity of particle (fig.6)}$$

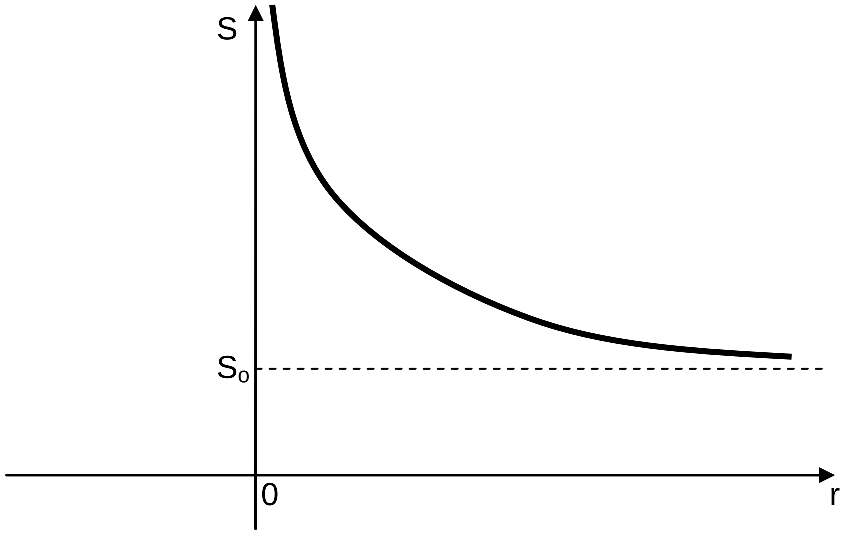

Fig.4 Graph of specific entropy of a gravitational system as function of distance. Specific entropy increases when the distance decreases. The decrease of the distance represents the natural evolution of a gravitational system.
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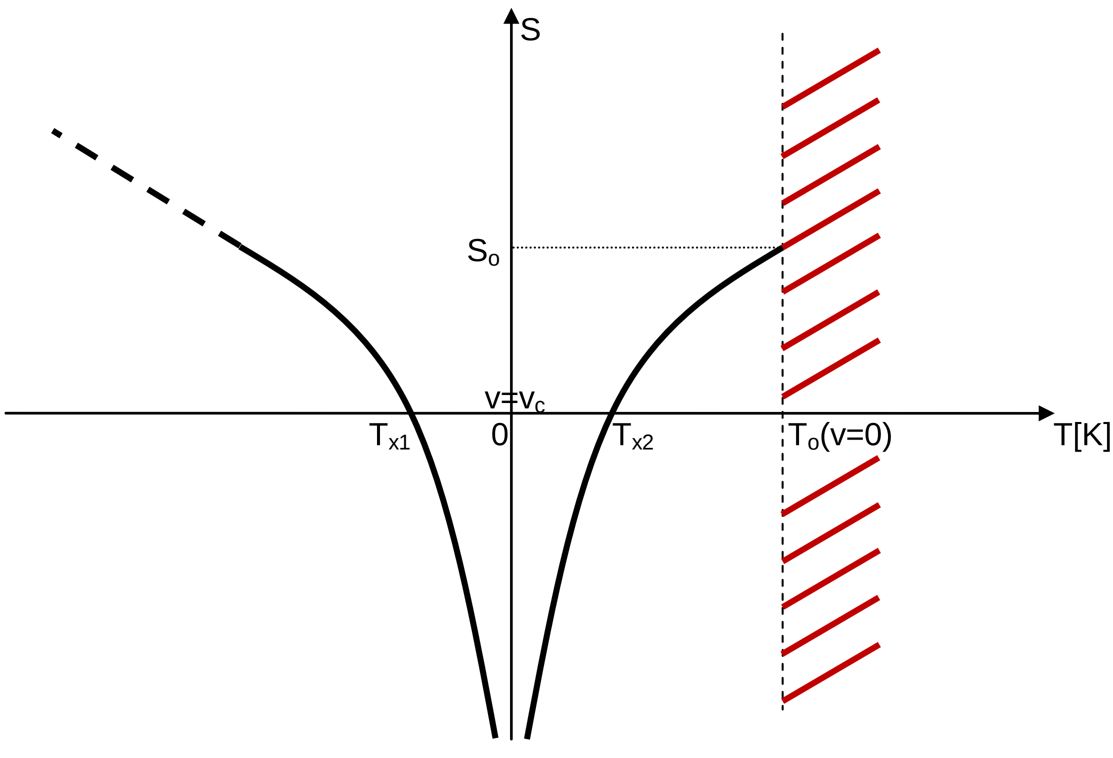

Fig.5 Graph of specific entropy of an elementary electrodynamic particle as function of the internal Kelvin temperature. Elementary particles can assume negative values of Kelvin temperature[2]

Comparing the graph of specific entropy of a thermodynamic system with the graph of an electrodynamic elementary particle we deduce the following considerations:

1. For thermodynamic systems there aren't upper limits for the Kelvin positive temperature while for electrodynamic particles there is an upper limit[2] represented by $T_o=2m_oc^2/3K$, in which m_o is electrodynamic mass of particle at rest and K is the Boltzmann constant.

2. For thermodynamic systems Kelvin negative temperatures aren't possible while it is possible for electrodynamic particles. Besides specific entropy of an electrodynamic particle decreases when the Kelvin temperature increases for negative values.

3. At the Kelvin temperature T=0 (in which particle has the critical velocity) entropy is - ∞ and hence electrodynamic particle would be theoretically into a state of maximum order.

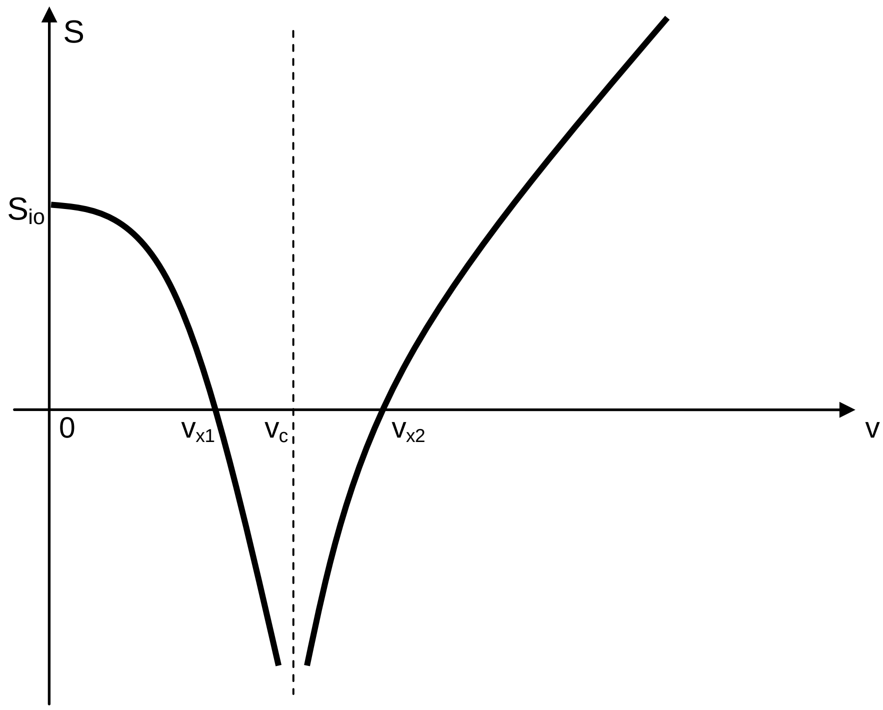

Fig.6 Graph of specific entropy of an electrodynamic elementary particle as function of velocity of particle

An interesting consideration regards astrophysical systems, known as black holes.
In fact according to the gravitational theory black holes would have a specific entropy that is practically infinite (fig.4) and according to the relativistic mechanical theory a specific entropy that is finite and dependent on velocity of celestial body (fig.3).

3. Entropy Gradient

Specific entropy has allowed to extend the concept of entropy from thermodynamic systems to all physical systems.
Let us define now the entropy gradient

$$G_s = \frac{dS}{dP} \qquad (9)$$

in which S is specific entropy and P is characteristic physical quantity of the considered system.

Entropy gradient allows to calculate the analytic trend of specific entropy while the relation (2) allows to calculate specific entropy.
Let us consider different cases:

3.1 for thermodynamic systems the specific entropy is given by

$$S = S_o \left(1 + \ln \frac{T}{T_o}\right) \qquad (3)$$

Hence the gradient of entropy is (fig.7)

$$G_s = \frac{dS}{dT} = \frac{S_o}{T} \qquad (10)$$

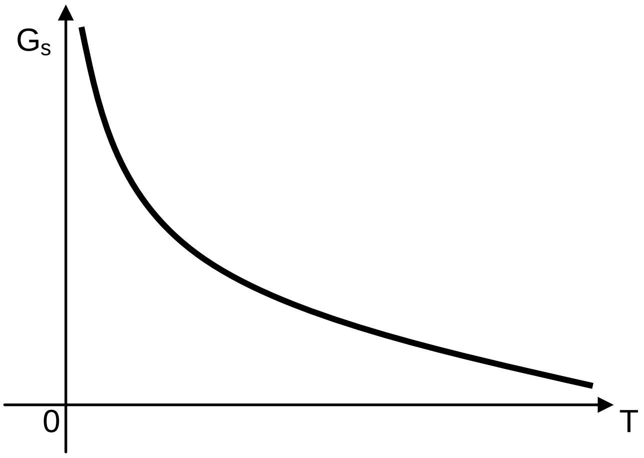

Fig.7 Entropy gradient for a thermodynamic system

3.2 for mechanical systems the specific entropy is

$$S = mv + S_o \qquad (4)$$

and the gradient of entropy (fig.8) is given by

$$G_s = \frac{dS}{dv} = m \qquad (11)$$

that is the gradient of entropy is constant and it coincides with mass of mechanical system

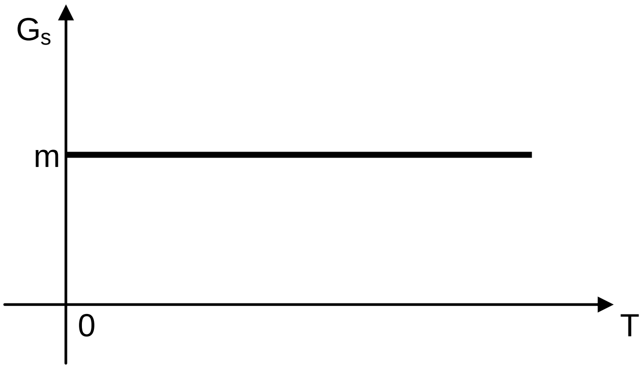

Fig.8 Entropy gradient for a mechanical system

3.3   for gravitational systems the specific entropy is

$$S = S_o + \frac{GMm}{2r^2} \qquad (6)$$

The gradient of entropy is (fig.9)

$$G_s = \frac{dS}{dr} = -\frac{GMm}{r^3} \qquad (12)$$

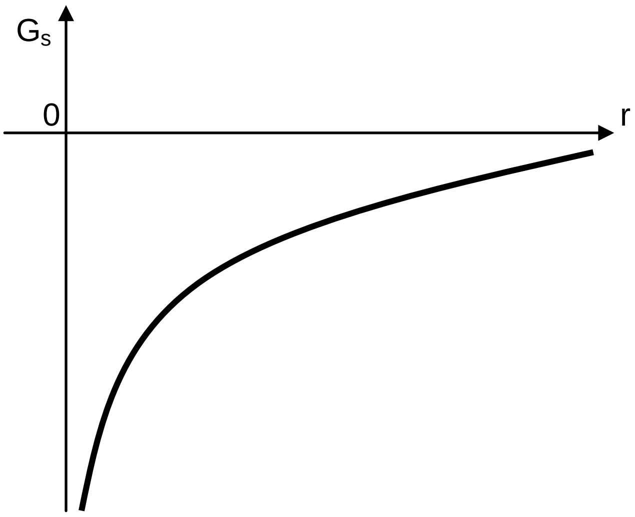

Fig.9 Entropy gradient for a gravitational system

3.4 for electrodynamic systems (elementary particles) the specific entropy is given by

$$S = S_o \left(1 + \ln \left|\frac{T}{T_o}\right|\right) \qquad (7)$$

The gradient of entropy is (fig.10)

$$G_s = \frac{dS}{dT} = -\frac{S_o}{|T|} \qquad (13) \qquad \text{for } T<0$$

and

$$G_s = \frac{dS}{dT} = \frac{S_o}{T} \qquad (14) \qquad \text{for } T>0$$

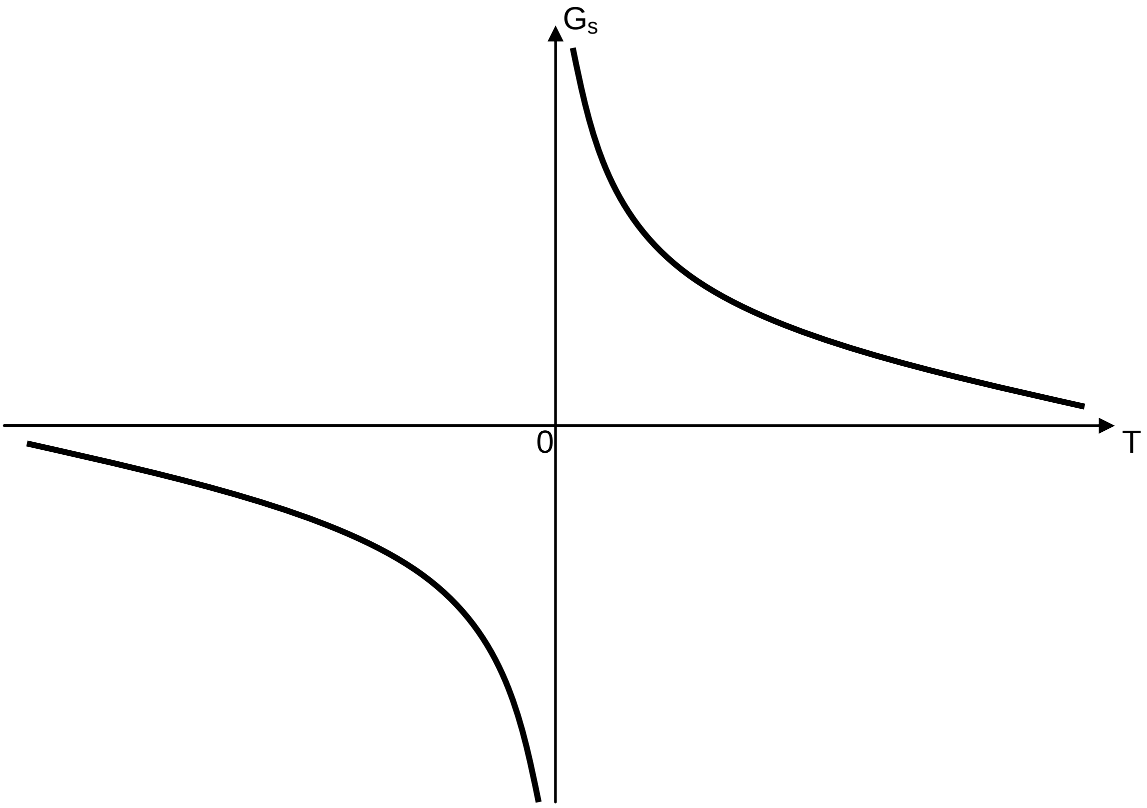

Fig.10 Entropy gradient for an electrodynamic system (elementary particles)

4. Relative Entropy

From the definition (2) of specific entropy we derive the definition of " relative entropy "

$$S_r = \frac{dE}{dP} \qquad (15)$$

in which the variation of energy isn't related to the specific physical quantity but to the variation of the same quantity.
For different types of physical systems the following relations of relative entropy are valid:

4.1 For thermodynamic systems it needs to consider E=Q and P=T.
Because

$$Q = Mc_sT \qquad (16)$$

in which M is mass of body, c_s is its specific heat and T is the Kelvin temperature, from (15) we derive the relative entropy of a thermodynamic system is constant and equal to

$$S_r = \frac{dQ}{dT} = Mc_s \qquad (17)$$

4.2 For mechanical systems energy E is the kinetic energy E_c and P is velocity v. Because

$$E_c = \frac{mv^2}{2} \qquad (18)$$

the relative entropy of a mechanical system is

$$S_r = \frac{dE_c}{dv} = mv \qquad (19)$$

Hence the relative entropy of a mechanical system increases linearly with the velocity and it coincides with the specific entropy (fig.10) excluding the initial entropy S_o.

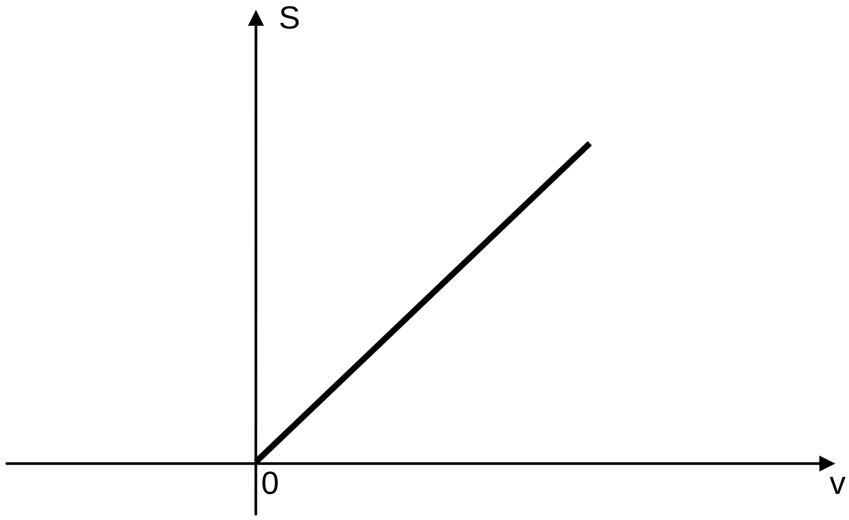

Fig.10 Trend of relative entropy for a mechanical system as function of velocity

4.3 For gravitational systems energy E is the kinetic energy E_c and for P we have assumed directly the distance r for which being

$$E_c = \frac{GMm}{r} \qquad (20)$$

the relative entropy of a gravitational system is (fig.11)

$$S_r = \frac{dE_c}{dr} = -\frac{GMm}{r^2} \qquad (21)$$

in which M is mass that generates the gravitational field and m is mass falling inside field.

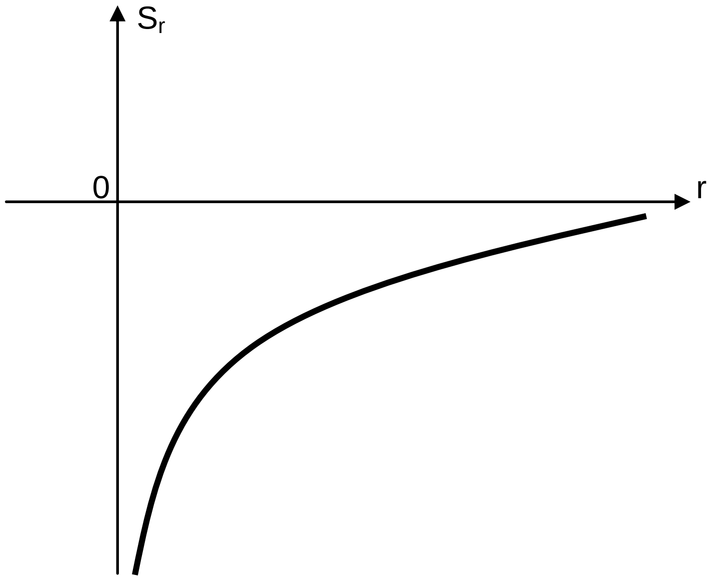

Fig.11 Graph of relative entropy for a gravitational system with mass m

4.4 For electrodynamic systems energy E is the intrinsic energy $E_i=mc^2$ in which $m=m_o(1-v^2/2c^2)$ is electrodynamic mass at the velocity v and the quantity P is temperature T for which

$$S_r = \frac{dE_i}{dT} \qquad (22)$$

Essendo in base alla equivalenza termodinamica[2]

$$E_i = mc^2 = \frac{3}{2} KT \qquad (23)$$

we obtain

$$S_r = \frac{3}{2} K \qquad (24)$$

Like thermodynamic systems also for elementary particles relative entropy is constant.

References

[1] D. Sasso, Specific Entropy of Physical Systems, viXra.org, 2018, id: 1801.0160

[2] D. Sasso, Thermodynamics of Elementary Particles, viXra.org, 2015, id: 1502.0009

[3] D. Sasso, Physics of Gravitational Fields, viXra.org, 2014, id: 1405.0028