# Transforming Candidate and Parties cardinal ratings into weak preference orderings

RICARDO ALVIRA

University of Murcia

# Contents

- **INTRODUCTION:**
  - SOME UNSOLVED PROBLEMS OF OUR ELECTORAL SYSTEMS

- **PROPOSAL:**
  - ALGORITHM TO GENERATE PREFERENCE ORDERINGS/ PROFILES

- **RESULTS:**
  - REVIEW OF TWO ELECTIONS: MADRID AND CATALONIA

- **CONCLUSIONS:**

**RICARDO ALVIRA** 2018

PART I

# INTRODUCTION:

3

# Introduction

- The importance of the **systems for electing our representatives** contrasts with the fact that their origin and evolution has been little studied, and that there are **some unsolved problems regarding their consistency**.

- In general terms, our current parliamentary model derives from the paradigm underlying the **Model Parliament** implemented in England at the end of the thirteen century [Simón de Monfort - Edward I]

# Introduction

- The system of election of representatives evolved, arriving to a widespread use of a system where countries were divided into multimember districts and **each citizen could cast as many votes as positions needed to be filled in each district** [Colomer, 2004; Urdánoz, 2009].

- This enabled **a well-organized group to get all the positions in a district** if that group was supported by the largest group of citizens, even if this support was well below 50% of the population.

# Introduction

- Some voices began to challenge **this system that enabled a very plural society being *represented* by monocolor parliament**; minorities were left without representation, and the "majority" of representatives had often been voted by a small percentage of citizens.

- Some alternatives were proposed:
  - **Condorcet, 1793**: proposed a *limited vote* [2 votes per citizen in districts of up to 18 deputies]
  - **Saint-Just, 1793**: proposed a *single vote* being the whole France a single district.

6

# Introduction

- In the nineteenth century numerous proposals for electoral reform were made, which can be grouped into **three different paths**:

  - One of them led to **systems based on quotas**, as the 'single transferable vote' [Gergonne, Andrae, Hare]

  - Another led to party lists **proportional representation systems** [d'Hondt, Sainte Lague, ...]

  - Another reduced the size of all the districts to one representative, leading to **majority systems** [First Past The Post].

# Introduction

- In the twentieth century we witnessed the evolution of these three paths, and some small modifications were proposed that allowed correcting some defects :

  - Systems were created with **two levels [eg, Germany]** to maximize the global proportionality votes seats.
  - **Legal thresholds [3%, 5% ...]**  were established to limit parliamentary fragmentation
  -  **Open lists** …
  - …

# Introduction

**What happened along the twentieth century with the Paradoxes / Impossibilities that are discovered in the field of Social Choice?**

Were not paradoxes found in the preference aggregation rules underlying the electoral systems?

9

# Introduction

- Surprisingly, **while in the Social Choice realm a large number of 'paradoxical' and impossibility results was published** [Nurmi, 2016]**, we find scarce studies reviewing this issue regarding the electoral systems:**

  - In 1949, Duncan Black published a first study that showed the electoral rules could be providing results not fitted to citizens preferences ...
  - In 1978, Colman & Pountney, detected Borda's paradoxes in the UK general election.
  - Van Deemen [1993] and Kurrild-Klitgaard [2008] published results showing prevalence of Borda paradox in elections in Netherlands and Denmark.

10

# Introduction

- Where are we now?

11

# Introduction

- Despite being a very important issue, the number of studies carried out is still very small, we hardly know the size of the problem.

- **In principle, any paradox found in the realm of Social Choice can be found in electoral systems**, since they use the same voting rules as a basis [van Deemen, 1993].

- However, since political preferences tend to be spatially structured, we cannot extrapolate random calculations, so **we do not know if it is a problem that occurs frequently or rarely.**

12

# Introduction

- Part of the difficulty in assessing it, derives from the **difficulty of obtaining the necesssary data for the analysis** [sometimes there is no data at all].

- In most of our societies, **single vote is used, so we do not know the actual citizens preference profile.**

# Introduction

- The present proposal seeks **to facilitate undertaking this type of evaluation of the electoral systems, by providing an easy method to generate citizens preference profile** based on information from citizens surveys that can be relatively easily accessed/found .

14

# Introduction

- Noteworthy, **our goal is not "determining the true preference profile of citizens" in an absolute sense, but generating a sufficiently 'likely' citizens preference profile, so we can contrast against it the allocation of seats that different electoral systems produce.**

PART II

# ALGORITHM TO GENERATE PREFERENCE PROFILES

16

# Proposal

- We review the information available in the surveys of the Spain Center for Sociological Research [CIS]

- We have found two type of information on voters preferences in CIS surveys:

# Proposal

- In some surveys, we find the average value of voters preference [0-10] for each candidate or political party:

**Pregunta 37**

Le voy a citar ahora los nombres de algunos/as líderes políticos/as. Le agradecería me indicara con respecto a cada política. Puntúelos/as de 0 a 10, sabiendo que el 0 significa que 'lo/la valora muy mal' y el 10 que 'lo/la valora mu

| | | | | | | | Recuerdo de voto en las elecciones gen | | |
| | TOTAL | PP | PSOE | Unidos Podemos | C's | En Comú Podem | Compromis-Podemos-EUPV | ERC | CDC |
|---|---|---|---|---|---|---|---|---|---|
| **Alberto Garzón** | | | | | | | | | |
| Valora | 74,8 | 78,8 | 79,2 | 91,1 | 81,7 | 85,5 | 92,7 | ,2 | 60 |
| No conoce | 10,9 | 9,8 | 10,6 | 4,4 | 7,5 | 3,8 | 3,2 | 6,6 | 21 |
| N.S. | 12,6 | 11,4 | 9,9 | 4,0 | 8 | 9,7 | 3,2 | 16,8 | 15 |
| N.C. | 1,8 | 0,1 | 0,3 | 0,5 | 1,0 | 1,1 | 0,8 | 1,5 | 1 |
| (N) | (6.175) | (1.196) | (1.089) | (654) | (518) | (186) | (124) | (137) | (6 |
| Media | 4,50 | 2,56 | 4,84 | 7,26 | 3,93 | 6,59 | 7,18 | 5,12 | 4, |
| Desviación típica | 2,94 | 2,40 | 2,48 | 2,23 | 2,49 | 2,38 | 2,07 | 2,51 | 2, |
| (N) | (4.616) | (942) | (863) | (596) | (423) | (159) | (115) | (103) | (3 |

**Pregunta 37**

Le voy a citar ahora los nombres de algur política. Puntúelos/as de 0 a 10, sabiendo

| | TOTAL | PP | P |
|---|---|---|---|
| **Alberto Garzón** | | | |
| Valora | 74,8 | 78,8 | |
| No conoce | 10,9 | 9,8 | |
| N.S. | 12,6 | 11,4 | |
| N.C. | 1,8 | 0,1 | |
| (N) | (6.175) | (1.196) | |
| Media | 4,50 | 2,56 | |
| Desviación típica | 2,94 | 2,40 | |
| (N) | (4.616) | (942) | |

18

# Proposal

- In other surveys we find lexical scales that we can transform to obtain an average value [e.g., CIS. ES 3087. Madrid]

**Pregunta 23**

Respecto a los siguientes partidos políticos, me gustaría que me dijera si se encuentra muy cercano, cercano, ni c... o muy distante de cada uno de ellos.

| | | | Recuerdo de voto en las elecciones autonómicas de la Comunidad de Madrid | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | TOTAL | PP | PSOE | Podemos | C's | IUCM-LV | UPyD | Otro partido | Voto nulo |
| **PP** | | | | | | | | | |
| Muy cercano | 3,4 | 17,4 | - | - | 1,1 | - | - | - | |
| Cercano | 15,7 | 52,7 | 2,3 | 0,5 | 14,6 | - | 20,0 | 24,1 | |
| Ni cercano ni distante | 16,0 | 22,2 | 5,7 | 4,9 | 41,6 | - | ,0 | 10,3 | 0,0 |
| Distante | 15,3 | 4,2 | 18,9 | 12,0 | ,5 | 8,7 | 50,0 | 31,0 | 10,0 |
| Muy distante | 43,5 | 1,8 | 6 | 1,0 | 20,2 | 91,3 | 20,0 | 31,0 | 40,0 |
| N.S. | 2,8 | 1,8 | 0,6 | 1,1 | - | - | - | 3,4 | - |
| N.C. | 3,3 | - | - | 0,5 | - | - | - | - | - |
| **(N)** | (973) | (167) | (175) | (184) | (89) | (23) | (10) | (29) | (10) |

**Pregunta 23**

Respecto a los siguientes partidos políticos, m... o muy distante de cada uno de ellos.

| | TOTAL | PP | PS |
|---|---|---|---|
| **PP** | | | |
| Muy cercano | 3,4 | 17,4 | |
| Cercano | 15,7 | 52,7 | |
| Ni cercano ni distante | 16,0 | 22,2 | |
| Distante | 15,3 | 4,2 | |
| Muy distante | 43,5 | 1,8 | |
| N.S. | 2,8 | 1,8 | |
| N.C. | 3,3 | - | |
| **(N)** | (973) | (167) | |

# Proposal

- From the above information **one way to build the preference profile is to consider that if the average preference value of some party voters for another party P$_1$ is greater than for another party P$_2$, then EVERY ONE AND ALL OF THAT PARTY VOTERS strictly prefer P$_1$ to P$_2$:**

$$v[P_1] > v[P_2] \leftrightarrow P_1 \succ P_2$$

20

# Proposal

- For example: If we review the data of "sympathy for the leaders of other political parties" in the 2016 post-election survey, the average of the sympathy value expressed by UP voters is as follows:

|  | UP | EH-BILDU | EAJ-PNV | PSOE | CCa | C's | PP |
|---|---|---|---|---|---|---|---|
| UP | 6.99 | 4.59 | 3.89 | 3.63 | 2.97 | 2.78 | 1.39 |

- We can then model UP voters' preference as:

$$UP \succ EH \succ PNV \succ PSOE \succ CCa \succ Cs \succ PP$$

# Proposal

- If we express the vote of a UP voter as a Pairwise Comparison Matrix [PCM], we obtain:

| | UP | EH-BILDU | EAJ-PNV | PSOE | CCa | C's | PP |
|---|---|---|---|---|---|---|---|
| UP | - | 1 | 1 | 1 | 1 | 1 | 1 |
| EH-BILDU | 0 | - | 1 | 1 | 1 | 1 | 1 |
| EAJ-PNV | 0 | 0 | - | 1 | 1 | 1 | 1 |
| PSOE | 0 | 0 | 0 | - | 1 | 1 | 1 |
| CCa | 0 | 0 | 0 | 0 | - | 1 | 1 |
| C's | 0 | 0 | 0 | 0 | 0 | - | 1 |
| PP | 0 | 0 | 0 | 0 | 0 | 0 | - |

22

# Proposal

- The issue with this approach is that it seems acceptable when the difference between the average rating for two policy options is very different, but it does not seem acceptable when the difference is very small.

- For example, if we check the preference of PSOE voters regarding the leaders of CDC, ERC and PP, they are practically equal:

| | PSOE | CP | CCa | C's | ECP | EM | UP | EAJ-PNV | CDC | ERC | PP | EH-BILDU |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **PSOE** | 6.77 | 4.67 | 4.53 | 4.23 | 3.51 | 3.40 | 3.07 | 2.50 | 2.09 | 2.08 | 2.07 | 2.00 |

23

# Proposal

- Data shows PSOE voters are mostly indifferent between these parties.

- How can we model this quasi-indifference of the voters regarding two parties?

# Proposal

- We have previously stated that

$$v[P_1] > v[P_2] \leftrightarrow P_1 \succ P_2$$

- But in some [exceptional] cases we can find that two options are equally preferred, which in turn would imply:

$$v[P_1] = v[P_2] \leftrightarrow P_1 \sim P_2$$

25

# Proposal

- From a binary paradigm it has been argued that the two previous equations are excluding that is:

$$v[P_1] > v[P_2] \leftrightarrow P_1 \succ P_2$$
$$\vee$$
$$v[P_1] = v[P_2] \leftrightarrow P_1 \sim P_2$$

- **This raises the problem of how to model preference orderings when the different valuation between the eligible options is almost zero.**

26

# Proposal

- Herein we propose an algorithm based on **considering that when the difference between the evaluation of two options is very small, it implies both a degree of strict preference and a degree of indifference:**

- We establish that **a preferential difference lower than 1 implies a degree of indifference**. We then have three possible situations:

27

# Proposal

- **Strict Indifference**

$$v(A) - v(B) = 0 \rightarrow A \sim B$$

- **Strict Preference**

$$v(A) - v(B) \geq 1 \rightarrow A \succ B$$

- **Partial Indifference and Partial Preference**

$$1 > v(A) - v(B) > 0 \rightarrow \left(1 - \left(v(A) - v(B)\right)\right)(A \sim B)$$
$$\wedge \; (v(A) - v(B))(A \succ B)$$

28

# Proposal

- The underlying paradigm is not adapting the order of preference according to the intensity of preference, but according to the degree to which the statement "X is preferred to Y" is fulfilled for each pair of eligible options.

- It is considered that **if the intensity differential of stated preference is greater than 1, then the statement is completely true.**

- But **if the intensity differential of stated preference is lower than 1, then the assertion is partially false.**

29

# Proposal

- That is, **if the average preference differential between two parties is less than 1, we assume that there is a percentage of voters with a strict preference between both parties, but that there is also a percentage of voters who are strictly indifferent among them.**

- This last percentage increases up to 100% of voters maintaining indifference [when the difference of Valuation approaches zero] while the first one increases up to 100% of the voters holding strict preference [if the difference of Valuation approaches1].

30

# Proposal

- Following this approach, we can easily obtain the PCM in two steps. Firstly, we draw a V Matrix expressing the difference of valuation between each pair of options:

| | UP | EH-BILDU | EAJ-PNV | PSOE | CCa | C's | PP |
|---|---|---|---|---|---|---|---|
| **UP** | - | 2.40 | 3.10 | 3.36 | 4.02 | 4.21 | 5.60 |
| **EH-BILDU** | | - | 0,70 | 0,96 | 1,62 | 1,81 | 3,20 |
| **EAJ-PNV** | | | - | 0,26 | 0,92 | 1,11 | 2.50 |
| **PSOE** | | | | - | 0,66 | 0,85 | 2,24 |
| **CCa** | | | | | - | 0,19 | 1,58 |
| **C's** | | | | | | - | 1.39 |
| **PP** | | | | | | | - |

31

# Proposal

- Building on V, we compute PCM using below formulas:
- Yellow cells: Equation 01:

$$Iij = \min\left[\max\left[Vij; \frac{[1 - Vij]}{2} + Vij\right]; 1\right]$$

- Grey cells: Equation 02 [complement]

$$Iji = 1 - Iij$$

| | UP | EH-BILDU | EAJ-PNV | PSOE | CCa | C's | PP |
|---|---|---|---|---|---|---|---|
| UP | - | | | | | | |
| EH-BILDU | | - | | | | | |
| EAJ-PNV | | | - | | | | |
| PSOE | | | | - | | | |
| CCa | | | | | - | | |
| C's | | | | | | - | |
| PP | | | | | | | - |

32

**RICARDO ALVIRA**  2018

# Proposal

- Using the above formulas, we obtain the PCM:

|           | UP   | EH-BILDU | EAJ-PNV | PSOE | CCa  | C's  | PP   |
|-----------|------|----------|---------|------|------|------|------|
| UP        | -    | 1,00     | 1,00    | 1,00 | 1,00 | 1,00 | 1,00 |
| EH-BILDU  | 0,00 | -        | 0,85    | 0,98 | 1,00 | 1,00 | 1,00 |
| EAJ-PNV   | 0,00 | 0,15     | -       | 0,63 | 0,96 | 1,00 | 1,00 |
| PSOE      | 0,00 | 0,02     | 0,37    | -    | 0,83 | 0,93 | 1,00 |
| CCa       | 0,00 | 0,00     | 0,04    | 0,17 | -    | 0,60 | 1,00 |
| C's       | 0,00 | 0,00     | 0,00    | 0,08 | 0,41 | -    | 1,00 |
| PP        | 0,00 | 0,00     | 0,00    | 0,00 | 0,00 | 0,00 | -    |

- Since we had arranged the options from the best to the worst valued, in the upper right half there are always higher or [in case of strict indifference] equal values than in the lower left half.

33

# Proposal

- To see the difference between both approaches, it is useful to compare the PCM generated for a UP typical voter, considering both binary and fuzzy preferences:

| | UP | EH-BILDU | EAJ-PNV | PSOE | CCa | C's | PP |
|---|---|---|---|---|---|---|---|
| UP | - | 1 | 1 | 1 | 1 | 1 | 1 |
| EH-BILDU | 0 | - | 1 | 1 | 1 | 1 | 1 |
| EAJ-PNV | 0 | 0 | - | 1 | 1 | 1 | 1 |
| PSOE | 0 | 0 | 0 | - | 1 | 1 | 1 |
| CCa | 0 | 0 | 0 | 0 | - | 1 | 1 |
| C's | 0 | 0 | 0 | 0 | 0 | - | 1 |
| PP | 0 | 0 | 0 | 0 | 0 | 0 | - |

| | UP | EH-BILDU | EAJ-PNV | PSOE | CCa | C's | PP |
|---|---|---|---|---|---|---|---|
| UP | - | 1 | 1 | 1 | 1 | 1 | 1 |
| EH-BILDU | 0 | - | 0,85 | 0,98 | 1 | 1 | 1 |
| EAJ-PNV | 0 | 0,15 | - | 0,63 | 0,96 | 1 | 1 |
| PSOE | 0 | 0,02 | 0,37 | - | 0,83 | 0,93 | 1 |
| CCa | 0 | 0 | 0,04 | 0,17 | - | 0,60 | 1 |
| C's | 0 | 0 | 0 | 0,08 | 0,41 | - | 1 |
| PP | 0 | 0 | 0 | 0 | 0 | 0 | - |

# Proposal

- Herein proposed algorithm **allows us to assess that often the preference of a group of voters between two parties can be similar, that is, it can imply a degree of indifference**.

- It is important to emphasize that this prodecure **is not based on considering interpersonal comparisons of utility but on the concepts of degree of truth and complementary value.**

35

# Proposal

- Once the procedure has been explained, we review two examples of real elections.

PART III

# REVIEW OF TWO ELECTIONS

37

# evaluation real elections

- First, we review **Community of Madrid elections 2015**

- We use **two contrasting criteria**:

  - We check whether the **seat assignment to parties satisfies the Condorcet winner criterion**.

  - We check the **correlation between allocation of seats and citizens collective preference for parties ...** [Miller & Stokes, 1963; Alvira, 2017]. Since we adapt this last criterion slightly, we explain it briefly below.

38

# evaluation real elections

- To model the collective preference, we first place the voters according to their self-location in each relevant dimension ...



LEFT.RIGHT [Es 3087.p. 34]

VOTERS SELF PLACEMENT [%]
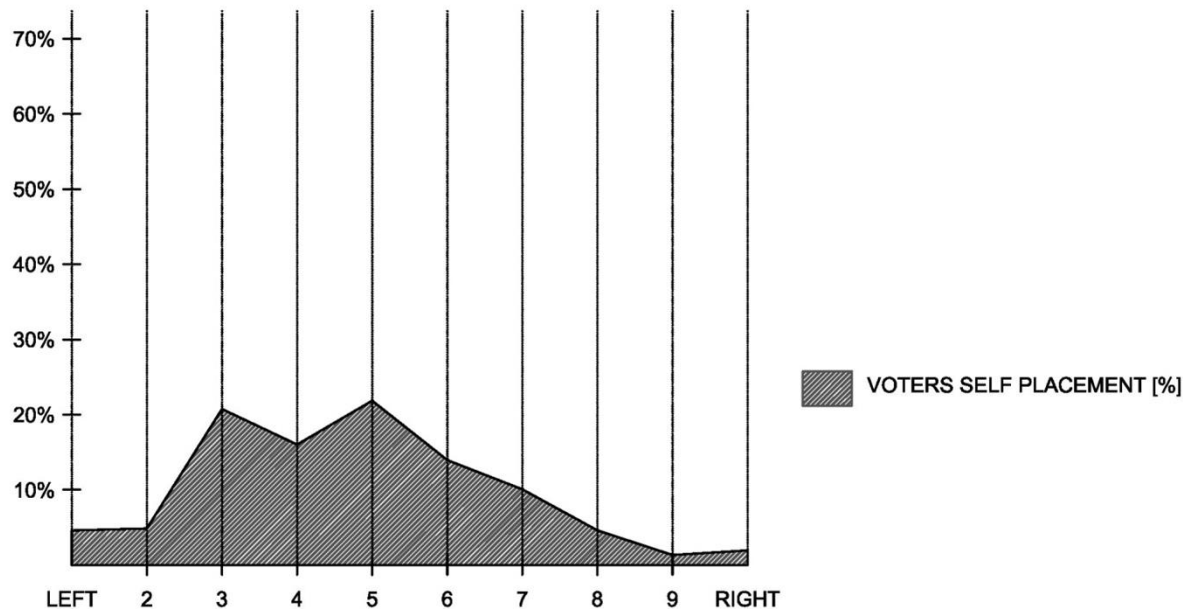
# evaluation real elections

- … secondly, we draw the collective preference for each point of the dimension, assuming preferences are single peaked, so the collective preference increases from the extremes to the median voter [Black, 1948]
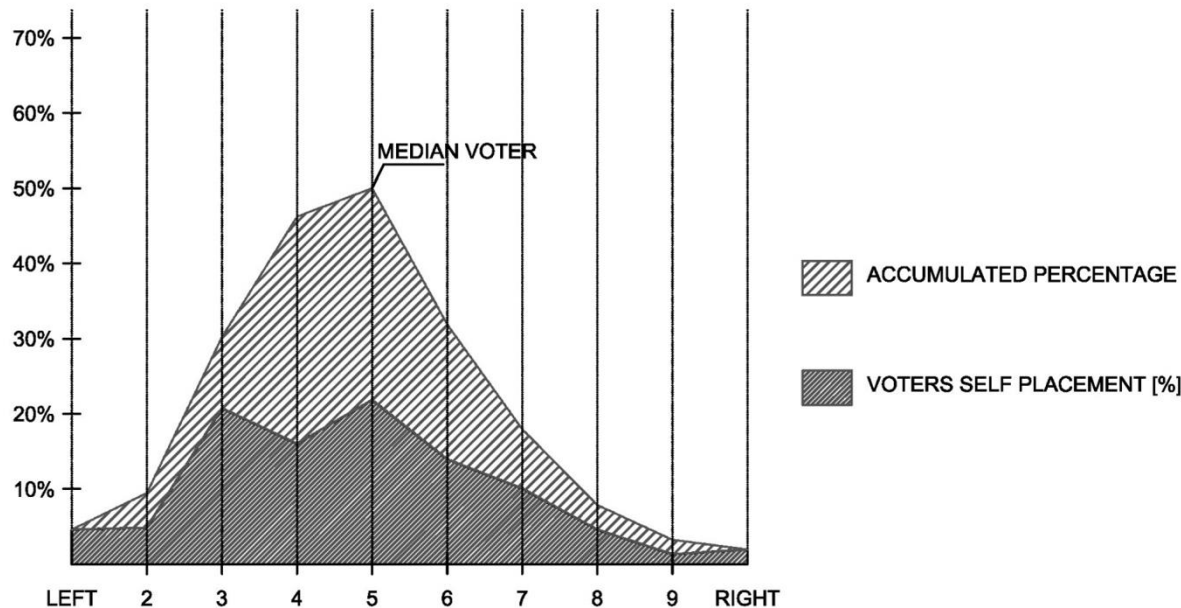
# evaluation real elections

- Finally, we locate the parties in the dimension, and compare the percentage of seats that each one receives with the collective preference of the citizenship for the location of said party in the ideological space.
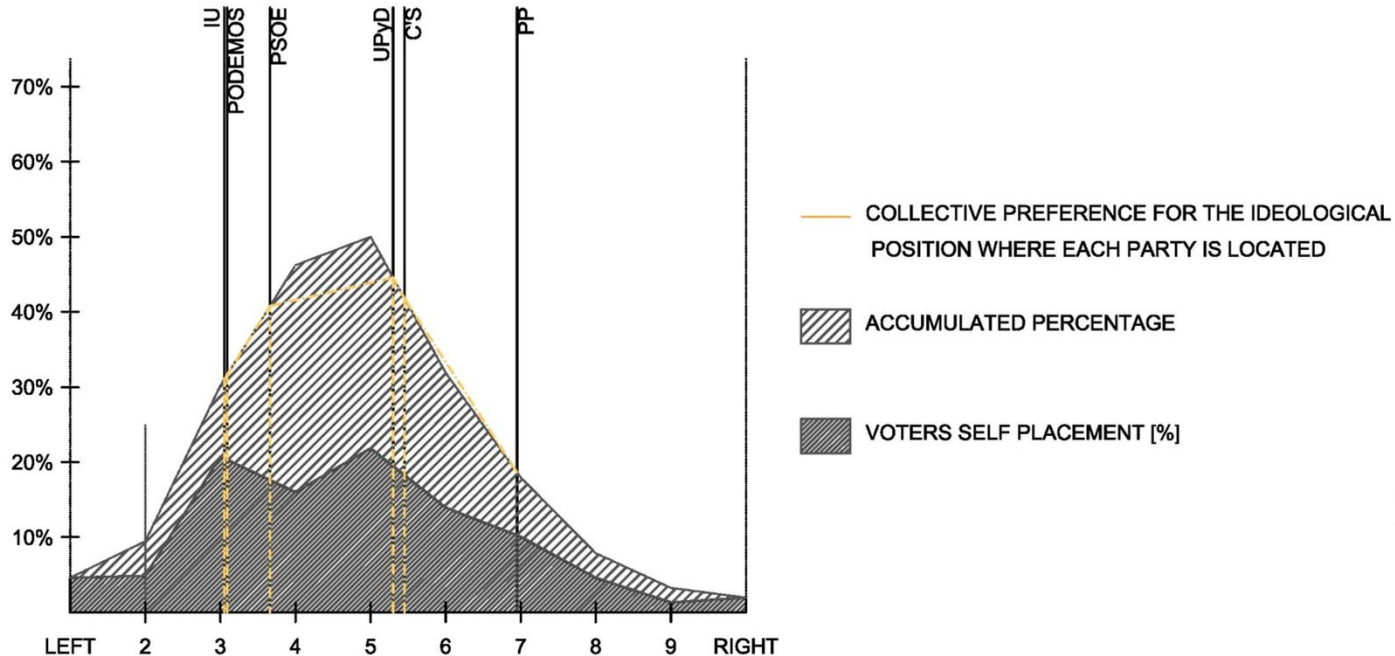
# evaluation real elections

- Once the criterion has been explained, let us review the elections. We also briefly review the current electoral system in the Community of Madrid.

42

# evaluation: Madrid 2015.

- The *Ley 11/1986, de 16 de diciembre, Electoral de la Comunidad de Madrid* defines the electoral system [Art 18]:

1. The electoral district is the Community of Madrid.

2. For the distribution of seats, only the lists that have obtained at least 5% of total valid votes are taken into account.

3. The allocation of seats shall be made in the manner established by the Ley Orgánica de Régimen Electoral General [LOREG] for the allocation of seats in the Congress at provincies level [i.e., using d'Hondt rule].

43

# evaluation: Madrid 2015.

- In 2015, this electoral system leads to the following allocation of seats:

| | | | SEATS | |
|---|---|---|---|---|
| | | | No. | % |
| PP | 1,050,256 | 33.08% | **48** | 37,21% |
| PSOE | 807,385 | 25,43% | **37** | 28,68% |
| PODEMOS | 591,697 | 18,64% | **27** | 20,93% |
| C's | 385,836 | 12,15% | **17** | 13,18% |
| IUCM - LV | 132,207 | 4,16% | - | - |
| UPyD | 64,643 | 2,04% | - | - |
| | | | 129 | |

- Therefore, according to Law 11/1986, collective preference ordering of political parties is:

$$PP \succ PSOE \succ PODEMOS \succ Cs \succ IUCM - LV \sim UPyD$$

44

# evaluation: Madrid 2015. Pr.01

- **Pr. 1:** We use the CIS survey ES.3087. Q23: "How close do you feel to each political party?". We assign to each position the following value:

| | |
|---|---|
| Very close | 10 |
| Close | 7.5 |
| Neither close nor distant | 5 |
| Distant | 2.5 |
| Very distant | 0 |

- We calculate for each group of voters the average valuation of each party. For example, the assessment of the parties according to UPyD voters:

| 8.00 | | 4.75 | | 3,25 | | 3,25 | | 2,50 | | 2,25 |
|---|---|---|---|---|---|---|---|---|---|---|
| UPyD | > | C's | > | PP | ~ | PSOE | > | IUCM - LV | > | PODEMOS |

45

# evaluation: Madrid 2015. Pr.01

- Consequently, we obtain for each party voters a PCM that implies only strict preference or strict indifference. For example, the matrix for a typical UPyD voter is:

| | UPyD | C's | PP | PSOE | IUCM - LV | PODEMOS |
|---|---|---|---|---|---|---|
| UPyD | - | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |
| C's | 0,00 | - | 1,00 | 1,00 | 1,00 | 1,00 |
| PP | 0,00 | 0,00 | - | 0,50 | 1,00 | 1,00 |
| PSOE | 0,00 | 0,00 | 0,50 | - | 1,00 | 1,00 |
| IUCM - LV | 0,00 | 0,00 | 0,00 | 0,00 | - | 1,00 |
| PODEMOS | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | - |

# evaluation: Madrid 2015. Pr.01

- If we multiply the number of voters of each party by the PCM defined for its voters, we obtain the following PCM [we rearrange the parties according to Left –Right Dimension and we reduce the legal threshold to 3%]

| | IUCM-LV | Podemos | PSOE | C's | PP |
|---|---|---|---|---|---|
| IUCM-LV | - | **2.440.327** | 723.904 | **1.531.289** | **1.531.289** |
| Podemos | 591.697 | - | 723.904 | **1.531.289** | **1.531.289** |
| PSOE | **2.308.120** | **2.308.120** | - | **1.531.289** | **1.563.611** |
| C's | 1.500.735 | 1.500.735 | 1.500.735 | - | **1.981.768** |
| PP | 1.500.735 | 1.500.735 | 1.468.414 | 1.050.256 | |

- Therefore, the collective preference ordering is:

$$PSOE \succ IUCM - LV \succ PODEMOS \succ Cs \succ PP$$

47

# evaluation: Madrid 2015. Pr.01

- Using above matrix, we can obtain the seat allocations [in percentage] that would produce other rules. For example, using **Borda Count** [Borda, 1771] and **Prll** [Alvira, 2016]:

|  | BORDA | Prll |
|---|---|---|
| **IUCM - LV** | 20,54% | 20,39% |
| **PODEMOS** | 14,44% | 20,00% |
| **PSOE** | 25,43% | 20,78% |
| **C's** | 21,38% | 19,61% |
| **PP** | 18,21% | 19,22% |

- According to these rules, the collective preference ordering of the parties is:

BORDA: $$PSOE \succ Cs \succ IUCM - LV \succ PP \succ PODEMOS$$

PRLL: $$PSOE \succ IUCM - LV \succ PODEMOS \succ Cs \succ PP$$

48

# evaluation: Madrid 2015. Pr.01

- On the other hand, we can calculate the allocation of seats using **Score Voting** [Heckscher, 1896], adding up the average valuations that each party voters make of the other parties and multiplying by the number of votes. We obtain the following allocation of seats [in percentage]:

| IUCM - LV | PODEMOS | PSOE | C's | PP |
|-----------|---------|--------|--------|--------|
| 18,09% | 18,96% | 22,63% | 22,10% | 18,22% |

- Therefore, according to this rule the collective preference ordering of the parties is:

$$PSOE > Cs > PODEMOS > PP > IUCM - LV$$

# evaluation: Madrid 2015. Pr.01

- Consistency of results: **collective preference orderings**

# evaluation: Madrid 2015. Pr.01

| | LOREG | SCORE VOTING | BORDA Pr 1 | PrII Pr 1 |
|---|---|---|---|---|
| IU | 0,00% | 18,09% | 20,54% | 20,39% |
| PODEMOS | 20,93% | 18,96% | 14,44% | 20,00% |
| PSOE | 28,68% | 22,63% | 25,43% | 20,78% |
| C's | 13,18% | 22,10% | 21,38% | 19,61% |
| PP | 37,21% | 18,22% | 18,21% | 19,22% |
| | 1,000 | 1,000 | 1,000 | 1,000 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **LAW 11/1986** | PP | > | PSOE | > | PODEMOS | > | C's | > | IUCM - LV |
| **SCORE VOTING** | PSOE | > | C's | > | PODEMOS | > | PP | > | IUCM - LV |
| **BORDA** | PSOE | > | C's | > | IUCM - LV | > | PP | > | PODEMOS |
| **PrII** | PSOE | > | IUCM - LV | > | PODEMOS | > | C's | > | PP |

51

# evaluation: Madrid 2015. Pr.01

- Consistency of results: **collective preference orderings**

- We see a **strong Borda Paradox: the most preferred option according to the** *Law 11/1986* **is the actual Condorcet Loser**

- However, it is striking that the Condorcet consistent rule [PrII] places 2nd and 3rd places away from the median voter, challenging the validity of the results …

52

# evaluation: Madrid 2015. Pr.01

- Consistency of results: **correlation between collective preference and allocation of seats to parties**

# evaluation: Madrid 2015. Pr.01

**Left- Right Dimension**
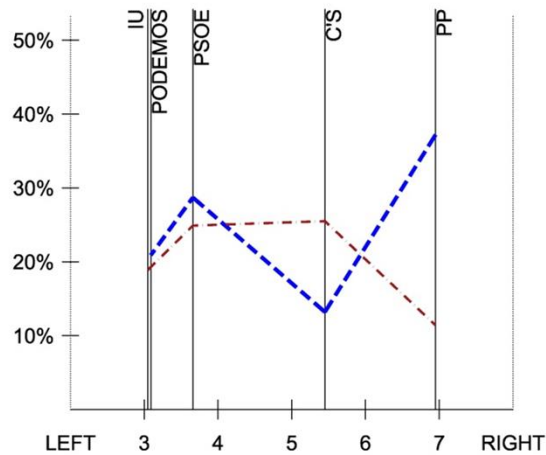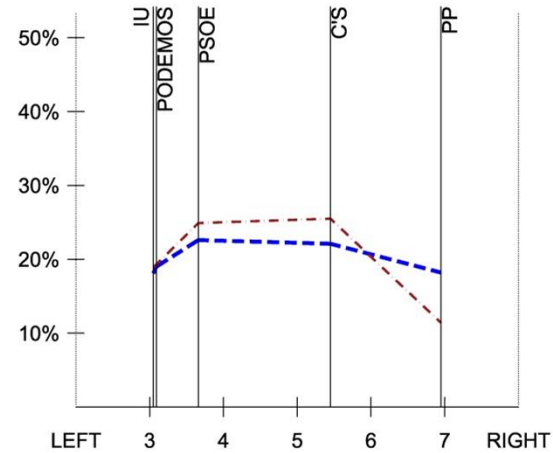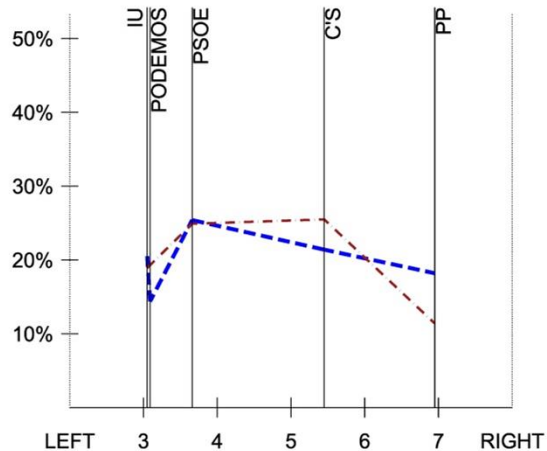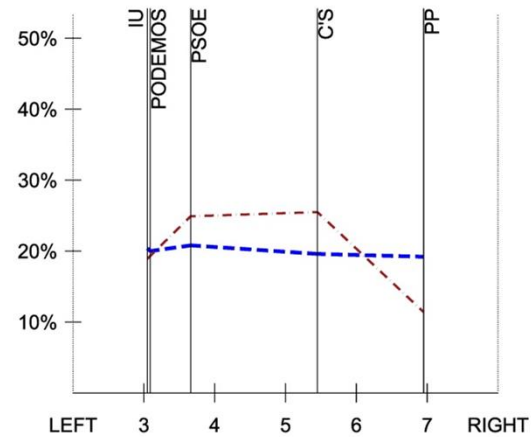


54

# evaluation: Madrid 2015. Pr.01

**Territorial Model**



LOREG
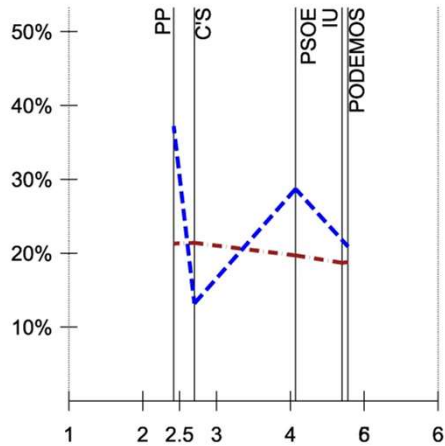
SCORE VOTING
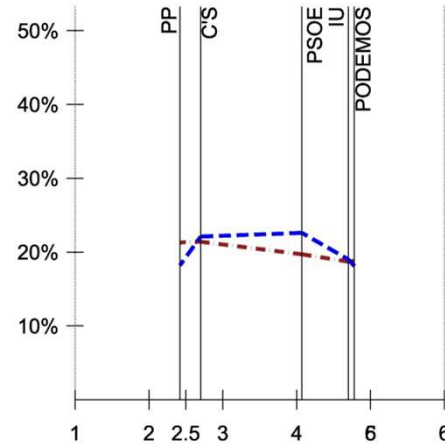
BORDA PR 1
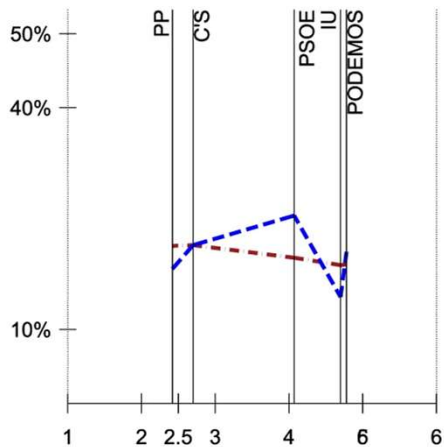
PRLL PR 1
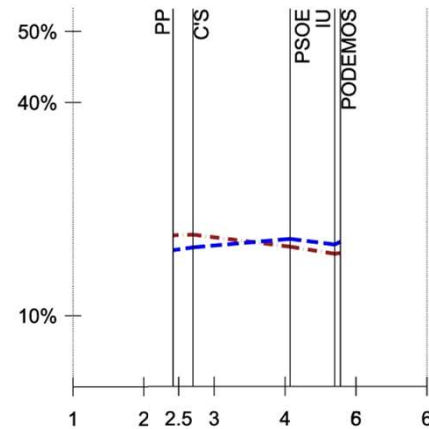
- - - CITIZENS COLLECTIVE PREFERENCE
- - - SEATS ALLOCATION TO PARTIES

55

# evaluation: Madrid 2015.

[correlation results are shown/reviewed later]

# evaluation: Madrid 2015.

- Next, we review the allocation of seats using the proposed algorithm [Pr 02] for modeling the preference profile:

57

# evaluation: Madrid 2015. Pr.02

- Using Pr02 small preference differentials are translated into some percentage of indifference. For example, the typical PSOE voter PCM according binary [Pr 01] an fuzzy [Pr02] are:

**Pr 01**

|  | PSOE | IUCM - LV | PODEMOS | C's | UPyD | PP |
|---|---|---|---|---|---|---|
| PSOE | - | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |
| IUCM - LV | 0,00 | - | 1,00 | 1,00 | 1,00 | 1,00 |
| PODEMOS | 0,00 | 0,00 | - | 1,00 | 1,00 | 1,00 |
| C's | 0,00 | 0,00 | 0,00 | - | 1,00 | 1,00 |
| UPyD | 0,00 | 0,00 | 0,00 | 0,00 | - | 1,00 |
| PP | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | - |

**Pr02**

|  | PSOE | IUCM - LV | PODEMOS | C's | UPyD | PP |
|---|---|---|---|---|---|---|
| PSOE | - | 1,00 | 1,00 | 1,00 | 1,00 | 1,00 |
| IUCM - LV | 0,00 | - | 0,52 | 1,00 | 1,00 | 1,00 |
| PODEMOS | 0,00 | 0,48 | - | 1,00 | 1,00 | 1,00 |
| C's | 0,00 | 0,00 | 0,00 | - | 0,88 | 1,00 |
| UPyD | 0,00 | 0,00 | 0,00 | 0,12 | - | 1,00 |
| PP | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | - |

58

# evaluation: Madrid 2015. Pr.02

- And the PCMs for the whole set of voters are:

## Pr 01 [binary]

|          | IUCM-LV   | Podemos   | PSOE      | C's       | PP        |
|----------|-----------|-----------|-----------|-----------|-----------|
| IUCM-LV  | -         | 2.440.327 | 723.904   | 1.531.289 | 1.531.289 |
| Podemos  | 591.697   | -         | 723.904   | 1.531.289 | 1.531.289 |
| PSOE     | 2.308.120 | 2.308.120 | -         | 1.531.289 | 1.563.611 |
| C's      | 1.500.735 | 1.500.735 | 1.500.735 | -         | 1.981.768 |
| PP       | 1.500.735 | 1.500.735 | 1.468.414 | 1.050.256 |           |

## Pr.02 [fuzzy]

|          | IUCM-LV   | Podemos   | PSOE      | C's       | PP        |
|----------|-----------|-----------|-----------|-----------|-----------|
| IUCM-LV  | -         | 1.501.088 | 731.984   | 1.531.289 | 1.539.369 |
| Podemos  | 1.530.936 | -         | 723.904   | 1.531.289 | 1.531.289 |
| PSOE     | 2.309.040 | 2.308.120 | -         | 1.494.102 | 1.626.309 |
| C's      | 1.500.735 | 1.500.735 | 1.537.922 | -         | 1.981.768 |
| PP       | 1.492.655 | 1.500.735 | 1.405.715 | 1.050.256 |           |

# evaluation: Madrid 2015. Pr.02

- Noteworthy, by modeling fuzzy preferences [Pr 02] **a cyclical majority relationship [Condorcet paradox] appears involving 4 options**

# evaluation: Madrid 2015. Pr.02

- We evaluate the consistency of the results. We begin with the preference orderings

# evaluation: Madrid 2015. Pr.02

| | LEY 11/1986 | SCORE VOTING | BORDA Pr 2 | Prll Pr 2 |
|---|---|---|---|---|
| IUCM - LV | - | 18,09% | 17,49% | 14,05% |
| PODEMOS | 20,93% | 18,96% | 17,54% | 14,33% |
| PSOE | 28,68% | 22,63% | 25,49% | 28,72% |
| C's | 13,18% | 22,10% | 21,51% | 29,12% |
| PP | 37,21% | 18,22% | 17,97% | 13,77% |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **LEY 11/1986** | PP | > | PSOE | > | PODEMOS | > | C's | > | IUCM - LV |
| **SCORE VOTING** | PSOE | > | C's | > | PODEMOS | > | PP | > | IUCM - LV |
| **BORDA** | PSOE | > | C's | > | PP | > | PODEMOS | > | IUCM - LV |
| **Prll** | C's | > | PSOE | > | PODEMOS | > | IUCM - LV | > | PP |

[……………………………………….…majority cycle …………………………………………..…]

62

# evaluation: Madrid 2015. Pr. 02

- Consistency of results: **collective preference orderings**

- Again, there is a strong **Borda Paradox: the most preferred option according to Law 11/1986 is the Condorcet Loser**

- The cyclical majority prevents us from establishing the Condorcet ordering of the other parties [PP would still be the least preferred option, since it is located below the Smith Set].

- The two closest parties to the median voter appear in 1st and 2nd position, showing more consistency than in the modeling of binary preferences, Pr. 01

63

# evaluation: Madrid 2015. Pr.02

- Consistency of results: **correlation between collective preference and allocation of seats to parties**

# evaluation: Madrid 2015. Pr.02

**Left-Right Dimension Dcha.**
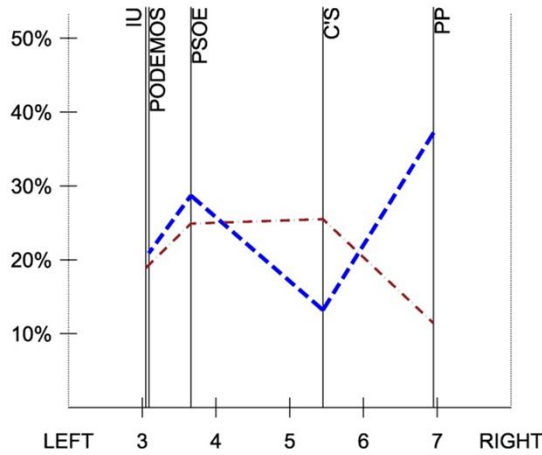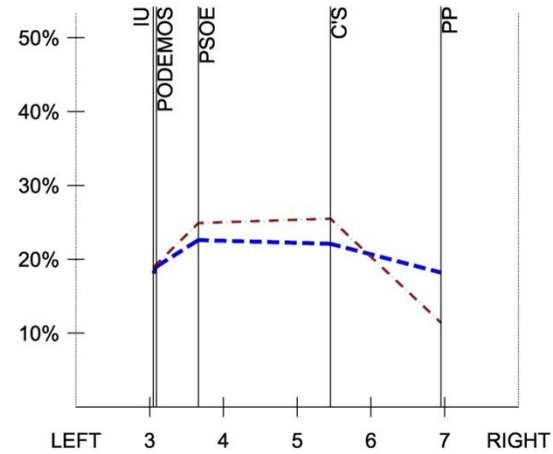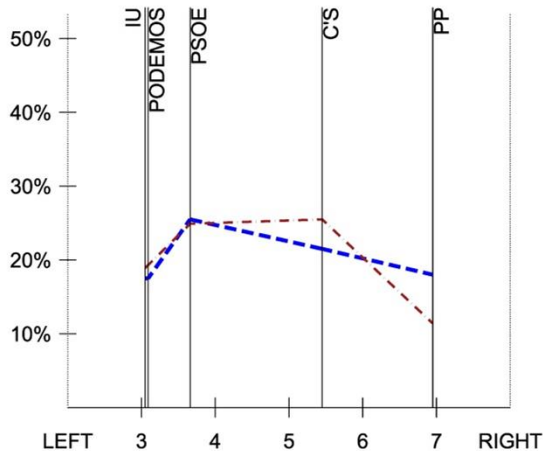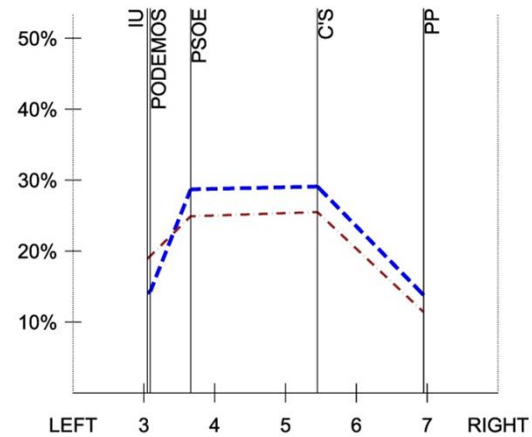


65

# evaluation: Madrid 2015. Pr.02

**Territorial Model**
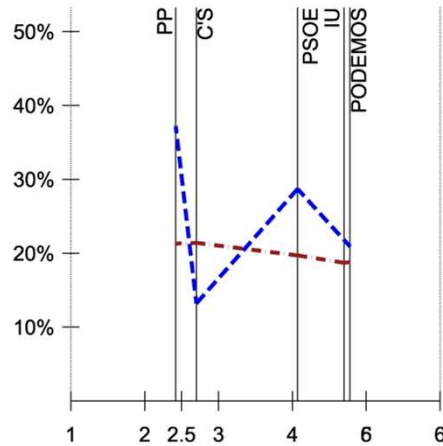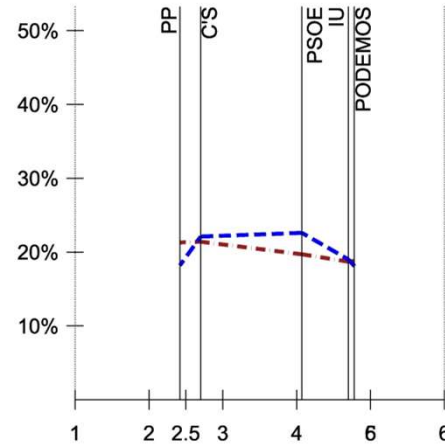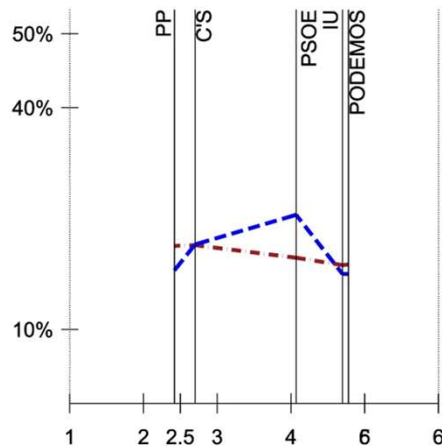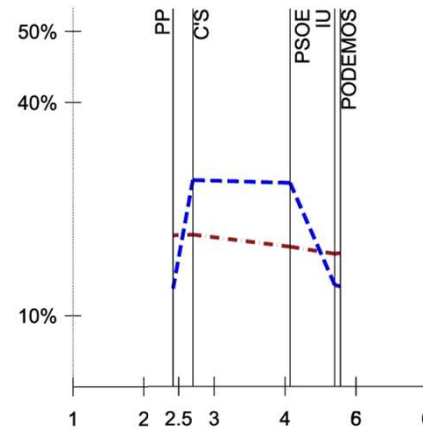


66

# evaluation: Madrid 2015.

- **Correlation between allocation of seats and collective preference:**

| | LEY 11/1986 | SCORE VOTING | BORDA | | Prll | |
|---|---|---|---|---|---|---|
| | | | Pr. 01 | Pr. 02 | Pr. 01 | Pr. 02 |
| LEFT RIGHT | -0,376 | 0,850 | 0,571 | 0,714 | 0,565 | 0,844 |
| TERRIT. MOD. | 0,461 | 0,287 | 0,162 | 0,193 | -0,731 | 0,366 |
| **MEDIA** | **0,043** | **0,043** | **0,366** | **0,454** | **-0.083** | **0.605** |

- **Correlation between seat allocation and Score Voting** [collective utility maximization]

| | LEY 11/1986 | BORDA | | Prll | |
|---|---|---|---|---|---|
| | | Pr. 01 | Pr. 02 | Pr. 01 | Pr. 02 |
| **SCORE VOTING** | 0,121 | 0,707 | 0,932 | 0,347 | 0,986 |

# evaluation: Madrid 2015.

- We see, **correlations between allocations of seats by Borda and PrlI and citizens collective preference notably improves by modelling preference profile following the fuzzy procedure** [Pr 02]. **Collective utility maximization also increases** for the fuzzy procedure. Both issues sustain the **validity of herein proposed procedure, Pr 02.**

- On the other hand, **we see current LEY 11/1986  leads to an almost nil correlation between allocation of seats and citizens collective preference, as well as to almost zero collective utility. This greatly challenges current electoral system.**

68

# evaluation: Catalonia 2017

- Next, we review the **regional elections of the Community of Catalonia in December 2017**

# evaluation: Catalonia 2017

- The current electoral law establishes the province as a circumscription, and allocation of seats through the d'Hondt rule. In the 2017 elections, results were:

| CANDIDATE | VOTES | | DEPUTIES | |
|---|---|---|---|---|
| | No. | % | No. | |
| CUP | 195.246 | 4,53% | 4 | 2,96% |
| ERC-CATSI | 935.861 | 21,72% | 32 | 23,70% |
| CATCOMU | 326,360 | 7,58% | 8 | 5,93% |
| PSC-PSOE | 606,659 | 14,08% | 17 | 12,59% |
| JUNTSXCAT | 948,233 | 22,01% | 34 | 25,19% |
| C's | 1,109,732 | 25,76% | 36 | 26,67% |
| PP | 185.670 | 4,31% | 4 | 2,96% |
| | 4,307,761 | 100,00% | 135 | |

- Therefore, the collective preference ordering of the parties is:
$$Cs \succ JxCat \succ ERC \succ PSC \succ CatComu \succ CUP \succ PP$$

70

# evaluation: Catalonia 2017

- For modeling the preference profile, we use CIS survey ES.3202. Q26: voters perceived distance to each political party.

# evaluation: Catalonia 2017 Pr. 01

- If we consider strict preferences we obtain the following PCM:

|  | CUP | CatComú-Podem | ERC-CatSí | PSC-PSOE | JUNTSxCAT | C's | PP |
|---|---|---|---|---|---|---|---|
| CUP | - | 2.079.340 | 195.246 | **2.444.443** | 521.606 | **2.444.443** | **3.051.102** |
| CatComú-Podem | **2.267.164** | - | **2.228.421** | **2.444.443** | 2.081.494 | **2.444.443** | **3.051.102** |
| ERC-CatSí | **4.151.258** | 2.118.083 | - | **2.444.443** | 1.496.210 | **2.444.443** | **3.051.102** |
| PSC-PSOE | 1.902.061 | 1.902.061 | 1.902.061 | - | **2.228.421** | **3.012.359** | **4.160.834** |
| JUNTSxCAT | **3.824.898** | **2.265.010** | **2.850.294** | 2.118.083 | - | **2.444.443** | **3.051.102** |
| C's | 1.902.061 | 1.902.061 | 1.902.061 | 1.334.145 | 1.902.061 | - | **4.160.834** |
| PP | 1.295.402 | 1.295.402 | 1.295.402 | 185.670 | 1.295.402 | 185.670 | - |

- Therefore, the collective preference ordering of the parties is:

$$[JxCat > ERC > PSC > CatComu > CUP] > Cs > PP$$

72

RICARDO ALVIRA    2018

# evaluation: Catalonia 2017. Pr. 1

- Therefore, **following the binary modeling of preferences, a cyclical majority relationship [Condorcet Paradox] appears involving 5 options**

# evaluation: Catalonia 2017. Pr. 1

- From the previous matrix, we obtain the following allocation of seats [in percentage] using **Borda and PrII rules**:

|  | BORDA | PrII |
|---|---|---|
| **CUP** | 11,76% | 15,64% |
| **ERC-CATSI** | 17,21% | 17,69% |
| **CATCOMU** | 15,90% | 18,11% |
| **PSC-PSOE** | 16,55% | 13,59% |
| **JUNTSXCAT** | 18,14% | 18,53% |
| **C's** | 14,36% | 11,54% |
| **PP** | 6,08% | 4,90% |

- Therefore, the collective preference ordering of the parties are:

BORDA: $JxCAT \succ ERC \succ PSC \succ CatComu \succ Cs \succ CUP \succ PP$

PRLL: $JxCAT \succ CatComu \succ ERC \succ CUP \succ PSC \succ Cs \succ PP$

74

# evaluation: Catalonia 2017. Pr. 1

- On the other hand, if we add up the average valuations that each party voters make of the other parties and multiply by the number of votes [**score voting**] we obtain the following allocation of seats [in percentage]:

| C's | JxCAT | ERC-CATSI | PSC-PSOE | CatComú | CUP | PP |
|---|---|---|---|---|---|---|
| 0,141 | 0,204 | 0,228 | 0,133 | 0,123 | 0,120 | 0,052 |

- Therefore, the collective preference ordering of the parties is:

$$ERC \succ JxCAT \succ Cs \succ PSC \succ CatComu \succ CUP \succ PP$$

75

# evaluation: Catalonia 2017. Pr. 1

- Consistency of results: **collective preference orderings**

# evaluation: Catalonia 2017. Pr. 1

|  | LOREG | SCORE | BORDA PR.1 | PRLL PR.1 |
|---|---|---|---|---|
| CUP | 2,96% | 12,01% | 11,76% | 15,64% |
| ERC-CATSI | 23,70% | 22,77% | 17,21% | 17,69% |
| CATCOMU | 5,93% | 12,30% | 15,90% | 18,11% |
| PSC-PSOE | 12,59% | 13,27% | 16,55% | 13,59% |
| JUNTSXCAT | 25,19% | 20,41% | 18,14% | 18,53% |
| C's | 26,67% | 14,09% | 14,36% | 11,54% |
| PP | 2,96% | 5,15% | 6,08% | 4,90% |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **LOREG** | C's > | JxCAT > | ERC > | PSC > | Catcomu > | CUP > | PP |
| **SCORE VOTING** | ERC > | JxCAT > | C's > | PSC > | Catcomu > | CUP > | PP |
| **BORDA** | JxCAT > | ERC > | PSC > | Catcomu > | C's > | CUP > | PP |
| **Prll** | JxCAT > | Catcomu > | ERC > | CUP > | PSC > | C's > | PP |

[.....................................majority cycle ..........................................]

77

# evaluation: Catalonia 2017. Pr. 1

- Consistency of results: **collective preference orderings**

- We see an almost strong **Borda Paradox: the most preferred option according to the LOREG would be in fact the preferred option in penultimate place** [although a cycle appears, C's is located below the Smith set, being only strictly preferred to the PP].

78

# evaluation: Catalonia 2017. Pr. 1

- Consistency of results: **correlation between collective preference and allocation of seats to parties**

# evaluation: Catalonia 2017. Pr. 1

**Left- Right Dimension**

# evaluation: Catalonia 2017. Pr. 1

**Territorial Model**
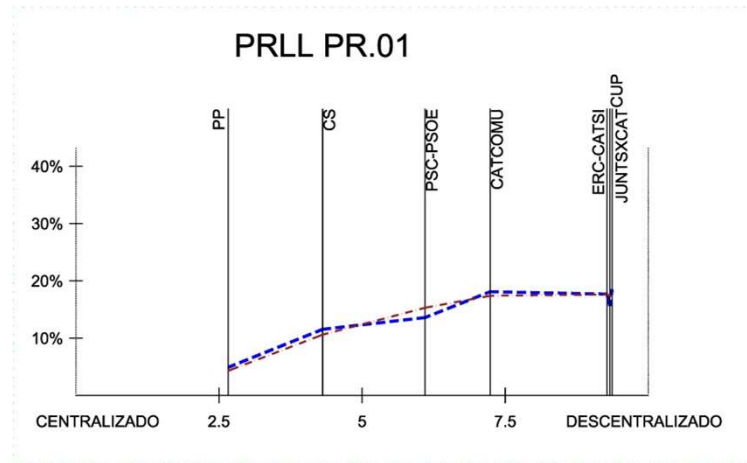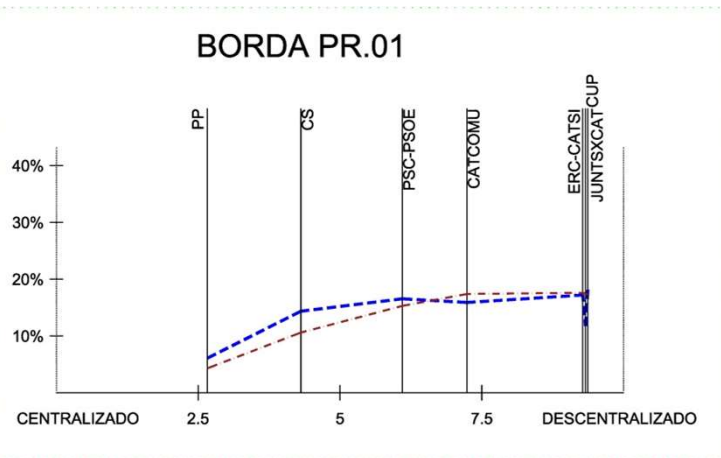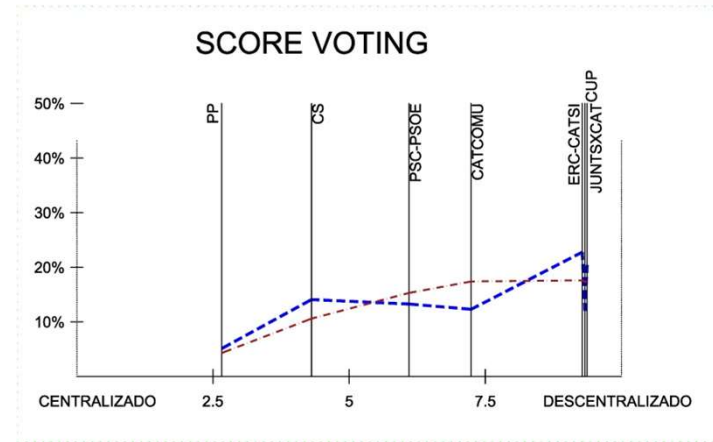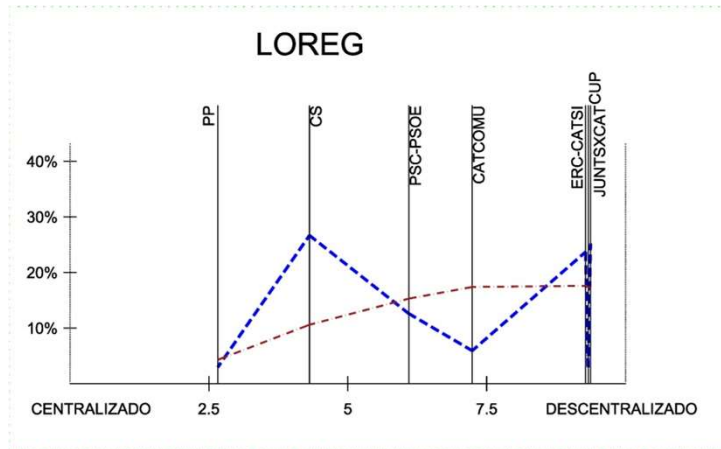
# evaluation: Catalonia 2017

[correlation results are shown/reviewed later]

# evaluation: Catalonia 2017

- Next, we review the allocation of seats using the proposed algorithm [Pr 02]

83

# evaluation: Catalonia 2017.  Pr.  2

- We obtain the following PCM:

| | CUP | CatComú-Podem | ERC-CatSí | PSC-PSOE | JUNTSxCAT | C's | PP |
|---|---|---|---|---|---|---|---|
| CUP | - | **2.226.363** | 913.157 | **2.363.520** | 935.898 | **2.434.515** | **2.788.044** |
| CatComú-Podem | 2.120.141 | - | 1.743.683 | **2.238.345** | 1.631.387 | **2.746.249** | **3.346.433** |
| ERC-CatSí | **3.433.347** | **2.602.821** | - | **2.397.685** | **2.288.980** | **2.598.476** | **3.009.751** |
| PSC-PSOE | 1.982.984 | 2.108.159 | 1.948.819 | - | 2.046.812 | **2.924.514** | **3.734.577** |
| JUNTSxCAT | **3.410.606** | **2.715.117** | 2.057.524 | **2.299.692** | - | **2.533.905** | **3.087.610** |
| C's | 1.911.989 | 1.600.255 | 1.748.028 | 1.421.990 | 1.812.599 | - | **3.318.903** |
| PP | 1.558.460 | 1.000.071 | 1.336.753 | 611.927 | 1.258.894 | 1.027.601 | - |

- Therefore, the collective preference ordering of the parties is:
$$ERC > JxCat > CUP > CatComu > PSC > Cs > PP$$

84

# evaluation: Catalonia 2017 Pr. 2

- Consistency of results: **collective preference orderings**

# evaluation: Catalonia 2017  Pr.  2

|  | LOREG | SCORE | BORDA PR.1 | PRLL PR.1 |
|---|---|---|---|---|
| **CUP** | 2,96% | 12,01% | 12,78% | 16,44% |
| **ERC-CATSI** | 23,70% | 22,77% | 17,89% | 18,24% |
| **CATCOMU** | 5,93% | 12,30% | 15,15% | 16,12% |
| **PSC-PSOE** | 12,59% | 13,27% | 16,16% | 15,74% |
| **JUNTSXCAT** | 25,19% | 20,41% | 17,64% | 17,56% |
| **C's** | 26,67% | 14,09% | 12,94% | 11,32% |
| **PP** | 2,96% | 5,15% | 7,44% | 4,58% |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **LOREG** | C's | > | JxCAT | > | ERC | > | PSC | > | Catcomu | > | CUP | > PP |
| **SCORE VOTING** | ERC | > | JxCAT | > | C's | > | PSC | > | Catcomu | > | CUP | > PP |
| **BORDA** | ERC | > | Borda | > | PSC | > | Catcomu | > | C's | > | CUP | > PP |
| **Prll** | ERC | > | Borda | > | CUP | > | Catcomu | > | PSC | > | C's | > PP |

86

# evaluation: Catalonia 2017 Pr. 2

- We see again an **almost strong Borda Paradox: the most preferred option according to the LOREG would actually be the preferred option in penultimate place.**
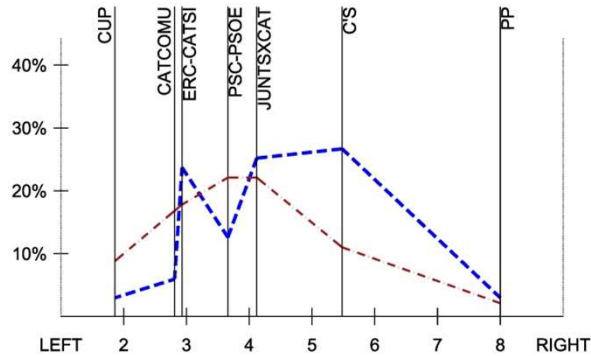
# evaluation: Catalonia 2017  Pr 2

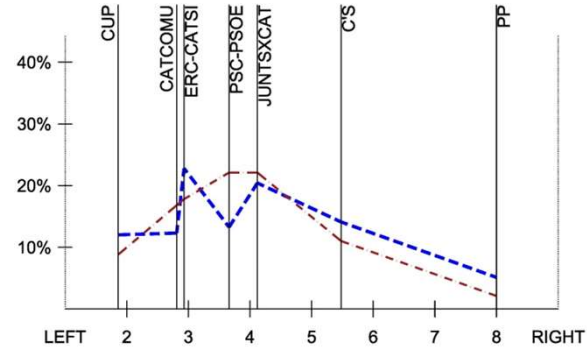- Consistency of results: **correlation between collective preference and allocation of seats to parties**
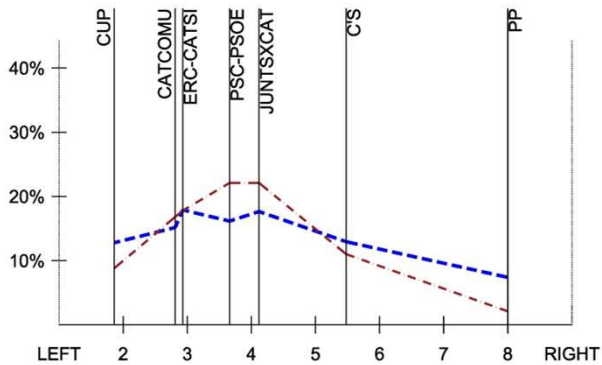
88

# evaluation: Catalonia 2017 Pr 2
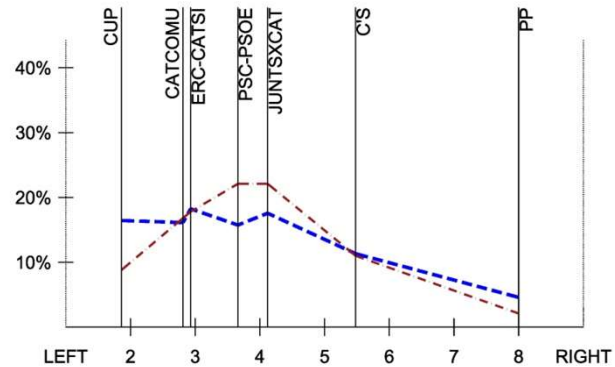
**Left- Right Dimension**



RICARDO ALVIRA 2018

89

# evaluation: Catalonia 2017  Pr 2

**Territorial Model**
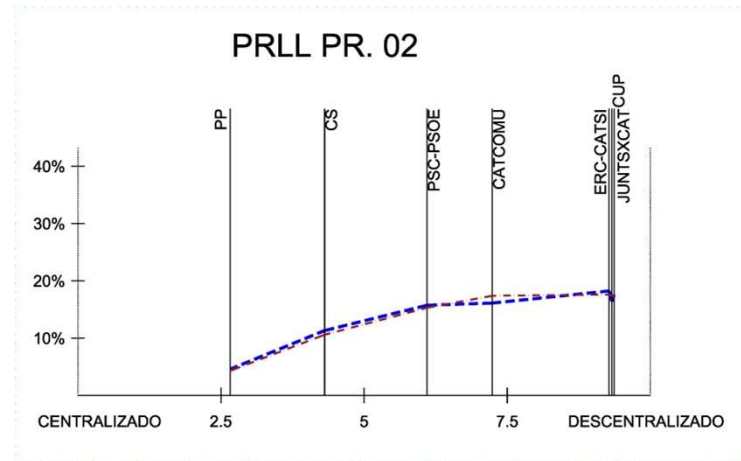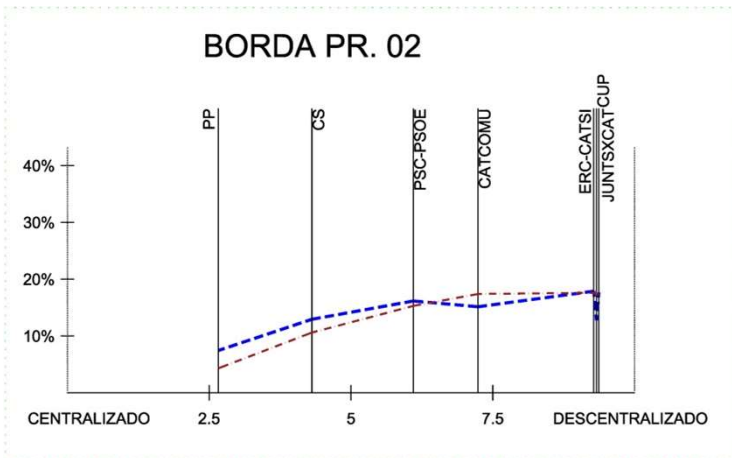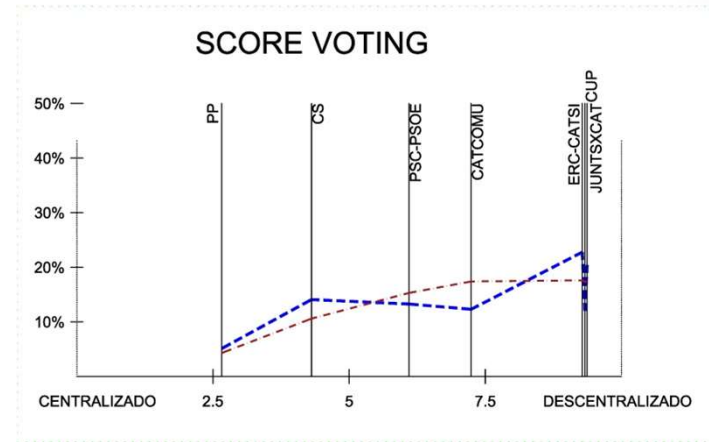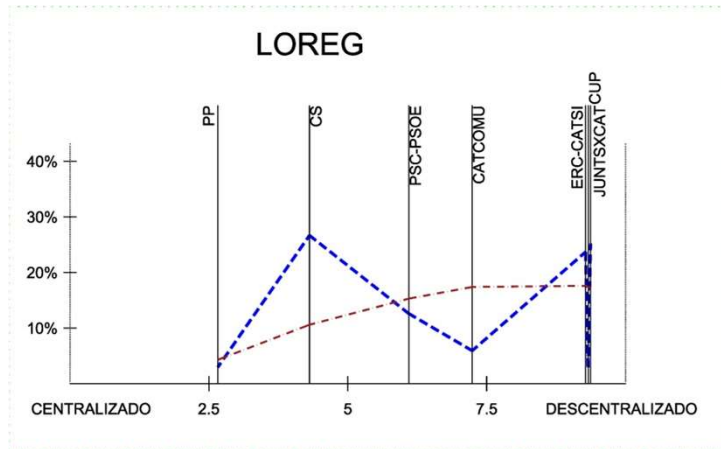


- – – – CITIZENS COLLECTIVE PREFERENCE
- - - - - SEATS ALLOCATION TO PARTIES

**RICARDO ALVIRA** **2018**

# evaluation: Catalonia 2017 Pr. 2

- **Correlation between allocation of seats and collective preference:**

|  | LOREG | SCORE VOTING | BORDA | | Prll | |
|---|---|---|---|---|---|---|
|  |  |  | Pr.01 | Pr.02 | Pr. 01 | Pr.02 |
| Left Right | 0,209 | 0,702 | 0,806 | 0,861 | 0,969 | 0,988 |
| TERRIT. MOD. | 0,520 | 0,744 | 0,951 | 0,944 | 0,776 | 0,808 |
| **AVERAGE** | **0,364** | **0.723** | **0.878** | **0.902** | **0,872** | **0,898** |

- **Correlation between seat allocation and score voting**
  [maximization of collective utility]

|  | LOREG | BORDA | | Prll | |
|---|---|---|---|---|---|
|  |  | Pr. 01 | Pr. 02 | Pr. 01 | Pr. 02 |
| **SCORE VOTING** | 0,779 | 0,846 | 0,895 | 0,772 | 0,789 |

91

# evaluation: Catalonia 2017  Pr.  2

- We see **correlation between allocations of seats by Borda and PrII and citizens collective preference improves by modelling preference profile following the fuzzy procedure** [Pr 02]. **Collective utility maximization also increases** for the fuzzy procedure. Both issues sustain again the **validity of herein proposed procedure, Pr 02.**

- On the other hand, **we see current electoral system leads to a much lower correlation between allocation of seats and citizens collective preference, and lower collective utility, which again challenges its validity.**

PART IV

# CONCLUSIONS:

93

# Conclusions

- While the properties and paradoxes of voting rules has been widely studied in the realm of Social Choice, we find scarce equivalent studies **referred to the systems for the election of representative chambers**.

- This scarce interest on the issue contrasts with the fundamental importance that Representative Chambers have acquired in our political systems, and may be due to the **difficulty of obtaining data to perform the analyzes.**

- In most cases, available data for reviewing past elections are not individuals' preference orderings but average preference values.

94

# Conclusions

- We are witnessing in Spain a situation of **increasing citizen disaffection** with current political system, **which many experts relate -at least in part- to the** *biased* **design of Spain current electoral system.**

- However, different experts disagree on the ultimate design the electoral system should have, leading to **several different proposals for electoral reform being made in recent times** [for a review of the latest ones, see Crespo et al, 2019]

# Conclusions

- As a way to move forward on the issue, it seems useful to evaluate the validity of both the current electoral system and its possible modifications by comparing allocation of seats and citizens collective preference. In order to do so, we need to review the **complete preference ordering of each voter**, not only first preferences expressed by single votes.

- However, this information is not usually accessible. As a way to fill this gap, **we have proposed a simple procedure to estimate citizens preference profile from usually available data**: average values which are frequently published by CIS.

# Conclusions

- Using that information, we have modeled two elections recently held in Spain.

- In the two reviewed examples, **modeling the preference profile using the fuzzy procedure [Pr.02] reflects more accuratelly the citizenship collective preference and maximixes collective utility compared to a profile based on strict preference/indifference [Pr. 01].**

- **Results suggests therefore the utility of herein proposed procedure [Pr02].**

97

# Conclusions

- Additionally, **the evaluation has revealed some important issues**:

# Conclusions

- **The ubiquity of the Borda Paradox** [there is a strong paradox in Madrid, and an almost strong one in Catalonia] **suggests current electoral systems [d'Hondt in both cases] may be creating an important ideological distance between representantives and citizens.**

- This departure is further increased by the fact regional governments are chosen by the representatives, and **could be the main cause of the current political instability and citizen discontent**, reinforcing the importance of reforming current Spanish [national and regional] electoral system[s].

99

# Conclusions

- The **great improvement in the correlation between citizenship preference and allocation of seats to the parties** provided by the rules that take into account voters' complete preference orderings [Score Voting, Borda and Prll] compared to current electoral systems, shows that **the path should start by abandoning single vote and adopting preferential vote.**

| AVERAGE CORRELATION BETWEEN CITIZENSHIP PREFERENCE AND ALLOCATION OF SEATS FOR THE TWO REVISED ELECTIONS | | | | |
|---|---|---|---|---|
| | SINGLE VOTE | PREFERENTIAL VOTE | | |
| | LOREG | SCORE VOTING | BORDA [PR 02] | Prll [PR 02] |
| LEFT RIGHT | -0,08 | 0,78 | 0,79 | 0,92 |
| MOD. TERR. | 0,49 | 0,52 | 0,57 | 0,59 |
| AVERAGE | **0.20** | **0.65** | **0,68** | **0,75** |

100

# Conclusions

- The case of Madrid 2015 is illustrative because **being a single district with 129 deputies, the allocation of seats is almost strictly proportional to single votes**, **and nevertheless a strong Borda Paradox is produced; ie, the most voted option is the least preferred.**

- **The reform needed by the Spanish electoral system should not be based on increasing the proportionality between votes and seats, but on replacing single vote by preferential vote, and using a voting rule that takes into account the entire voters preference orderings.**

101

# Conclusions

- Finally, it should be noted that **the frequent appearance [50% of cases] of the Condorcet Paradox** , highlights that the preference aggregation rule must be able to satisfactorily solve cyclical majority relationships .

Part V

# REFERENCES

103

# References

Alvira, R. (2016). *Mnll, Psll and Prll*

Alvira, R. (2017). 'Borda and Condorcet Paradoxes in Proportional representation. Evidence from Spain General Elections, June 2016', communication at XIV Meeting REES, Barcelona

Black, D. (1949). Some Theoretical Schemes of Proportional Representation. *Canadian Journal of Political Science*, 15 (3)

Borda, JC (1781). Mémoire sur les élections au scrutin, *Mémoires de L'Académie Royale des Science*: 657-664

Colman, A. & Pountney, I. (1978). Borda's voting paradox: theoretical likelihood and electoral occurrences, *Behavioral Science*, Volume 2 (1).

104

# References

Colomer, JM (2004). *Handbook of Electoral System Choice*. New York, Palgrave-Macmillan.

Crespo, I.; Nohlen, D. y Alvira, R. (2019). 'El sistema electoral español' in *Política y gobierno en España*. Ed: Tirant lo Blanch

Hägele, G. & Pukelsheim, F. (2008). 'The electoral systems of Nicholas of Cusa in the Catholic Concordance and beyond' in *The Church, the Councils, & Reform: The Legacy of the Fifteenth Century*, ed. Christianson et Al, Washington, DC

Kurrild - Klitgaard, P. (2008). Voting Paradoxes under Proportional Representation: Evidence from Eight Danish Elections.

# References

Lagerspetz, E. (2014). Albert Heckscher on collective decision-making.

Miller, W., & Stokes, D. (1963). Constituency Influence in Congress. *Journal of American Political Science*, 57 (1), 45-56.

Nurmi, H. (2016) *Voting Procedures under Uncertainty*, Springer

Urdánoz, J. (2009). La representación proporcional: génesis histórica de un paradigma confuso. *Revista de Estudios Políticos*, (139), 65-96

Van Deemen, A. (1993). Paradoxes of Voting in List Systems of Proportional Representation

106

# Thank you for attention !!

107