Electromagnetic wave functions of CMB and Schwarzschild space-time

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ABSTRACT

In the general relativity theory, we find electro-magnetic wave functions of Cosmic Microwave Background and Schwarzschild space-time. Specially, this article is that electromagnetic wave equations are treated by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

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1. Introduction

In the general relativity theory, our article's aim is that we find electro-magnetic wave equations and functions by gauge fixing equations in Robertson-Walker space-time and Schwarzschild space-time.

At first, Electro-magnetic field equations are in general relativity theory

$$F^{\mu\upsilon}{}_{;\upsilon} = \frac{4\pi}{c} j^{\mu} \tag{1}$$

$$F_{\mu\upsilon;\lambda} + F_{\upsilon\lambda;\mu} + F_{\lambda\mu;\upsilon} = 0 \tag{2}$$

The electro-magnetic field is

$$F_{\mu\upsilon} = A_{\upsilon;\mu} - A_{\mu;\upsilon} = \frac{\partial A_{\upsilon}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\upsilon}}$$

The gauge fixing equation in general relativity theory

$$A^{\mu}{}_{;\mu} = \frac{\partial A^{\mu}}{\partial x^{\mu}} + \Gamma^{\mu}{}_{\mu\rho} A^{\rho}$$

$$\rightarrow \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + \partial^{\rho}\Lambda)$$

$$= \partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + g^{\rho\rho}\partial_{\rho}\Lambda) \tag{3}$$

2. electro-magnetic wave equations in Robertson-Walker space-time

Because the gauge fixing equation is the electro-magnetic wave equation, the electro-magnetic wave equation is in Robertson-Walker space-time.

The Robertson-Walker solution is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{4}$$

In this time, 2-dimensional solution is

$$d\Omega = 0$$

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\frac{dr^2}{1-kr^2} \tag{5}$$

The gauge fixing equation is in 2-dimensional solution

$$\partial_{\mu}(A^{\mu} + g^{\mu\upsilon}\partial_{\upsilon}\Lambda) + \Gamma^{\mu}{}_{\mu\rho}(A^{\rho} + g^{\rho\rho}\partial_{\rho}\Lambda)$$

$$= \partial_{\mu}A^{\mu} + \Gamma^{1}{}_{10}A^{0} + \Gamma^{1}{}_{11}A^{1} + \partial_{\mu}g^{\mu\upsilon}\partial_{\upsilon}\Lambda + g^{\mu\upsilon}\partial_{\mu}\partial_{\upsilon}\Lambda + \Gamma^{1}{}_{10}g^{00}\frac{1}{c}\frac{\partial\Lambda}{\partial t} + \Gamma^{1}{}_{11}g^{11}\frac{\partial\Lambda}{\partial r} \tag{6}$$

Hence, we can find electro-magnetic wave equation in 2-dimentional Robertson-Walker space-time.

$$\partial_\mu g^{\mu\upsilon}\partial_\upsilon(\sin\Phi)+g^{\mu\upsilon}\partial_\mu\partial_\upsilon(\sin\Phi)+\Gamma^1{}_{10}g^{00}\frac{1}{c}\frac{\partial}{\partial t}(\sin\Phi)+\Gamma^1{}_{11}g^{11}\frac{\partial}{\partial r}(\sin\Phi)$$

$$=[\frac{-2kr}{\Omega^2(t)}\frac{\partial}{\partial r}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}+\frac{1-kr^2}{\Omega^2(t)}\frac{\partial^2}{\partial r^2}-\frac{\dot{\Omega}}{c\Omega}\frac{1}{c}\frac{\partial}{\partial t}+\frac{kr}{\Omega^2(t)}\frac{\partial}{\partial r}]\sin\Phi=0\,,$$

$$\Gamma^1{}_{10}=\frac{\dot{\Omega}}{c\Omega}\ ,\ \Gamma^1{}_{11}=\frac{kr}{1-kr^2} \tag{7}$$

In this time, we can think the shape of electro-magnetic wave function from 2-dimetional Robertson-Walker space-time. In this case, light is

$$d\tau^2=dt^2-\frac{1}{c^2}\Omega^2(t)\frac{dr^2}{1-kr^2}=0$$

$$\frac{dt}{\Omega(t)}=\frac{dr}{\sqrt{1-kr^2}} \tag{8}$$

$$\vec{E}=\vec{E}_0\sin\Phi,\vec{B}=\vec{B}_0\sin\Phi$$

$$\Phi=\omega_0[\int\frac{dt}{\Omega(t)}-\frac{1}{c}\int\frac{dr}{\sqrt{1-kr^2}}]$$

i) $k=1,\Phi=\omega_0[\int\frac{dt}{\Omega(t)}-\frac{1}{c}\sin^{-1}r]$

ii) $k=0,\Phi=\omega_0[\int\frac{dt}{\Omega(t)}-\frac{1}{c}r]$

iii) $k=-1,\Phi=\omega_0[\int\frac{dt}{\Omega(t)}-\frac{1}{c}\sinh^{-1}r]$ (9)

The electro-magnetic wave equation –Eq(7) is satisfied by the electro-magnetic wave function-Eq(9).

3.electro-magnetic wave equations in Schwarzschild space-time

The gauge fixing equation is the electro-magnetic wave equation, the electro-magnetic wave equation is in Schwarzschild space-time.

The Schwarzschild solution is

$$d\tau^2=(1-\frac{2GM}{rc^2})dt^2-\frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}}+r^2d\Omega^2] \tag{10}$$

In this time, 2-dimensional solution is

$$d\Omega=0$$

$$d\tau^2=(1-\frac{2GM}{rc^2})dt^2-\frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} \tag{11}$$

The gauge fixing equation is in 2-dimensional solution

$$\partial_\mu (A^\mu + g^{\mu\upsilon}\partial_\upsilon \Lambda) + \Gamma^\mu{}_{\mu\rho}(A^\rho + g^{\rho\rho}\partial_\rho \Lambda)\,1$$

$$= \partial_\mu A^\mu + \Gamma^0{}_{01}A^1 + \Gamma^1{}_{11}A^1 + \partial_\mu g^{\mu\upsilon}\partial_\upsilon \Lambda + g^{\mu\upsilon}\partial_\mu\partial_\upsilon \Lambda + \Gamma^0{}_{01}g^{11}\frac{\partial \Lambda}{\partial r} + \Gamma^1{}_{11}g^{11}\frac{\partial \Lambda}{\partial r}$$

$$= \partial_\mu A^\mu + \partial_\mu g^{\mu\upsilon}\partial_\upsilon \Lambda + g^{\mu\upsilon}\partial_\mu\partial_\upsilon \Lambda$$

$$\Gamma^0{}_{01} = \frac{GM}{r^2c^2}\frac{1}{1-\frac{2GM}{rc^2}},\ \Gamma^1{}_{11} = -\frac{GM}{r^2c^2}\frac{1}{1-\frac{2GM}{rc^2}} \qquad (12)$$

Hence, we can find electro-magnetic wave equation in 2-dimentional Schwarzschild space-time.

$$\partial_\mu g^{\mu\upsilon}\partial_\upsilon(\sin\Phi) + g^{\mu\upsilon}\partial_\mu\partial_\upsilon(\sin\Phi)$$

$$= [\frac{2GM}{r^2c^2}\frac{\partial}{\partial r} - \frac{1}{(1-\frac{2GM}{rc^2})}\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + (1-\frac{2GM}{rc^2})\frac{\partial^2}{\partial r^2}]\sin\Phi = 0 \qquad (13)$$

In this time, we can think the shape of electro-magnetic wave function from 2-dimetional Schwarzschild space-time. In this case, light is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} = 0$$

$$dt = \frac{dr}{1-\frac{2GM}{rc^2}} \qquad (14)$$

$$\vec{E} = \vec{E}_0 \sin\Phi, \vec{B} = \vec{B}_0 \sin\Phi$$

$$\Phi = \omega_0[t - \frac{1}{c}\int\frac{dr}{1-\frac{2GM}{rc^2}}] = \omega_0[t - \frac{1}{c} - \frac{2GM}{c^3}\ln|\,r - \frac{2GM}{c^2}\,|\,] \qquad (15)$$

The electro-magnetic wave equation –Eq(13) is satisfied by the electro-magnetic wave function-Eq(15).

4. Conclusion

We find electro-magnetic wave (CMB) equation, function of 2-dimentional Robertson-Walker solution and electro-magnetic wave function, equation in 2-dimentional Schwarzschild space-time.

Reference

[1]S.Yi, “Electromagnetic Field Equation and Lorentz Gauge in Rindler space-time”, The African review of physics,**11,**33(2016)-INSPIRE-HEP
[2]S.YI,”Electromagnetic Wave Function and Equation, Lorentz Force in Rindler spacetime”, International Journal of Advanced Research In Physical Science 5(9):46-56
[3]S.Weinberg,Gravitation and Cosmology(John wiley & Sons,Inc,1972)
[4]W.Rindler, Am.J.Phys.**34**.1174(1966)
[5]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V
[6]C.Misner, K,Thorne and J. Wheeler, Gravitation(W.H.Freedman & Co.,1973)
[7]S.Hawking and G. Ellis,The Large Scale Structure of Space-Time(Cam-bridge University Press,1973)
[8]R.Adler,M.Bazin and M.Schiffer,Introduction to General Relativity(McGraw-Hill,Inc.,1965)
[9]A.Miller, Albert Einstein’s Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)
[10]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)
[11]Massimo Pauri, Michele Vallisner, "Marzke-Wheeler coordinates for accelerated observers in special relativity":Arxiv:gr-qc/0006095(2000)
[12]A. Einstein, “ Zur Elektrodynamik bewegter K¨orper”, Annalen der Physik. 17:891(1905)