# Deviation of a ray of light in a gravitational field

Considering the problem of deflecting a beam light in a gravitational field is possible within the framework of the classical theory gravity, according to the formalism of which anybody or physical object located in a gravitational field will experience acceleration depending on the distance to source of gravity and regardless of the mass of this object. Thus, we can present light as the sum of infinitely small physical objects with a mass of each close to zero.

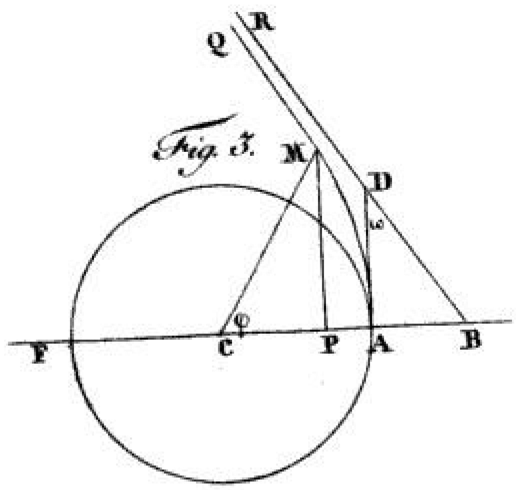
If we consider light as a stream of corpuscular particles, as viewed by Newton, then, proceeding from The Law of the Worldwide Gravity, the light should deviate from its original direction near massive bodies in the direction of the center of gravity.

I. Zoldner in 1801 published an article in an astronomical yearbook, where he provided a solution to the problem of deflecting a ray of light in a gravitational field Sun.

According to Zoldner's decision, the deflection angle of the beam was supposed to be 0"87 , but later observations showed the result 1",74-2" that was more than Zoldner's calculations at least 2 times.

A later verification of Zoldner's calculations revealed some errors in his calculations. Zoldner's faultless solution gives a deviation for the ray of light 1",74 , which corresponds to the available observations, and not less, as previously thought.

Below is Zoldner's decision and his critical analysis.




Here:

RB - direct beam,
QA - refracted ray, CM=r, CP=x, MP=y, CA=R=1, углы MCP=φ, ADB=ω.

Acceleration of free fall of the body at each point of its path Zoldner defined by the expression $\frac{2g}{r^2}$.

Projection of this acceleration on the axis:

$$\frac{ddx}{dt^2} = -\frac{2g}{r^2}cos\phi, \qquad (I)$$

$$\frac{ddy}{dt^2} = -\frac{2g}{r^2} sin\phi. \qquad \text{(II)}$$

Multiplying (I) by $-sin\phi$, and (II) by $cos\phi$ and adding them together, we get:

$$\frac{ddy \cos\phi - ddx \sin\phi}{dt^2} = 0. \qquad \text{(III)}$$

Multiplying (I) by $cos\phi$, and (II) by $sin\phi$ and adding them together, we get:

$$\frac{ddx\ cos\phi + ddy\ sin\phi}{dt^2} = -\frac{2g}{r^2}. \qquad \text{(IV)}$$

Denote:

$$x = r\ cos\phi \quad and \quad y = r\ sin\phi,$$

and differentiate:

$$dx = dr \cos\phi - r d\phi \sin\phi,$$
$$dy = dr \sin\phi + r d\phi \cos\phi.$$

Let's differentiate again:

$$ddx = \cos\phi\ ddr - 2\sin\phi\ d\phi dr - r\sin\phi\ dd\phi - r\cos\phi\ d\phi^2,$$
$$ddy = \sin\phi\ ddr + 2\cos\phi\ d\phi dr + r\cos\phi\ dd\phi - r\sin\phi\ d\phi^2.$$

Substitute **(III)** :

$$\frac{ddy\ cos\phi - ddx\ sin\phi}{dt^2} = \frac{2d\phi\ dr + r\ dd\phi}{dt^2},$$

$$\frac{2d\phi\ dr + r\ dd\phi}{dt^2} = 0. \qquad \text{(V)}$$

From **(IV)** :

$$\frac{ddr - r\ d\phi^2}{dt^2} = -\frac{2g}{r^2}. \qquad \text{(VI)}$$

Multiply **(V)** by $rdt$ :

$$\frac{2r\ d\phi\ dr + r^2\ dd\phi}{dt} = 0.$$

Integrating, we get:

$$r^2 d\phi = C\ dt.$$

Next, Zoldner seems to make a mistake and claims that constant C = cR = v where c is speed of light, although it is clear that $r^2\ \partial\varphi$ corresponds to the area of the right-angled triangle $Rr\sin\phi$, the legs of which are cdt and R and constant integration must be C=cR/2. This error does not affect the further conclusion of formulas, but only on the final formula.

From here:

$$d\phi = \frac{v\ dt}{r^2}. \qquad \textbf{(VII)}$$

Substitute **(VII)** in **(VI)**:

$$\frac{ddr}{dt^2} - \frac{v^2}{r^3} = -\frac{2g}{r^2}.$$

Multiply by: $2dr$

$$\frac{2dr\ ddr}{dt^2} - \frac{2v^2\ dr}{r^3} = -\frac{4g\ dr}{r^2}.$$

After integration:

$$\frac{dr^2}{dt^2} + \frac{v^2}{r^2} = \frac{4g}{r} + D,$$

$$dt = \frac{dr}{\sqrt{D + \frac{4g}{r} - \frac{v^2}{r^2}}}.$$

From **(VII)** :

$$d\phi = \frac{vdr}{r^2\sqrt{D + \frac{4g}{r} - \frac{v^2}{r^2}}}.$$

To integrate this equation, we will bring it to the form:

$$d\phi = \frac{v\ dr}{r^2\sqrt{D + \frac{4g^2}{v^2} - \left(\frac{v}{r} - \frac{2g}{v}\right)^2}}.$$

Denote: $\frac{v}{r} - \frac{2g}{v} = z.$

Then we will have:

$$\frac{v\ dr}{r^2} = -dz,$$

$$d\phi = \frac{-dz}{\sqrt{D + \frac{4g^2}{v^2} - z^2}}.$$

Integrating:

$$\phi = \arccos \frac{z}{\sqrt{D + \frac{4g^2}{v^2}}} + \alpha,$$

where α - some constant.

$$\cos(\phi - \alpha) = \frac{z}{\sqrt{D + \frac{4g^2}{v^2}}}.$$

$$\cos(\phi - \alpha) = \frac{v^2 - 2gr}{r\sqrt{v^2 D + 4g^2}}.$$

For $\alpha = 0$:

For $\varphi = 0$ and $r = 1$:

$$\cos\phi = \frac{v^2 - 2gr}{r\sqrt{v^2 D + 4g^2}}.$$

$$\sqrt{v^2 D + 4g^2} = v^2 - 2gr,$$

$$cos\phi = \frac{v^2 - 2gr}{r(v^2 - 2g)},$$

$$r + \left[\frac{v^2 - 2g}{2g}\right] r\ cos\phi = \frac{v^2}{2g}. \qquad \textbf{(VIII)}$$

Denote:

$$x = 1 - r\ cos\phi,$$

$$y = r\ sin\phi,$$

$$r = \sqrt{(1-x)^2 + y^2}.$$

From **(VIII)**:

$$y^2 = \frac{v^2(v^2-4g)}{4g^2}[1-x]^2 - \frac{v^2(v^2-2g)}{2g^2}[1-x] + \frac{v^2}{4g^2}.$$

Transforming:

$$y^2 = \frac{v^2}{g}x + \frac{v^2(v^2-4g)}{4g^2}x^2. \qquad \textbf{(IX)}$$

Equation for all canonical sections:

$$y^2 = px + \frac{p}{2a}x^2.$$

Need to find the angle ω:

$$tang\ \omega = \frac{AB}{AD}.$$

$$p = \frac{2b^2}{a}.$$

From the general properties of hyperbola we know:

Substitute this value into the general equation of hyperbola: $y^2 = px + \frac{p}{2a}x^2,$

then we get: $y^2 = \frac{2b^2}{a}x + \frac{b^2}{a^2}x^2.$

If we compare now the coefficients at x and x² with those that are in the equation **(IX)** we get a horizontal leg:

$$a = \frac{2g}{v^2-4g} = AB,$$

and vertical leg:

$$b = \frac{v}{\sqrt{v^2-4g}} = AD.$$

Substituting these values in tang ω:

$$\tan\omega = \frac{2g}{v\sqrt{v^2-4g}}.$$

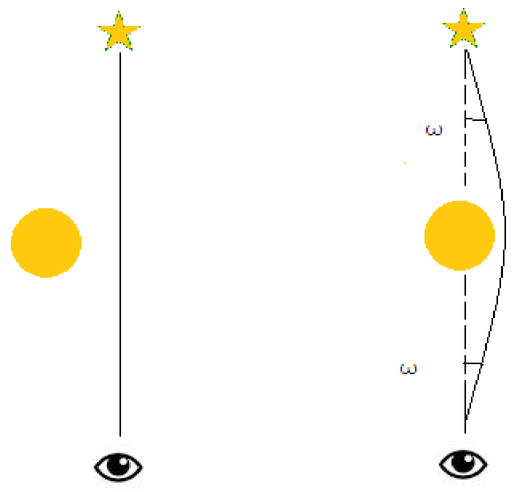
Remembering that in the numerator with g stands for R = 1, determining from the initial conditions that g = MG/2 and substituting the Zoldner constant v = cR, multiplying the angle by 2 times, we obtain the final angle according to Zoldner:

$$\Theta = \frac{2gR}{c^2}.$$

If we replace the Zoldner constant with the corrected constant $v = \frac{cR}{2}$, then the final angle for the corpuscular beam:

$$\Theta = \frac{8gR}{c^2}.$$

The latter result was obtained only if the angle ω was doubled, but it is easy to show that the observed deflection angle of the beam should not be doubled, but single.




Based on this condition, the last formula for deflecting a ray of light can be finally written down as follows:

$$\Theta = \frac{4gR}{c^2}.$$

## Observational data

This effect was first discovered in 1919 and subsequently tested many times. However, even now, the accuracy of optical measurements remains low. The measured values of the deflection of light rays lie in the range of 1,6"—2,2". During the expedition carried out in 1973 by the University of Texas and the Royal Greenwich Observatory, a deviation of 1,66" ± 0,18" was observed. Systematic errors in this experiment were not taken into account.

Measurement accuracy is significantly improved when moving from the optical to the radio range. Radio base interferometers with a large base (VLBI) have now made it possible to bring the measurement accuracy of the effect for radio waves to 2–3%. Radio measurements were carried out on two groups of sources: quasars 3G 273 and AP 279, one of which passes every year on October 8 after the Sun, and radio sources 0116 + 08, 0119 + 11 and 0111 + 02. [2]

It seems that the main reason for the deviation of the results from the calculated value may be the inhomogeneity of the density of space near the Sun, caused, for example, by the emission of a transparent gas from the Sun. For radio waves, the refractive index of such a gas may be smaller than for light. In addition, the heterogeneity of the magnetic field of the sun can affect the change in the speed of light near the sun.

## Comment

Due to the high probability of the absence of gravity in galaxies and stars, the above calculation should be applied only to planets. [3]
The coincidence of the calculated values of the observed must be understood as random

## Source

1. *http://de.wikisource.org/wiki/Ueber_die_Ablenkung_eines_Lichtstrals_von_seiner_geradlinigen_Bewegun*

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2. *1977 /. December Vol 123, no. 4, Advances in Physical Sciences, Gravitational Experiments in Space, N. L. Konopleva*
3. *http://vixra.org/abs/1912.0086 «Law of Planetary Gravity»*

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