

Una Integral Elemental

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Resumen

Esta nota muestra una integral elemental.

Integral

$$\int_0^{\infty} \left\{ 1 - \left(\sqrt[3]{\sqrt{\frac{1}{4} + \frac{1}{27x^6}} + \frac{1}{2}} - \sqrt[3]{\sqrt{\frac{1}{4} + \frac{1}{27x^6}} - \frac{1}{2}} \right)^{3/2} \right\} dx = \frac{3\sqrt{3}}{2\pi\sqrt[3]{4}} (\Gamma(2/3))^3 \quad (1)$$

El cambio de variable: $\frac{2}{3\sqrt{3}x^3} = \sinh y$, transforma la integral (1) en:

$$\int_0^{\infty} \left\{ 1 - \left((\cosh x)^{2/3} - (\sinh x)^{2/3} \right)^{3/2} \right\} \frac{\cosh(2x)}{(\sinh(2x))^{4/3}} dx = \frac{27}{8\pi} (\Gamma(2/3))^3 \quad (2)$$

Observación:

- $\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.1415\dots$
- $\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt$, $x > 0$, función gamma.

Referencias

1. Gradshteyn, I.S. and Ryzhik, I.M.: Table of Integrals, Series and Products. seventh edition, Edited by Alan Jeffrey and Daniel Zwillinger, Academic Press, 2007.