Fractales y Fórmulas

Edgar Valdebenito

27-12-2018 20:07:34

Resumen
Esta nota muestra una colección de fractales.

1. Introducción

- La constante Pi:

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 4\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+...\right) \tag{1}$$

- La constante de Catalan:

$$G = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2} = 1-\frac{1}{3^2}+\frac{1}{5^2}-\frac{1}{7^2}+... \tag{2}$$

- La constante $\ln 2$:

$$\ln 2 = \sum_{n=0}^{\infty}\frac{(-1)^n}{n+1} = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+... \tag{3}$$

- La constante $\sqrt{2}$:

$$\sqrt{2} = 2\sum_{n=0}^{\infty}\binom{2n}{n}2^{-2n} = 2\left(1-\frac{1}{2}+\frac{1\cdot 3}{2\cdot 4}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}+...\right) \tag{4}$$

- La constante e de Euler:

$$e = \sum_{n=0}^{\infty}\frac{1}{n!} = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+... \tag{5}$$

- Tres fórmulas:

$$\pi\ln 2 = \int_0^{\infty} e^{-(x/2)\sqrt{1-e^{-x}\sqrt{1-e^{-x}\sqrt{1-e^{-x}...}}}}\,dx \tag{6}$$

$$\frac{\pi\ln 2}{2} - G + \ln 2 = \int_0^{\sqrt{2}\ln 2} e^{-(x/2)\sqrt{1-e^{-x}\sqrt{1-e^{-x}\sqrt{1-e^{-x}...}}}}\,dx \tag{7}$$

$$\frac{\pi \ln 2}{2} + G - \ln 2 = \int_{\sqrt{2}\ln 2}^{\infty} e^{-(x/2)\sqrt{1-e^{-x}\sqrt{1-e^{-x}\sqrt{1-e^{-x}\cdots}}}}\, dx \tag{8}$$

2. La función: $f(z) = z - e^{-\sqrt{1-z}}$, $z \in \left(-0.1 - 0.9\sqrt{-1}, 1.7 + 0.9\sqrt{-1}\right)$.

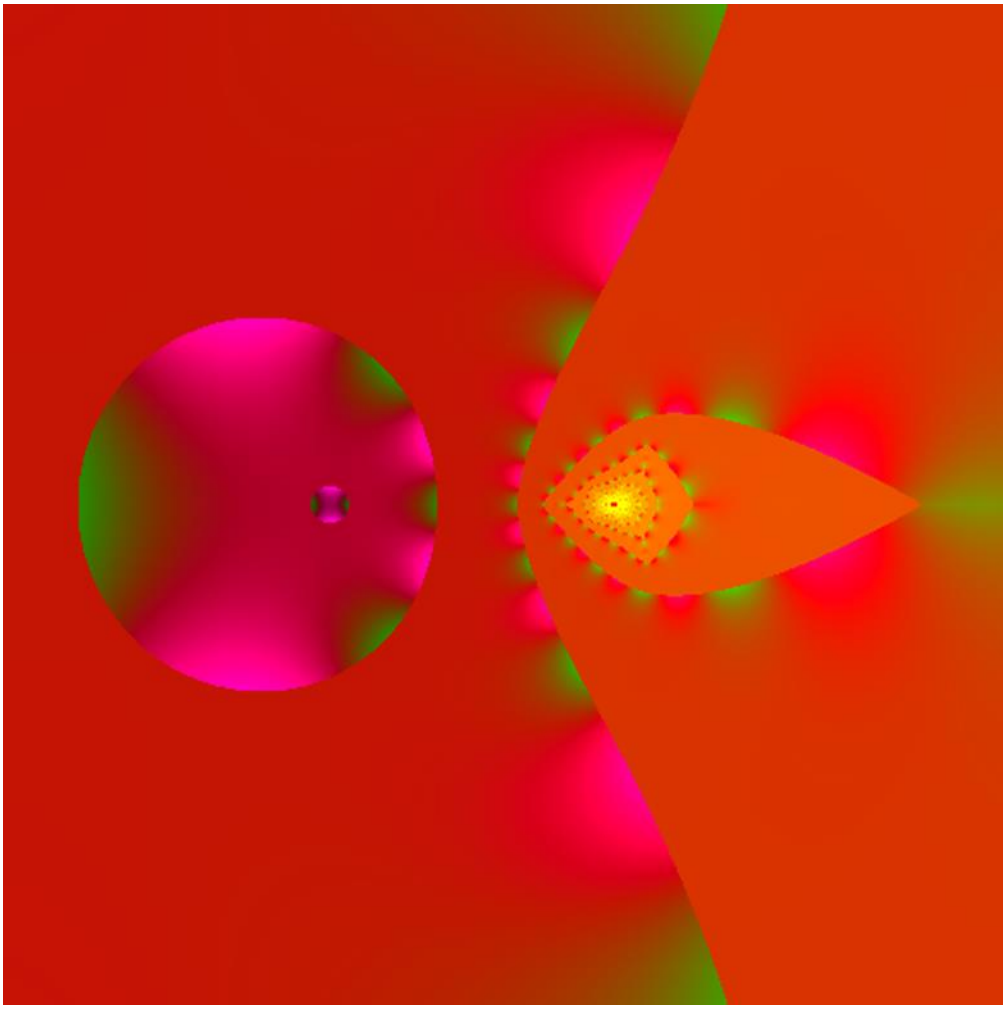

Figura 1.

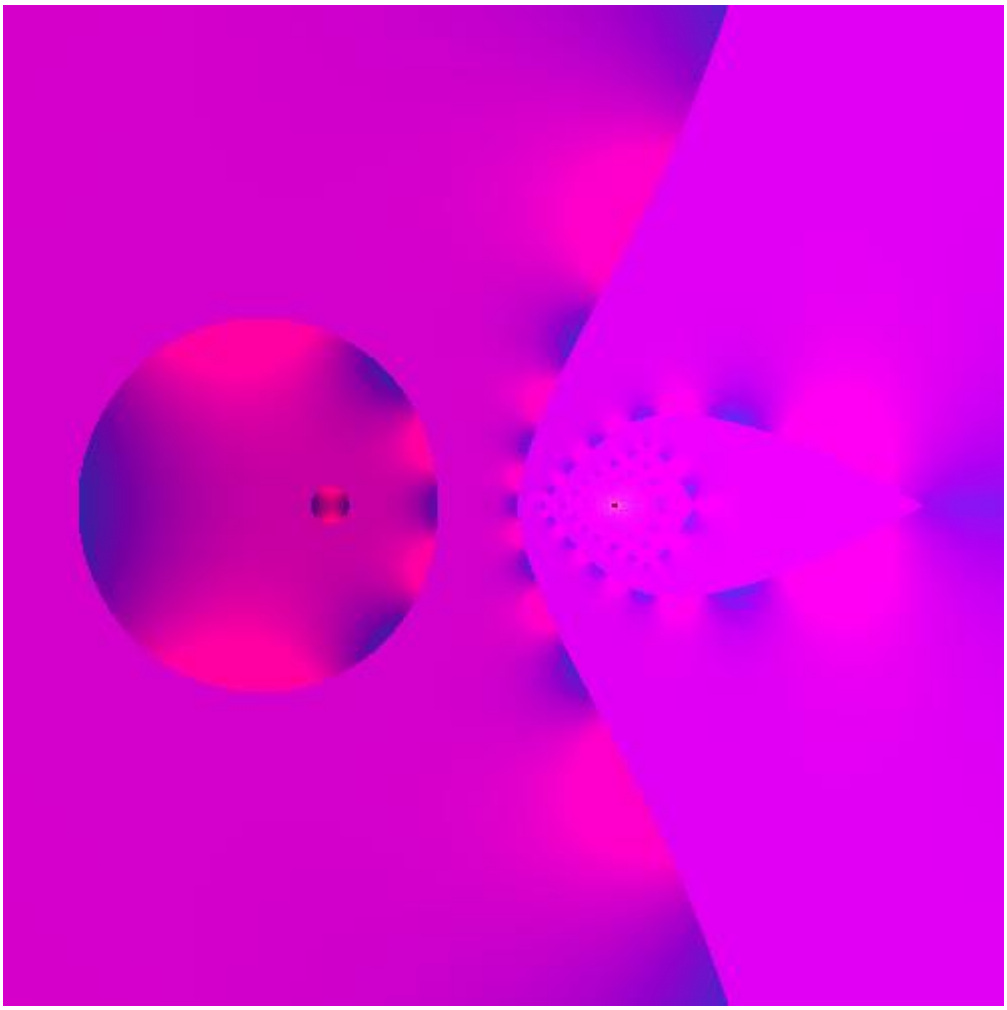

Figura 2.

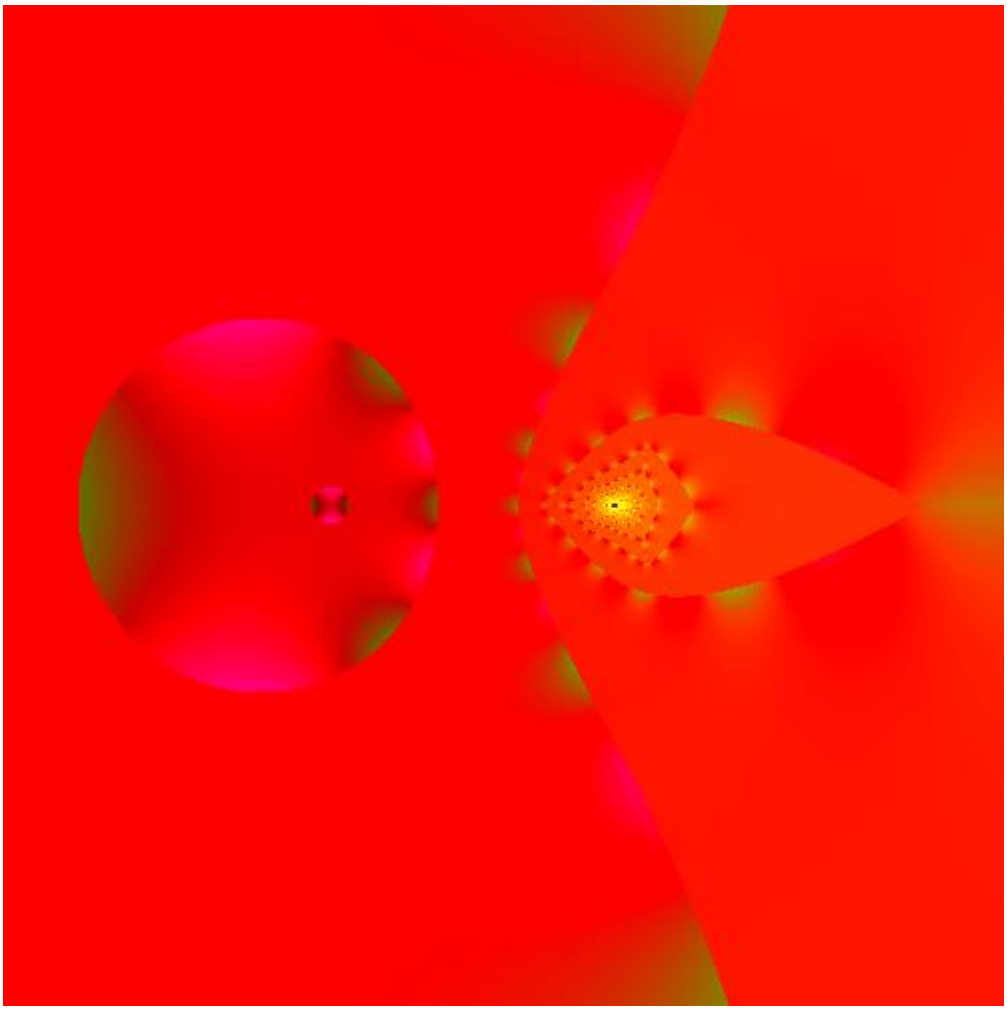

Figura 3.

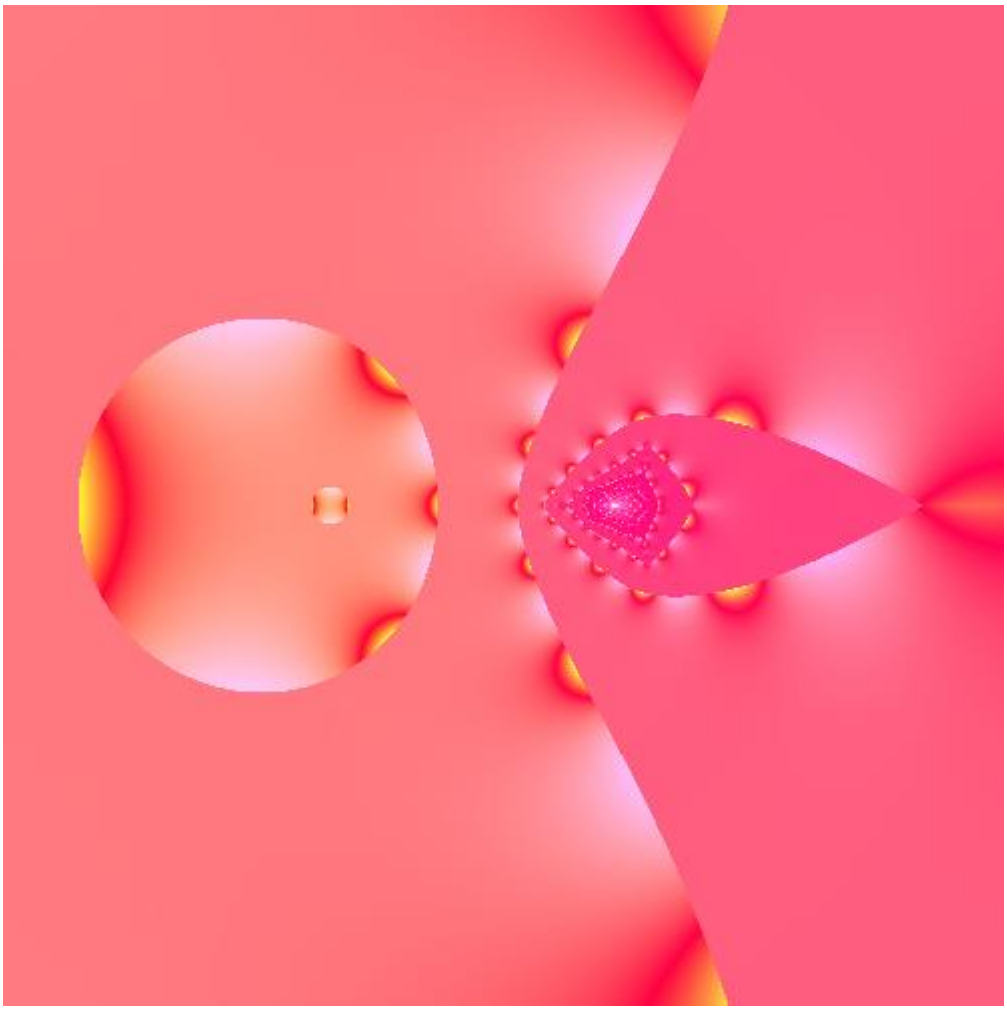

Figura 4.

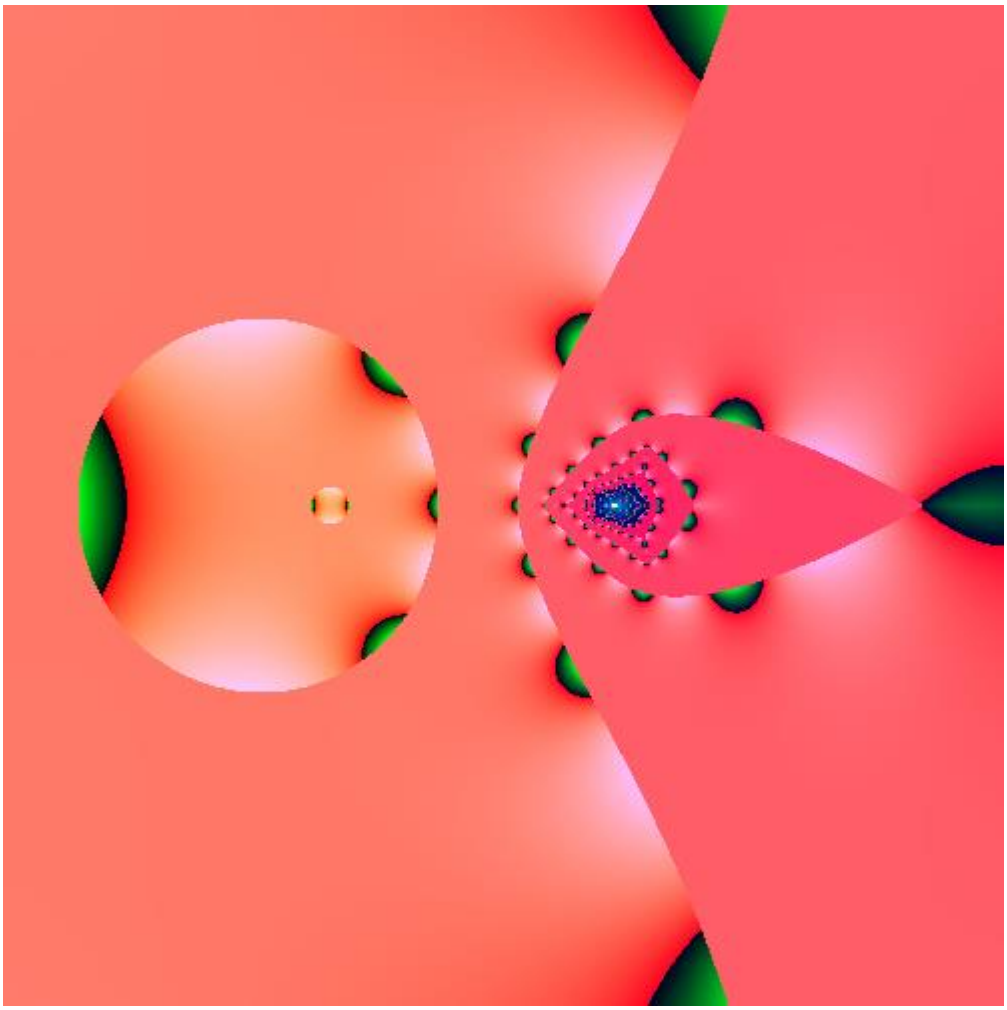

Figura 5.

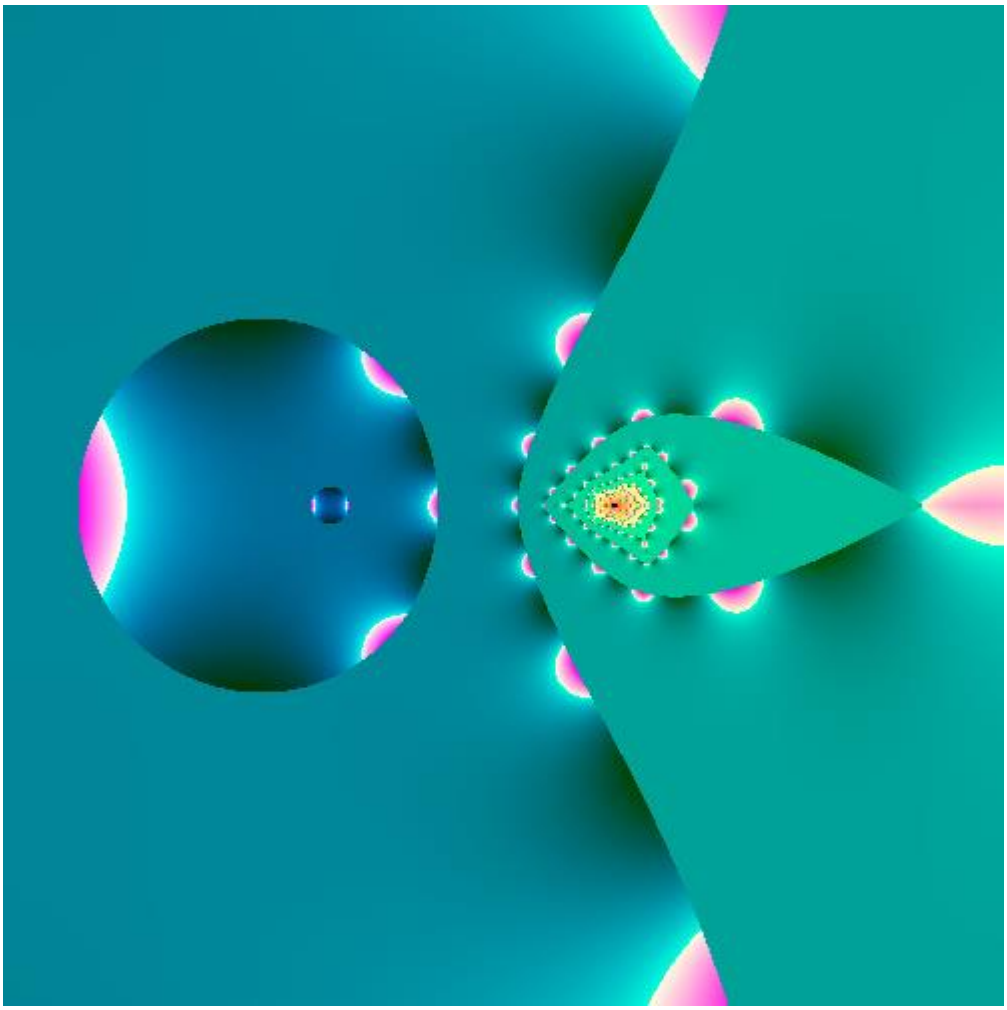

Figura 6.

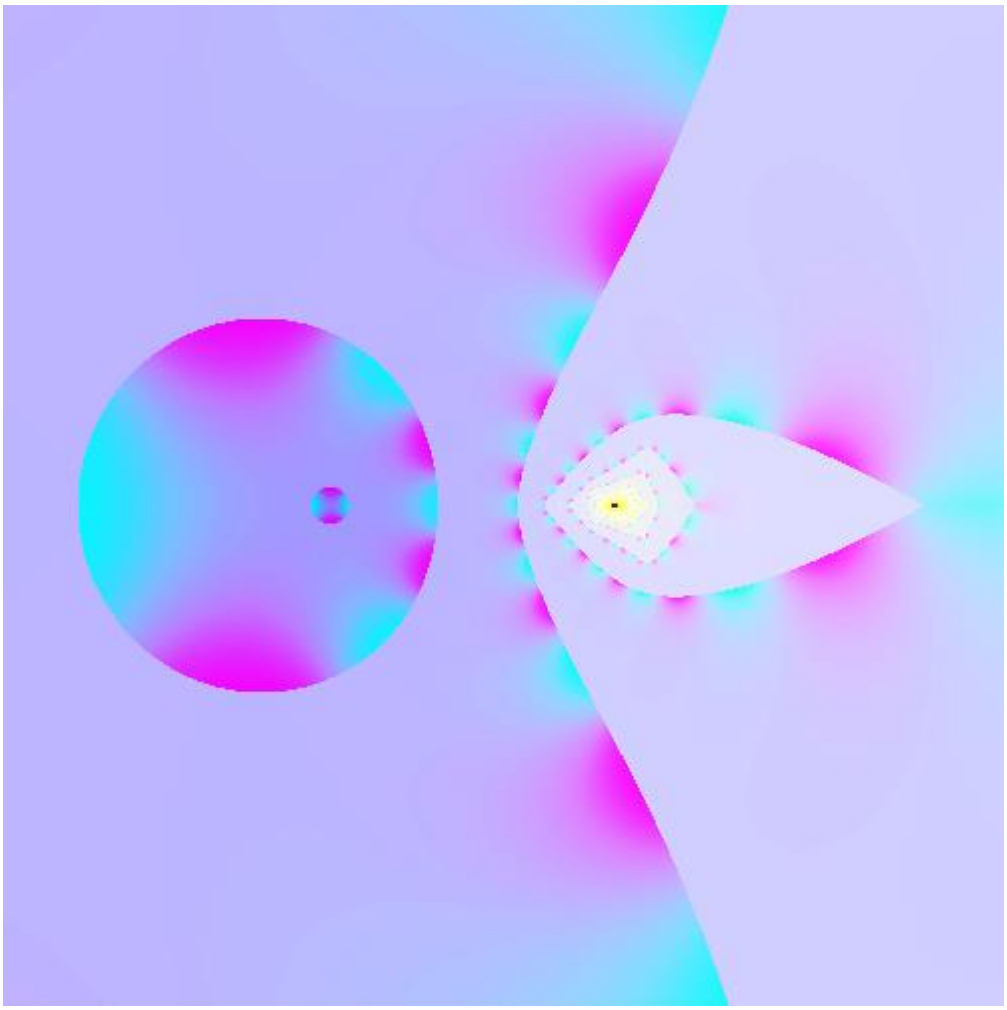

Figura 7.

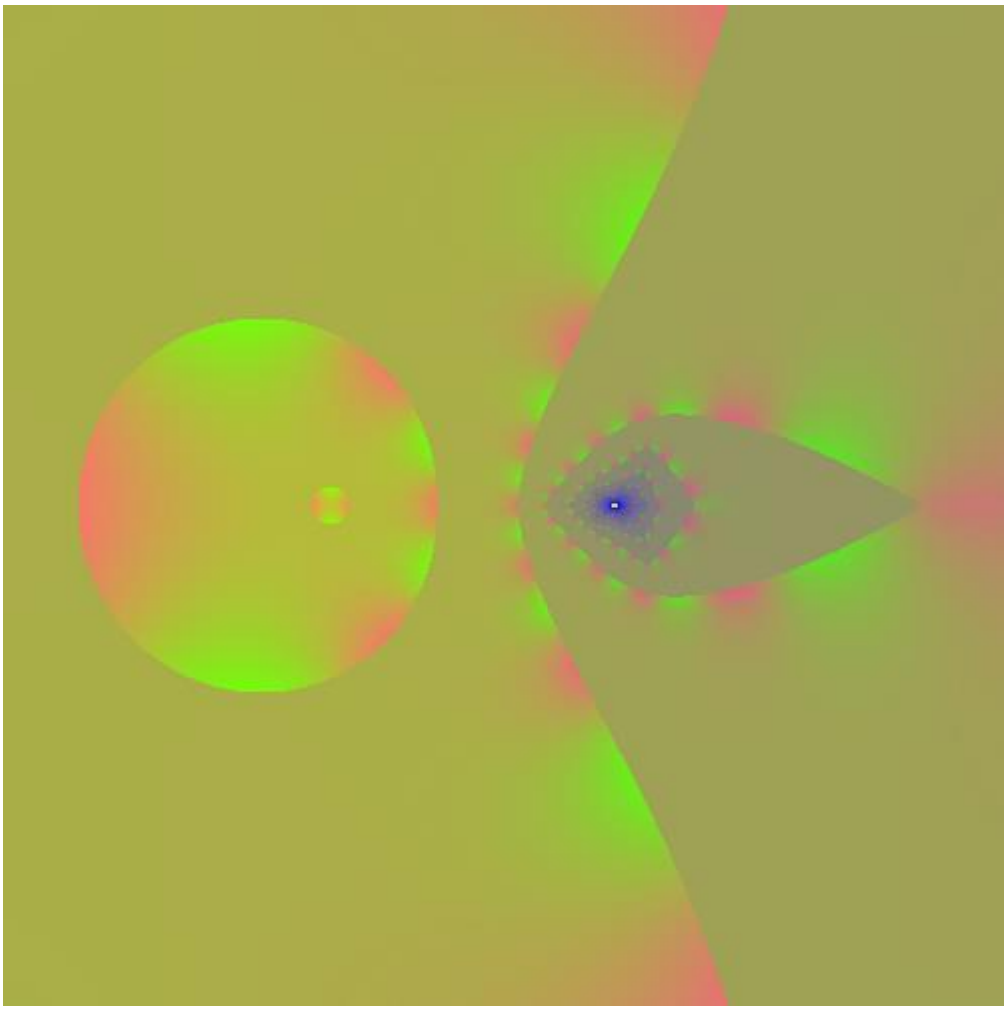

Figura 8.

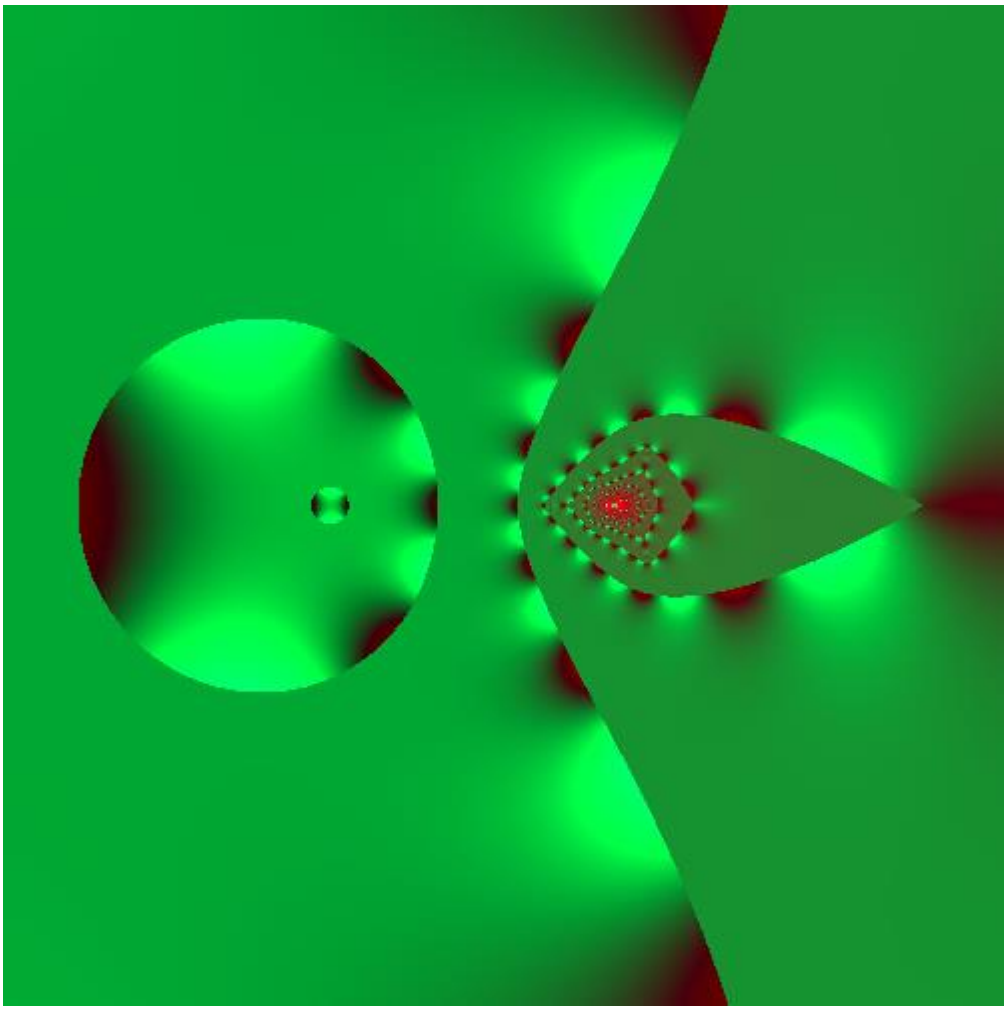

Figura 9.

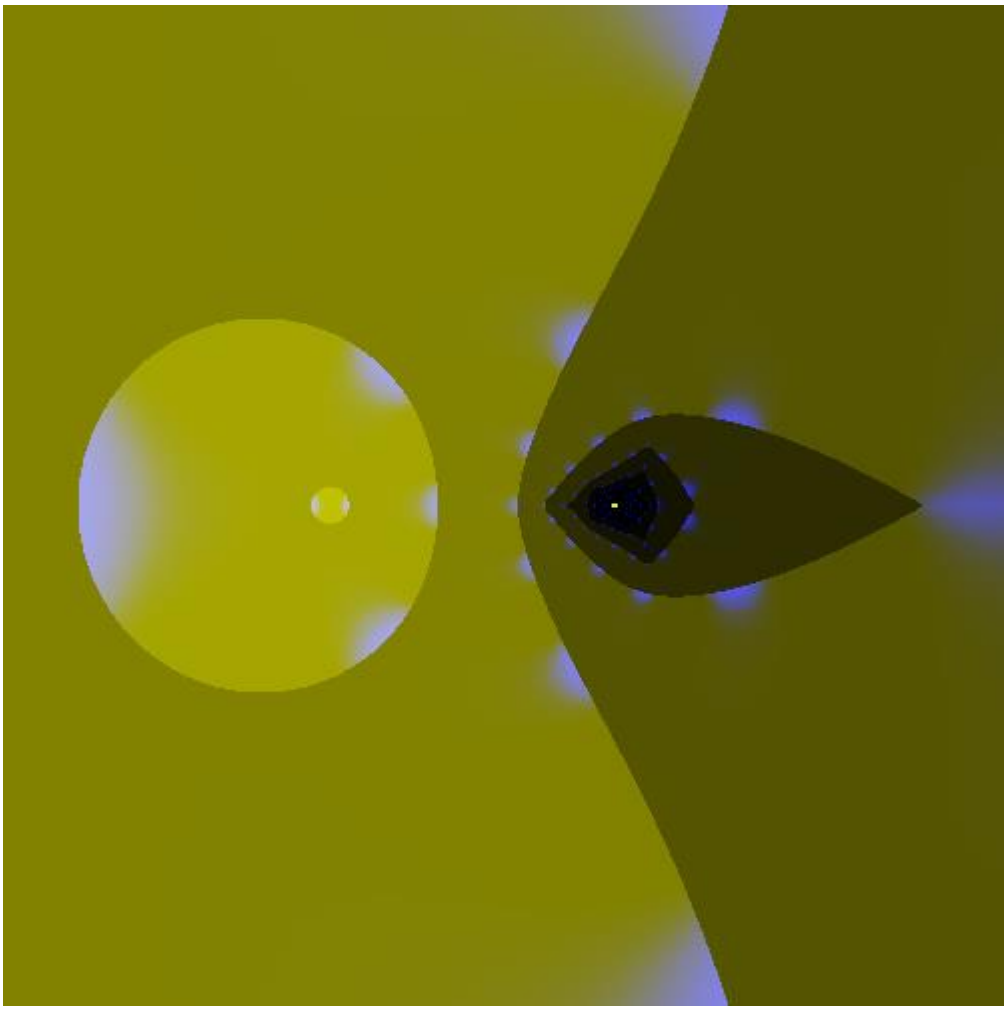

Figura 10.

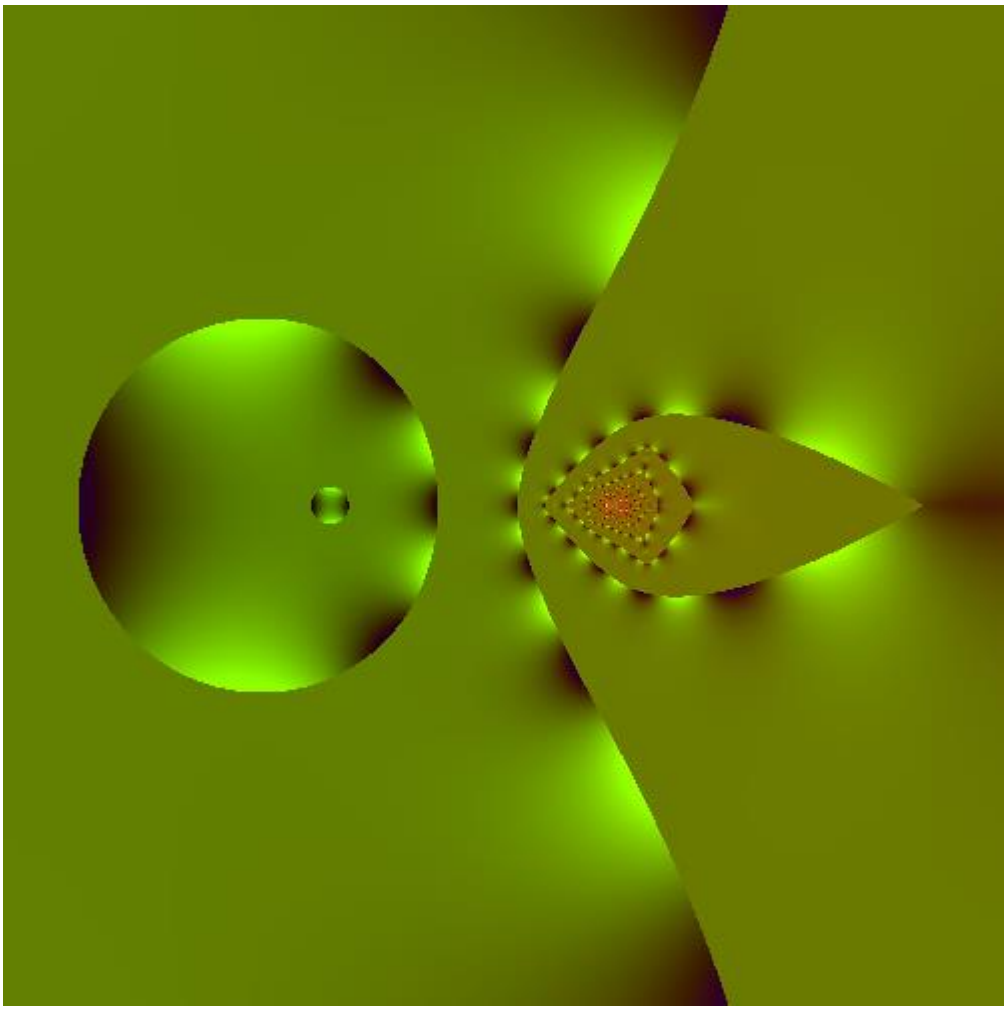

Figura 11.

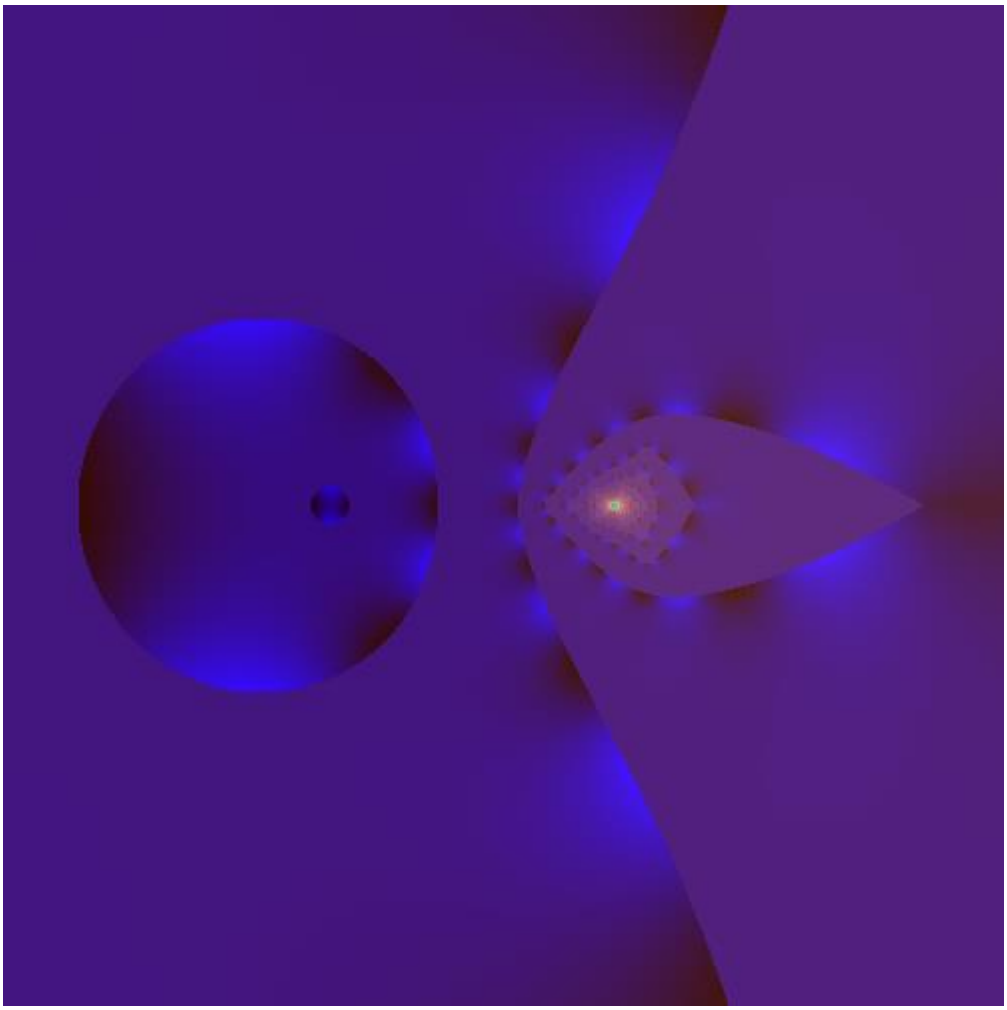

Figura 12.

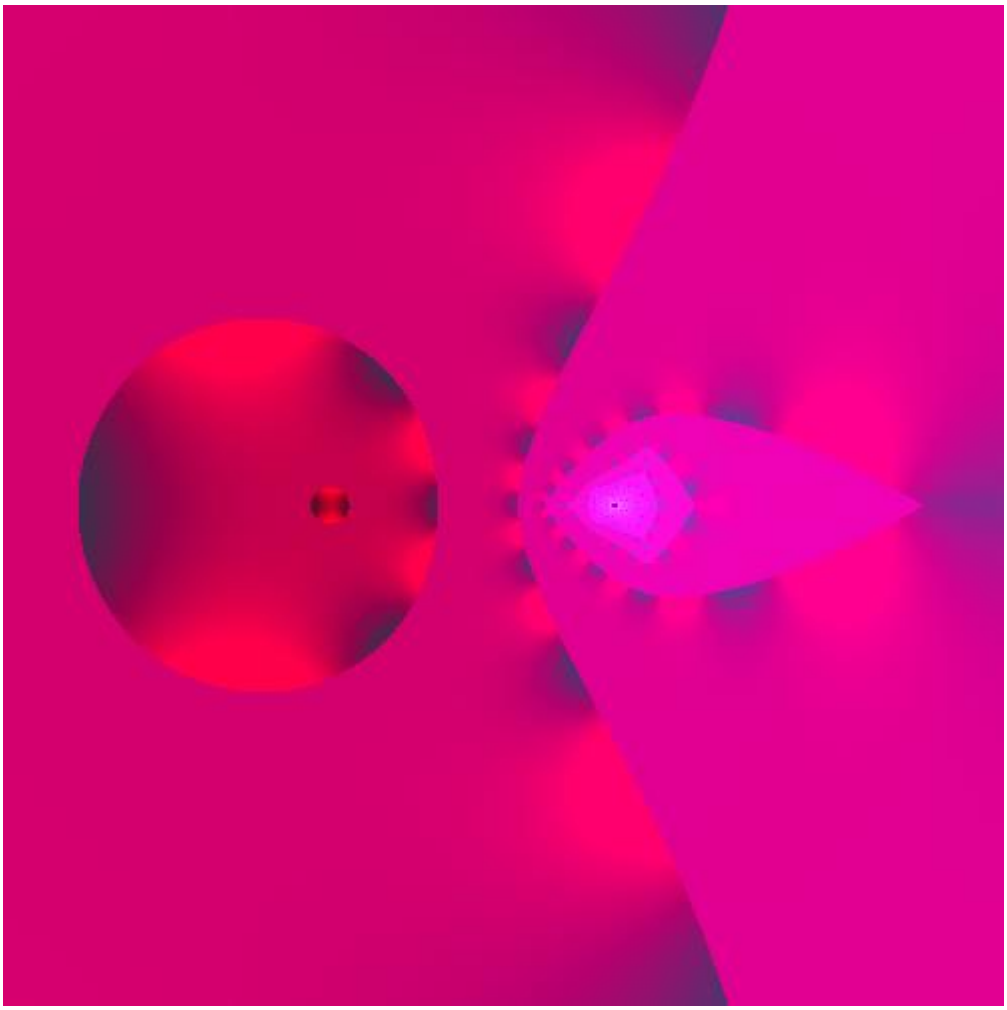

Figura 13.

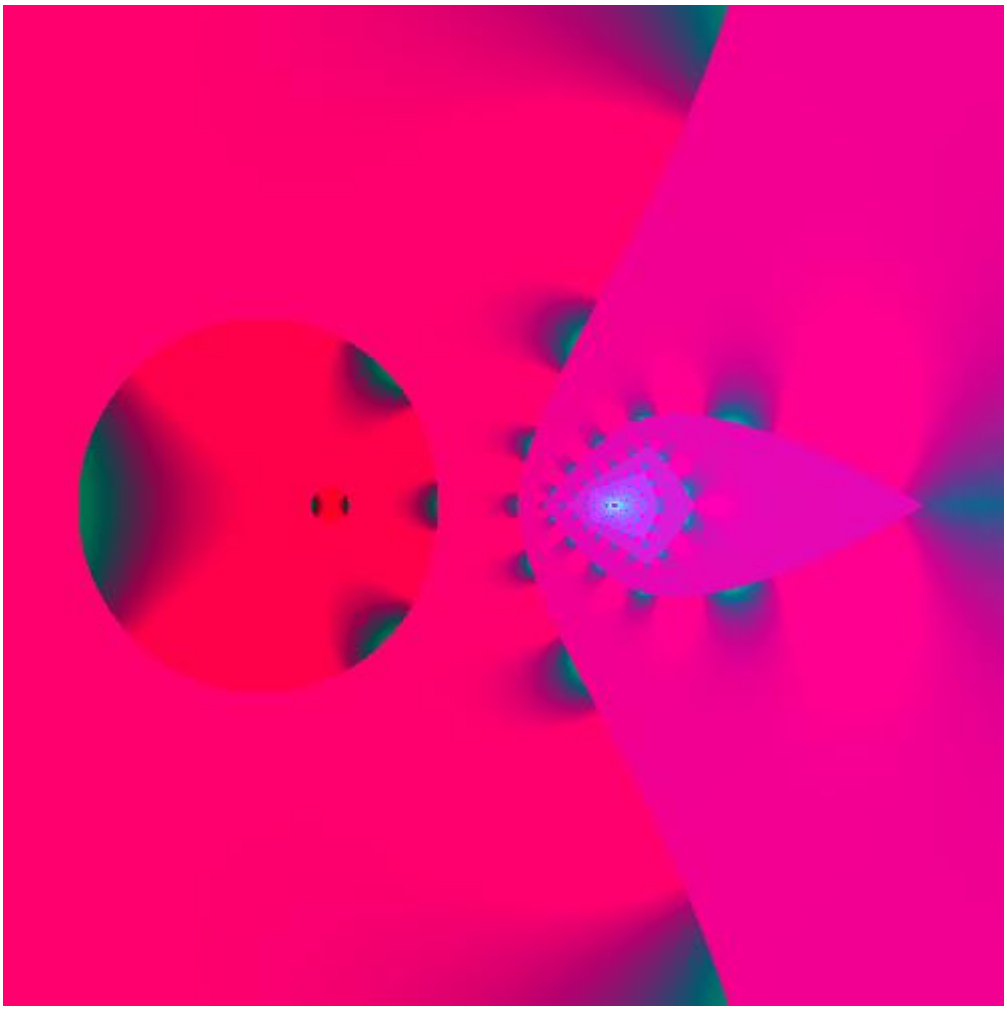

Figura 14.

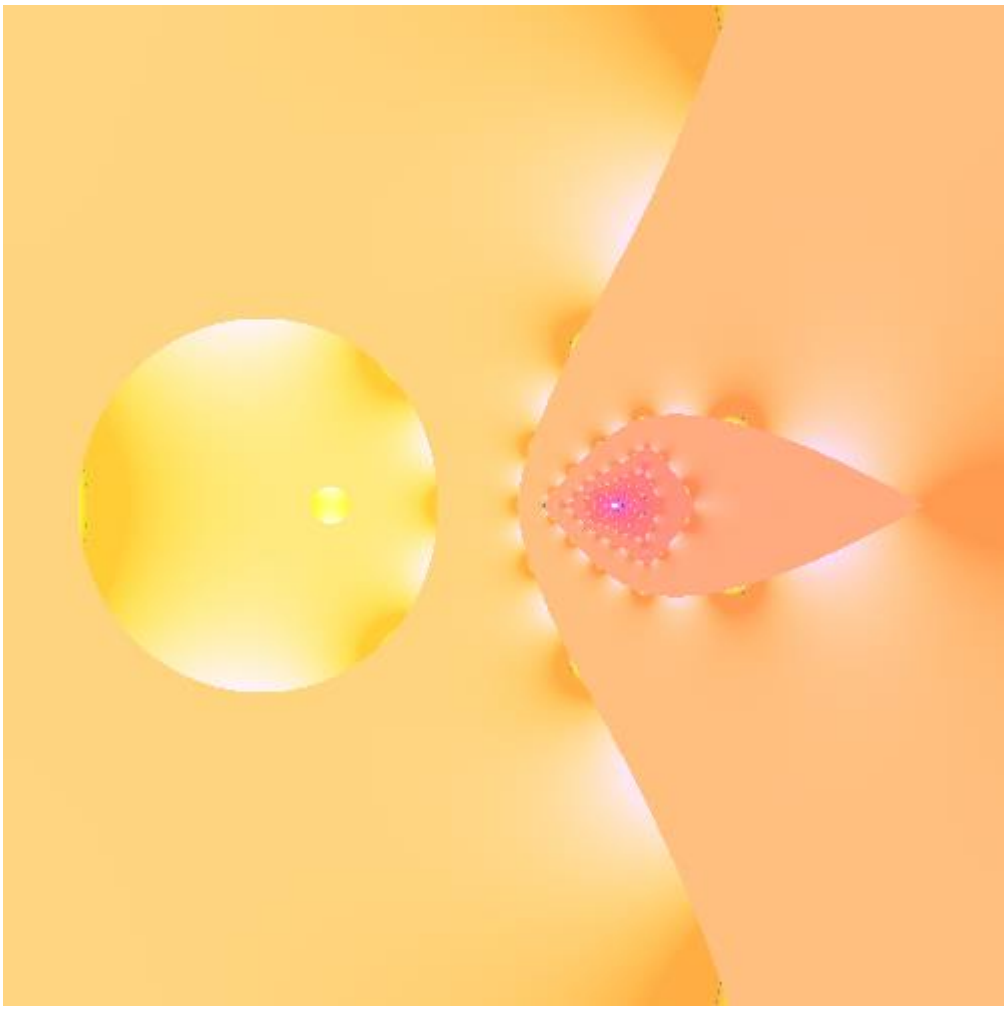

Figura 15.

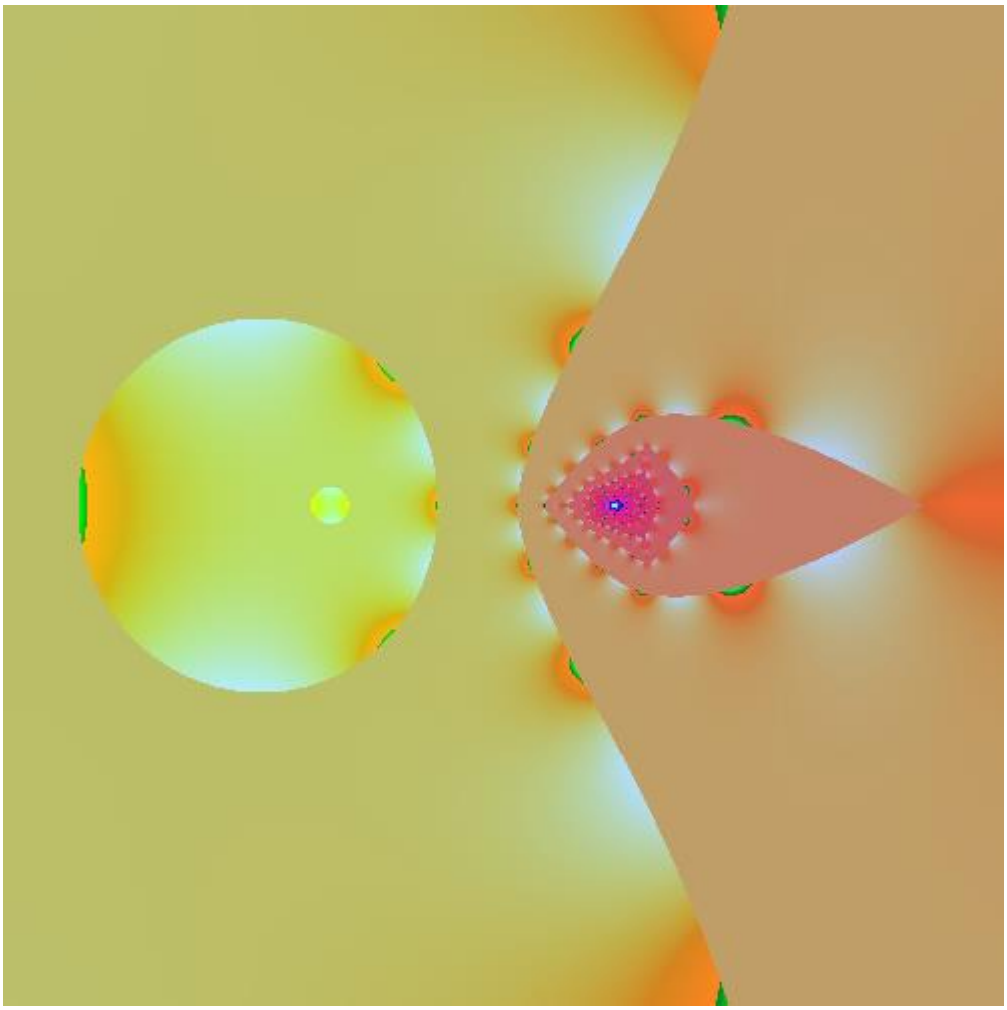

Figura 16.

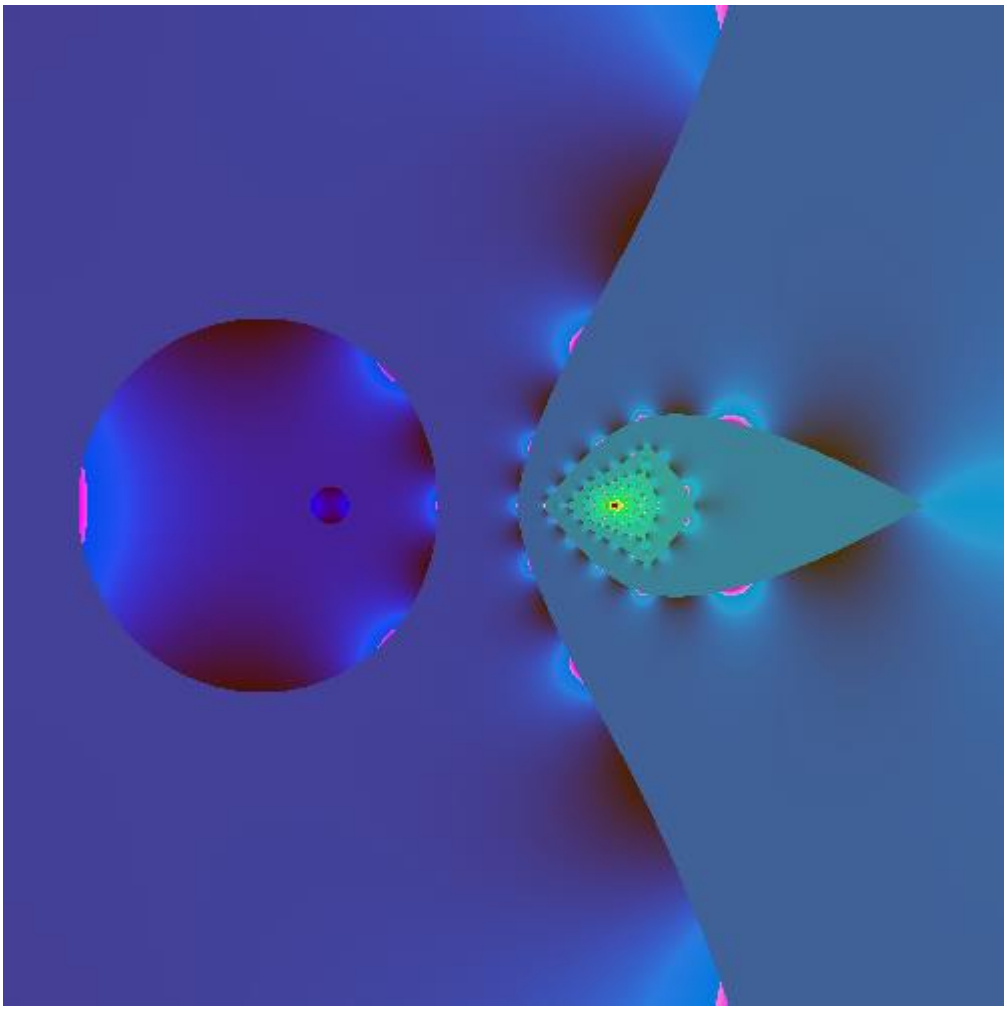

Figura 17.

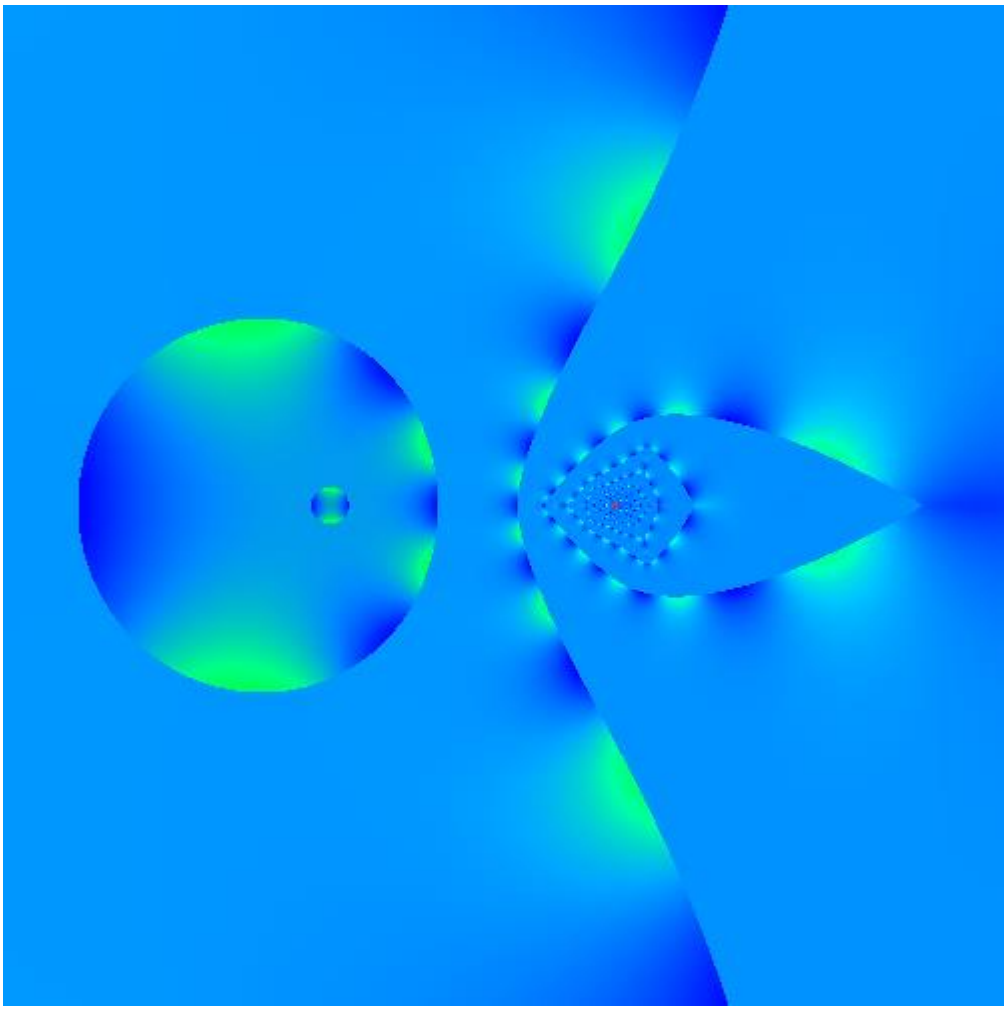

Figura 18.

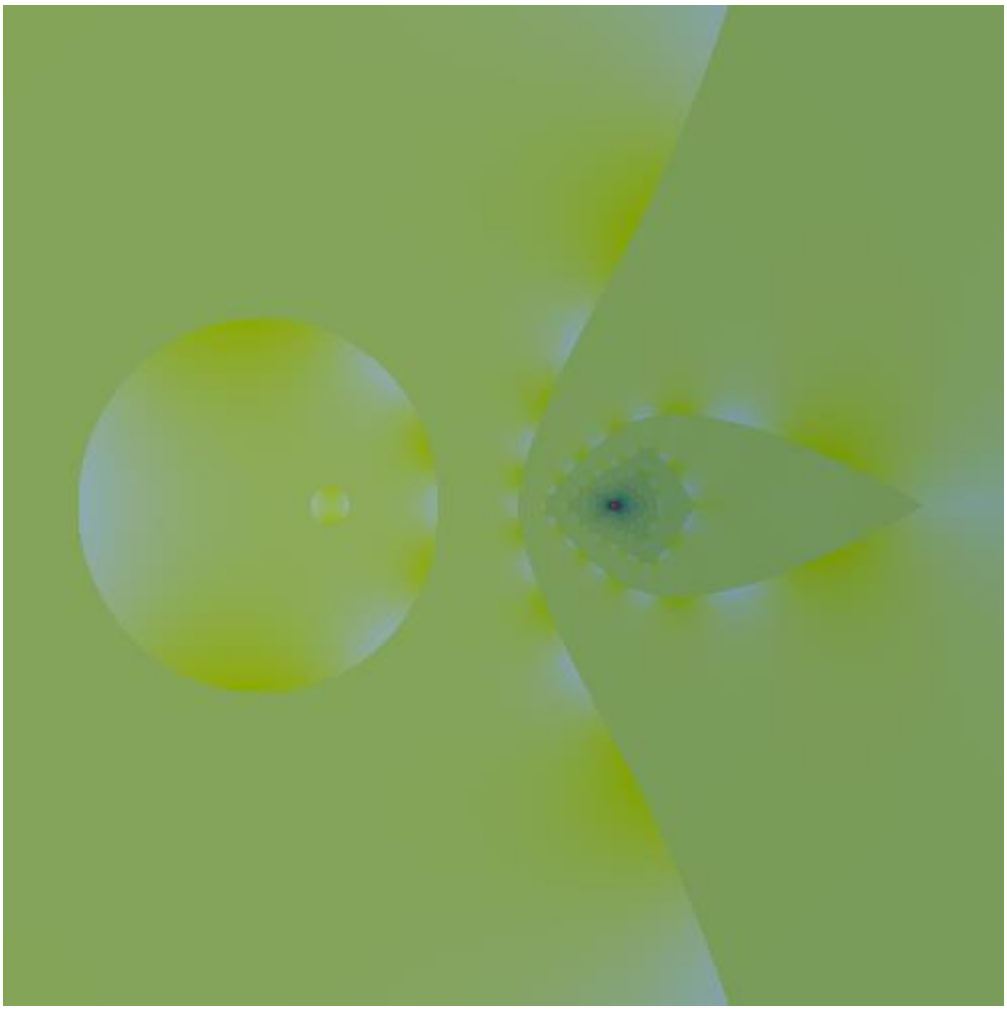

Figura 19.

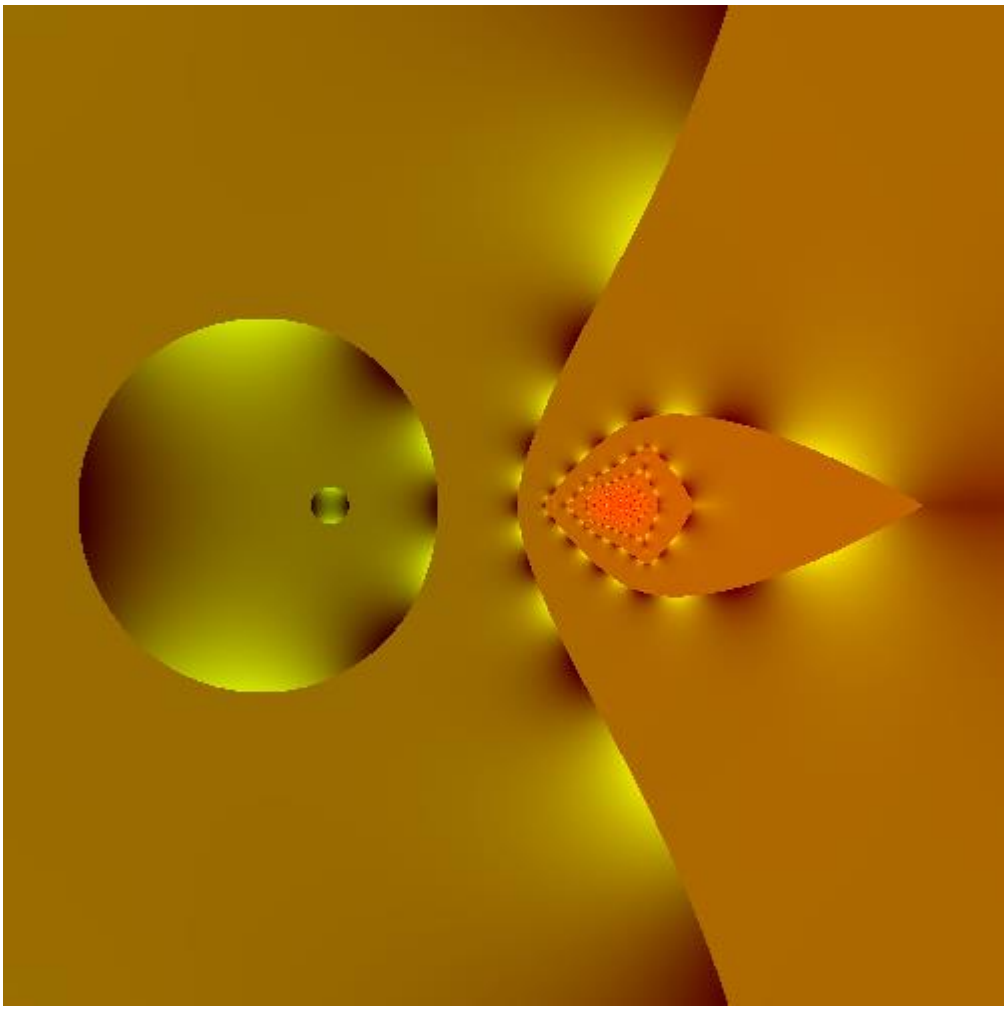

Figura 20.

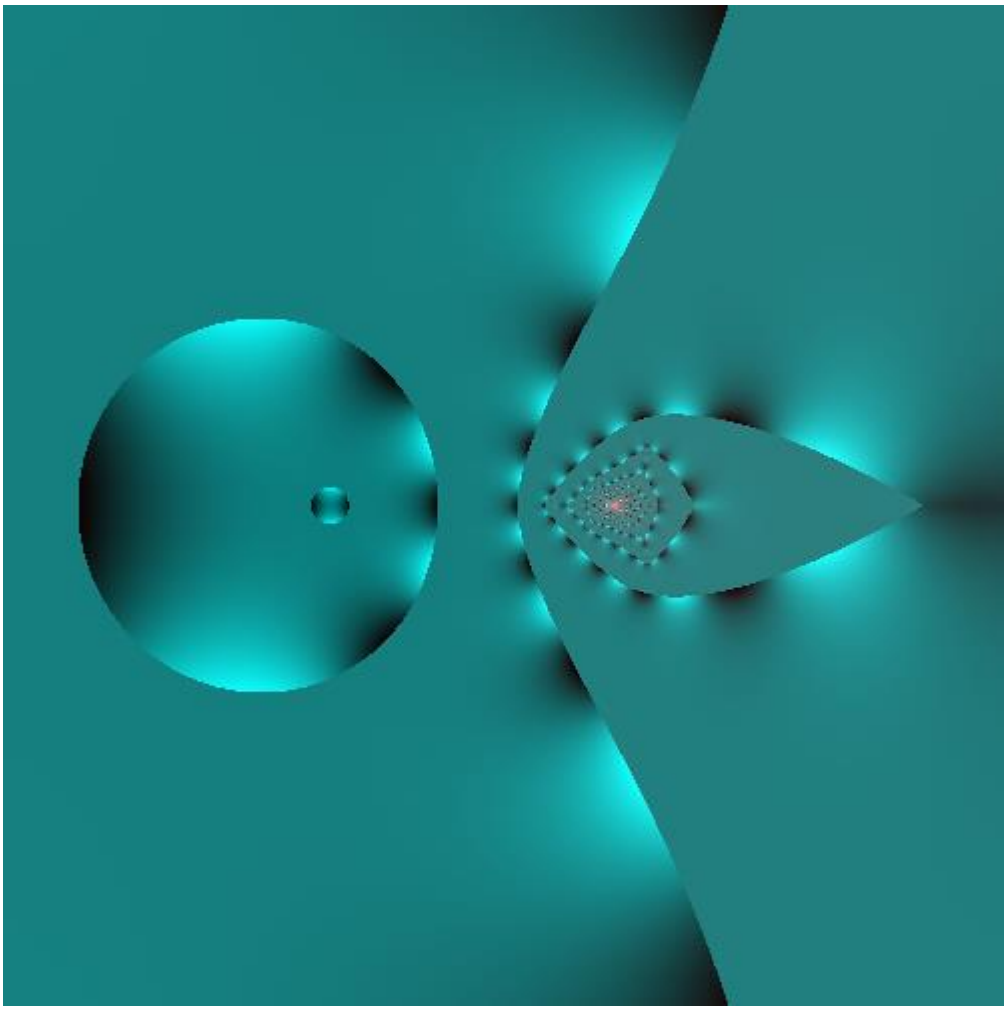

Figura 21.

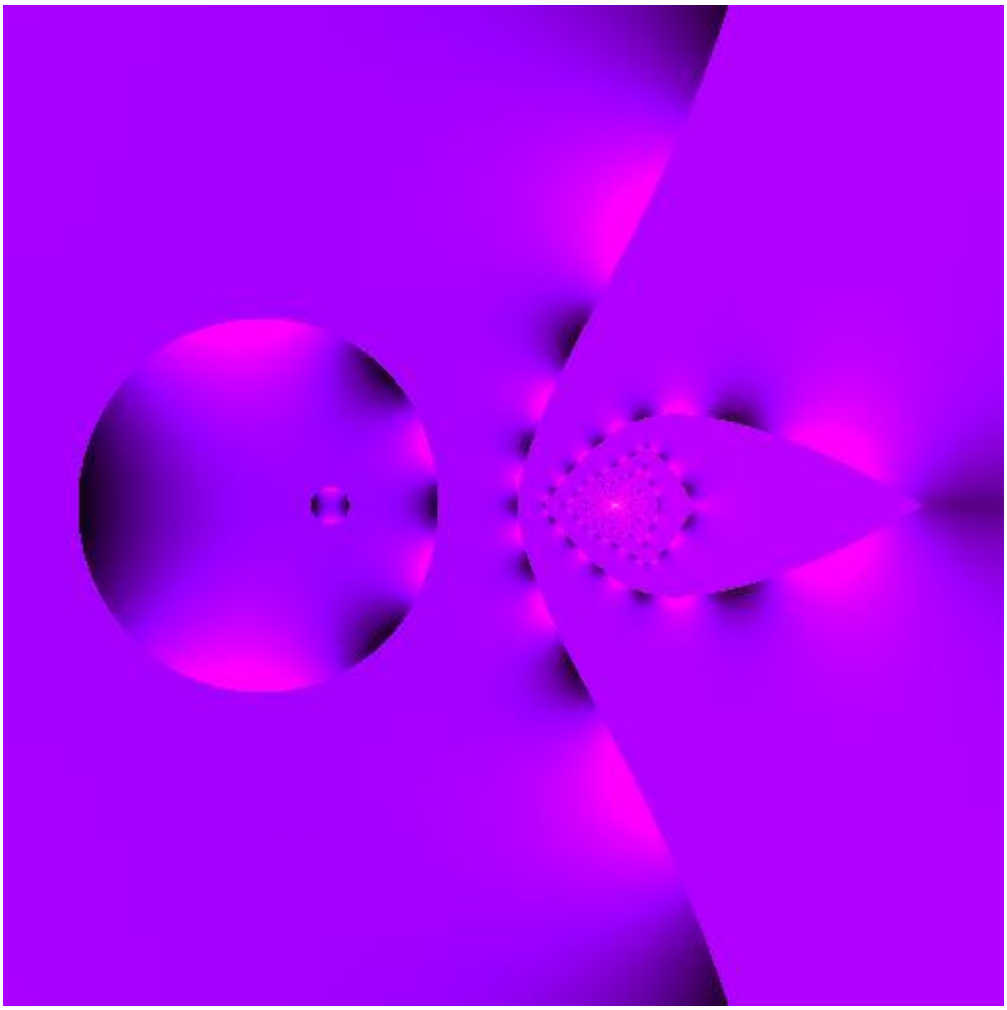

Figura 22.

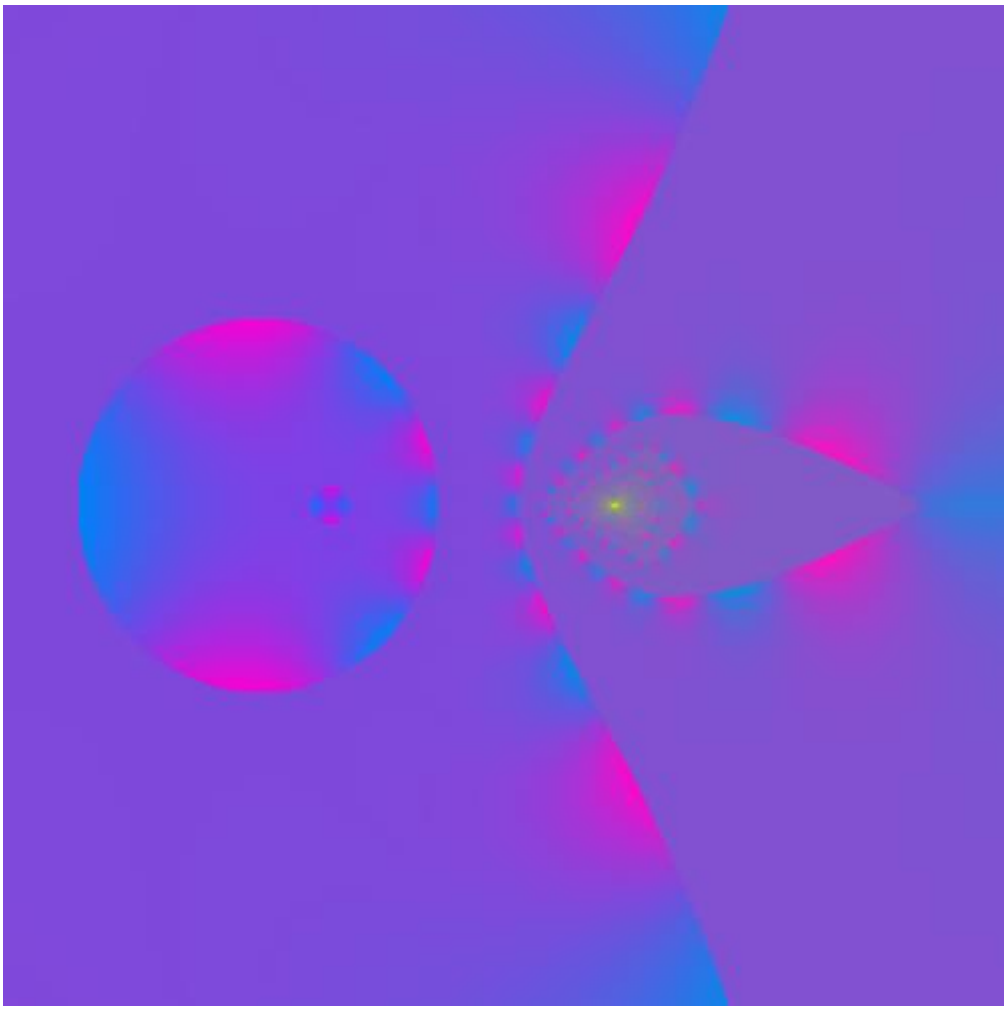

Figura 23.

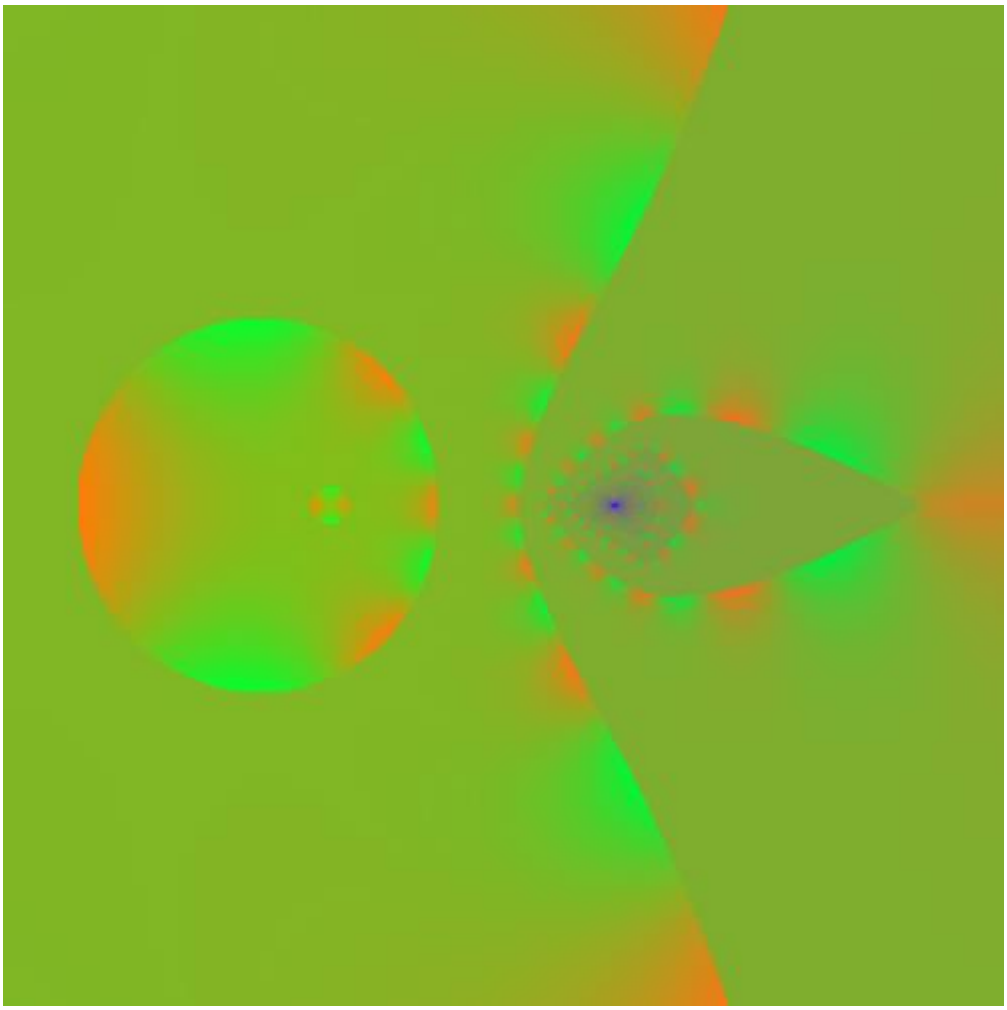

Figura 24.

3. La función $f(z)=z-e^{-\sqrt{1-z}}$, $z\in\left(-0.1-0.46\sqrt{-1},0.82+0.46\sqrt{-1}\right)$.

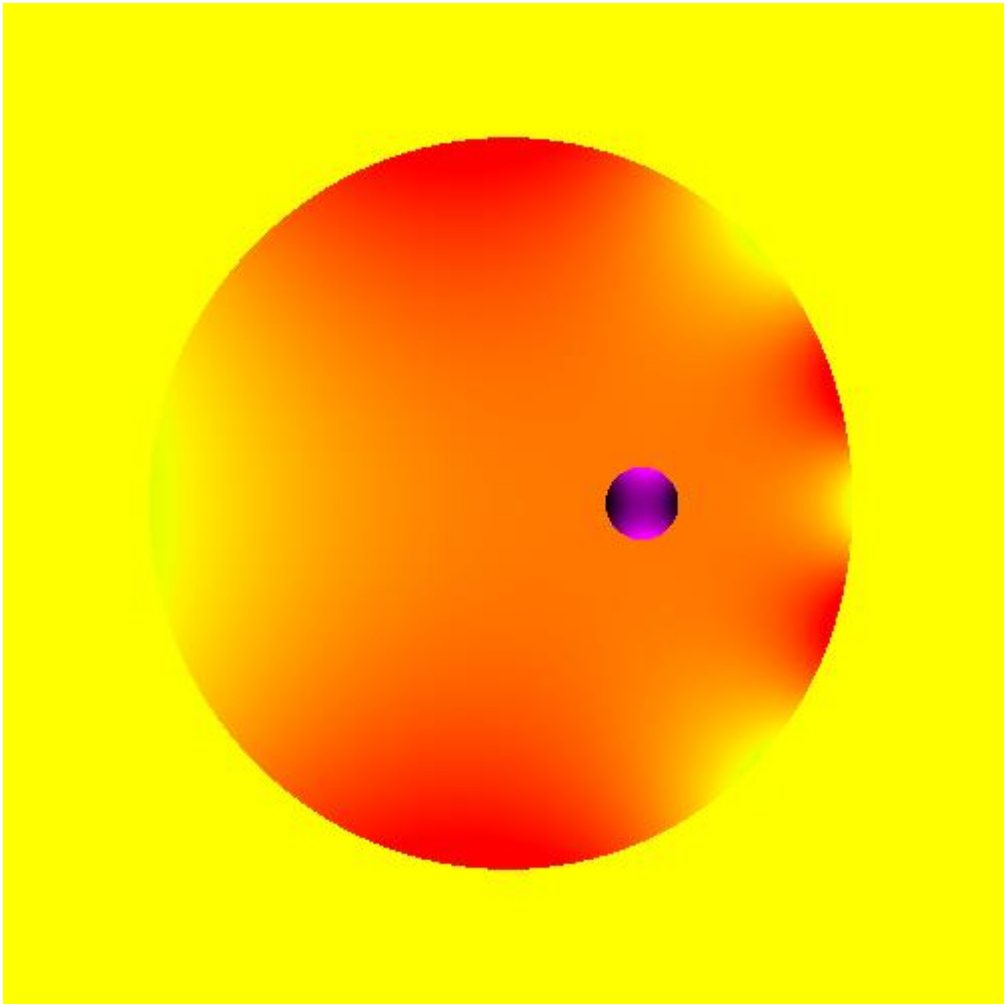

Figura 25.

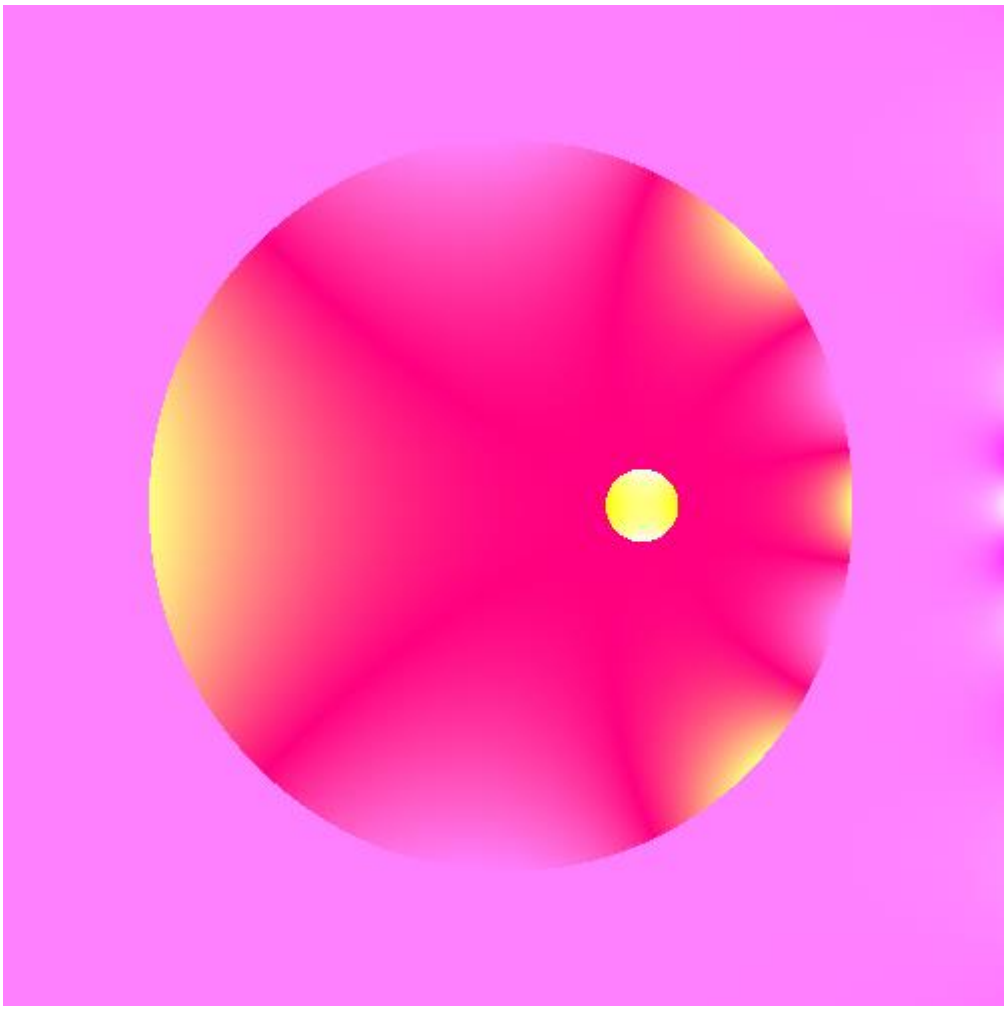

Figura 26.

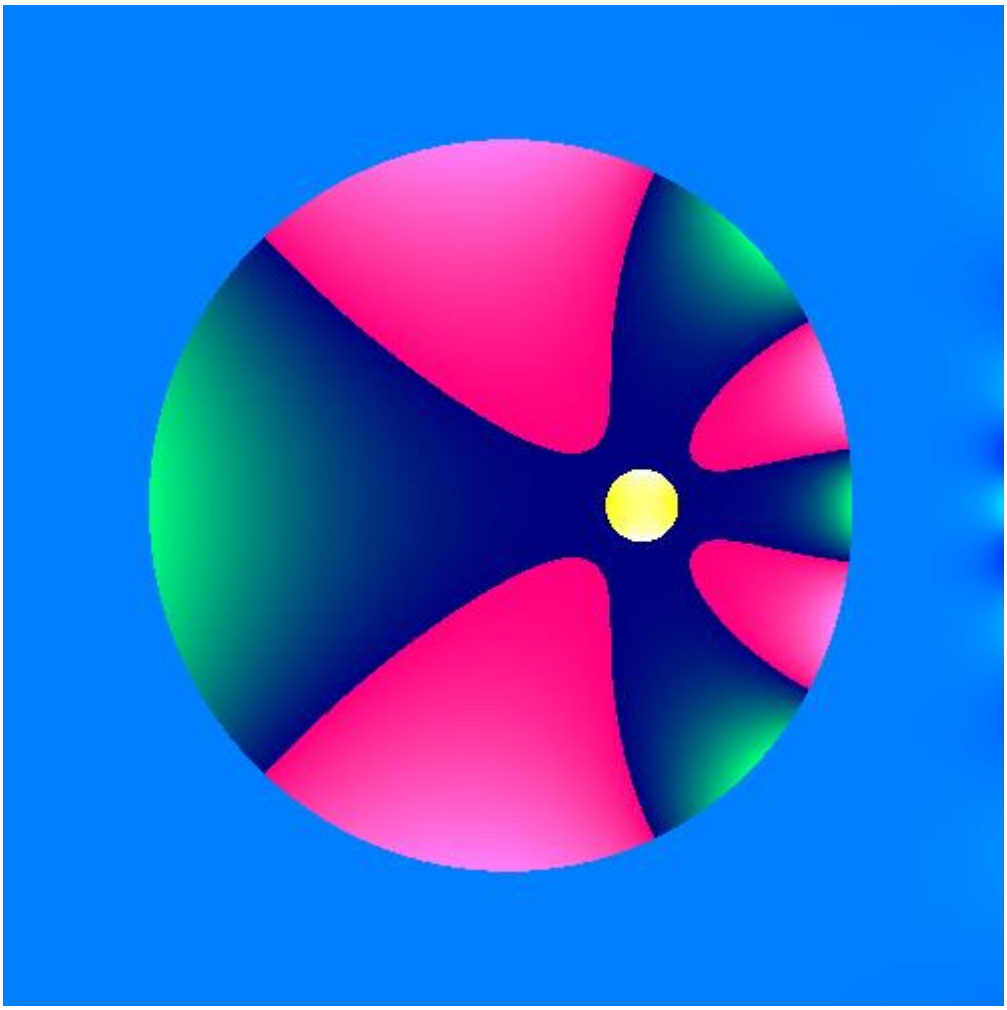

Figura 27.

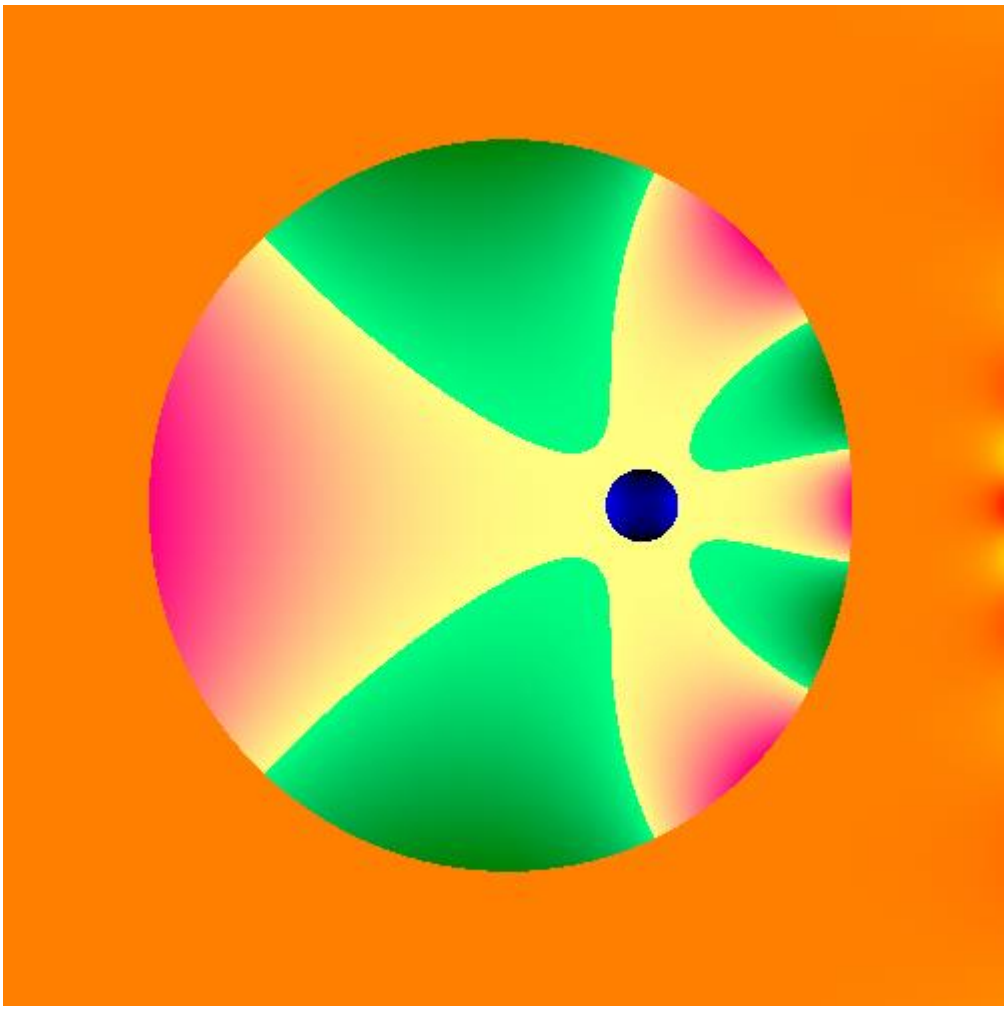

Figura 28.

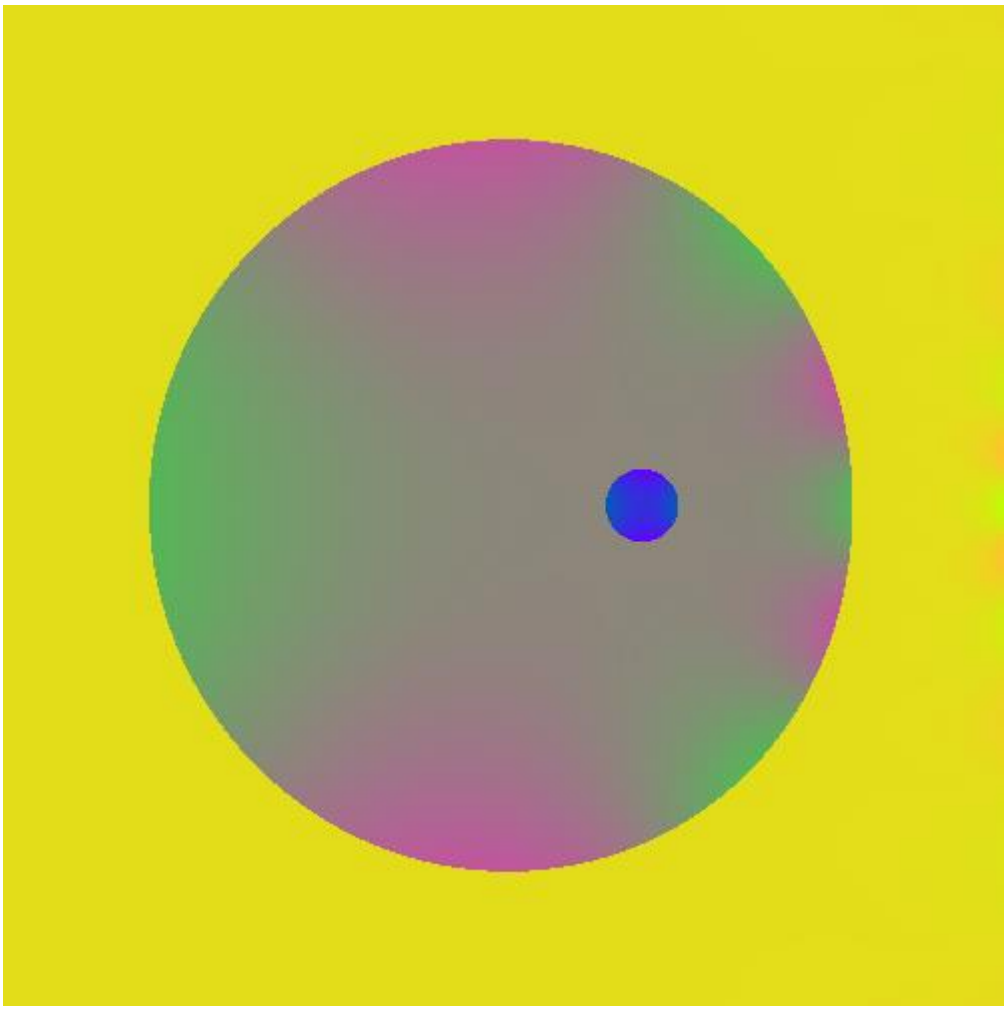

Figura 29.

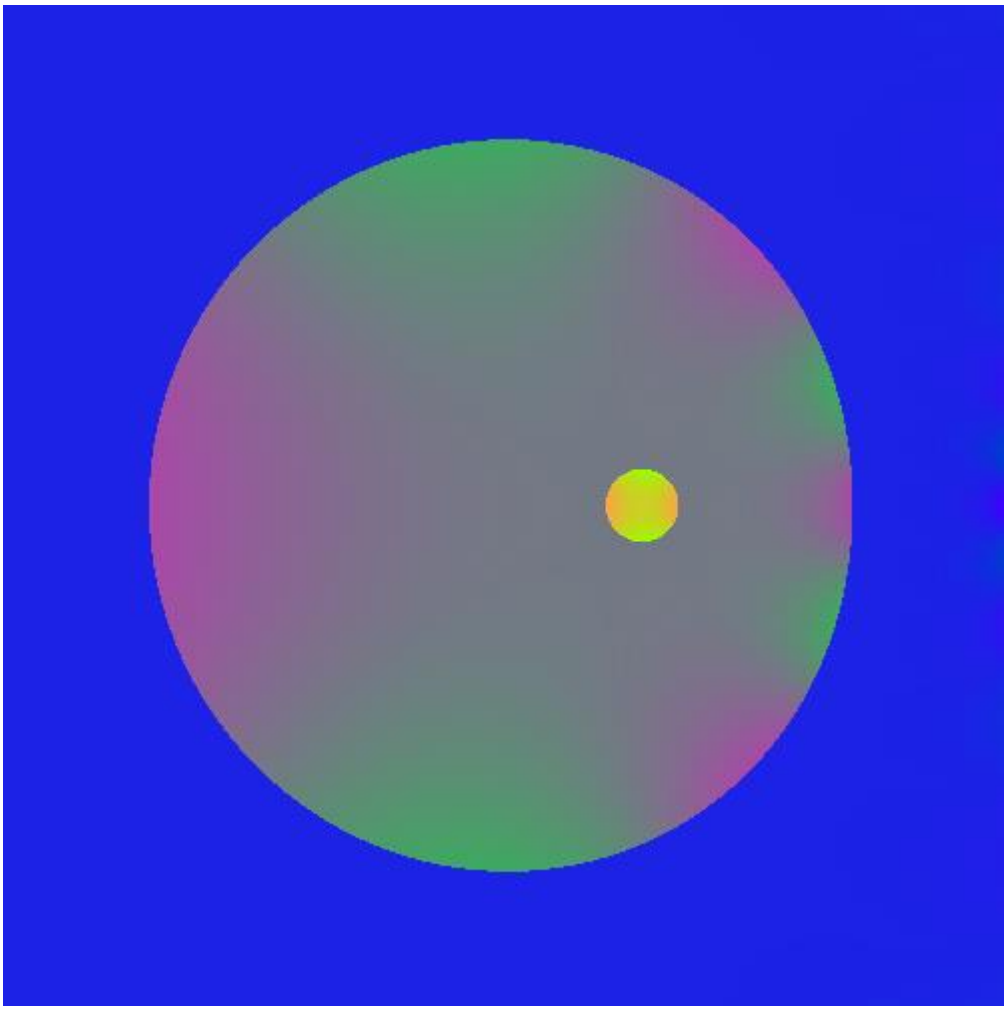

Figura 30.

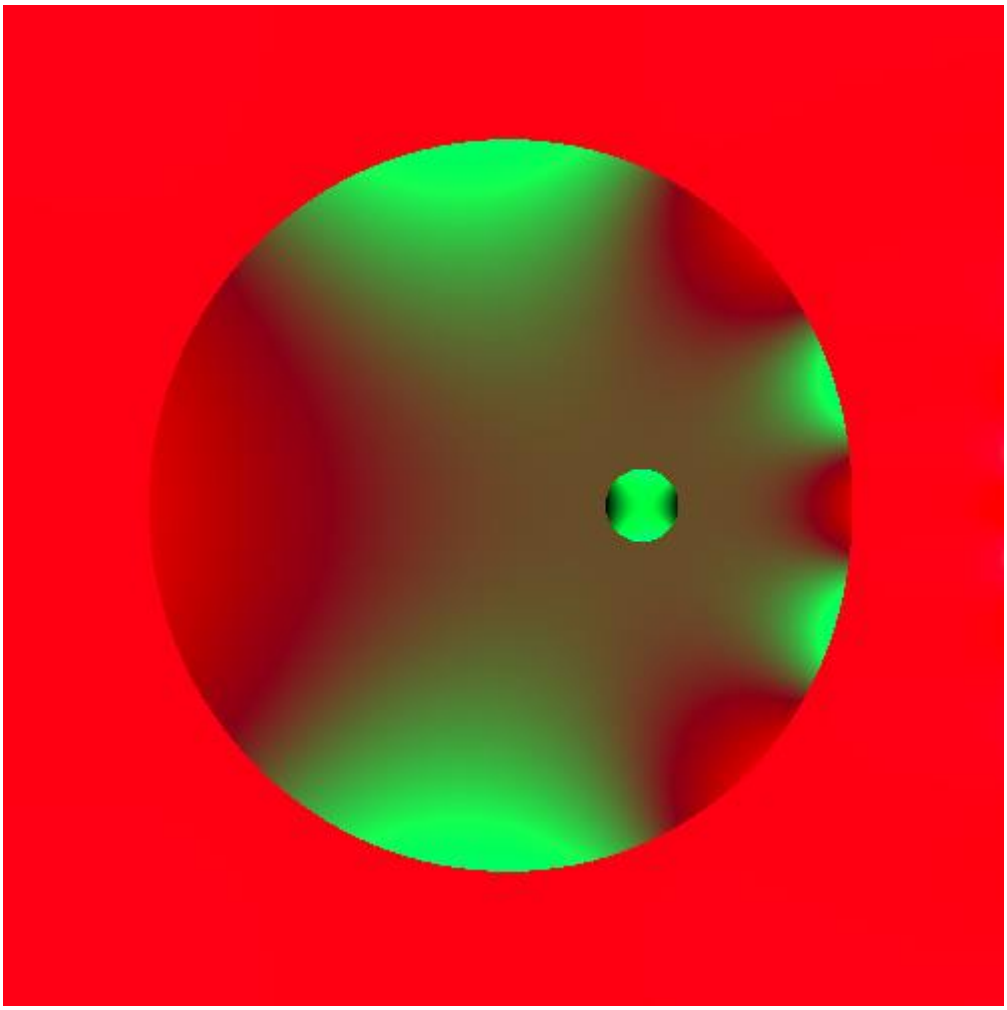

Figura 31.

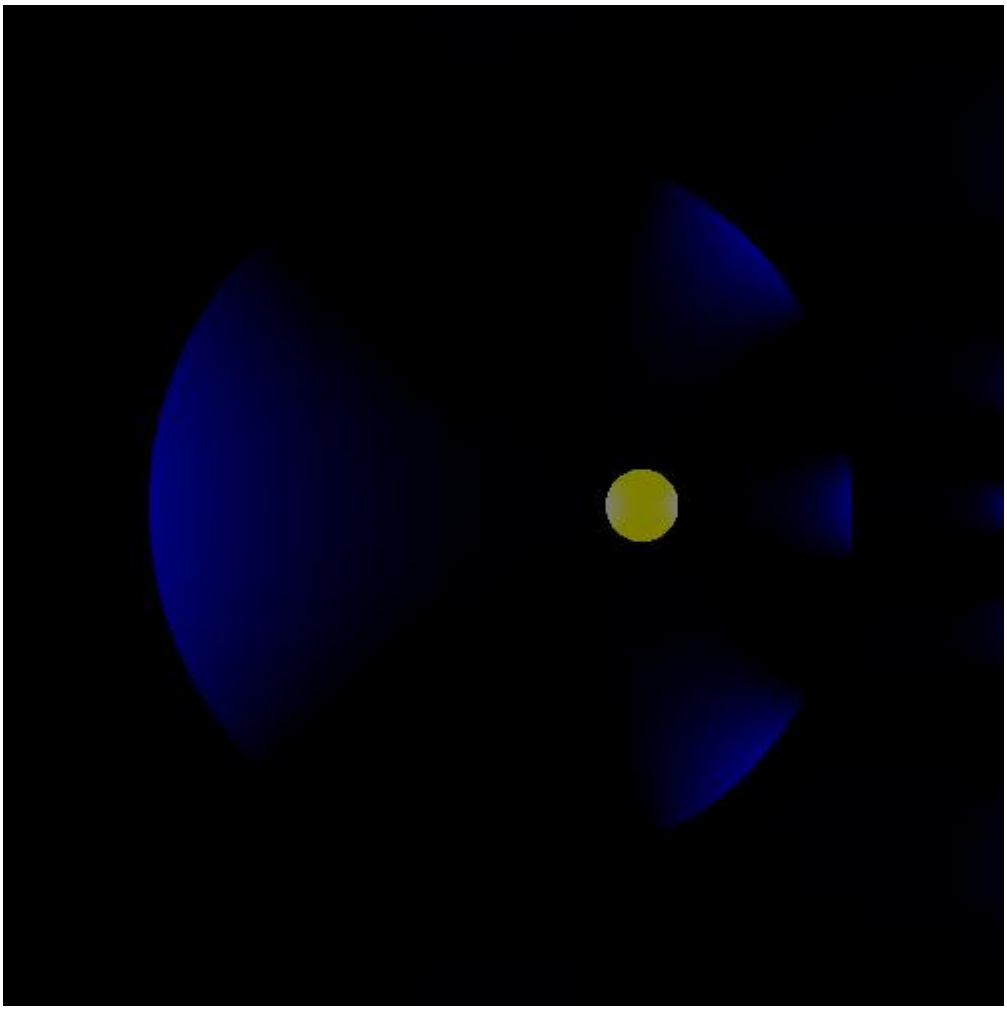

Figura 32.

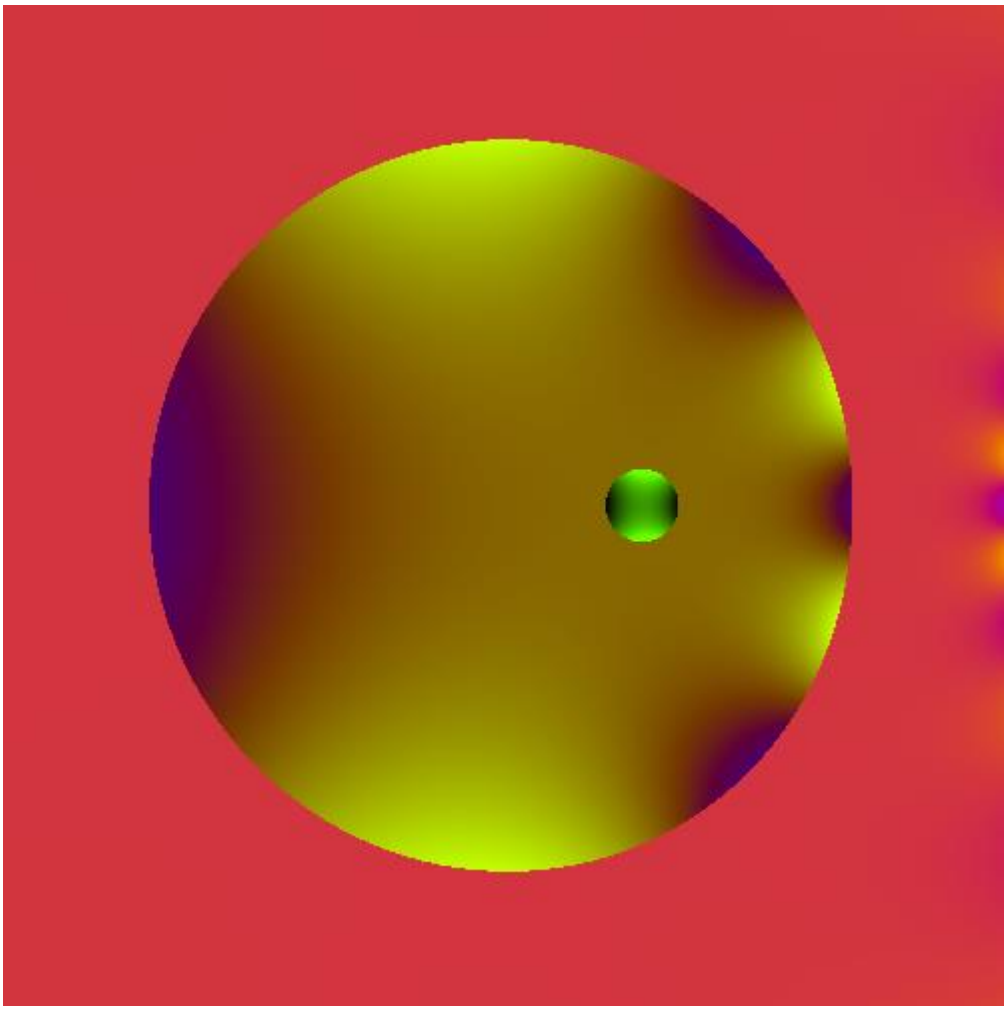

Figura 33.

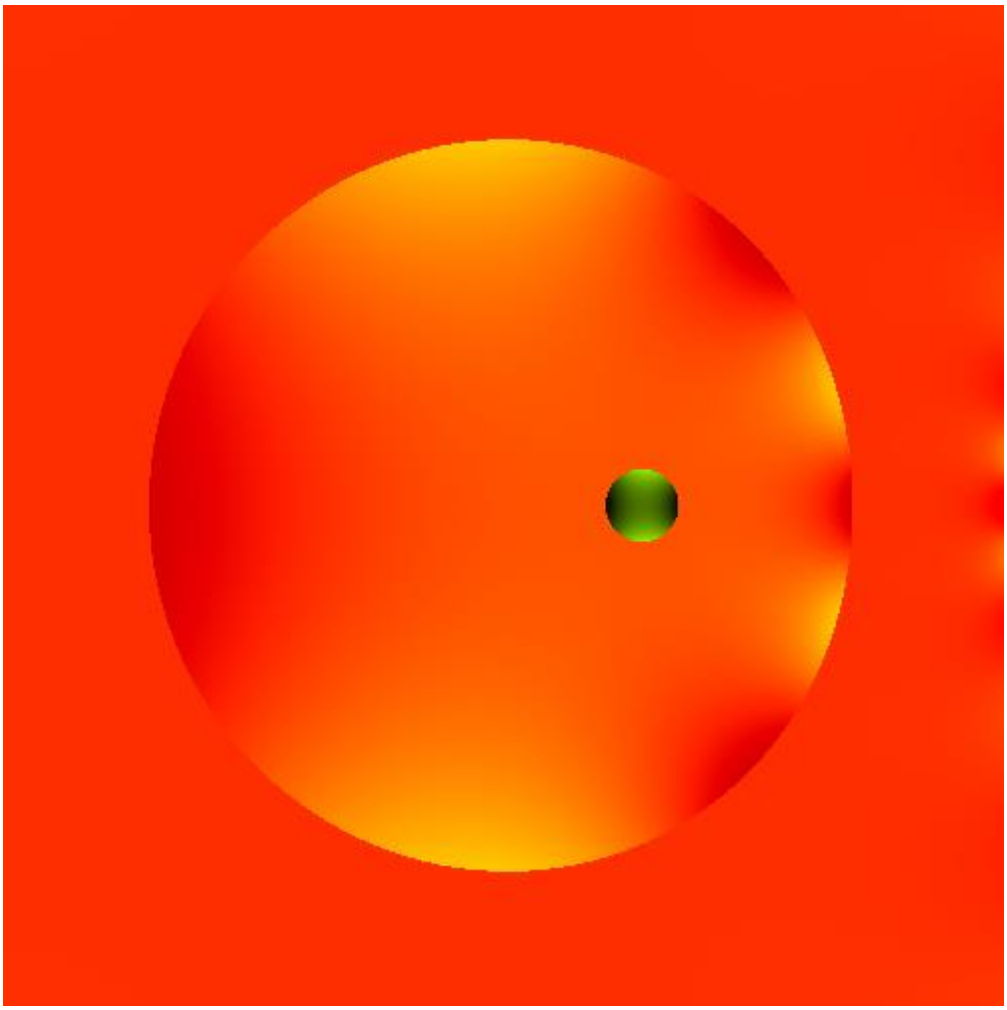

Figura 34.

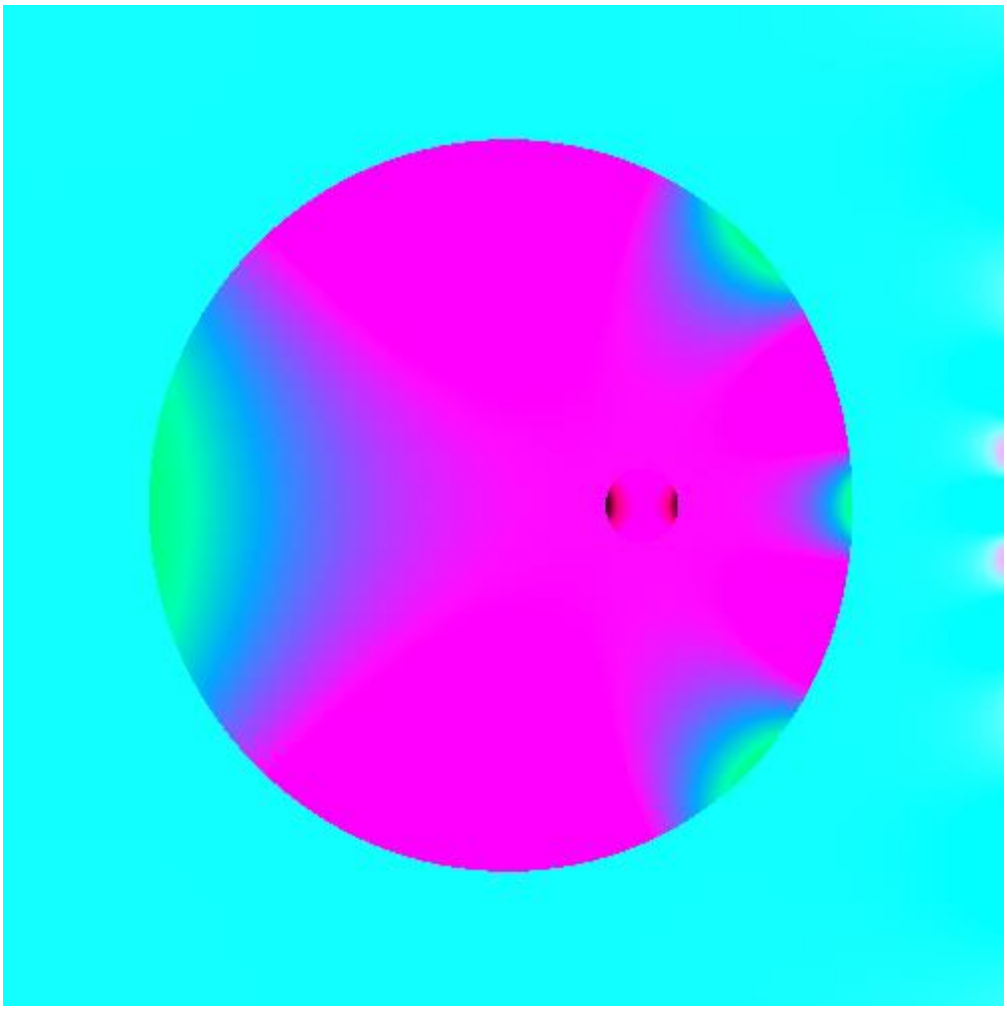

Figura 35.

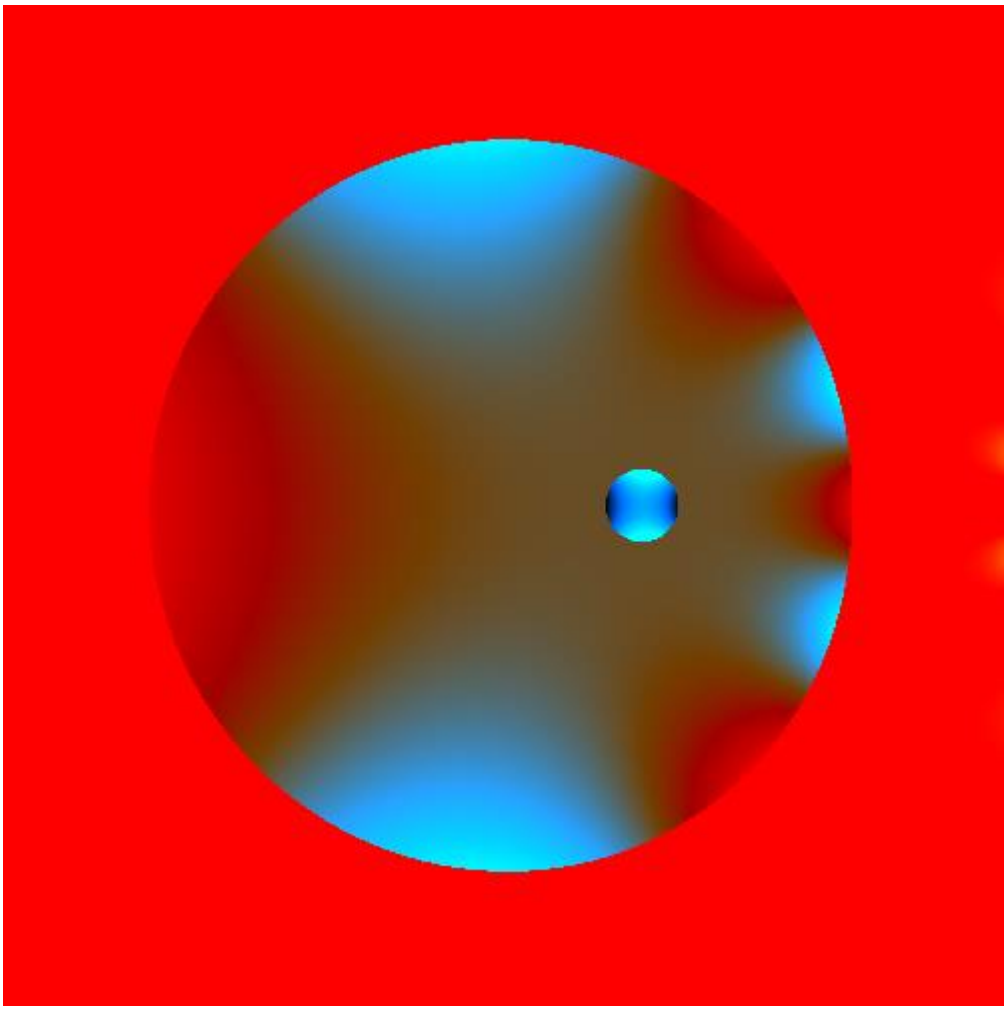

Figura 36.

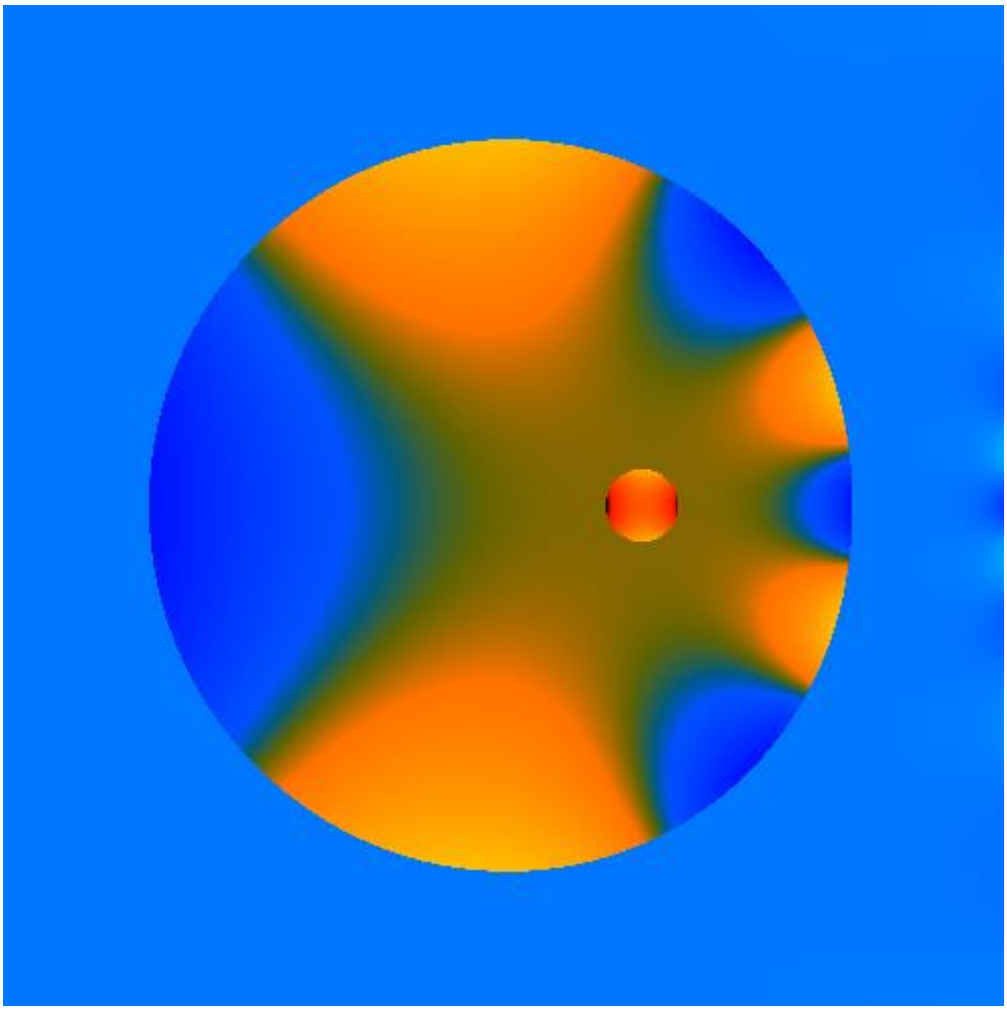

Figura 37.

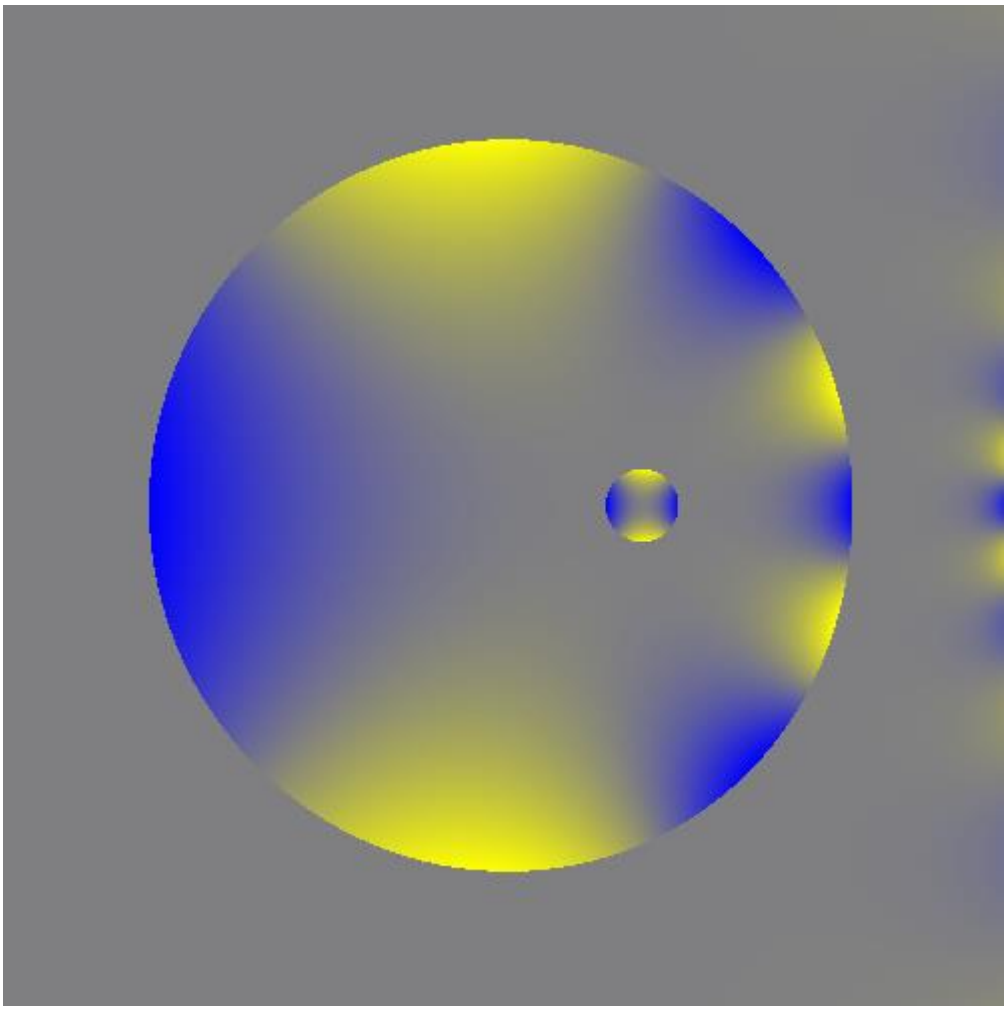

Figura 38.

4. La función: $f(z) = z - e^{-\sqrt{1-z}}$, $z \in \left(0.8 - 0.4\sqrt{-1}, 1.6 + 0.4\sqrt{-1}\right)$.

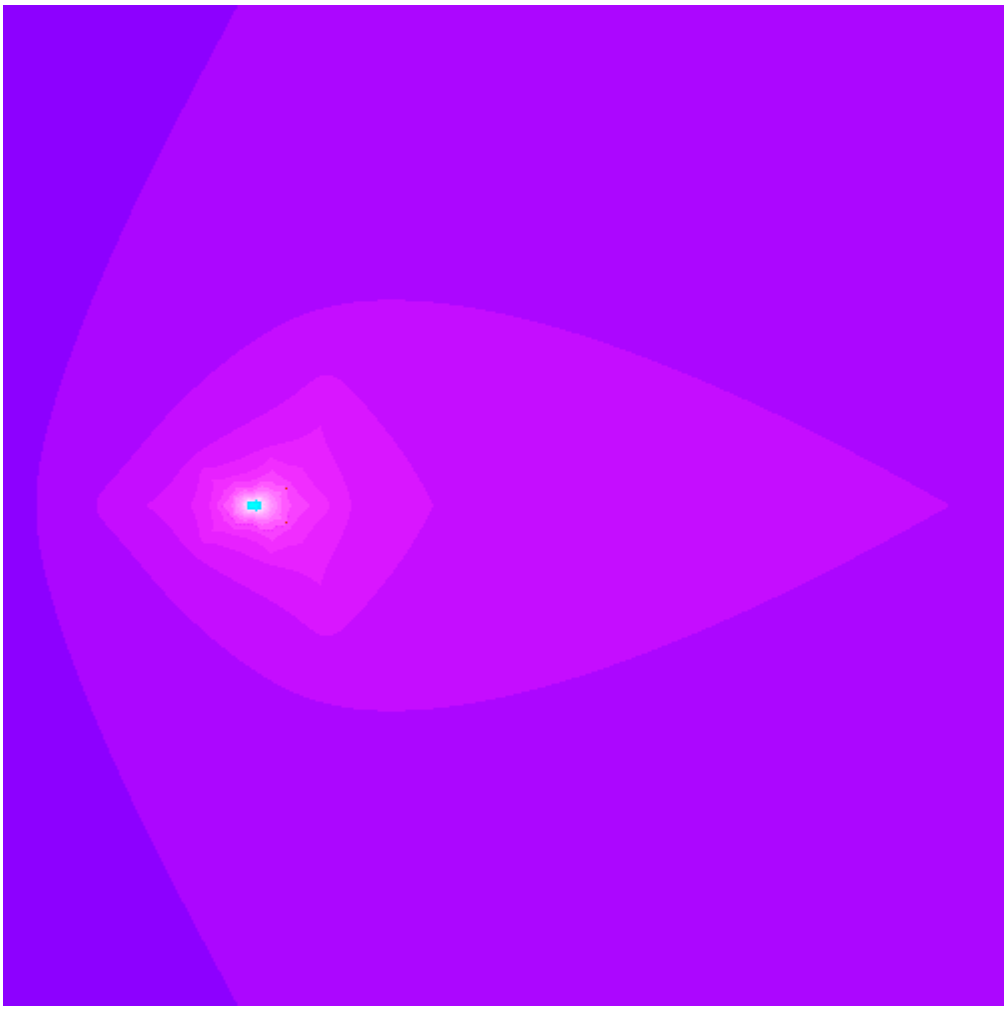

Figura 39.

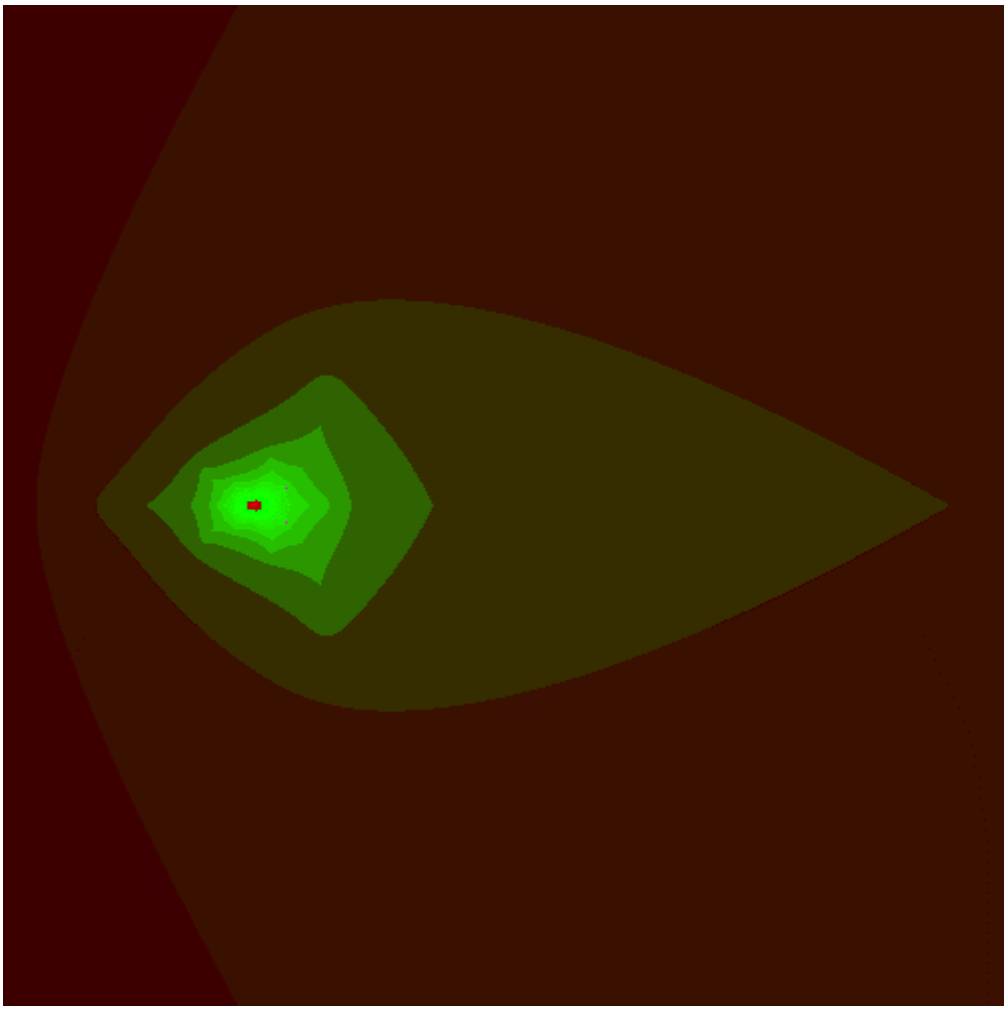

Figura 40.

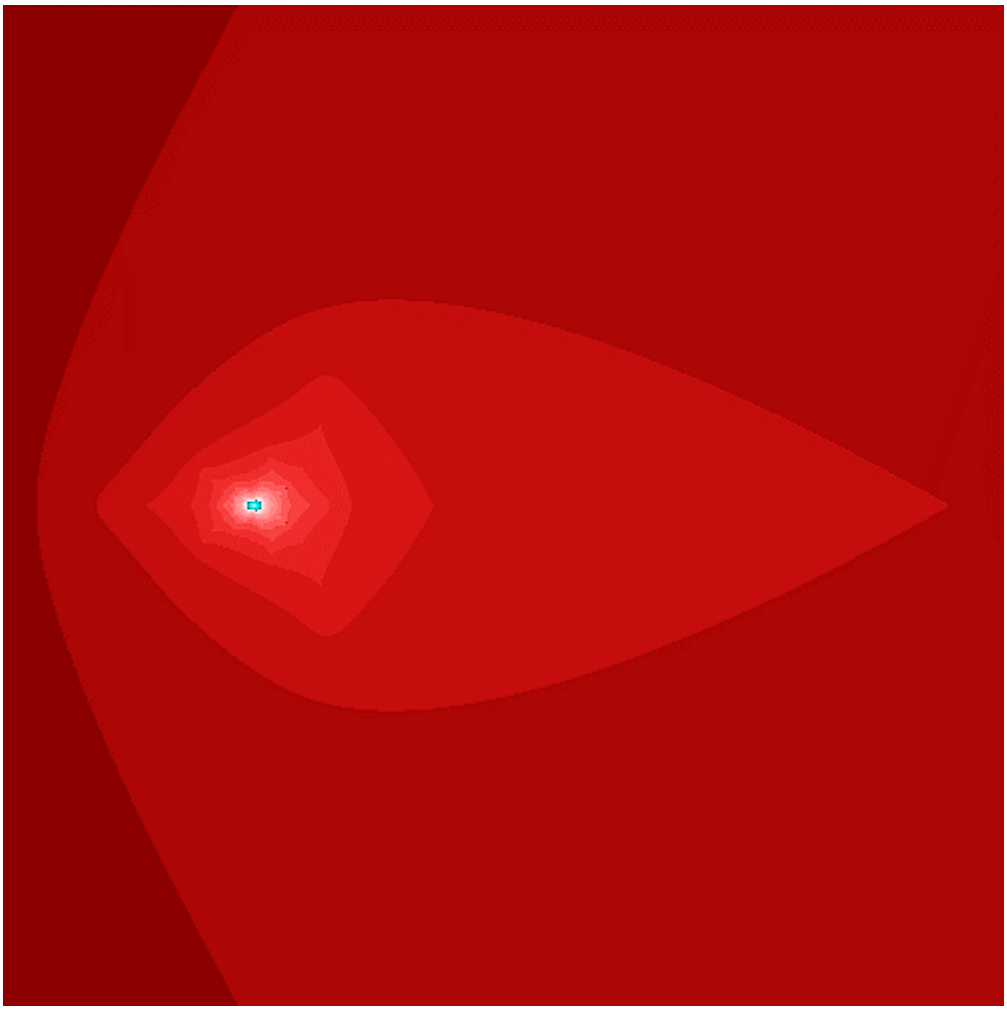

Figura 41.

5. La función: $f(z) = z - e^{-\sqrt{1-z}}$, $z \in \left(0.7 - 0.25\sqrt{-1}, 1.2 + 0.25\sqrt{-1}\right)$.

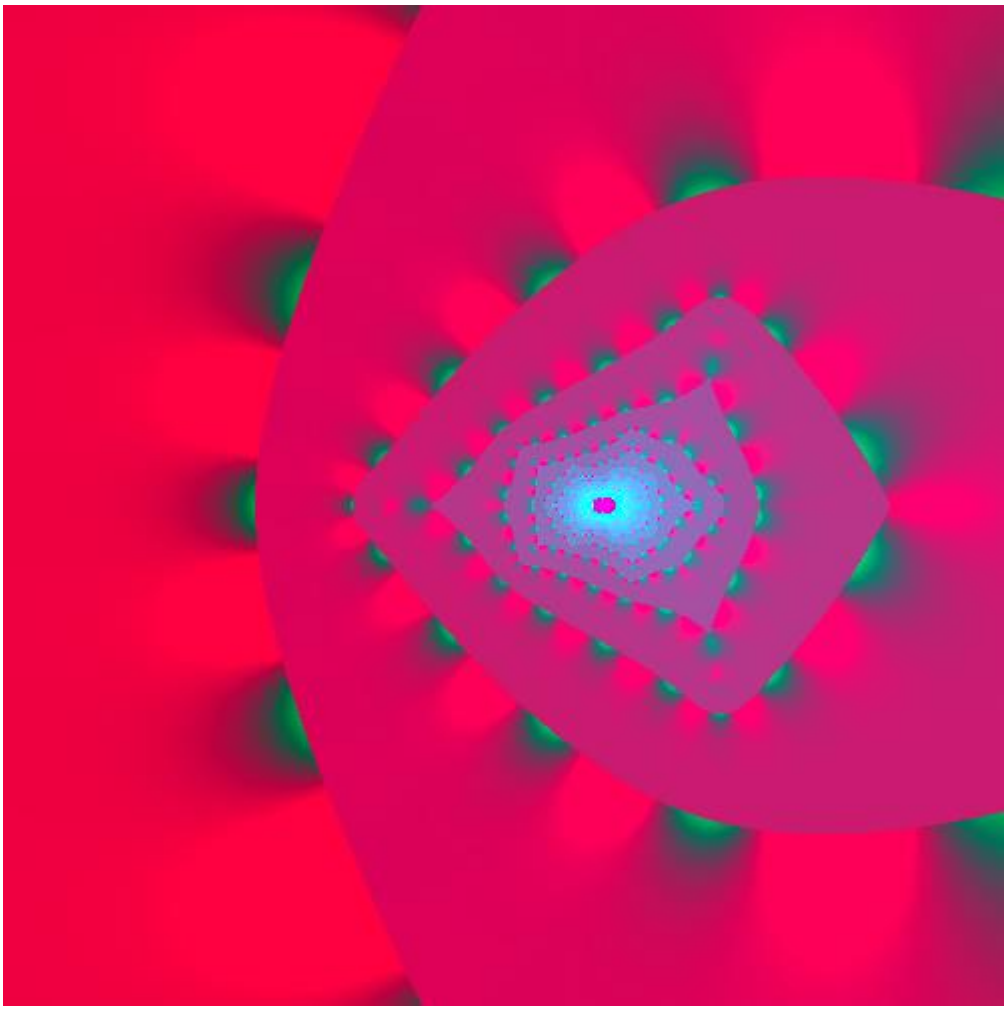

Figura 42.

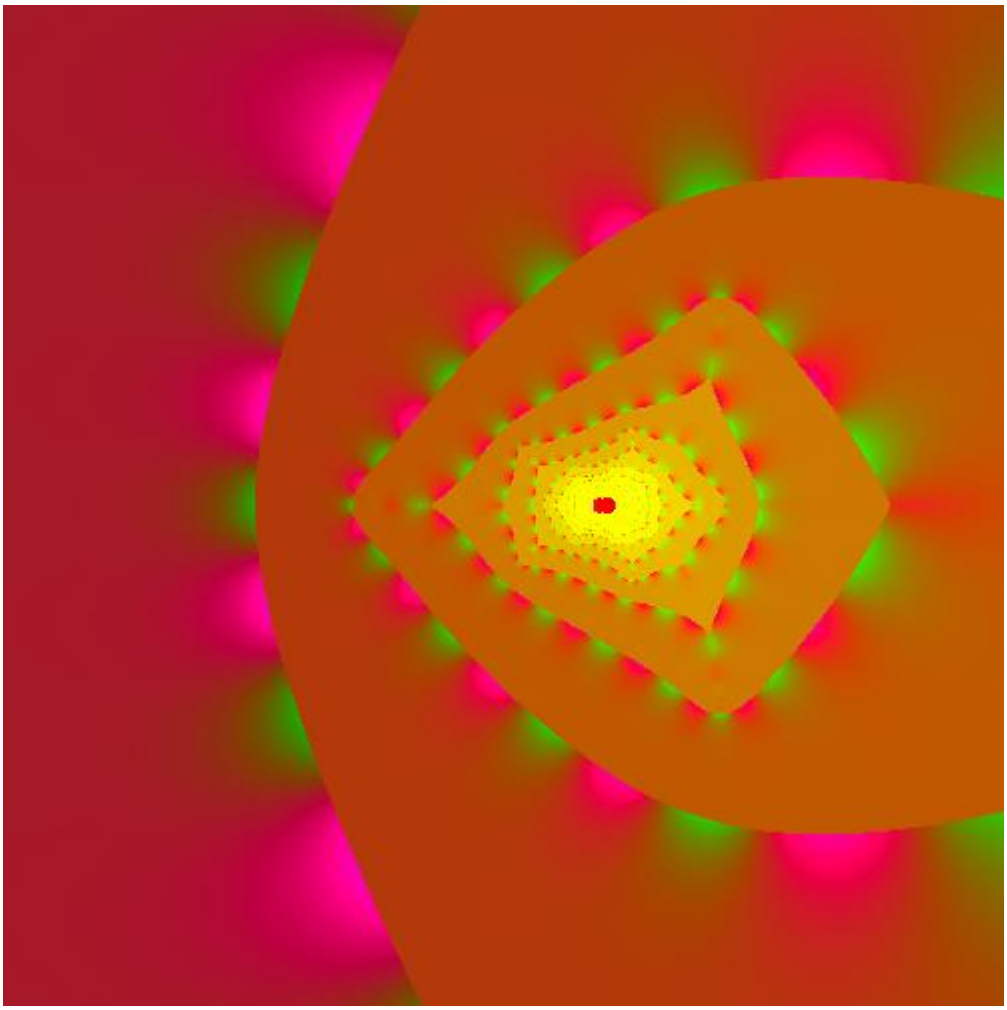

Figura 43.

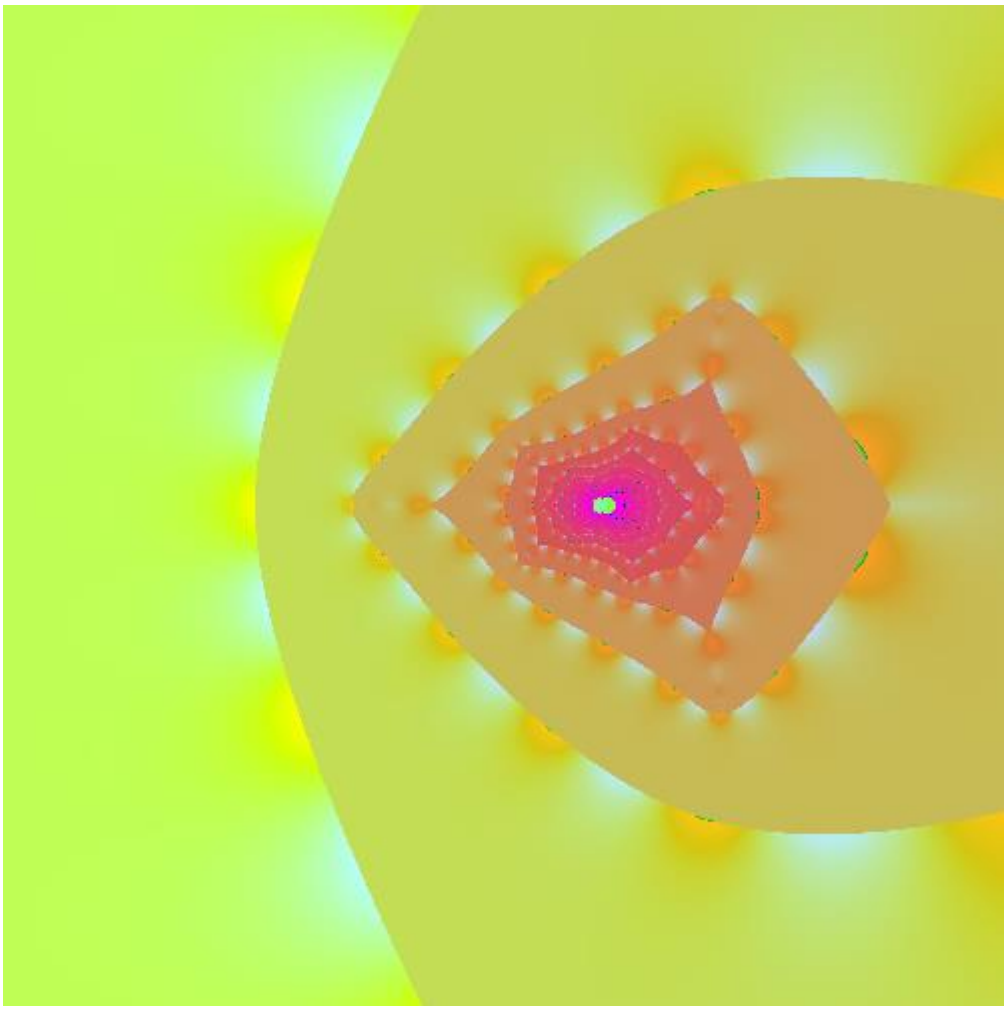

Figura 44.

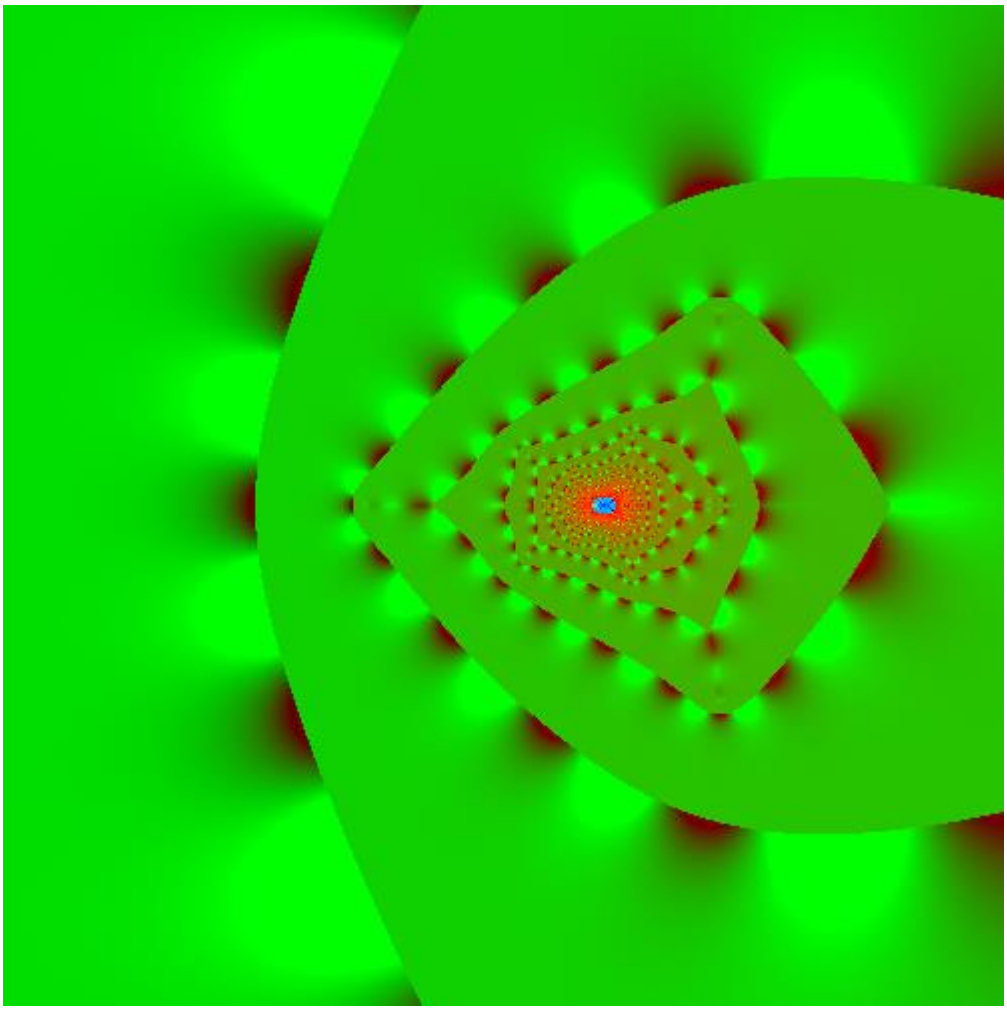

Figura 45.

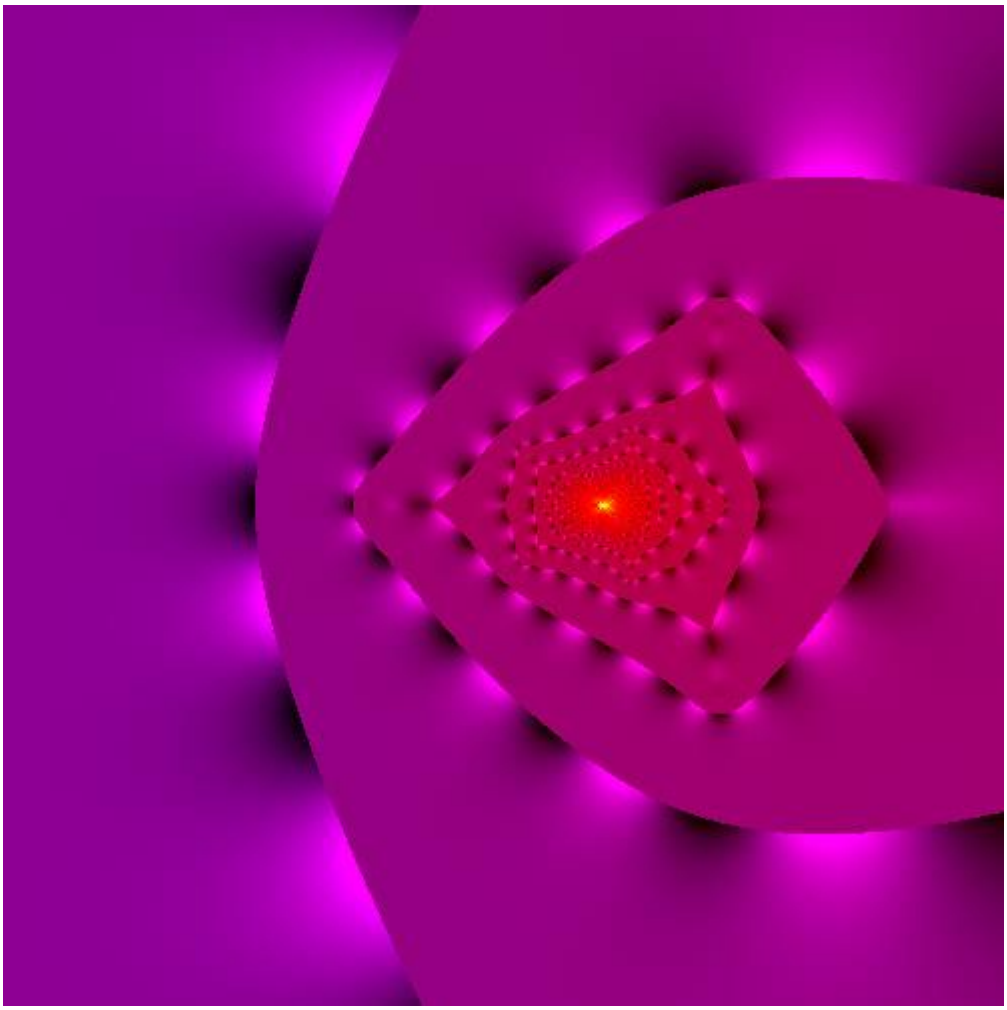

Figura 46.

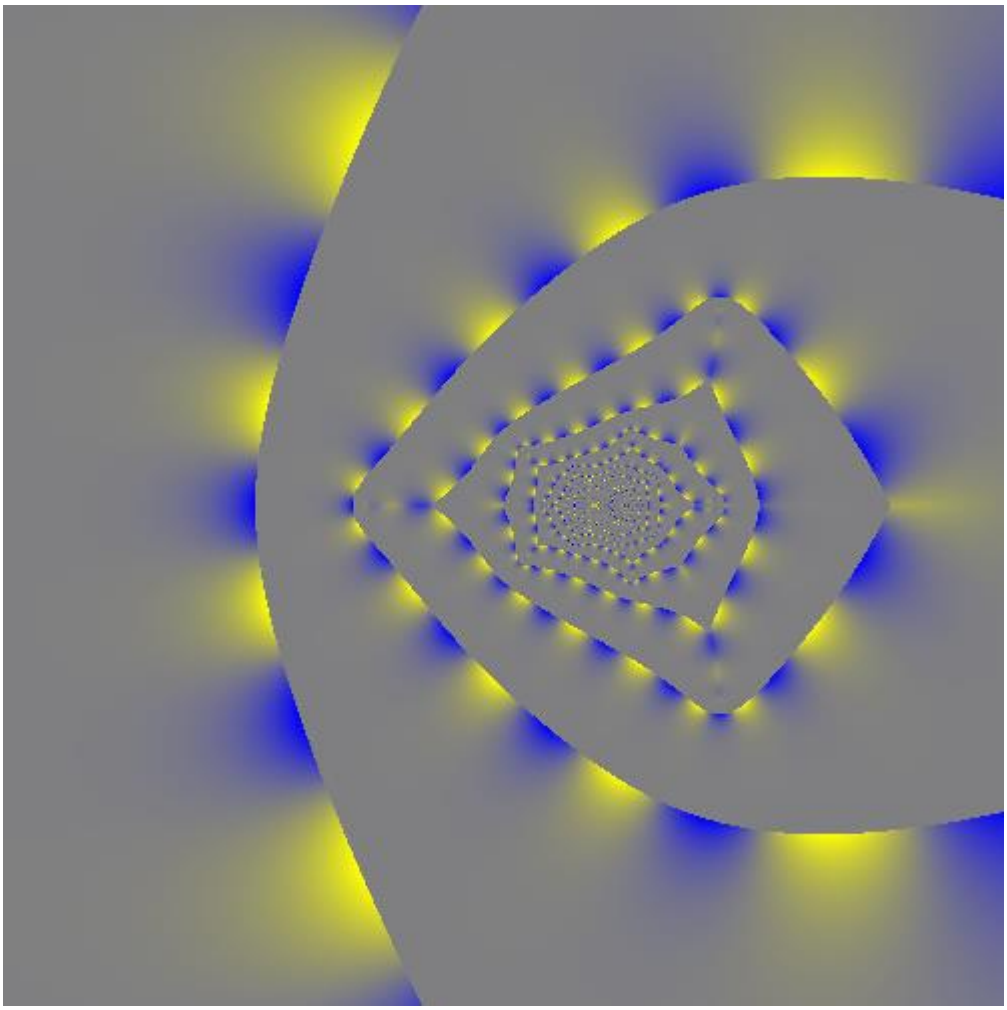

Figura 47.

6. La función: $f(z) = z - e^{-\sqrt{1-z}}$, $z \in \left(-2-2.5\sqrt{-1}, 3+2.5\sqrt{-1}\right)$.

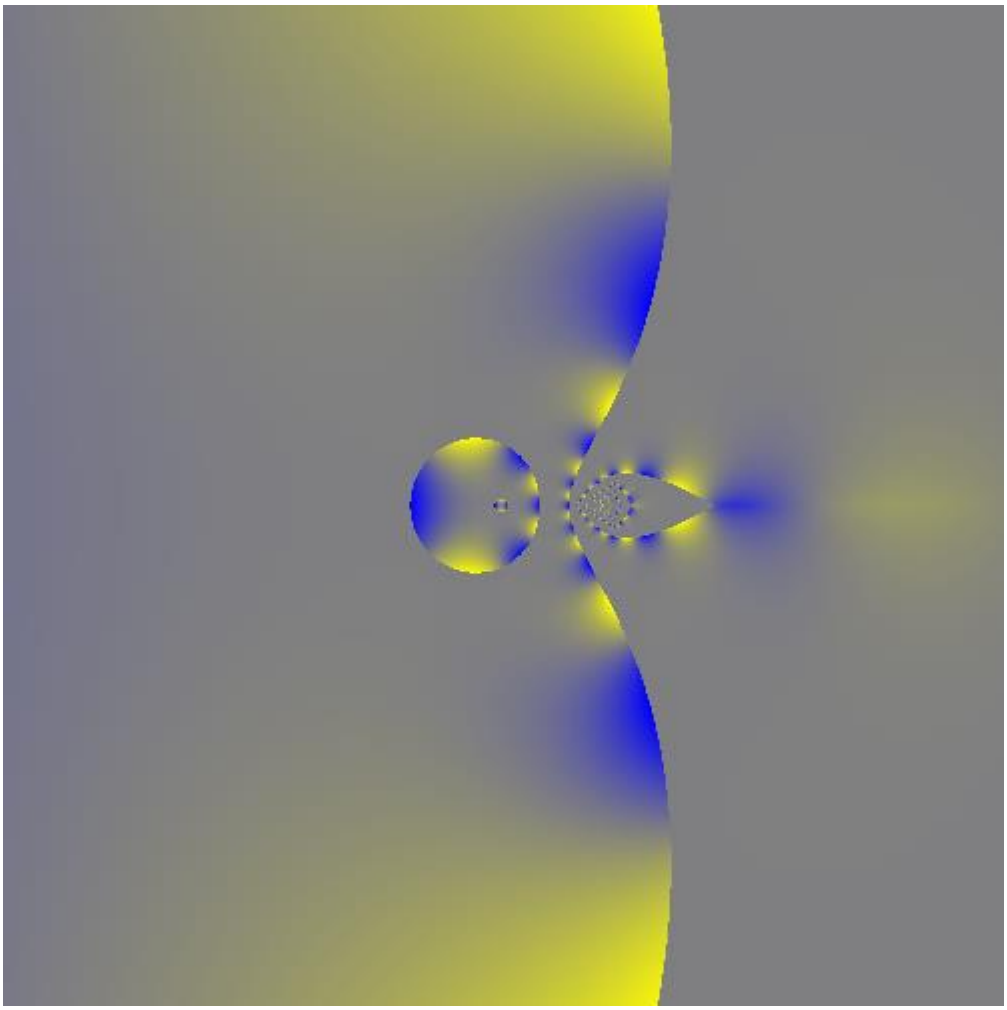

Figura 48.

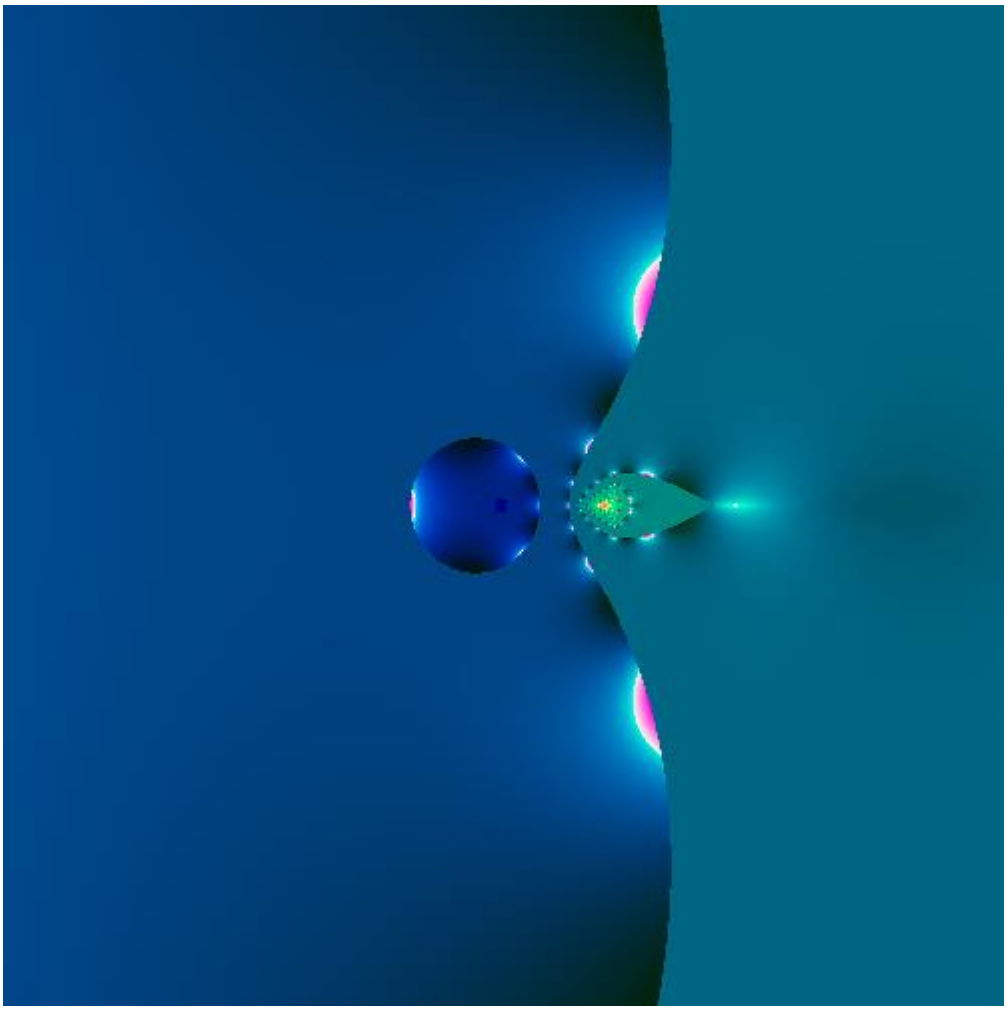

Figura 49.

7. La función: $f(z)=z-e^{-\sqrt{1-z}}$, $z\in\left(-16-16\sqrt{-1},16+16\sqrt{-1}\right)$.

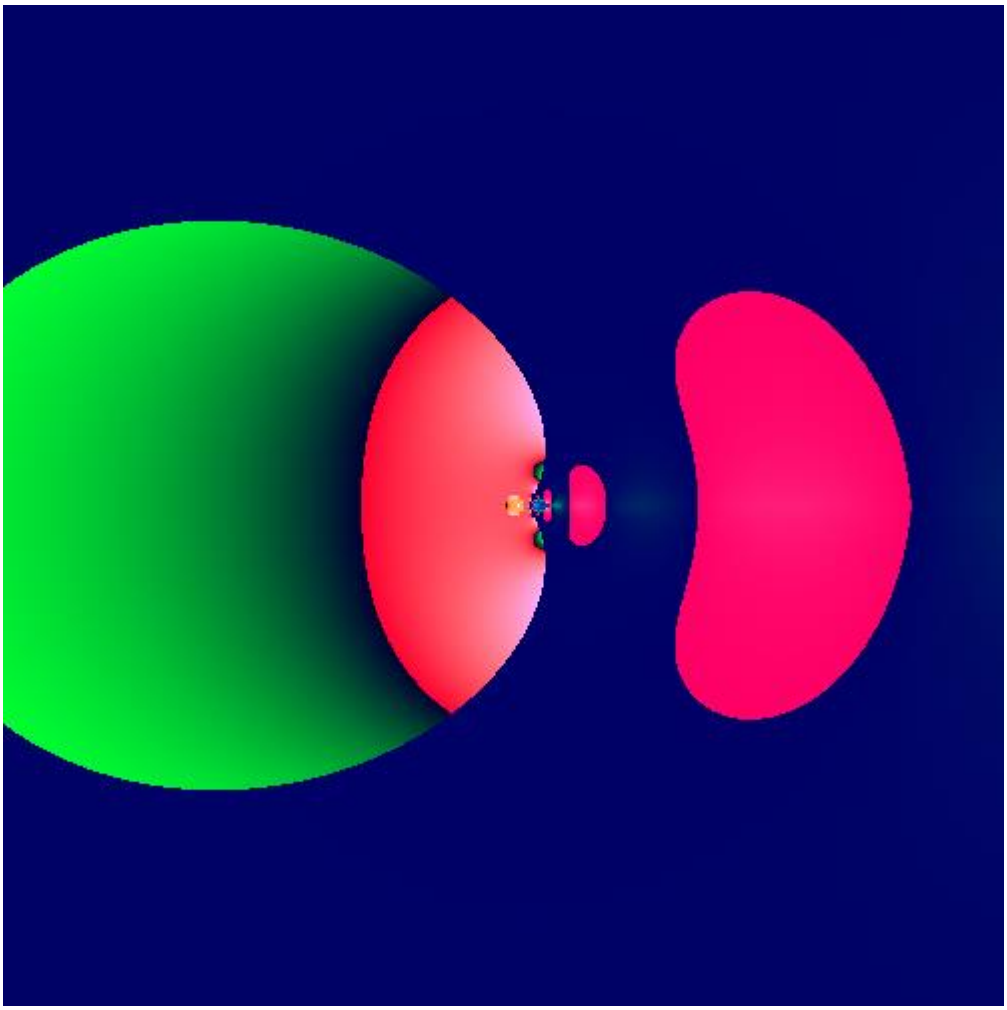

Figura 50.

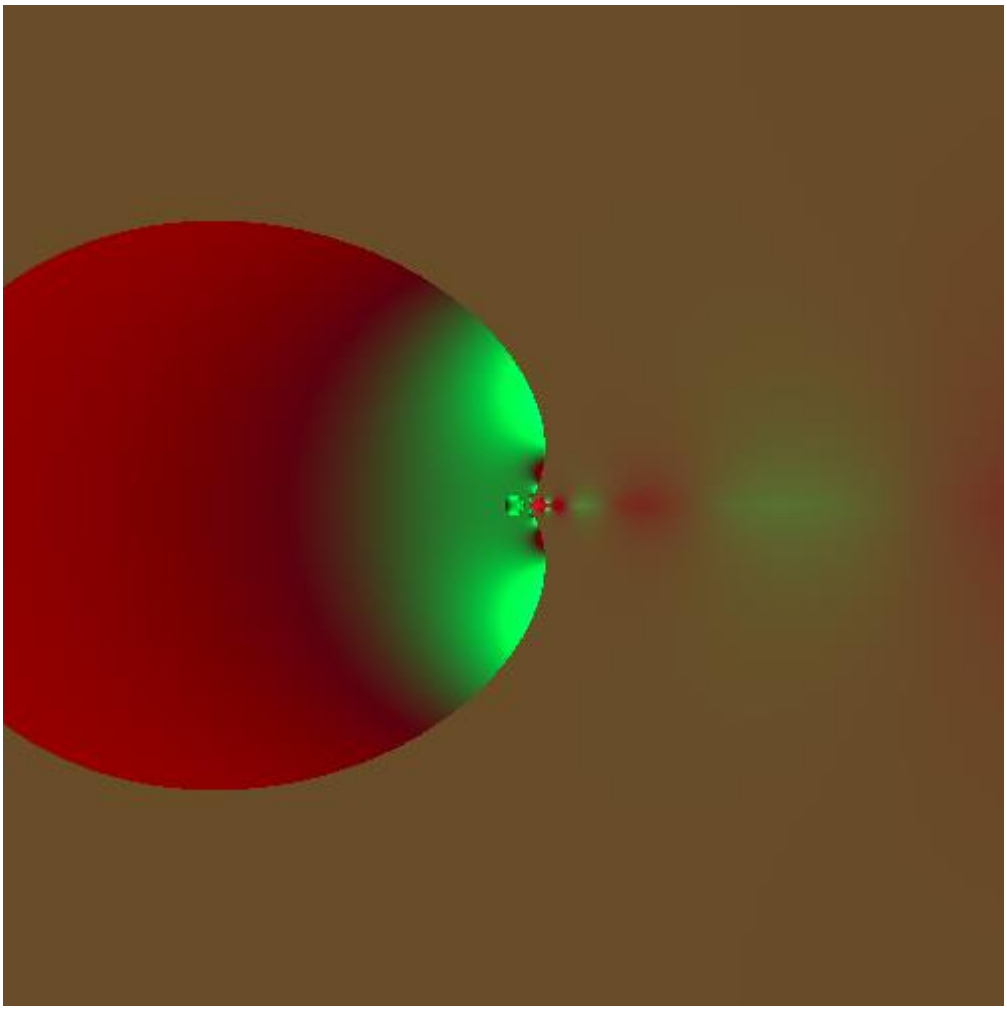

Figura 51.

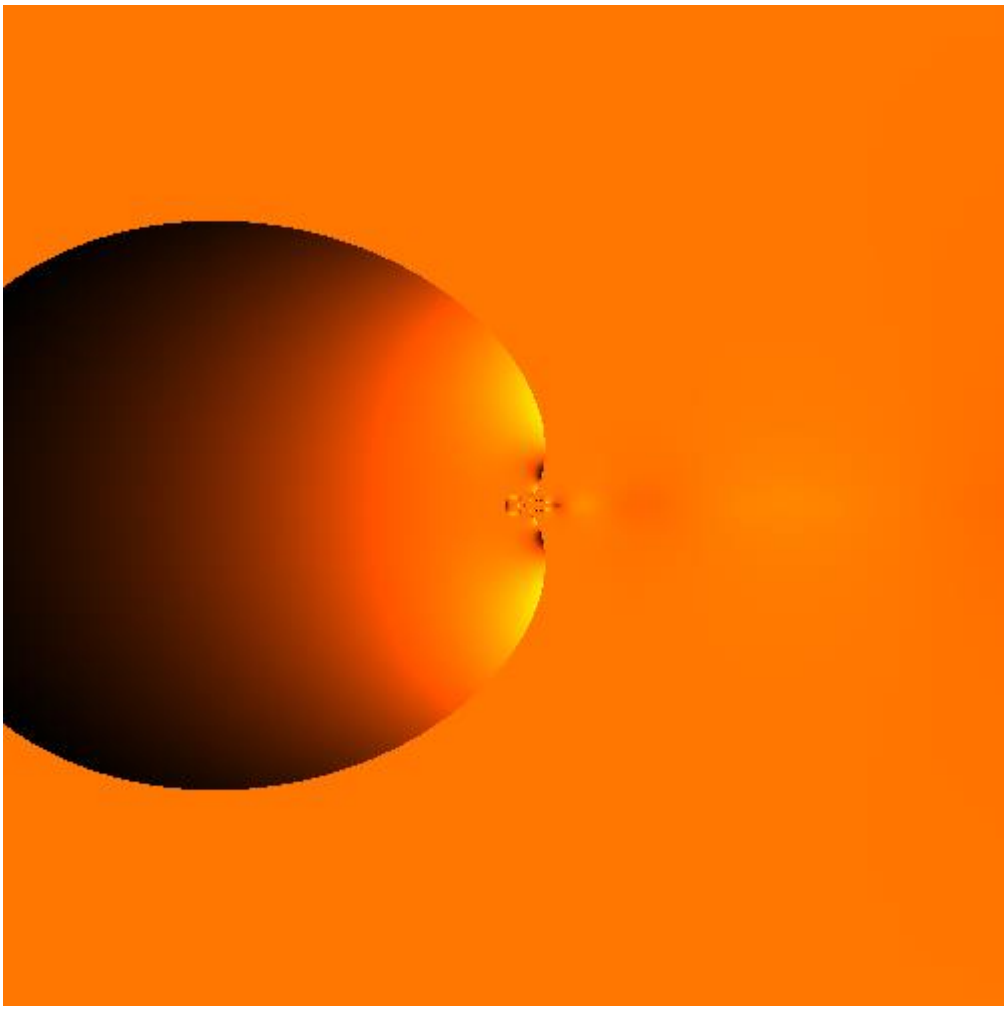

Figura 52.

8. La función: $f(z) = z - e^{-\sqrt{1-z}}$, $z \in \left(-56 - 56\sqrt{-1}, 56 + 56\sqrt{-1}\right)$.

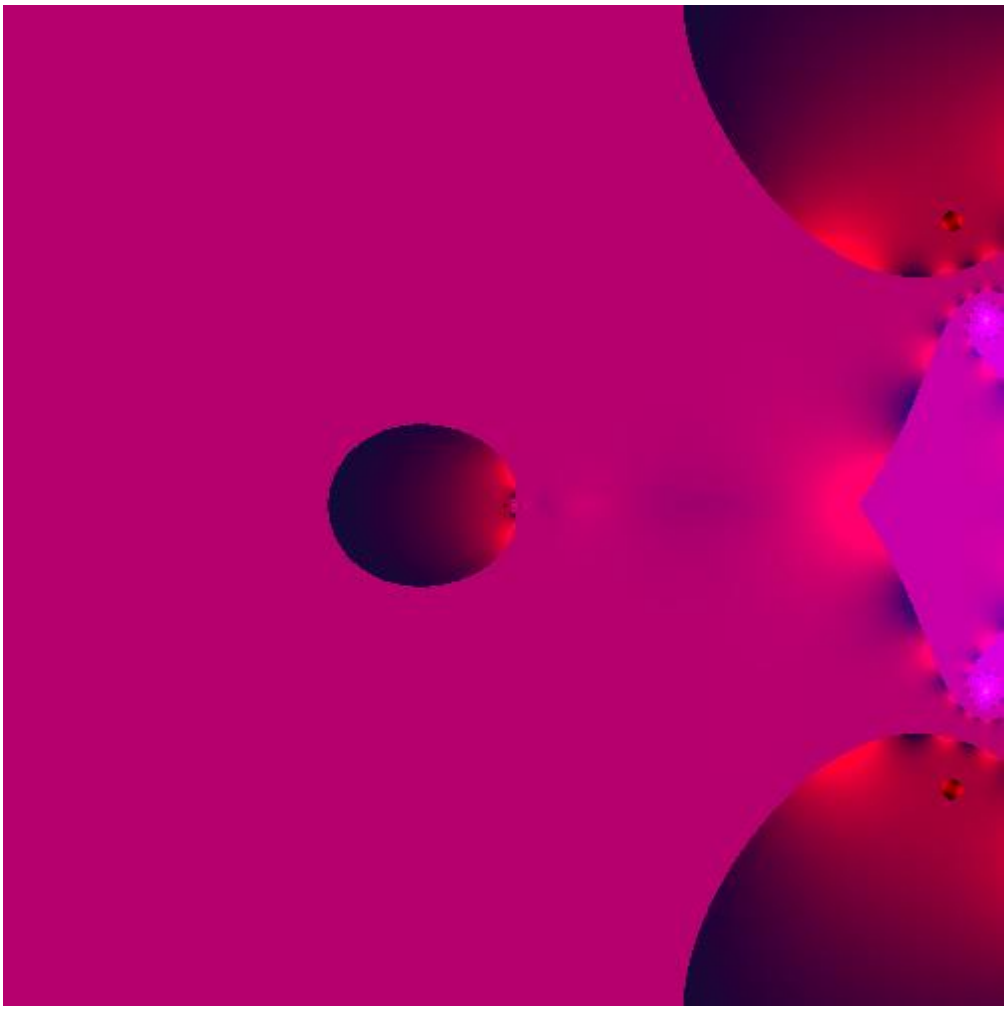

Figura 53.

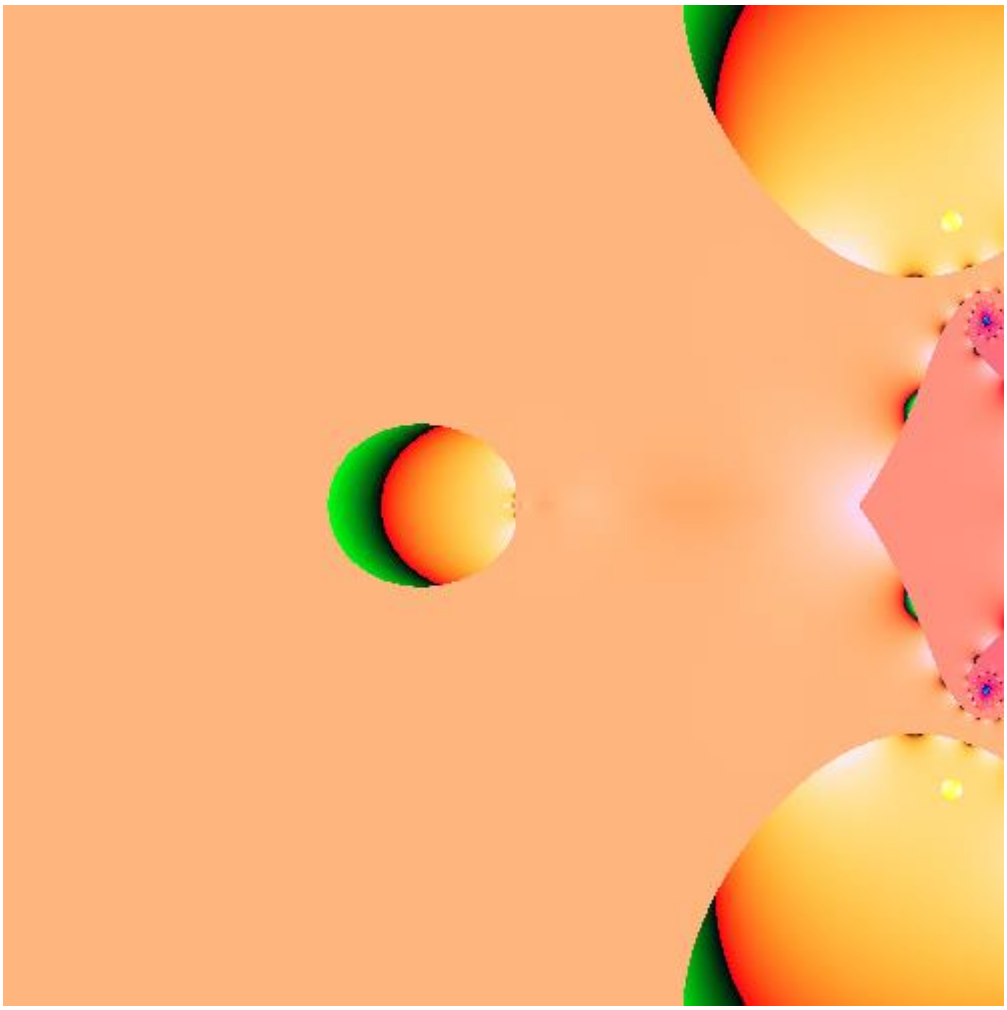

Figura 54.

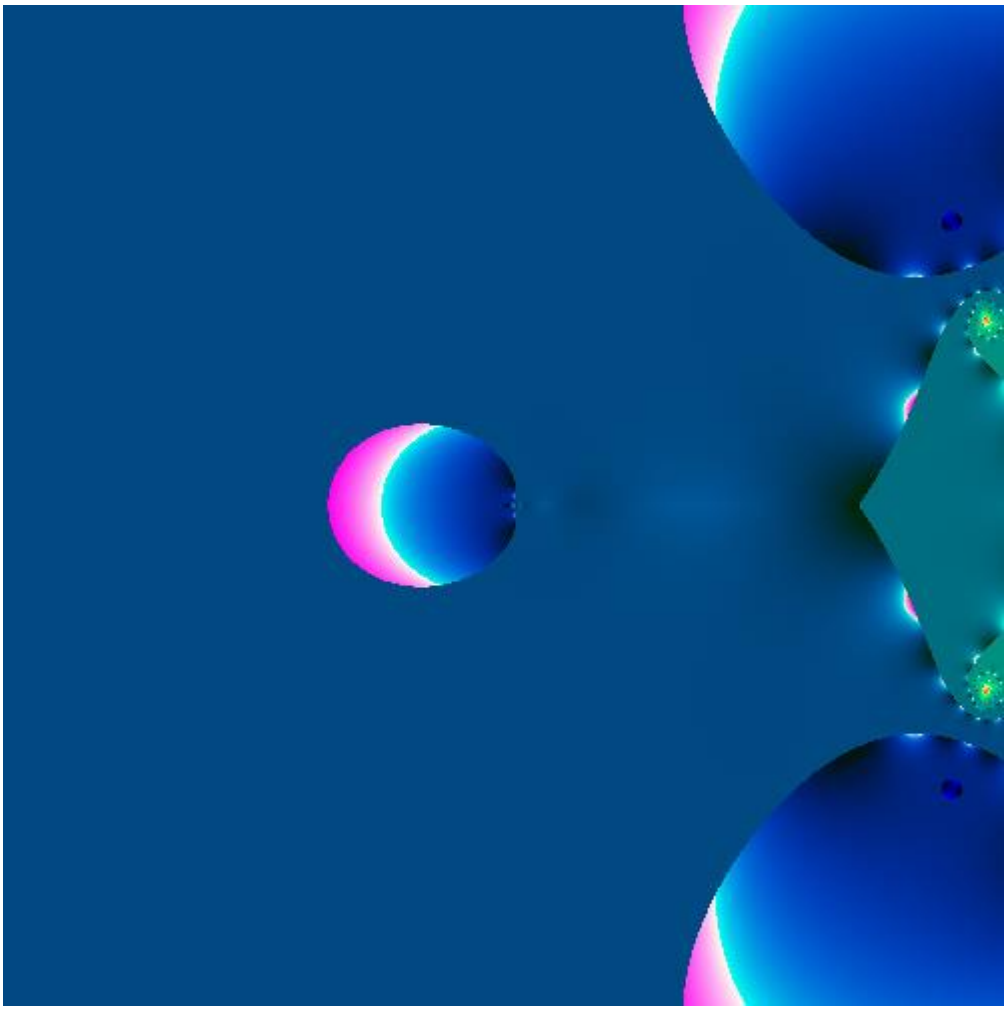

Figura 55.

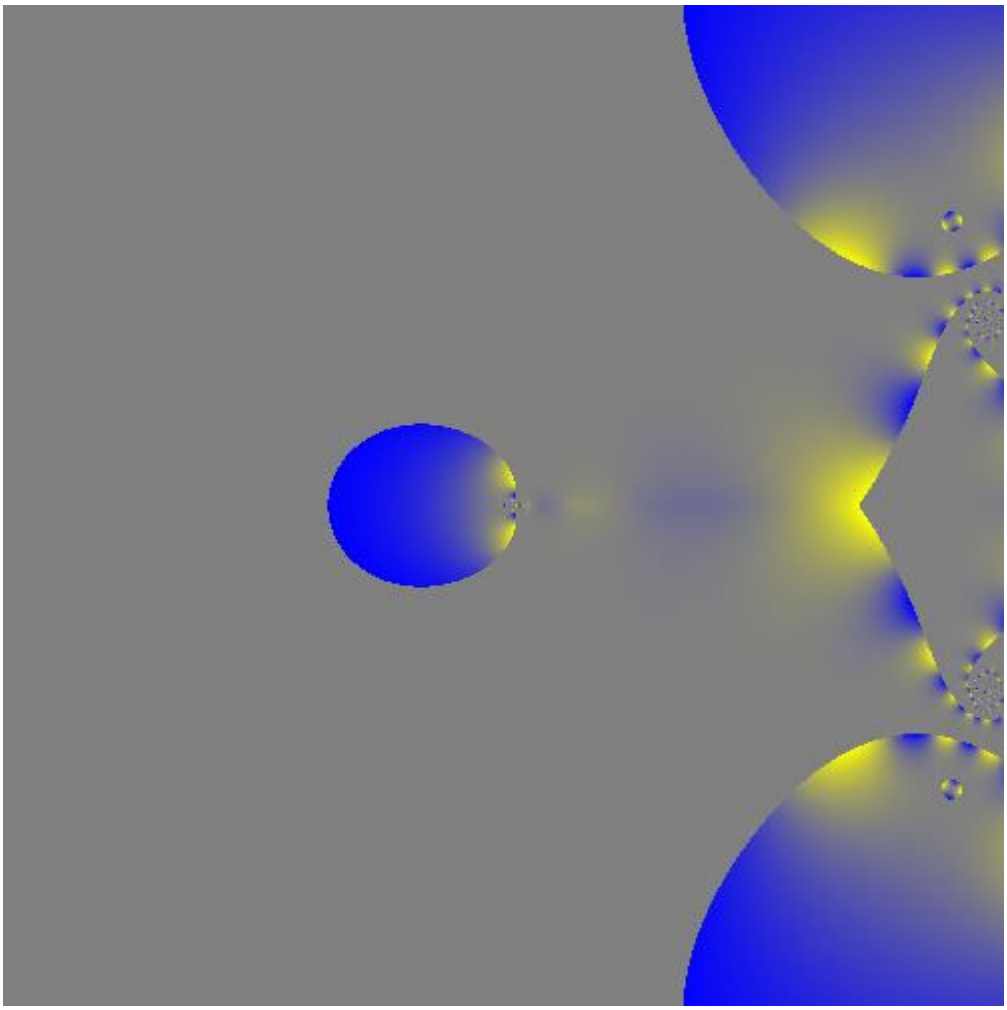

Figura 56.

9. La función: $f(z) = z - e^{-\sqrt{1-z}}$, $z \in \left(-40 - 90\sqrt{-1}, 90 + 90\sqrt{-1}\right)$.

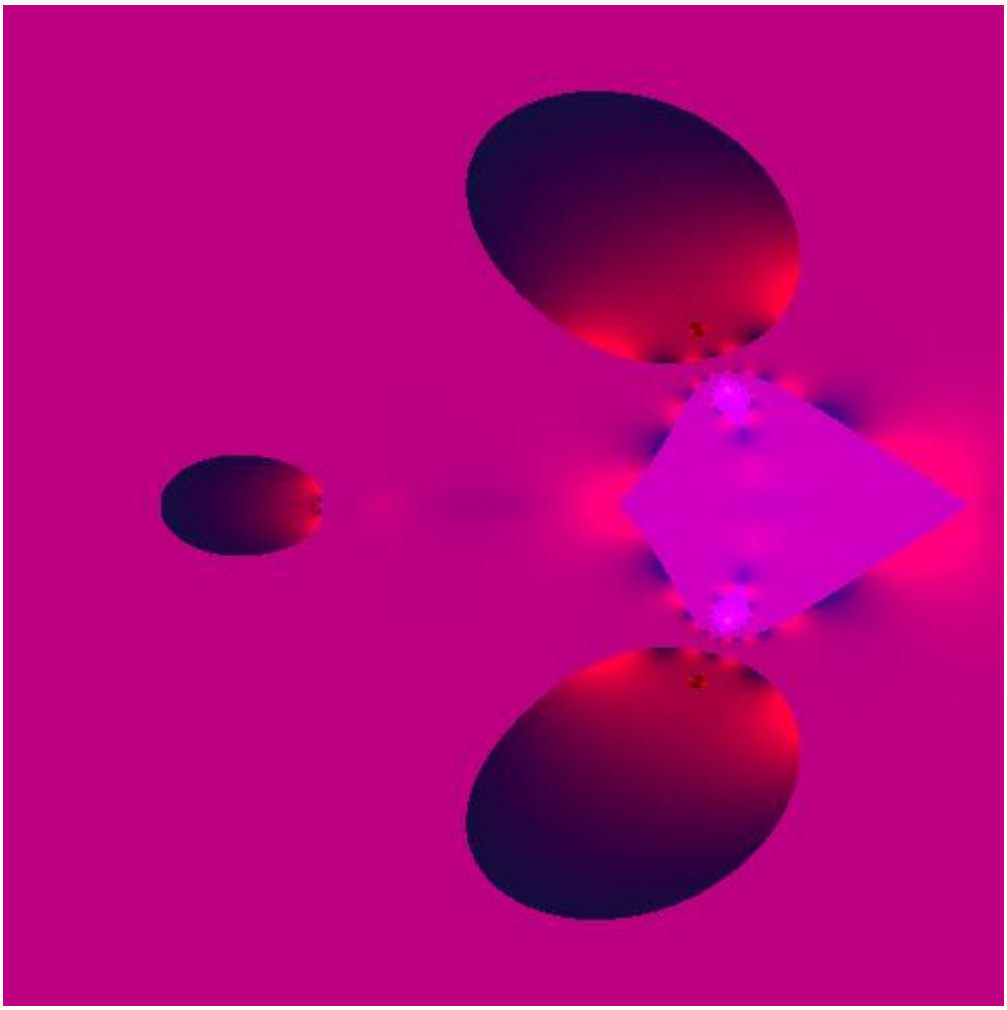

Figura 57.

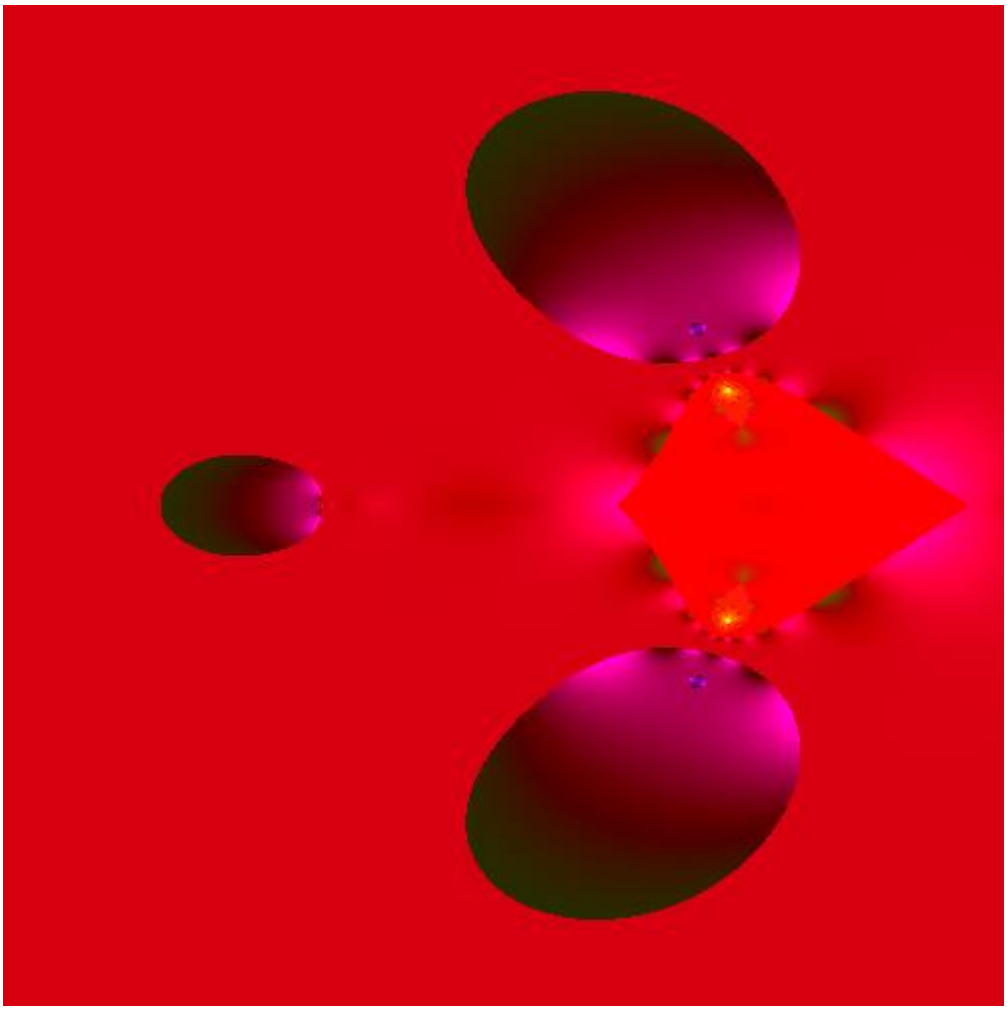

Figura 58.

Figura 59.

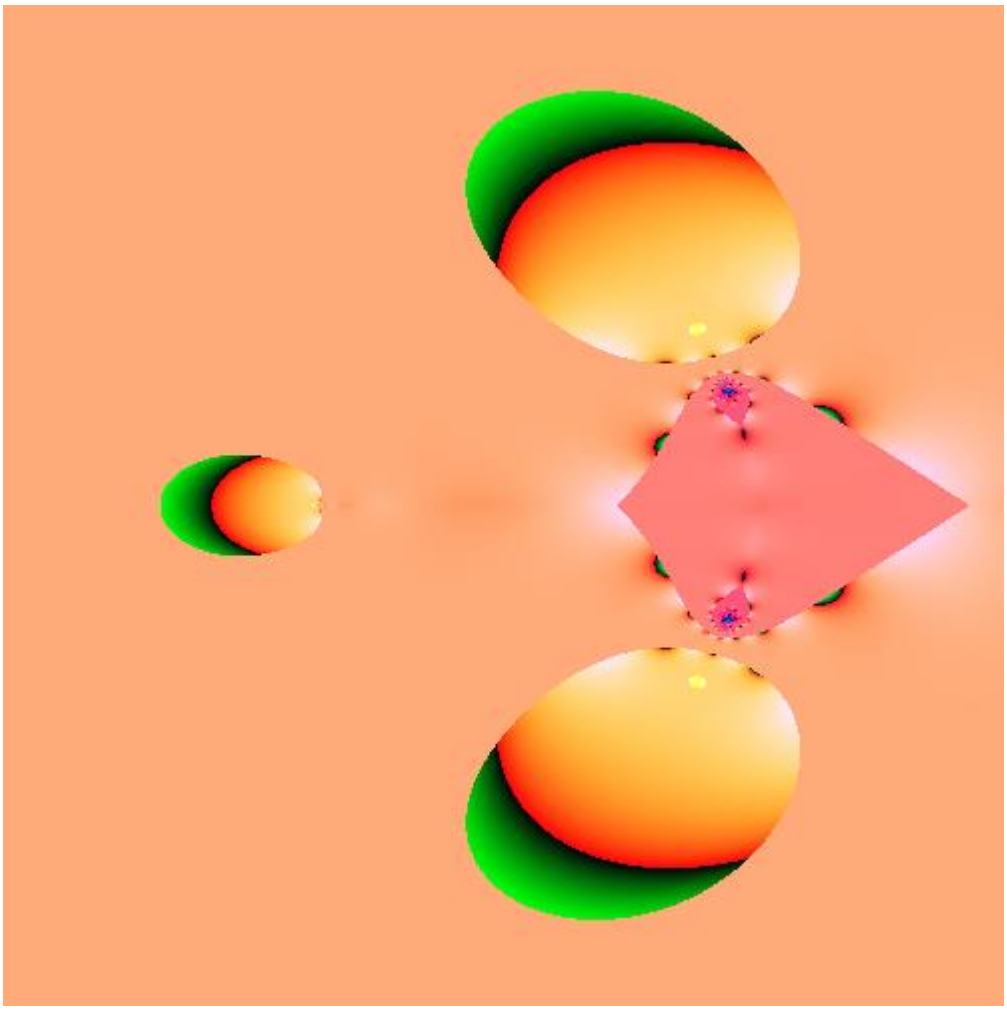

Figura 60.

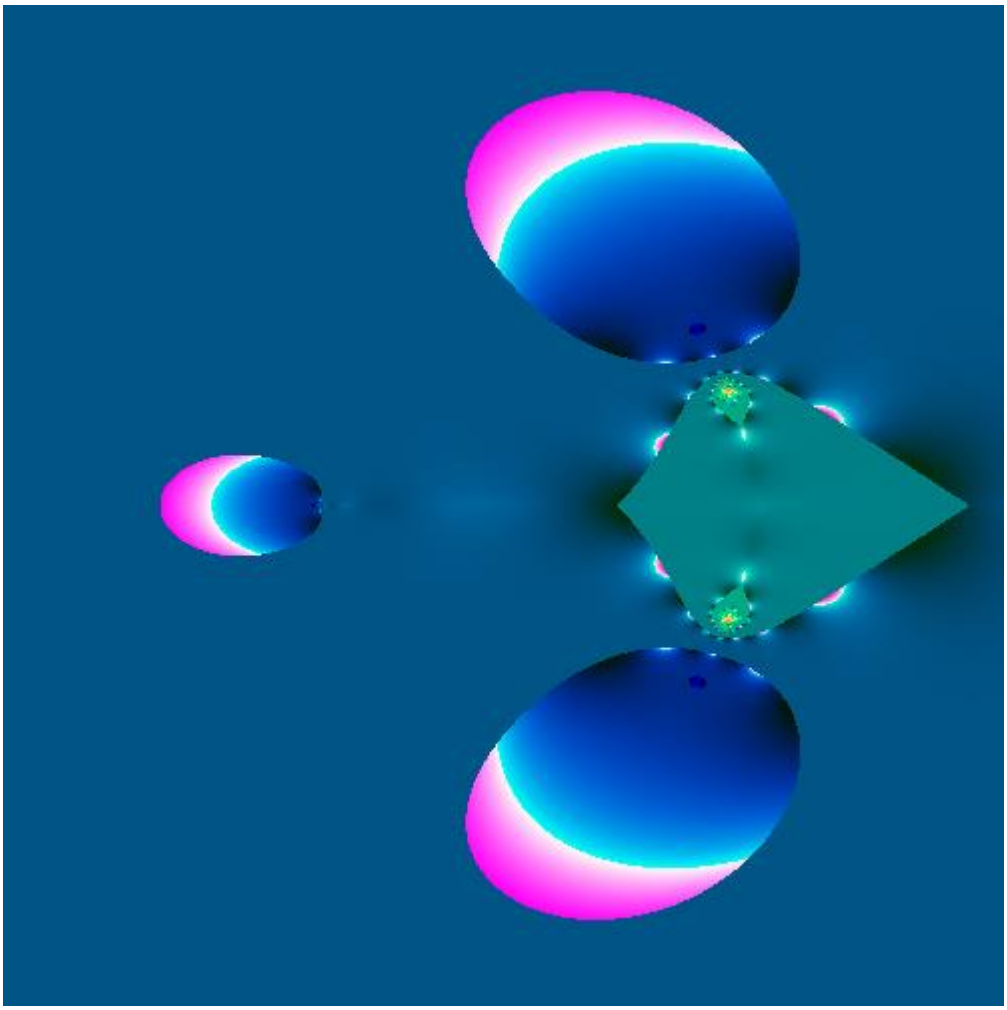

Figura 61.

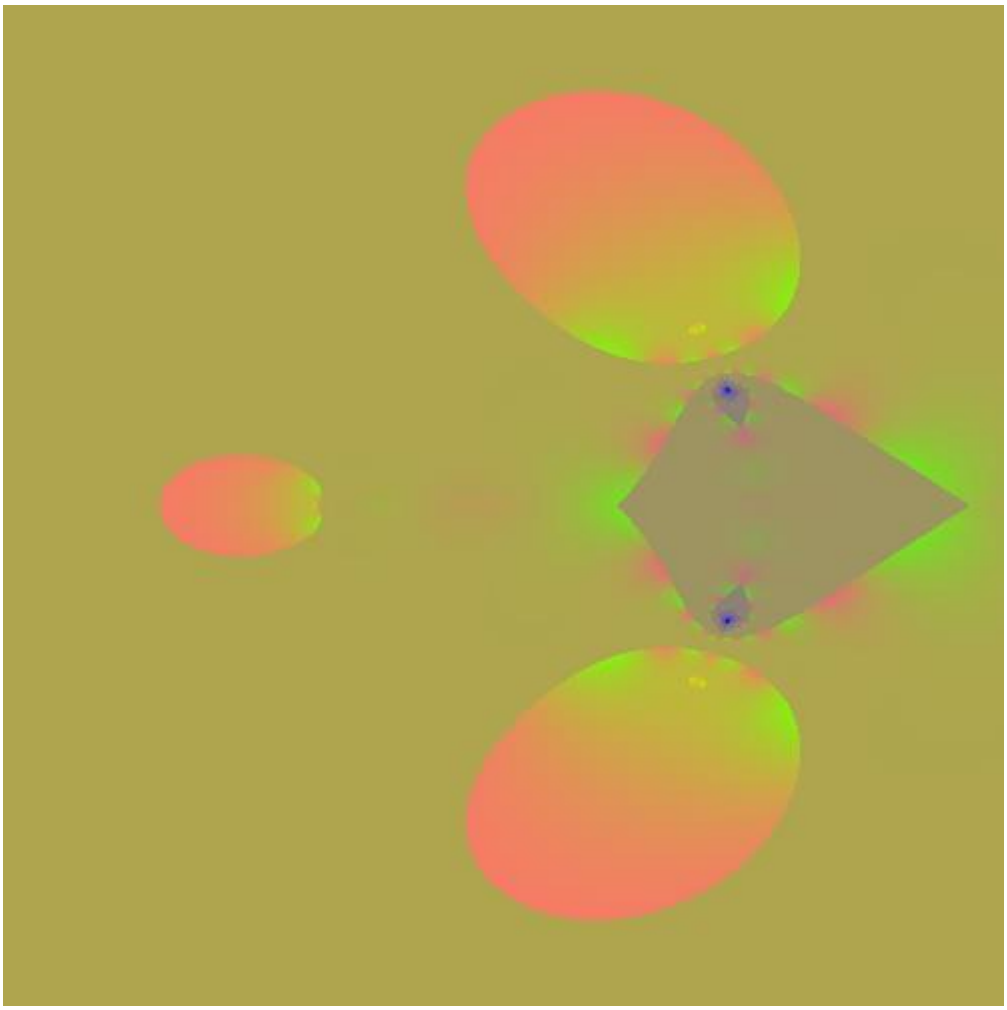

Figura 62.

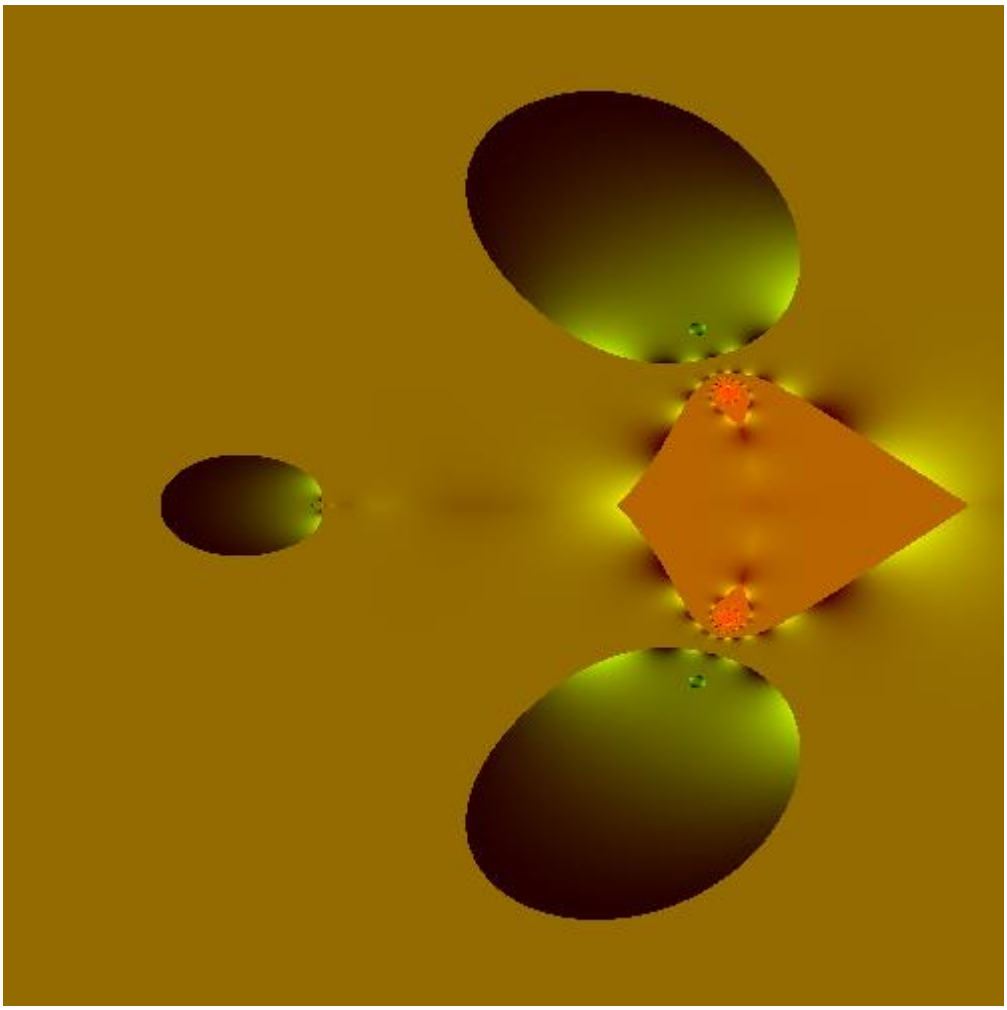

Figura 63.

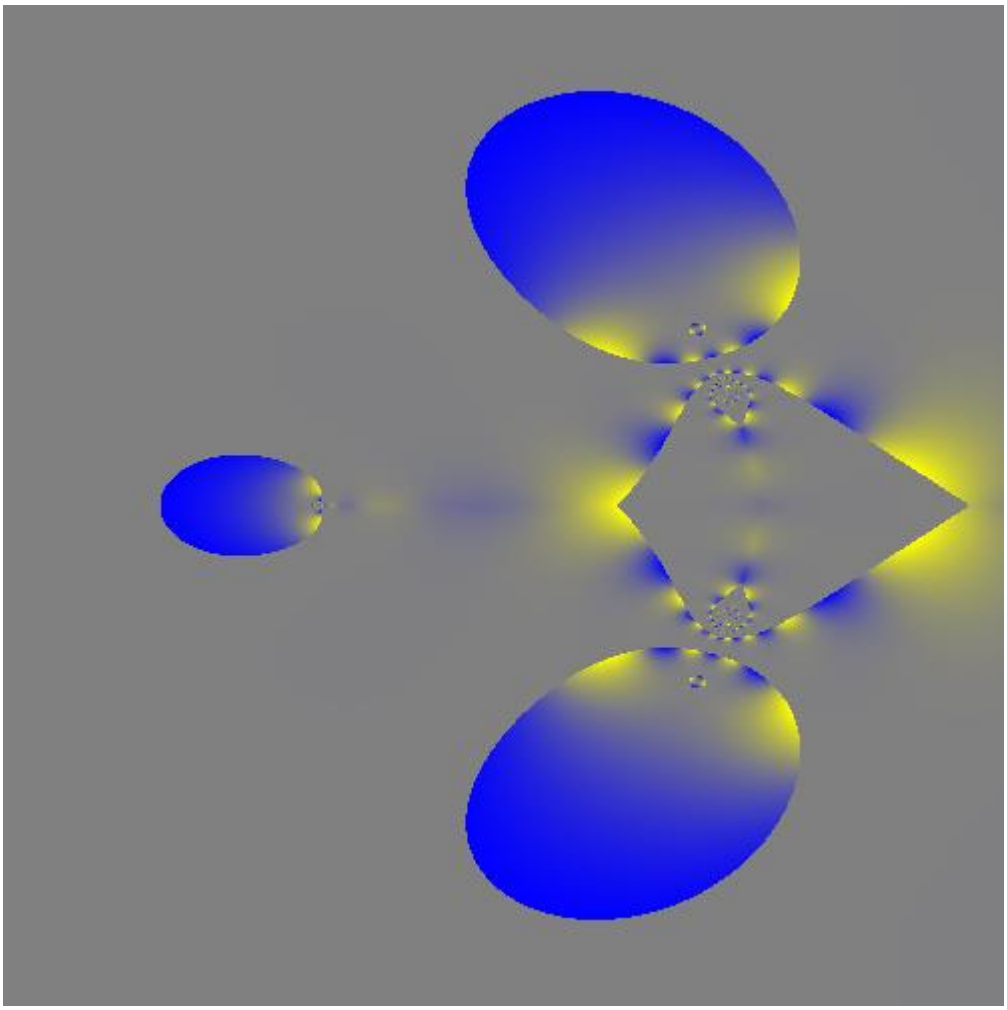

Figura 64.

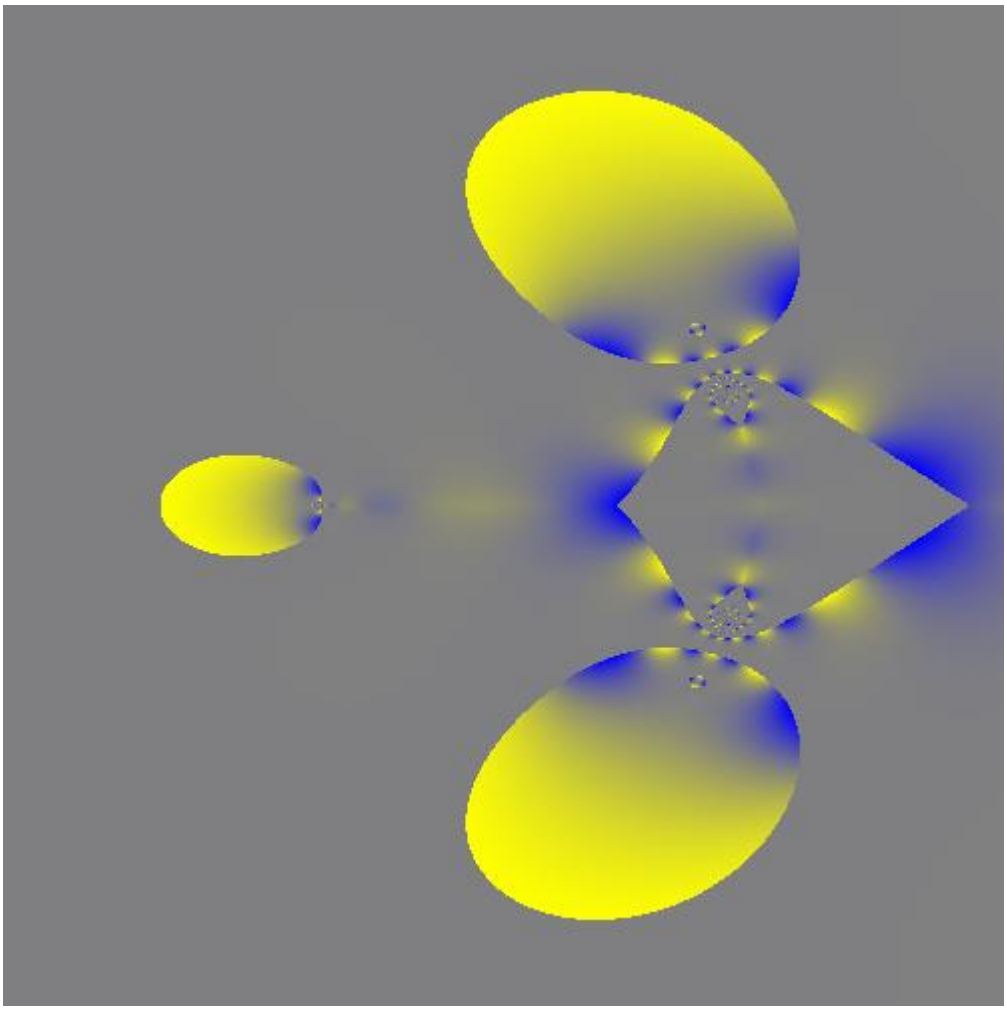

Figura 65.

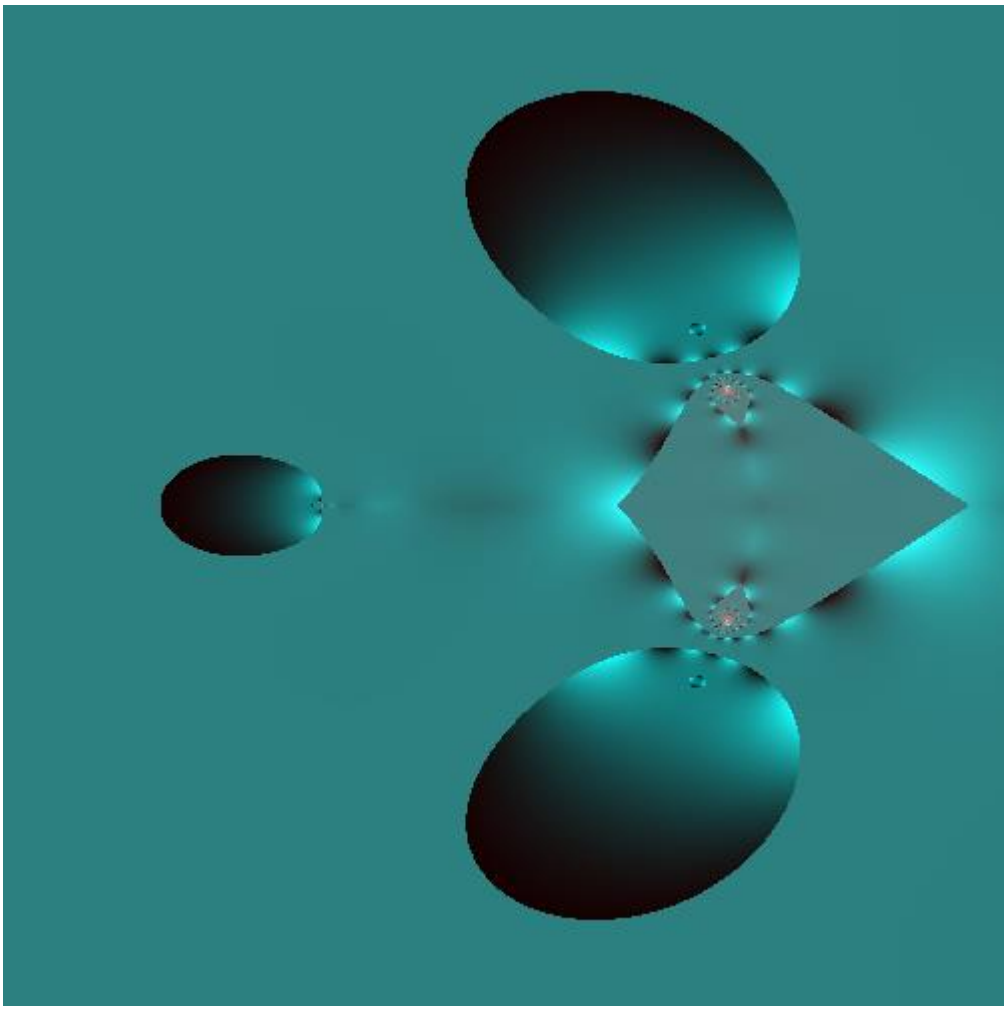

Figura 66.

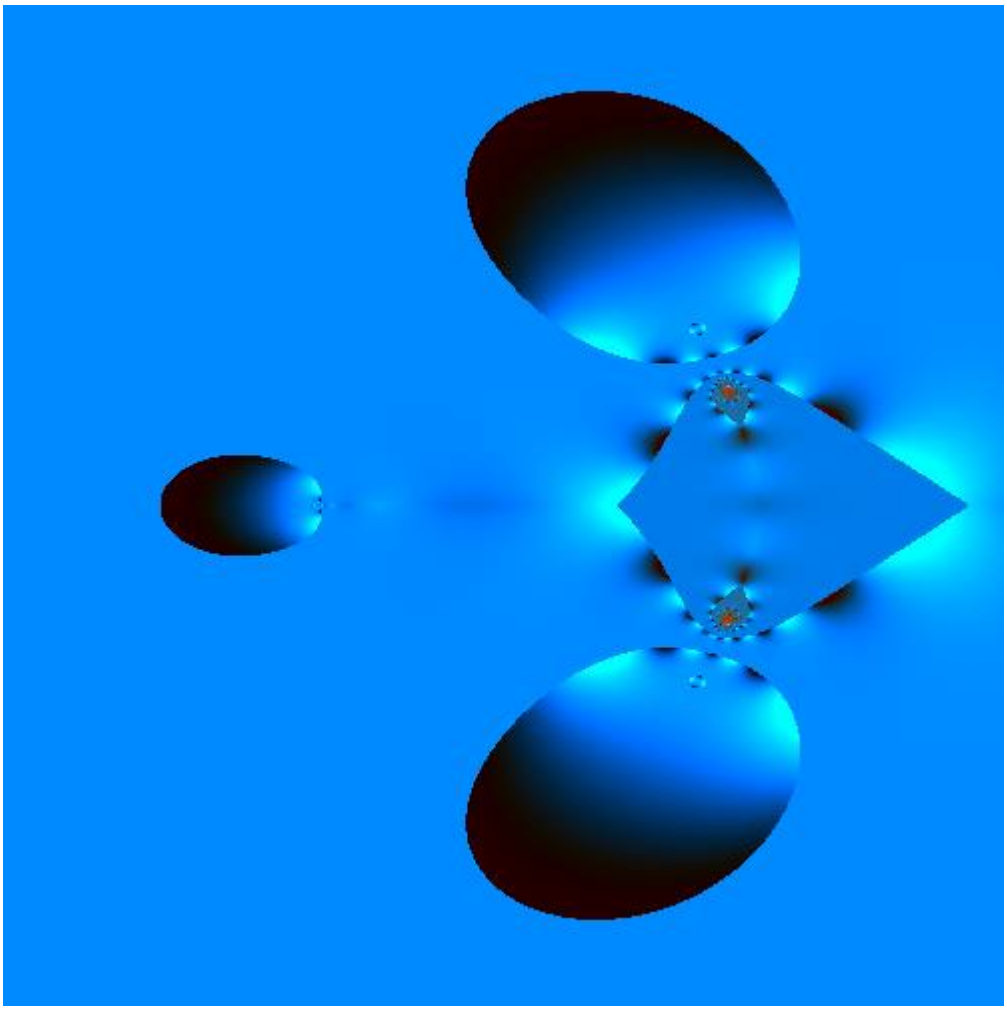

Figura 67.

Referencias

1. Barnsley, M.F. and Rising, H.: Fractals Everywhere, 2nd ed. Boston, MA: Academic Press, 1993.
2. Bunde, A. and Havlin, S.: Fractals and Disordered Systems, 2nd ed. New York: Springer-Verlag, 1996.
3. Bunde, A. and Havlin, S.: Fractals in Science. New York: Springer-Verlag, 1994.
4. Cayley, A.: The Newton – Fourier imaginary problema, Amer. J. Math. 2, 1879, 97.
5. Cayley, A.: Applications of the Newton – Fourier Method to an imaginary root of an equation, Quart. J. of Pure and Applied Math. 16, 1879, 179-185.
6. Devaney, R.L.: Complex Dynamical Systems: The Mathematics Behind the Mandelbrot and Julia Sets. Providence, RI: Amer. Math. Soc., 1994.
7. Fatou, P.: Sur les équations fonctionnelles, Bull. Sci. Math. France, 47,1919,161-271.
8. Fatou, P.: Sur les équations fonctionnelles (Deuxiéme mémoire), Bull. Sci. Math. France, 48,1920,33-94.
9. Fatou, P.: Sur les équations fonctionnelles (Deuxiéme mémoire), Bull. Sci. Math. France, 48,1920,208-314.
10. Mandelbrot, B: Fractals: Form, Chance,& Dimension. San Francisco, CA: W.H. Freeman, 1977.
11. Mandelbrot, B.: The Fractal Geometry of Nature. New York: W.H. Freeman, 1983.
12. Milnor, J.: Dynamics in One Complex Variable: Introductory Lectures, Vieweg,1999.
13. Peitgen, H.-O., Jurgens, H. and Saupe, D.: Chaos and Fractals: New Frontiers of Science. New York: Springer-Verlag, 1992.
14. Pickover, C.A.: Fractal Horizons: The Future Use of Fractals. New York: St. Martin's Press, 1996.
15. Schröder, E.: Über unendlich viele Algorithmen zur Auosung der Gleichungen, Math. Ann. 2, 1870 , 317-365.
16. Schröder, E.: Über iterite funktionen, Math. Ann. 3, 1871, 296-322.
17. Valdebenito, E.: Elementary Fractals: Part I, http://vixra.org/pdf/1811.0214v1pdf .
18. Valdebenito, E.: Elementary Fractals: Part II, http://vixra.org/pdf/1811.0324v1pdf .
19. Valdebenito, E.: Elementary Fractals: Part III, http://vixra.org/pdf/1811.0435v1pdf .
20. Valdebenito, E.: Elementary Fractals: Part IV, http://vixra.org/pdf/1812.0061v1pdf .