Spin½ 'Plane' & Simple

Abstract: **To fully characterize any spin requires identification of its spin vector *and* its plane of rotation. Classical presumptions obscure both for "intrinsic" spin. Here, Euclidean interval-time coordinates literally *lift the veil* of space to reveal it. Probability amplitude is also physically realized.**

Retreat or Conceit?

Generations of unsurpassed success in Quantum Mechanics has transformed "quantum weirdness" from a set of *mysteries* to be solved, to hollow *cornerstones*, precariously built upon. Foremost among these is quantum spin. Cautions abound:

"...spin is an intrinsic property of a particle, unrelated to any sort of motion in space."[1]

"Physically, this means it is ill-defined what axis a particle is spinning about"[2]

"…any attempt to visualize it [spin½] classically will badly miss the point."[3]

"[Quantum spin] has nothing to do with motion in space...but is somewhat analogous to classical spin"[4]

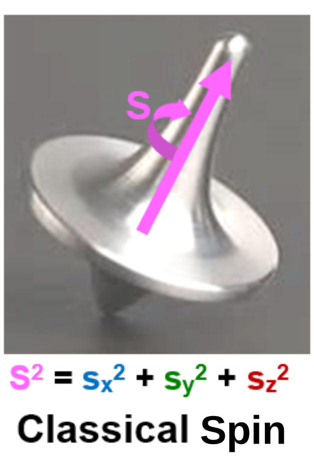
"the spin…of a fundamental fermion…with no classical analog, is…abstract…with no possibility of intuitive visualization."[5]

Is this resignation to a fundamental reality or complacency accompanying conceit? To avoid the latter, all feasible models must be exhausted. Indeed, fermion spin is *abstractly* modeled with spinors[6], but physics isn't physics unless it's about the physical! A real, physical model is presented here.

Top Performer

Because fermions have mass, "spin½" may be conveniently considered in a rest frame. Having *real* angular momentum, with measurable spin components, a *primary spin vector* (σ) is presumed to correspond to a *real* axis. The goal is to identify it, based upon these facts:

1. σ is *not* classical. Such a spin vector would decompose to ordinary spatial spin components and this is not the case (Fig. 1).
2. σ is *not* orthogonal to a spatial plane. That would yield zero-magnitude spatial spin components, contrary to those observed.
3. σ makes *equal* angles with *every* spatial direction, as revealed by equal size spin components (ħ/2), measured in any direction.[7]
4. σ represents a conserved quantity, angular momentum.[8]
5. σ has a 720° horizon. It takes *two* classical rotations to return a fermion to its *exact* original state[7] (accounting for probability amplitude).
6. σ spin components correlate probabilistically. Having measured one, the sign (±) of any other component relates to the half angle between them.[9]



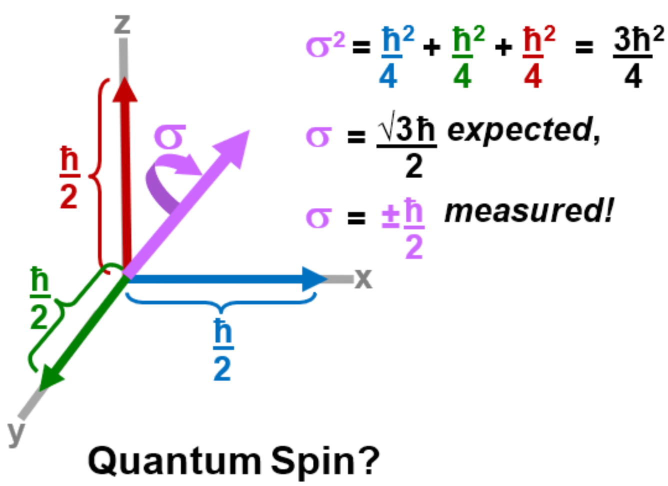


Fig. 1. Left: Classical spin vector (**S**) decomposes to coordinate projections. Right: If spin components ħ/2 are found on three spatial coordinates, a resultant spin vector (σ) of $\sqrt{3}\hbar/2$ is *expected*. Instead, ħ/2 (or -ħ/2) is *measured* there as well! (reduced Planck constant $\hbar = h/2\pi$.)

Putting a Noether Spin on It

Conservation laws are of two types.[10] "Exact" laws are never expected to fail while "approximate" laws hold within restrictions. For example, *conservation of mass-energy* is exact but divides into approximate laws: *conservation of mass* and *conservation of energy*, which are true, barring interchanges (exhibiting $E = mc^2$).

By Noether's theorem, each conservation law is now associated with a symmetry (i.e. *transformational invariance*).[11] For example, *conservation of mass-energy* is "time invariant". The mass-energy of an isolated system is unaltered by temporal displacement (Fig. 2).

Two *omissions* in the exact laws are conspicuous:

1. Laws exist for both temporal *and* spatial **translation** but only about spatial axes for **rotation**.

2. A *non-spatial,* primary "intrinsic" spin axis (σ) *must* list with the spatial axes for angular momentum.

Exact Law	Noether Symmetry Invariance		Number of Dimensions		
Conservation of mass-energy	Time invariance	Lorentz invariance symmetry	1	**translation** along time	4D
Conservation of linear momentum	Translation symmetry		3	**translation** along x, y, z	
Conservation of angular momentum	Rotation invariance		3	**rotation** about x, y, z, σ	3D?
CPT symmetry	Lorentz invariance		1+1+1	charge ($q \rightarrow -q$) + position ($r \rightarrow -r$) + time ($t \rightarrow -t$) inversions	
Conservation of electric charge	Gauge invariance		1⊗4	scalar field (1D) in 4D	
Conservation of color charge	SU(3) Gauge invariance		3	r, g, b	
Conservation of weak isospin	$SU(2)_L$ Gauge invariance		1	weak charge	
Conservation of probability	Probability invariance		1⊗4	Σ probabilities = 1 in x,y,z	

Fig. 2 It is inconsistent to purport 4D of translation (rows 1+ 2) and yet pretend only 3D are available for rotation (row 3). Widely acknowledged to be non-spatial,[1-5] the primary fermion spin axis must be recognized by a fourth dimension for σ (added). Angular momentum is, after all, conserved.

The 'Plane' Truth

To understand intrinsic spin, both the *primary spin axis* and the *plane of rotation* must be identified. A classical plane of rotation entails two dimensions. In 4D, full rotation occurs in a 3-plane (Fig. 3). A spin vector is defined perpendicular to a flat plane of rotation, but such a plane is obscured in non-Euclidean spacetime. Non-zero spatial spin components mean σ is *not* orthogonal to a spatial plane.

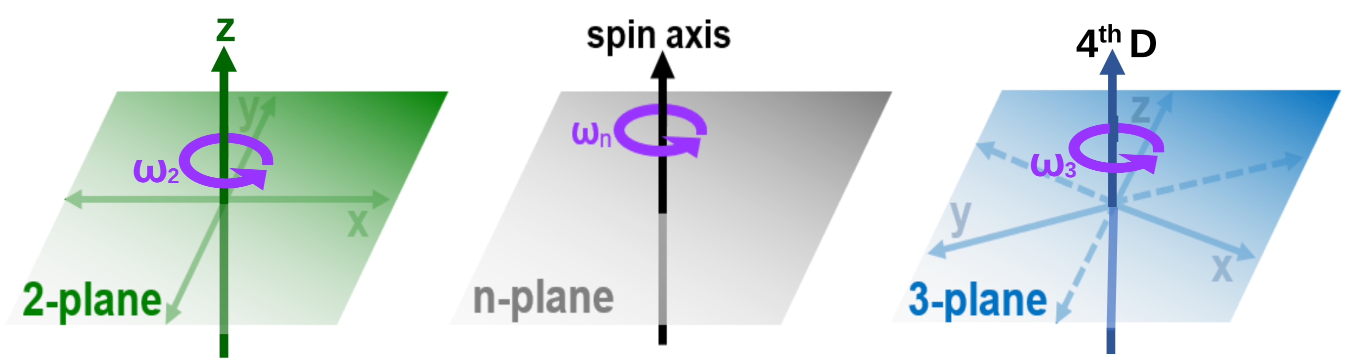


Fig. 3. An orthogonal spin axis makes no projections on a flat plane of any dimensionality.

Put Space in Its Place

To accommodate survival (e.g. bow hunting), our brains are hardwired for space and time, which have become default coordinates. But Minkowski spacetime has a hyperbolic geometry,[12] which is depicted with significant distortions.

"… the best we can do for figures in Minkowski space is to map them onto Euclidean space, … Such maps necessarily distort metric relations and one has to compensate for this distortion."[13]

Adapting the balloon analogy of cosmic expansion, to contain a central Big Bang event, gives rise to a *curved-space, radial-time* model.[14] Time emanates from the center as a 4D temporal field, enclosed at any radius by a 3-sphere, representing a spatial simultaneity in the rest frame of the cosmos. The radius corresponds to the age of the universe. All locations on a 3-sphere find Euclidean coordinates, with time normal to space and intervals tangent to it (Fig. 4).

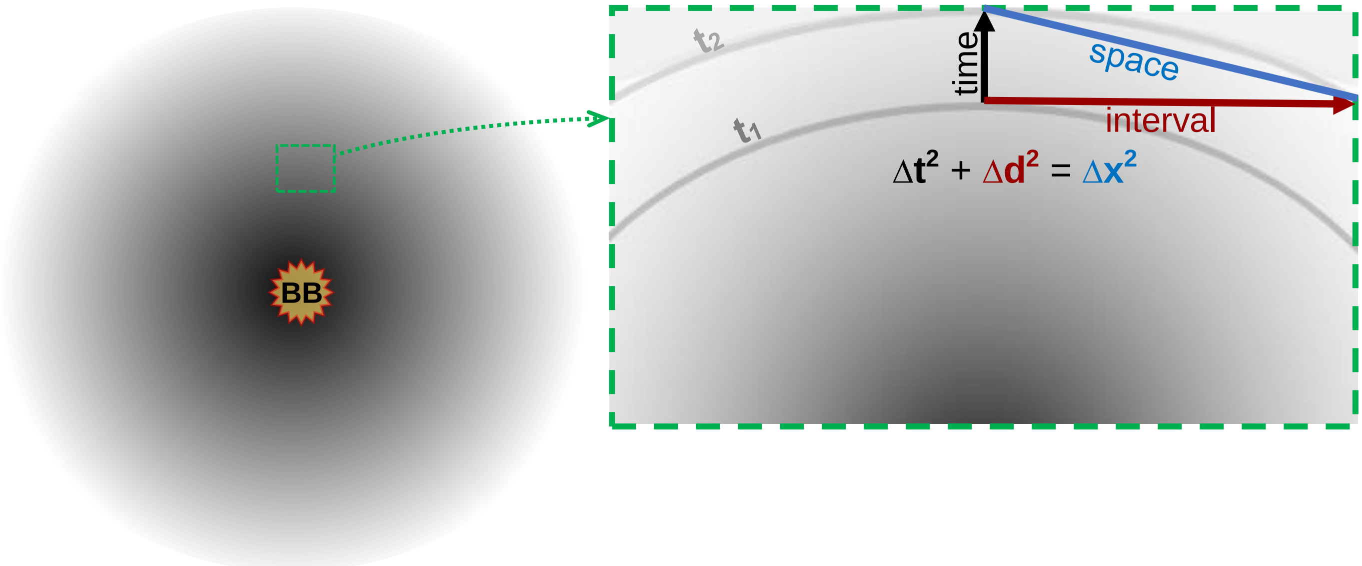


Fig. 4 Left: A temporal 4-field, centered on the Big Bang (BB) yields a curved-space, radial-time model. Right: A 2D slice of the indicated region, between earlier (t_1) and later (t_2) simultaneities, illustrates Euclidean, interval-time coordinates, allowing for local spatial flatness.

The Pythagorean theorem applies uniquely to Euclidean geometry. A spacelike convention provides interval formula[15]: $\Delta d^2 = \Delta x^2 - \Delta t^2$, which rearranges simply as $\Delta x^2 = \Delta d^2 + \Delta t^2$. This implies interval-time coordinates corresponding to the legs of a right triangle.

Time to Turn Things Around!

With that Euclidean lens, σ can be seen as *chronaxial spin*, in an *interval 3-plane*. Intrinsic spin is *classical* spin about a *non-classical* axis, time. More generally, it is spin about a particle's worldline which, in its rest frame, is its *timeline*. No longer a coordinate, space instead arcs past σ like an umbrella over its handle (Fig. 5). A field of spin components thus projects equally in *every* spatial direction, consistent with an underlying *curved-space, radial-time* structure.

Relativity makes a 4th dimension of spin axes unsurprising. All fermions age (e.g. muons decay) so, time undeniably supports *translation*. There is thus, no basis to deny that time also supports *rotation*. An objection might be that chronaxial spin is effectively instantaneous, easily developing circumferential speeds exceeding universal limit c. However, a fermion "point particle" of zero radius invokes no such restriction. In fact, instantaneous chronaxial spin may be the origin of quantum indeterminism.

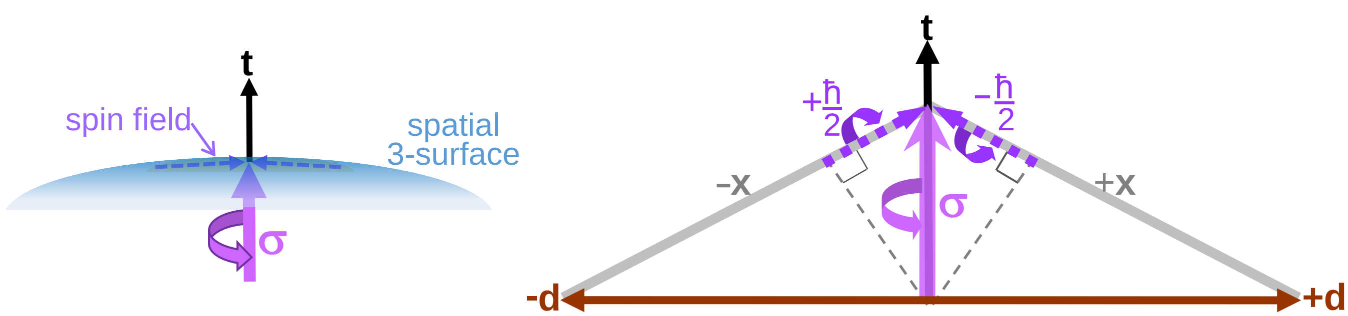


Fig. 5. Left: Arching past time, space exhibits a symmetric field of spin projections in all directions from the primary *chronaxial* spin vector (σ). Right: For clarity, a 2D slice in Euclidean interval-time coordinates shows spatial arc (±**x**) locally flat and highly inclined. σ projects symmetric $\pm\hbar/2$ components on space.

Solid Reasoning

Just as *conservation of mass-energy* divides to approximate laws for mass and energy, *conservation of angular momentum* divides into *approximate* laws (barring interchanges), distinguished by the dimension n of the plane of rotation. Time is excluded from any *n-plane* of rotation since its unidirectionality denies the required oscillatory freedom. The three approximate laws are:

1. *Conservation of **Linear**-Angular Momentum* - vibration, simple harmonic motion. $\boldsymbol{\omega_1 = 0\pi f}$
2. *Conservation of **Planar**-Angular Momentum* - classical spin & orbits. $\boldsymbol{\omega_2 = 2\pi f}$
3. *Conservation of **Solid**-Angular Momentum* - quantum spin & *orbitals* (both chronaxial). $\boldsymbol{\omega_3 = 4\pi f}$

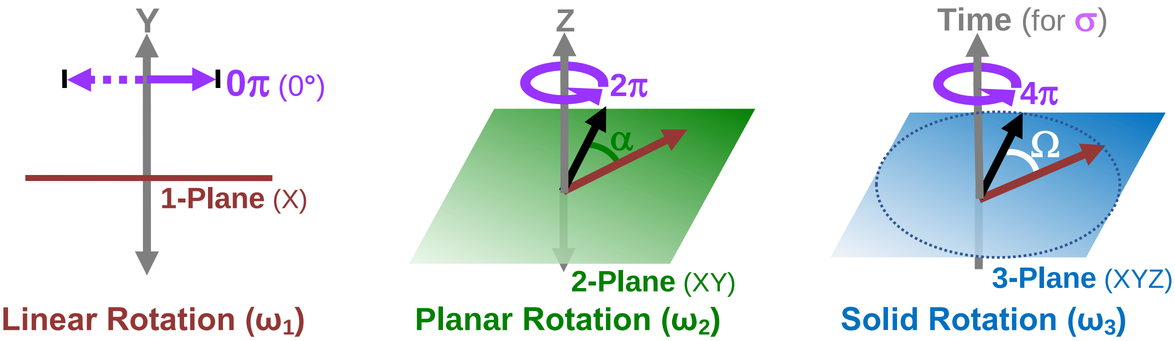
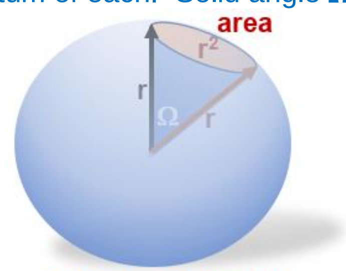


Fig. 6. Dimensional Spin Progression: Each rotation occurs in a flat n-plane about an orthogonal axis. Angular velocity ($\boldsymbol{\omega_n}$) relates to the approximately-conserved, angular momentum of each. Solid angle **Ω** appears circular in a 3-plane (above right) and within a sphere (below).



Solid Angle (Ω = 1sr)

Planar rotation entails 2π radians. Going up one dimension, chronaxial spin may be depicted in a 3-plane about a timeline, where a sphere's volume is flatly exposed (Fig. 6). This entails a *solid angle* of **4π** steradians (sr).[16]

Easy as Pi

A "half quantum" is an oxymoron because a "quantum" is *"the minimum amount of any physical entity involved in an interaction."*[17] Yet "spin½" implies such a halving, arising from inadequate *reduction* of the Planck constant (h).

"In applications where it is natural to use the angular frequency (i.e. …in terms of radians per second…) it is often useful to absorb a factor of 2π into the Planck constant…called the reduced Planck constant …equal to the Planck constant divided by 2π, and is denoted ħ (pronounced 'h-bar')"[18]

Division by 2π is fine for *classical* rotation, but there is *no basis* to apply that to quantum spin. Solid-angular, *chronaxial* spin must instead be reduced by 4π. Applied to fermions, spin is not "½" but quite *whole* at h/4π (i.e. ħ/2), exactly as measured (Fig. 7). Further, QED rightly boasts 12 digits of precision for the electron magnetic moment, but mysteriously remains off by a factor of two!

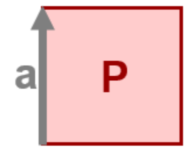
"... one famous triumph of the Quantum Electrodynamics theory is the accurate prediction of the electron g-factor. The magnetic moment of an electron is approximately twice what it should be in classical mechanics. The factor of two [g_S] implies that the electron appears to be twice as effective in producing a magnetic moment as the corresponding classical charged body. …a correction term [a_e]… takes account of …interaction…with the magnetic field"[19]

$$\mu_S = -\frac{g_S \mu_B S}{\hbar} = -\frac{\cancel{g_S}\, \mu_B S}{\dfrac{h}{\cancel{2\pi}}} \quad \leftarrow 4\pi$$

Fig. 7 Denominator ħ is only *half* reduced, as h/2π. Correcting with *solid-angular* range 4π is equivalent to having a factor of 2 in the numerator. Thus, g_S does not mysteriously need to be *"twice"* the classical g-factor g_L.[20] The anomalous magnetic moment (a_e)[21] is then accommodated at *half* the conventional value in: $g_S = 1 + a_e = 1.001159652181643$.

Both **S** (electron spin angular momentum) and $\boldsymbol{\mu}_B$ (Bohr magneton)[23] incorporate ħ/2 which, in that form, is fully reduced (i.e. h/4π). Sufficiently reducing denominator ħ as well makes *fudge factor* g_s obsolete.

Probable Cause



Two related mysteries of fermion spin remain.

1. While any two spin components have equal magnitude, their signs (±) vary, correlating probabilistically with the angle separating them. An essential, but so far *abstract*, "probability amplitude" (a) strangely appears to be the *square root* of that probability (P).

"The probability of an event is represented by the square of an arrow [probability amplitude]."[22]

"The [probability] amplitude arrows are fundamental to the description of the world given by quantum theory. No satisfactory reason has been given for why they are needed."[23]

"There have been many attempts to derive the Born rule from the other assumptions of quantum mechanics, with inconclusive results. ... probability is equal to the amplitude-squared"[24]

"These [probability amplitudes] are extremely abstract, and it is not at all obvious what their physical significance is."[25]

2. The probability amplitude sees a 720° horizon. In this case, 720° relates to 4π *radians* of arc length, rather than the 4π *steradians* noted earlier for solid angles (another reason to reduce h by 4π).

"[The] physical effects of the difference between the rotation of a spin-½ particle by 360° as compared with 720° have been experimentally observed in classic experiments in neutron interferometry."[26]

Experimentally, the sign of a prepared spin will correlate with that of a subsequently-measured spin, at angle θ, with probability (P) such that: $P = \cos^2(\theta/2)$. If amplitude (a) has a physical realization, it will be confined in a boundary condition from which to generalize (Fig. 8).

For example, amplitude a *must* coincide with the prepared spin when the subsequently measured spin has the same axis (i.e. $\theta = 0$). More generally, as the *half-angle* specification suggests, a occurs on the angle *bisector*. Instead of amplitude as a *square root* of probability, it is better viewed as a probability in its own right, the projection of 100% self-correlation onto the bisector, i.e. $a = \mathbf{1}\cos(\theta/2)$. That value is in turn, projected onto the subsequently-measured component, which results in the observed correlation probability: $P = \mathbf{1}\cos(\theta/2)\cos(\theta/2) = \cos^2(\theta/2)$. Thus, the Born rule is born.

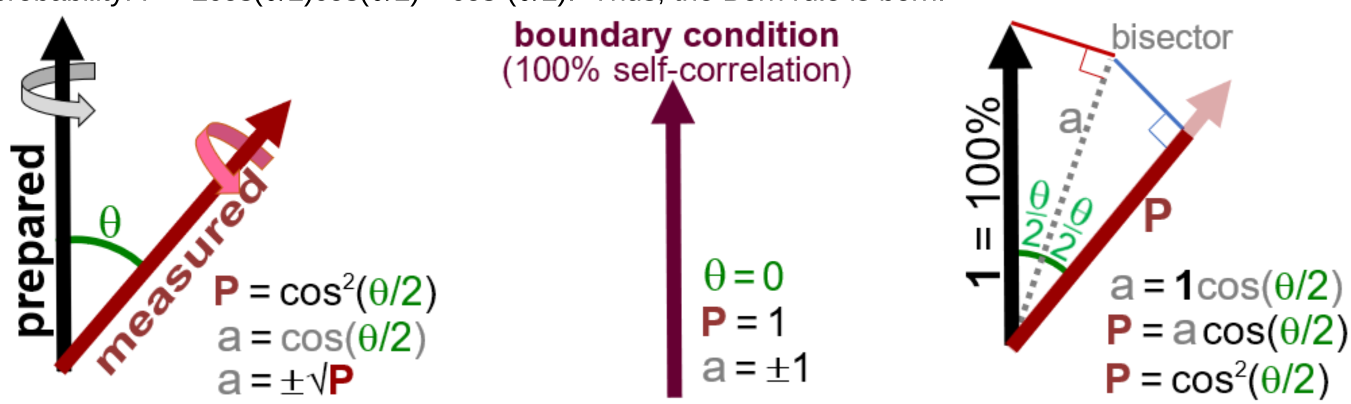


Fig. 8. Left: The half-angle correlation, of prepared and subsequently-measured spin components, entails a 720° range for a yet *unidentified* "probability amplitude" (a). Center: Amplitude a is pinned down in the boundary condition of **100%** self-correlation. Right: a is that spin probability **1**, projected on the bisector. It is the *unmeasured* correlation probability of the bisector. Probability **P** is in turn, the projection of a on the subsequently-*measured* axis.

Being on the angle bisector, probability amplitude a has the mysterious property of existing, but *never* directly measurable. To do so would make it the subsequently-measured component, which immediately redefines the bisector. The amplitude is thus always out of reach, as is the half-way point of Zeno's dichotomy paradox.[27]

With equal magnitude spin components as sides, the triangle they describe is isosceles. Its altitude (on the bisector) is the probability amplitude. Both change sign when angle θ crosses becomes convex (excedes180°). At 360° the value is -1, which continues back to +1 at 720° (Fig. 9).

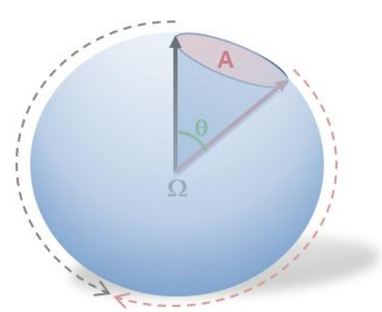


Fig. 9 An equator of solid angle Ω is described by θ, for which altitude turns negative after 180° (i.e. $\cos(\theta/2)$).

Simple as Riding a Bi-Cycle

The Stern-Gerlach device is known for sorting a beam of spin½ particles in two rather than a continuous spread. Convention suggests that observation itself inexplicably constrains the spin axis to the *one* being observed. By contrast, the present model projects a spin field in *every* spatial direction at once (Fig. 10).

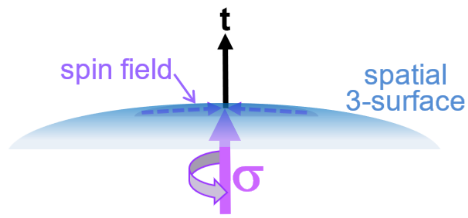


Fig. 10 A 3-field entails 4π thus, invoking the half angle formula.

An objection might be that for a given axis (say x), the resulting forward (+x) and rearward (-x) magnetic moments should cancel. But fermions see a 720° (4π) horizon, so the spin projection observed 180° from +x is that of the half angle (90°), which has no effect. Further, at 360° the half angle provides the *negative* of the expected "pointing away" polarity thus, *doubling* rather than cancelling all observed magnetic moments. This is equivalent to accommodating 4π as in Fig. 7.

"The magnetic moment of an electron is approximately twice what it should be…"[19]

Having previously dealt with bosons,[14] interval-time coordinates here provide a Euclidean lens which clearly reveals the fermion spin axis and 3-plane of rotation. With that, 4π is unavoidable, whether as 4π *steradians* of solid angular rotation, the combined ranges (2π each) of two spherical coordinates or the 4π *radians* of probability amplitude. Though awkward at first, it becomes second nature with practice, like riding a bike.

Bohr stated, *"...however far [quantum] phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms."*[28]

Quantum spin is ***classical*** spin about a ***non-classical*** axis. It is ***chronaxial spin***.

Wheeler said, *"Behind it all is surely an idea so simple, so beautiful, that when we grasp it - in a decade, a century, or a millennium - we will all say to each other, how could it have been otherwise? How could we have been so stupid for so long?"*[29]

We don't have to be *"stupid"* about quantum spin anymore.

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Related Animations - by the author (as "Faradave"), in sequence

Getting Coordinated (the world via Euclidean Lens)
Contact Sport (*c*-ing the light)
The "Hole" Shebang (fishing for wormholes)
Applying a Light Touch (unmasking photons)
What Time Is It? (leaving Lorentz transforms)
3-Ring Circus (dark energy and speed c):
Spin½ "Plane" & Simple (classically modeled)
Riding a Bi-Cycle (720° rotation)
Probable Cause (probability amplitude)
A Noether Round (new symmetry)