Bootstrapping some structure of the Standard Model via Supersymmetry

Alejandro Rivero*

July 17, 2017

Abstract

Aiming to select special sections of the string landscape, we suggest a simple self-referencing postulate in the Chan-Paton charges that fixes some of the freedom of model building.

This is a draft to clarify this idea and eventually incorporate it to the literature in some letter journal; The initial proposal is ten years old, so probably I am assuming a lot of details not written, and writing a lot of details too trivial for readers so any feedback is specially welcome

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Bootstrapping, as it is known, refers to the possibility of determining some parameters -usually of an scattering process- via some set of consistency conditions and a self-referential mechanism -usually a duality-. This was expected to generate some non-linearity pinpointing special places of the space of parameters. Back in 1970, such program was still considered unable to pinpoint the quantum numbers of a model, as Chew himself remarked in an article in Physics Today: *The unfriendly question raised most often ... is how self-consistency can possibly be expected to generate "internal quantum numbers" such as hypercharge and baryon number..., but how can you bootstrap a symmetry? A conceivable response is that symmetries (or the associated quantum numbers) are related to particle multiplicities, and the non-linear unitarity condition responds to the number of different particles.*

Nowadays the progress on the analysis of anomalies makes us more knowledgeable: we know that we need complete generations of the model, that there is a restriction in the total charge, and that if we want to use more sophisticated model, as in string theory, we are limited in our options to choose a gauge group. Today it is more likely to fix a gauge group via consistency conditions than to bootstrap mass relations or coupling constants, but even here the path has been barred: from the bottom we are puzzled by the number of generations, and from the top by a landscape of possible options.

In this note we want to point out a possible principle, in the frame of open superstrings, with a self-referential argument not far from the old idea of nuclear democracy, where all the particles were considered equally elementary and composite. Here is is going to be enough to demand all the bosonic particles to be composite, in the sense of having a Chan-Paton group, and in a way such that the labeling references to the same model.

Lets start by noticing that an unoriented bosonic string with SO(10) Chan-Paton labels contains a precursor of the susy scalars of the standard model, already with three generations. The 54 of SO(10) goes down to *flavour* $SU(5) \times U_1(1)$

$$54 = 15(4) + \bar{15}(-4) + 24(0)$$

And each representation goes down to (flavour) $SU(3) \times SU(2) \times U_2(1)$

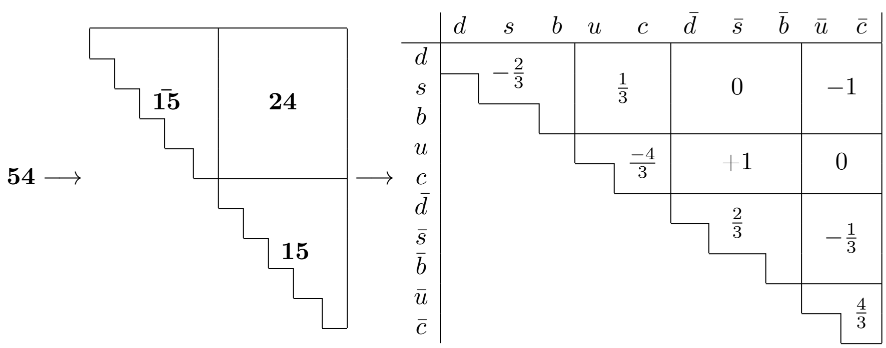
$$\begin{aligned} 15 = & \quad (1,3)(-6) + (3,2)(-1) + (6,1)(4) \\ 24 = & \quad (1,1)(0) + (1,3)(0) + (3,2)(5) + (\bar{3},2)(-5) + (8,1)(0) \end{aligned}$$

*arivero@unizar.es

In this model we look only to electric charge. Setting the two $U(1)$ charges to $Q_1 + Q_2 = -\frac{1}{6}$, $Q_2 = -1/5$ we can produce a total combination Q as in the following table. Note that N here is the number of states.

irrep	N	Y_1	Y_2	$Q = \frac{1}{30}Y_1 - \frac{1}{5}Y_2$
$(6,1)$	6	4	4	$-2/3$
$(3,2)$	6	4	-1	$+1/3$
$(1,3)$	3	4	-6	$+4/3$
$(\bar{6},1)$	6	-4	-4	$+2/3$
$(\bar{3},2)$	6	-4	1	$-1/3$
$(1,\bar{3})$	3	-4	6	$-4/3$
$(\bar{3},2)$	6	0	-5	$+1$
$(3,2)$	6	0	5	-1
$(1,1)$				
$(1,3)$	12	0	0	0
$(8,1)$				

The table contains, as promised, three generations of scalars with the right electric charge; plus three "half-generations" of an extra quark[1]. To propose an identification for the components of the 10 of $SO(10)$ we can also branch the 10-plet of the group first to $SU(5)$, with $10 = 5(2) + \bar{5}(-2)$ and then to $SU(3) \times SU(2)$, via $5 = (3,1)(2) + (1,2)(-3)$. We see that the 10-plet has three elements of $Q = \frac{2}{30} - \frac{2}{5} = -\frac{1}{3}$, two elements of $Q = \frac{2}{30} + \frac{3}{5} = \frac{2}{3}$ and then the corresponding opposite charges. Abusing a bit on traditional notation, we can call d, s, b to the elements of the triplet and u, c to the doublet, and draw the whole construction as [in figure]. It can be said that the bosons are actually being generated from pairing "quarks" or "antiquarks".



Such bosons as they stand are decolorated, and sort of deconstructed and chiral-less. We do not plan to address this latter problem, but we will speak about recovering the colour later. First we want to address the question of if this structure is boostrappable, this is, if it is possible to single it out from some postulates.

We need to use both these "quarks" and the opposite "antiquarks" as labels of the Chan-Paton group, and to request unorientability to produce the 54 states (in principle, we discard the singlet) that reproduce a theory of mesons, quarks and diquarks. Of course we could then again discard the unoriented states to reduce to a theory of mesons with SU(5) group, which in this case would be the three generations of scalar leptons.

Looking for a postulate for this structure, the idea that we propose is to assume that there exists a supersymmetric theory containing this same set of scalar states, and that there is a nuclear democracy in the labeling: the Chan-Paton charges of this theory are a subset of the quarks. We can call them the light quarks or, indistinctly, the preons of the theory.

So, consider r quarks of $+2/3$ charge and s quarks of charge $-1/3$. Our postulate is that they combine pairwise -say, at the extremes of an open string- to form consistent generations of squarks and sleptons.

We can formulate this requisite in two ways:

- we can ask that the combination must build the same number of "up type" and "down type" diquarks. This is, that $rs = s(s+1)/2 = 2n$ and so $s = 2r - 1$. Plus, we can ask r to be even, to

[1] it could be interesting to set also the Q of this extra (1,3) to zero, but in order to do this we would need to grant some baryon number to (6,1) and (3,2)

make sure we build a pair number of scalars of each charge (we want to be able to promote them to a supersymmetric theory in the future).

- or, we can go for a more general condition, that we will see implies the former one: we can ask that the total number of combinations must be an integer multiple of a single set, this is a multiple of rs. In this case we look for integer positive solutions of:

$$rs + \frac{s^2+s}{2} + \frac{r^2+r}{2} + (r+s)^2 - 1 = Krs$$

and then $K = 9$. This in turns implies that one generation is composed of $(K-1)/2$ Dirac-like tuples -including the neutrino-, plus one extra state, with its antiparticle. This extra state can be either neutral or charged with $+4/3$.

In any case, either requirement fixes $s = 2r - 1$ and then the allowed groups are $SO(2s+2r) = SO(6r-2)$, with r even. The smallest possible group is SO(10), but bigger groups are possible.

We need an extra postulate to fix $n_g = 3$. To look for it, lets examine a table of solutions, with the corresponding total of "standard model generations" of scalars and its extra content, that we see adds always to n_g. so one extra scalar in each generation.

r	s	n_g	$\frac{r(r+1)}{2}$	$\frac{r^2+s^2-1}{2} - 2n_g$
2	3	3	3	0
4	7	14	10	4
6	11	33	21	12
8	15	60	36	24
...				

From the table, it seems that the simplest extra postulate is to ask for absence of excess neutral bosons. An equivalent requisite is to ask that the number of neutral bosons to be equal to the number of charged ones, given that the later is always $2rs$ as in the "quark" sector.

As for colour; it can be recovered if one considers to assign a SU(3) singlet to the 24 of SU(5) and SU(3) triplets and anti-triplets to the 15 and $\bar{1}5$. The decomposition should then be promoted to something under $SO(10) \otimes SU(3)$ such as

$$? \longrightarrow (\mathbf{15}, \mathbf{3}) + (\bar{\mathbf{1}}\mathbf{5}, \bar{\mathbf{3}}) + (\mathbf{24}, \mathbf{1})$$

In the sense of being strings, and thus related to a large $SU(N)$ limit, all the states are colored; the triplet or singlet state being associated to the planarity of the string. The pairing particle/antiparticle is used to recover an oriented sector. What we need is some argument to pinpoint exactly $SU(3)$, either as a preferred labeling or extracting it out of a $SU(3+N)$ in large N.

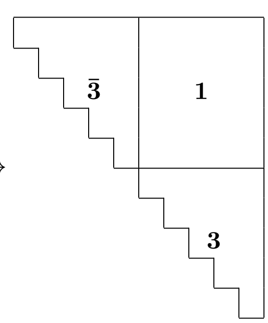
We argue that a bootstrap of $N = 3$ is derived from the fact that

$$\mathbf{3} \times \mathbf{3} = \mathbf{6} + \mathbf{3}$$

so in some sense a composite 3 is composed of itself. The mesonic strings, on the other hand, go to the singlet from

$$\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{8} + \mathbf{1}$$

and the colouring paints the matrix as desired:

$? \longrightarrow$ 

After adding colour, it can be seen more clearly the process

$$\textit{oriented} \rightarrow \textit{unoriented} \rightarrow \textit{reoriented}$$

that allow to extract out a set of mesons with Chan-Paton flavour charges under $SU(r+s)$. This technique is used by Armori et al under the market name "planar orientifold" for the calculation of meson masses in a large N theory generating an "Hadronic Susy". Note that the diquark, unoriented part, can not survive in the large N, making already a sort of confinement.

Amusingly, colouring the scheme has expanded the original 10-plet to a 30-plet, ans so from $SO(10)$ to $SO(30)$, and we are really only one "uncolored preon" out of reaching $SO(32)$.

Even if we had no results about consistency of gauge theories in strings, we could have considered, by symmetry reasons, to leave the colour boxes to overflow the diagonal by adding singlets, this is, to consider $U(3)$ instead of $SU(3)$ and the $\mathbf{54}+\mathbf{1}$ of $SO(10)$ so that the decomposition now should be something as

$$\begin{aligned}(\mathbf{15},\mathbf{3})+(\mathbf{15},\mathbf{6})+(\bar{\mathbf{15}},\bar{\mathbf{3}})+(\bar{\mathbf{15}},\mathbf{6})+(\mathbf{24},\mathbf{8})+(\mathbf{24},\mathbf{1})+(\mathbf{1},\mathbf{8})+(\mathbf{1},\mathbf{1})\\ =45+90+45+90+192+24+8+1\\ =495\end{aligned}$$

So we see that a super-bootstrapped three family structure supplemented with three colours has about the right size to potentially reach the Green-Schwarz cancellation. Note that honestly we are still in the world of the bosonic string; we can take this number as sheer coincidence or as signal that the "sBoostrap" postulate actually has some susy in it. In any case, it isolates a pathway for model building with exactly three generations.

Actually the sum is a bit of luck; a more accurate decomposition, for instance, is to branch out from $SO(32)$ to $SU(5)\times SU(3)$ via say $SO(30)$ and $SU(15)$. Such pathway decomposes:

$$\begin{aligned}\mathbf{496}=(\mathbf{435})_{\mathbf{0}}\oplus(\mathbf{30})_{\mathbf{2}}\oplus(\mathbf{30})_{-\mathbf{2}}\oplus(\mathbf{1})_{\mathbf{0}}\\ =(\mathbf{224})_{\mathbf{0},\mathbf{0}}\oplus(\mathbf{105})_{\mathbf{0},\mathbf{4}}\oplus(\mathbf{105})_{\mathbf{0},-\mathbf{4}}\oplus(\mathbf{1})_{\mathbf{0},\mathbf{0}}\oplus(\mathbf{15})_{\mathbf{2},\mathbf{2}}\oplus(\mathbf{15})_{\mathbf{2},-\mathbf{2}}\oplus(\mathbf{15})_{-\mathbf{2},\mathbf{2}}\oplus(\mathbf{15})_{-\mathbf{2},-\mathbf{2}}\\ =(\mathbf{24},\mathbf{8})_{\mathbf{0},\mathbf{0}}\oplus(\mathbf{24},\mathbf{1})_{\mathbf{0},\mathbf{0}}\oplus(\mathbf{1},\mathbf{8})_{\mathbf{0},\mathbf{0}}\oplus\\ (\mathbf{15},\mathbf{3})_{\mathbf{0},\mathbf{4}}\oplus(\mathbf{10},\mathbf{6})_{\mathbf{0},\mathbf{4}}\oplus(\mathbf{15},\mathbf{3})_{\mathbf{0},-\mathbf{4}}\oplus(\mathbf{10},\mathbf{6})_{\mathbf{0},-\mathbf{4}}\oplus(\mathbf{1})_{\mathbf{0},\mathbf{0}}\oplus\\ (\mathbf{5},\mathbf{3})_{\mathbf{2},\mathbf{2}}\oplus(\mathbf{5},\mathbf{3})_{\mathbf{2},-\mathbf{2}}\oplus(\mathbf{5},\mathbf{3})_{-\mathbf{2},\mathbf{2}}\oplus(\mathbf{5},\mathbf{3})_{-\mathbf{2},-\mathbf{2}}\end{aligned}$$

In the current context, as we are not including the weak force in the model, there is no value on locating a concrete descent. Also, we ignore if there is some more direct way to relate $SO(10)$ to $SO(32)=SO(2^{D/2})$; is is a bit intriguing that adding colour seems to be enough.

In conclusion, lets review what we have got. We have started from the postulate that a bosonic unoriented string could be constructed such that:

- The "preons" labeling the string are of two kinds according electric charge: "up" and "down".
- In the diquark sector, the number of "up" diquarks is equal to the number of "down" diquarks.
- In the meson sector, the number of charged mesons is equal to the number of neutral mesons

We call this arrangement "a supersymmetrical bootstrap" because in a field theory of quarks and mesons the preons should be a subset of the quarks of the theory, assuming that such quarks are recovered via susy from the bosonic sector.

The main conclusion is that there is a consistent assignment of charges, and the only possible number of generations is three.

Then we add colouring compatible with the construction: a meson must be a singlet of colour, a diquark must be in the same representation of colour than a (anti-)quark. This isolates $SU(3)$ and strongly signals that the full group must be $SO(32)$

The complete program to build such model should imply to start from this bosonic "hadron-diquark" string model, upgrade to superstrings then going to D=10, and then analyze the branes corresponding to Chan-Paton matrices. Note that it would still be true that removing coloured sectors the hadronic mesons of the standard model are recovered, so as a minimum the approach meets the goals of the original program of 1970!

It is interesting that the number of quarks actually involved in the construction is one less than the number of quarks obtained in the process, and that the one odd out is of "up" type. So it could be expected that the process of building the model allows, or even forces, a mass for the top quark separated from the other five.

The following pages are a revised version of the previous note, to be emailed to some friends and colleagues.

Bootstrapping charges via Supersymmetry

Alejandro Rivero*

July 27, 2017

Abstract

A simple self-referencing postulate in the Chan-Paton factors of a string allows to fix some of the freedom in the landscape of standard-model-like models.

The view from above

Consider $SO(32)$. We can factor it as flavour times colour, $G^F \times SU(3)$, following the procedure of [2] where it is listed as case 4, equation 2.18:

$$
\begin{aligned}
(32 \times 32)_A = [1,1,1^c] + (\mathbf{1},\mathbf{24},\mathbf{1^c}) + (1,1,1^c) + (1,1,8^c) + (2,5,3^c) + (2,\bar{5},\bar{3}^c) \\
+[\mathbf{1},\mathbf{15},\mathbf{\bar{3}^c}] + [\mathbf{1},\mathbf{\bar{15}},\mathbf{3^c}] + [1,10,6^c] + [1,\bar{1}0,\bar{6}^c]
\end{aligned}
$$

where the derived flavour group is $G^F = SO(2) \times U(5) \times U(1)$. In fact the same structure plus an extra $(1, 28, 8^c)$ can be derived branching down via

$$SO(32) \supset SO(30) \times U(1) \supset SU(15) \times U(1) \times U(1) \supset SU(5) \times SU(3) \times U(1) \times U(1)$$

from where, following the branching rules in [4] we get

$$
\begin{aligned}
\mathbf{496} = (24,8^c)_{0,0} \oplus (1,1^c)_{0,0} \oplus (\mathbf{24},\mathbf{1^c})_{\mathbf{0},\mathbf{0}} \oplus (1,1^c)_{0,0} \\
\oplus(1,8^c)_{0,0} \oplus (5,3^c)_{2,2} \oplus (5,3^c)_{2,-2} \oplus (5,3^c)_{-2,2} \oplus (5,3^c)_{-2,-2} \\
\oplus(\mathbf{15},\mathbf{3^c})_{\mathbf{0},\mathbf{4}} \oplus \oplus(\mathbf{15},\mathbf{3^c})_{\mathbf{0},-\mathbf{4}} \oplus (10,6^c)_{0,4} \oplus (10,6^c)_{0,-4}
\end{aligned}
$$

In this case a second $U(1)$ from the flavour part is explicitly raided out. Still another $U(1)$ charge can be got by further breaking to $SU(5) \supset SU(3) \times SU(2) \times U(1)$. Then the **24** descend as

$$24 = (1,1)(0) + (1,3)(0) + (3,2)(5) + (\bar{3},2)(-5) + (8,1)(0)$$

and with $Q = \frac{1}{5}Y$ it looks as three generations of scalar leptons. Furthermore, the **15** descends as

$$15 = (1,3)(-6) + (3,2)(-1) + (6,1)(4)$$

and then extending the assignment of the $U(1)$ charges the total $\mathbf{15} + \mathbf{\bar{15}} + \mathbf{24}$ can be interpreted as three generations of scalar quarks and leptons:

irrep	N	Y_1	Y_2	$Q = \frac{1}{30}Y_1 - \frac{1}{5}Y_2$
$(6,1)$	6	4	4	$-2/3$
$(3,2)$	6	4	-1	$+1/3$
$(1,3)$	3	4	-6	$+4/3$
$(\bar{6},1)$	6	-4	-4	$+2/3$
$(\bar{3},2)$	6	-4	1	$-1/3$
$(1,\bar{3})$	3	-4	6	$-4/3$
$(\bar{3},2)$	6	0	-5	$+1$
$(3,2)$	6	0	5	-1
$(1,1)$				
$(1,3)$	12	0	0	0
$(8,1)$				

*arivero@unizar.es

In this sense, this is possibly the most straightforward way to obtain three generations of standard model colour and electric charge out of a string motivated group. This is probably left unnoticed because one wants to get also the chiral charge, and then one looks for spinors in complex representations by exploring groups $SO(4n+2)$, which excludes $SO(32)$.

Isolated, 15+15+24 can be considered as a 54 of SO(10) or a 54+1 in Sp(10) or O(10). Such representations are occasionally considered to organize "hidden mesons", for example recently in [1, 4] but the charge assignment in such cases is aimed to obtain colour and weak charges as usual.

In our case the table is interpreted three generations of scalars with the right electric charge; plus three "half-generations" of an extra quark[1]. To propose an identification for the components of the 10 of $SO(10)$ we can also branch the 10-plet of the SO(10) group first to $SU(5)$, with $10 = 5(2) + \bar{5}(-2)$ and then to $SU(3) \times SU(2)$, via $5 = (3,1)(2) + (1,2)(-3)$. We see that the 10-plet has three elements of $Q = \frac{2}{30} - \frac{2}{5} = -\frac{1}{3}$, two elements of $Q = \frac{2}{30} + \frac{3}{5} = \frac{2}{3}$ and then the corresponding opposite charges. Abusing a bit on traditional notation, we can call d, s, b to the elements of the triplet and u, c to the doublet, and draw the whole construction as [in figure]. It can be said that the bosons are actually being generated from pairing "quarks" or "antiquarks".

The view from below

The goal of this paper is to point out a possible principle, in this frame of open superstrings, to bootstrap this particular group and charge from a self-referential argument.

Lets consider here the $54 = \mathbf{15} + \mathbf{\bar{15}} + \mathbf{24}$ as produced by an unoriented bosonic string with SO(10) Chan-Paton labels, and in a first step we delegate the function of colour to the string itself. We request unorientability to produce all the the 54 states forming a set of mesons, quarks and diquarks. Of course we could then again discard the unoriented states to reduce to a theory of mesons with SU(5) group, which in this case would be the three generations of scalar leptons.

Looking for a postulate for this structure, the idea that we propose is to assume that there exists a supersymmetric theory containing this same set of scalar states, and that there is a nuclear democracy in the labeling: the Chan-Paton charges of this theory are a subset of the quarks. We can call them the light quarks or, indistinctly, the preons of the theory.

So, consider r quarks of $+2/3$ charge and s quarks of charge $-1/3$. Our postulate is that they combine pairwise -at the extremes of an open string- to form consistent generations of squarks and sleptons.

We can formulate this requisite in two ways:

- we can ask that the combination must build the same number of "up type" and "down type" diquarks. This is, that

$$rs = s(s+1)/2 = 2n$$

and so $s = 2r - 1$. Plus, we can ask r to be even, to make sure we build a pair number of scalars of each charge (we want to be able to promote them to a supersymmetric theory in the future).

- or, we can go for a more general condition, that we will see implies the former one: we can ask that the total number of combinations must be an integer multiple of a single set, this is a multiple of rs. In this case we look for integer positive solutions of:

$$rs + \frac{s^2+s}{2} + \frac{r^2+r}{2} + (r+s)^2 - 1 = Krs$$

and then $K = 9$. This in turns implies that one generation is composed of $(K-1)/2$ Dirac-like tuples -including the neutrino-, plus one extra state, with its antiparticle. This extra state can be either neutral or charged with $+4/3$.

In any case, either requirement fixes $s = 2r - 1$ and then the allowed groups are $SO(2s + 2r) = SO(6r - 2)$, with r even. The smallest possible group is SO(10), but bigger groups are possible.

We need an extra postulate to fix $n_g = 3$. To look for it, lets examine a table of solutions, with the corresponding total of "standard model generations" of scalars and its extra content, that we see adds always to n_g. so one extra scalar in each generation.

[1] it could be interesting to set also the Q of this extra (1,3) to zero, but in order to do this we would need to grant some baryon number to (6,1) and (3,2)

r	s	n_g	$\frac{r(r+1)}{2}$	$\frac{r^2+s^2-1}{2} - 2n_g$
2	3	3	3	0
4	7	14	10	4
6	11	33	21	12
8	15	60	36	24
...				

From the table, it seems that the simplest extra postulate is to ask for absence of excess neutral bosons. An equivalent requisite is to ask that the number of neutral bosons to be equal to the number of charged ones, given that the later is always $2rs$ as in the "quark" sector.

With this extra requisite, the solution is unique:

$$r = 2, s = 3, n_g = 3, G = SO(10)$$

Lets now address colour. We already know that it can be recovered if one considers to assign a SU(3) singlet to the 24 of SU(5) and SU(3) triplets and anti-triplets to the 15 and $\bar{1}5$. The decomposition should then be promoted to something under $SO(10) \otimes SU(3)$ such as

$$? \longrightarrow (\mathbf{15}, \mathbf{3}) + (\bar{\mathbf{1}}\mathbf{5}, \bar{\mathbf{3}}) + (\mathbf{24}, \mathbf{1})$$

What we need is some argument to pinpoint exactly $SU(3)$, either as a preferred labeling or extracting it out of a $SU(3 + N)$ in large N.

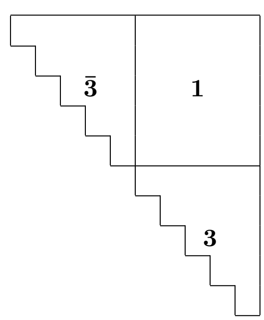
We can argue, again self-referencially, that $N = 3$ is derived from the fact that

$$\mathbf{3} \times \mathbf{3} = \mathbf{6} + \mathbf{3}$$

so in some sense a composite 3 is composed of itself. The mesonic strings, on the other hand, go to the singlet from

$$\mathbf{3} \times \bar{\mathbf{3}} = \mathbf{8} + \mathbf{1}$$

and the colouring paints a matrix



where it can be seen more the process

$$oriented \rightarrow unoriented \rightarrow reoriented$$

that allows to extract out a set of mesons with Chan-Paton flavour charges under $SU(r + s)$, the upper right box. This technique is used by Armori et al under the market name "planar orientifold" for the calculation of meson masses in a large N theory generating an "Hadronic Susy". Note that the diquark, unoriented part, can not survive in the large N, making already a sort of confinement.

From bottom to the top

Now we notice that colouring the scheme has expanded the solution 10-plet

$$dsb\ uc\ \bar{d}\bar{s}\bar{b}\ \bar{u}\bar{c}$$

to a 30-plet $d_{rgb}s_{rgb}$..., and so from $SO(10)$ to $SO(30)$, and we are really only one "uncolored preon" out of reaching $SO(32)$.

Even if we had no results about consistency of gauge theories in strings, we could have considered, by symmetry reasons, to leave the colour boxes to overflow the diagonal by adding singlets, this is, to consider $U(3)$ instead of $SU(3)$ and the $\mathbf{54}+\mathbf{1}$ of $SO(10)$ so that the full composition, colour+flavour, of $U(3)\times SO(10)$ should be something adding as

$$\begin{aligned}(\mathbf{15},\mathbf{3})+(\mathbf{15},\mathbf{6})+(\bar{\mathbf{15}},\bar{\mathbf{3}})+(\bar{\mathbf{15}},\mathbf{6})+(\mathbf{24},\mathbf{8})+(\mathbf{24},\mathbf{1})+(\mathbf{1},\mathbf{8})+(\mathbf{1},\mathbf{1})\\ =(15,9)+(\bar{15},9)+(25,9)\\ =(45+90)+(45+90)+(192+24+8+1)\\ =495\end{aligned}$$

So we see that our super-bootstrap of a three family structure when supplemented with three colours has about the right size to potentially reach the Green-Schwarz cancellation.

The chiral basis

$$d^L=d+\bar{d},d^R=d-\bar{d},....$$

(TO BE DONE)

Conclusion

In conclusion, lets review what we have got. We have started from the postulate that a bosonic unoriented string could be constructed such that:

- The "preons" labeling the string are of two kinds according electric charge: "up" and "down".
- In the diquark sector, the number of "up" diquarks is equal to the number of "down" diquarks.
- In the meson sector, the number of charged mesons is equal to the number of neutral mesons

We call this arrangement "a supersymmetrical bootstrap" because in a field theory of quarks and mesons the preons should be a subset of the quarks of the theory, assuming that such quarks are recovered via susy from the bosonic sector.

The main conclusion is that there is a consistent assignment of charges, and the only possible number of generations is three.

Then we add colouring compatible with the construction: a meson must be a singlet of colour, a diquark must be in the same representation of colour than a (anti-)quark. This isolates $SU(3)$ and strongly signals that the full group must be $SO(32)$

The complete program to build such model should imply to start from this bosonic "hadron-diquark" string model, upgrade to superstrings then going to D=10, and then analyze the branes corresponding to Chan-Paton matrices. Note that it would still be true that removing coloured sectors the hadronic mesons of the standard model are recovered, so as a minimum the approach meets the goals of the original program of 1970!

It is interesting that the number of quarks actually involved in the construction is one less than the number of quarks obtained in the process, and that the one odd out is of "up" type. So it could be expected that the process of building the model allows, or even forces, a mass for the top quark separated from the other five.

Bootstrapping, as it is known, refers to the possibility of determining some parameters -usually of an scattering process- via some set of consistency conditions and a self-referential mechanism -usually a duality-. This was expected to generate some non-linearity pinpointing special places of the space of parameters. Back in 1970, such program was still considered unable to pinpoint the quantum numbers of a model, as Chew himself remarked in an article in Physics Today: *The unfriendly question raised most often ... is how self-consistency can possibly be expected to generate "internal quantum numbers" such as hypercharge and baryon number..., but how can you bootstrap a symmetry? A conceivable response is that symmetries (or the associated quantum numbers) are related to particle multiplicities, and the non-linear unitarity condition responds to the number of different particles.* Nowadays the progress on the analysis of anomalies makes us more knowledgeable so that today it is more likely to fix the particle content of gauge group via consistency conditions. What we have exemplified here is that a combination of attacks from below and from above can allow us to select some particular vacuum out of the landscape of string, and then specific generations of standard model like families.

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The following three pages were a revised version of the previous note, to be incorporated in a blog entry

The view from above

Consider $SO(32)$. We can factor it as flavour times colour, $G^F \times SU(3)$, following GellMann 1976, case 4, equation 2.18.

$$(32 \times 32)_A = [1,1,1^c] + (\mathbf{1},\mathbf{24},\mathbf{1^c}) + (1,1,1^c) + (1,1,8^c) + (2,5,3^c) + (2,\bar{5},\bar{3}^c)$$
$$+[\mathbf{1},\mathbf{15},\bar{\mathbf{3}}^{\mathbf{c}}] + [\mathbf{1},\bar{\mathbf{15}},\mathbf{3^c}] + [1,10,6^c] + [1,\bar{1}0,\bar{6}^c]$$

where $G^F = SO(2) \times U(5) \times U(1)$. Or, if you prefer, branch down

$SO(32) \supset SO(30) \times U(1) \supset SU(15) \times U(1) \times U(1) \supset SU(5) \times SU(3) \times U(1) \times U(1)$

to get

$$\mathbf{496} = (24,8^c)_{0,0} \oplus (1,1^c)_{0,0} \oplus (\mathbf{24},\mathbf{1^c})_{\mathbf{0},\mathbf{0}} \oplus (1,1^c)_{0,0}$$
$$\oplus (1,8^c)_{0,0} \oplus (5,3^c)_{2,2} \oplus (5,3^c)_{2,2} \oplus (5,3^c)_{-2,2}(5,3^c)_{-2,2}$$
$$\oplus (\mathbf{15},\mathbf{3^c})_{\mathbf{0},\mathbf{4}} \oplus (\mathbf{15},\mathbf{3^c})_{\mathbf{0},\mathbf{4}} \oplus (10,6^c)_{0,4} \oplus (10,6^c)_{0,4}$$

Our interest, as we will explain in a moment is in the 15 and 24 multiplets. Breaking them further via $SU(5) \supset SU(3) \times SU(2) \times U_Y(1)$. Then the **24** descends to

$$24 = (1,1)_0 + (1,3)_0 + (3,2)_5 + (\bar{3},2)_5 + (8,1)_0$$

and with $Q = \frac{1}{5}Y$ it looks as three generations of scalar leptons. It starts to be interesting, do you agree?

Furthermore, the **15** descends as

$$15 = (1,3)_6 + (3,2)_1 + (6,1)_4$$

and then extending the assignment of the $U(1)$ charges the total $\mathbf{15} + \bar{\mathbf{15}} + \mathbf{24}$ can be interpreted as three generations of scalar quarks and leptons:

$irrep$	N	Y_1	Y_2	$Q = \frac{1}{30}Y_1 - \frac{1}{5}Y_2$
$(6,1)$	6	4	4	$-2/3$
$(3,2)$	6	4	-1	$+1/3$
$(1,3)$	3	4	-6	$+4/3$
$(\bar{6},1)$	6	-4	-4	$+2/3$
$(\bar{3},2)$	6	-4	1	$-1/3$
$(1,\bar{3})$	3	-4	6	$-4/3$
$(\bar{3},2)$	6	0	-5	$+1$
$(3,2)$	6	0	5	-1
$(1,1)$				
$(1,3)$	12	0	0	0
$(8,1)$				

The table is interpreted as three generations of scalars with the correct electric charge; plus three "half-generations" of a +4/3 object. it could be interesting to set also the Q of this extra (1,3) to zero, but in order to do this we would need to grant some baryon number to (6,1) and (3,2).

This is possibly the most straightforward way to obtain three generations of standard model colour and electric charge out of a string motivated group. I have never seen it mentionated in the literature, but I have never been very conversant with string literature. From GUT point of view, this also left unnoticed because one wants to get also the chiral charge, and then one looks for spinors in complex representations by exploring groups $SO(4n+2)$, which excludes $SO(32)$.

Isolated, 15+15+24 can be considered as a 54 of SO(10) or a 55 in Sp(10) or O(10). To propose an identification for the components of the 10 of $SO(10)$ we can also branch the 10-plet of the SO(10) group first to $SU(5)$, with $10 = 5_2 + \bar{5}_{-2}$ and then to $SU(3) \times SU(2)$, via $5 = (3,1)_2 + (1,2)_{-3}$. We see that the 10-plet has three elements with $Q = \frac{2}{30} - \frac{2}{5} = -\frac{1}{3}$, two elements with $Q = \frac{2}{30} + \frac{3}{5} = \frac{2}{3}$ and then the corresponding opposite charges. Abusing a bit on traditional notation -or perhaps not!-, we can call d, s, b to the elements of the triplet and u, c to the doublet, and draw the whole construction as [in figure]. It can be said that the bosons are actually being generated from pairing "quarks" or "antiquarks".

The view from below

To upgrade the above view to something similar to the standard model, we assume that there exists a supersymmetric theory containing this same set of scalar states, and that there is a nuclear democracy in the labeling: the "Chan-Paton charges" of this theory are a subset of the quarks.

We can call them the light quarks or, indistinctly, the preons of the theory. The model itself can have more quarks, even more generations, but only the "light ones" can bin into scalar quarks and scalar leptons.

So, consider r quarks of $+2/3$ charge and s quarks of charge $-1/3$. Our postulate is that they combine pairwise -at the extremes of an open string- to form consistent generations of squarks and sleptons.

We can formulate this requisite in two ways:

- we can ask that the combination must build the same number of "up type" and "down type" diquarks. This is, that

$$rs = s(s+1)/2 = 2n$$

and so $s = 2r - 1$. Plus, we can ask r to be even, to make sure we build a pair number of scalars of each charge (we want to be able to promote them to a supersymmetric theory in the future).

- or, we can go for a more general condition, that we will see implies the former one: we can ask that the total number of combinations must be an integer multiple of a single set, this is a multiple of rs. In this case we look for integer positive solutions of:

$$rs + \frac{s^2+s}{2} + \frac{r^2+r}{2} + (r+s)^2 - 1 = Krs$$

and then $K = 9$. This in turns implies that one generation is composed of $(K-1)/2$ Dirac-like tuples -including the neutrino-, plus one extra state, with its antiparticle. This extra state can be either neutral or charged with $+4/3$.

In any case, either requirement fixes $s = 2r - 1$ and then the allowed groups are $SO(2s + 2r) = SO(6r - 2)$, with r even. The smallest possible group is, as expected, SO(10), but bigger groups are possible.

When we need now is an extra postulate to fix $n_g = 3$

The simplest extra postulate is to ask for absence of excess neutral bosons.

With this extra requisite, the solution is unique:

$$r = 2, s = 3, n_g = 3$$

and $G = SO(10)$ or similar.

Now, we can try to add $SU(3)$ colour and in this path we scale up the group size. We notice that a full representation of $O(10) \times U(3)$ would have a size $55 \times 9 = 495$ and then we suspect we can get the needed representations by breaking some group of about this number of components. Immediate candidates are $SO(32)$ and $E_8 \times E_8$. We have got a good match to the non-chiral quantum numbers of the standard model using the former, it is work in progress to see if using the later we get some match to the chiral numbers.

And the global view

A peculiar thing is that the bosons of this theory appear in equal number that the different kind of mesons known in the experimental spectroscopy. This is, of course, because the top quark does not hadronize. On the other hand, the diquarks here are not the "good quarks" but the bad ones; they have been paired with the adequate colour during the branching.

An interesting idea is that we are actually seen, as mesons and diquarks, the remmants of the scalars of the supersymmetric standard model.

Remmants or hidden, what we are telling is that the scalars are composites of the quarks. So we have a theory of preons, but without preons.

But of course, we could ask what is the result of applying the susy transformation operator to such scalars. Is it a composite fermion? I do not know, but if it is, it is a composite containing quarks. The standard model particles bootstrap, themselves, as composites of themselves.

Still, the previous document failed to do a complete blog entry. Instead, in July 19, 2017 it seems I uploaded the following **blog entry**

Families from SO(32)

$$
\begin{array}{llll}
496 = & & & \\
(\mathbf{1},\mathbf{24},\mathbf{1^c}) & +[\mathbf{1},\mathbf{15},\mathbf{\bar{3}^c}] & +[\mathbf{1},\mathbf{\bar{15}},\mathbf{3^c}] & + \\
1,24,8^c & +[1,10,\bar{6}^c] & +[1,\bar{10},6^c] & + \\
(1,1,8^c) & & & + \\
 & (2,5,3^c) & +(2,\bar{5},\bar{3}^c) & + \\
 & (1,1,1^c) & +[1,1,1^c] &
\end{array}
$$

(or from SO(30), or perhaps just O(10)xU(3) or U(5)xU(3))

Point is, the first three lines seem to contain three generations with electric and colour charge. It is possible to break the 24 and 15 from su5 to su3 + su2, and then identify the electric charge.

	Q_1	Q_2	$su3+su2$	Q_3	N
$(1,24,1^c)$	0	0	$(8,1)+(1,3)+(1,1)$	0	12
	0	0	$(3,2)$	5	6
	0	0	$(\bar{3},2)$	-5	6
$(1,15,\bar{3}^c)$	0	4	$(\bar{6},1)$	4	6
	0	4	$(3,2)$	-1	6
	0	4	$(1,3)$	-6	3
$(1,\bar{15},3^c)$	0	-4			
$(1,24,8^c)$	0	0			
$(1,10,\bar{6}^c)$	0	4	$(\bar{3},1)$	4	
	0	4	$(3,2)$	-1	
	0	4	$(1,1)$	-6	
$(1,\bar{10},6^c)$	0	-4			
$(1,1,8^c)$	0	0			
$(2,5,3^c)$	2	± 2	$(3,1)$	2	
	2	± 2	$(1,2)$	-3	
$(2,\bar{5},\bar{3}^c)$	-2	± 2			
$(1,1,1^c)$	0				
$(1,1,1^c)$	0				

We can choose 4(Q2+Q3)=-2/3 and Q3=-1/5 or 4(Q2+Q3)=1/3 and Q3=1/5

Honest problem is, I do not know how to identify weak isospin, nor weak hypercharge, given that we are looking, I guess, to scalars. Perhaps the repr is too big, doubled?

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so, umm, either Q2 1/5 = -1/6 or Q2 +1/5 = 1/12, so Q2=1/30 or Q2= -7/60,. Preons could then be wither, hmm -1/3,+2/3 or, aggg, 1/6, -5/6

Later in `https://a.rivero.nom.es/recopilando-y-poniendo-orden-en-84/` it seems I did an attempt to relate the representations to SO(9), but it is a different topic. The idea of looking for a 84 irrep is to have two dual theories of mass, one where the neutrinos are the only massive particles, via Majorana, and other where the top quarks are the only massive particles, via Dirac mass.