Relation between

The Euler Totient,

the counting prime formula

and the prime generating Functions

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1/ Introduction:

The theory of numbers is an area of mathematics which deals with the properties of whole and rational numbers… In this paper I will introduce relation between Euler phi function and prime counting and generating formula, as well as a concept of the possible operations we can use with them. There are four propositions which are mentioned in this paper and I have used the definitions of these arithmetical functions and some Lemmas which reflect their properties, in order to prove them.

2/Definitions:

Here are some definitions to illustrate how the functions work and describe some of their most useful properties.

The Euler totient:

The Euler totient function is defined to be the number of positive integers which are less or equal to an integer and are relatively prime to that integer: for n ≥ 1, the Euler totient φ(n) is:

$$\varphi(n) = \sum_{k=1}^{n} 1$$

There is a formula for the divisor sum which is one of the most useful properties of the Euler totient:

Lemma:

for n ≥ 1 we have $\sum_{d/n} \varnothing(d)$ = n. Since the Euler totient is the number of positive integers relatively prime to n.

$$\sum_{d/n} \varnothing(d) = \sum_{\frac{p}{n}} \varnothing(P) + \sum_{\frac{Q}{n}} \varnothing(Q) = n \quad ; \ldots.(1)$$

$$d = \sum p + \Sigma q \ , P\epsilon\, P\, c\, N\, , Q \in N - \{P\}$$

$$\sum \varnothing(p) = \sum p - 1 \ \ldots.(2)$$

From (1) & (2):

$$\sum_{d/n} \varnothing(d) = \sum_{\frac{p}{n}} p - 1 + \sum_{\frac{Q}{n}} \varnothing(Q) = n$$

And $\pi(n) = \sum_{p \leq n} 1 = \sum_{\frac{p}{n}} 1 + \sum_{\frac{p}{\text{¿}n}} 1$ (symbol :// p is not divisor of n)

$$\sum_{\frac{p}{n}} p - \sum_{\frac{p}{n}} 1 + \sum_{\frac{Q}{n}} \varnothing(Q) = \sum_{\frac{p}{n}} p - \sum_{p \leq n} 1 + \sum_{\frac{p}{\text{¿}n}} 1 + \sum_{\frac{Q}{n}} \varnothing(Q)$$

$$\sum_{\frac{p}{n}} p - \pi(n) + \sum_{\frac{p}{\text{¿}n}} 1 + \sum_{\frac{Q}{n}} \varnothing(Q) = n$$

So:

$$\pi(n) = \sum_{\frac{p}{n}} p + \sum_{\frac{p}{\text{¿}n}} 1 + \sum_{\frac{Q}{n}} \varnothing(Q) - n$$

Exp:

$$\pi(9) = \sum_{\frac{p}{9}} p + \sum_{\frac{p}{9}} 1 + \sum_{\frac{Q}{9}} \varnothing(Q) - 9$$

and divisors of 9: $\{1,3,9\}$ and 3 is prime, there 3 primes are not divisors of 9: $\{2,5,7\} \leq 9$

= 3+3+ $\phi(1)$+$\phi(9)$-9 =3+3+1+6-9=4

And there are 4 primes less than 9: $\{2,3,5,7\}$