Mach's Principle: the origin of the inertial mass (II)

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Abstract. We find that the force of inertia acting on an accelerated body is the result of the action of the gravitational induction force produced by the relative movement of the Universe as a whole, which fully confirms the Mach's Principle. The calculations are developed with the linearized theory of General Relativity.

1. Introduction

In a previous paper we have shown that the forces of gravitational induction of the whole of the Universe can be identified with the forces of inertia [1]. We have obtained this result for a vector gravitational theory compatible with Special Relativity.

The techniques developed in the previous paper will be applied to the linearized theory of General Relativity. Again we find that the forces of cosmic induction are responsible for the phenomena of the inertia.

In a subsequent investigation we will analyze the conclusions that derive from our theory, and we will devise mechanisms to experimentally verify the conclusions of the theory.

We recommend the previous reading of the reference [1].

2 Forces of inertia in Special Relativity

The equation of motion of a free particle in Special Relativity is a geodesic that has the equation

$$\frac{d^2x^r}{d\tau^2}+\Gamma^r_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau}=0 \tag{1}$$

$d\tau$ is the proper time of the particle and the symbols of Christoffel Γ^r_{ik} arise for three reasons: by using curvilinear coordinates instead of Cartesian, by the non-inertial character of the chosen reference system and by the gravitational force. In the following calculations the Christoffel symbols are different from zero because the reference system we are considering is not inertial, that is, the effect of the inertial forces will appear in the resulting equations of motion.

We consider an example. K' is a reference system that rotates with respect to an inertial system K, with a angular velocity ω directed towards the positive part of the z axis, which is common to both systems and whose origin moves with velocity $\mathbf{V}$ along the z-axis. The transformation equations between the two reference systems are

$$\begin{aligned}
t&=t'\\
x&=x'\cos\omega t-y'\sin\omega t\\
y&=x'\sin\omega t+y'\cos\omega t\\
z&=z'+\int V dt
\end{aligned}$$

and the space-time line element in the reference system K' is

$$ds^2=\left[1-\frac{\omega^2\left(x'^2+y'^2\right)}{c^2}-\frac{V^2}{c^2}\right]cdt'^2+2\frac{\omega y'}{c}cdt'dx'-2\frac{\omega x'}{c}cdt'dy'-2\frac{V}{c}dz'cdt'-dx'^2-dy'^2-dz'^2,$$

g_{ik} are the components of the metric tensor

$$g'_{00} = 1 - \frac{\omega^2\left(x'^2 + y'^2\right)}{c^2} - \frac{v^2}{c^2}; \quad g'_{01} = \frac{\omega y'}{c}; \quad g'_{02} = -\frac{\omega x'}{c}; \quad g'_{03} = -\frac{V}{c}; \quad g'_{\alpha\beta} = -\delta_{\alpha\beta}. \tag{2}$$

These components are put as

$$g'_{00} = 1 + h_{00}; \quad g'_{0\alpha} = h_{0\alpha}; \quad g'_{\alpha\beta} = -\delta_{\alpha\beta}.$$

where h_{ik} is the difference between the metric tensor g_{ik} of the system K' and the metric tensor of Minkowski η_{ik} of the system *K*. From these quantities h_{ik} the inertial forces are derived.

In developing the Christoffel symbols, we find from equation (1)

$$\frac{d}{d\tau}\left(g_{rl}\frac{dx^l}{d\tau}\right) = \frac{1}{2}\partial_r g_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau}$$

we have eliminated the apostrophe to simplify. Using (2) we obtain the equation of movement of a particle that moves with velocity **w** with respect to the rotating system K'

$$-\frac{d^2x^\alpha}{dt^2} + c\frac{dh_{0\alpha}}{dt} = c\partial_\alpha h_{0\beta}\frac{dx^\beta}{dt} + \frac{1}{2}c^2\partial_\alpha h_{00} \tag{3}$$

where we have assumed small the velocity of the particle and therefore the coordinate time coincides with the proper time. Developing the second term of the first member of (3) results

$$\frac{d^2x^\alpha}{dt^2} = c\left[\partial_\beta h_{0\alpha} - \partial_\alpha h_{0\beta}\right]\frac{dx^\beta}{dt} + c\partial_t h_{0\alpha} - \frac{1}{2}c^2\partial_\alpha h_{00} \tag{4}$$

or in vector notation

$$\frac{d\mathbf{w}}{dt} = -\dot{\mathbf{V}} - \boldsymbol{\omega}\wedge\left(\boldsymbol{\omega}\wedge\mathbf{r}'\right) - \dot{\boldsymbol{\omega}}\wedge\mathbf{r}' - 2\boldsymbol{\omega}\wedge\mathbf{w} \tag{5}$$

$d\mathbf{w}/dt$ and **w** are the acceleration and velocity of the free particle with respect to the K' system. The equation (5) is the same as that found in classical mechanics.

Multiplying (5) by the inertial mass m_i of the body, we find the second law of Newtonian dynamics in a non-inertial reference system

$$\mathbf{F} + \mathbf{F}_i = m_i\frac{d\mathbf{w}}{dt}$$

F is the applied force (zero in our example) and $\mathbf{F}_i$ is the sum of all the fictitious forces of inertia that by (5) is

$$\mathbf{F}_i = -m_i\dot{\mathbf{V}} - m_i\boldsymbol{\omega}\wedge\left(\boldsymbol{\omega}\wedge\mathbf{r}'\right) - m_i\dot{\boldsymbol{\omega}}\wedge\mathbf{r}' - m_i 2\boldsymbol{\omega}\wedge\mathbf{w},$$

these forces of inertia are not real and are introduced to maintain the form of the second law of dynamics in a non-inertial system. But there are real forces of inertia, are those that oppose the change of the state of movement of a body, ie the forces that arise from the phenomenon of inertia. These forces are real and therefore measurable and their value is $-m\mathbf{a}$, where **a** is the acceleration with respect to an inertial reference system. These forces always oppose the change of movement, that is, they are opposed to the applied force and have their same value, as expressed by the law of dynamic equilibrium

$$\mathbf{F} + \mathbf{F}_i = 0,$$

in the following, when we speak of force of inertia we will be referring to this force not to the aforementioned fictitious forces.

3 Linearized Theory of General Relativity

The gravitational field equations of General Relativity

$$R_{ik} - \frac{1}{2}g_{ik}R + \Lambda g_{ik} = -\chi T_{ik} \tag{6}$$

are non-linear differential equations, this tells us that the sum of two solutions of the above equation is not, in general, a new solution.

We can always decompose the metric tensor as

$$g_{ik} = \eta_{ik} + h_{ik}$$

where η_{ik} is the metric tensor of Minkowski, that is to say, the metric tensor in the absence of gravity, which in Cartesian coordinates has the components

$$\eta_{ik} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$

with the tensor h_{ik} we represent the gravitational part of the metric tensor.

We say that a gravitational field is weak if the components of h_{ik} and of its first derivatives are small, so that its products ($h \cdot h, h \cdot h_{,i}, h_{,i} \cdot h_{,j}$) can be neglected. In the presence of a weak gravitational field, the geometry is the space-time of Minkowski on which a perturbation given by h_{ik} is superimposed. Under these circumstances we can put

$$g^{ik} = \eta_{ik} - h_{ik}$$

in effect

$$g_{ik} g^{kj} = (\eta_{ik} + h_{ik})(\eta_{kj} - h_{kj}) \approx \eta_{ik}\eta_{kj} - \eta_{ik} h_{kj} + \eta_{kj} h_{ik}$$

and since η_{ik} and h_{ik} are symmetric we find that

$$\eta_{ik} h_{kj} = \eta_{kj} h_{ik} \quad \Rightarrow \quad g_{ik} g^{kj} \approx \eta_{ik}\eta_{kj} = \delta_i^j$$

as it should be.

For the weak gravitational field is permissible to eliminate all the non-linear terms that appear in the field equation. If we also use the harmonic gauge condition and we suppose null the cosmological constant, we find that (26) is reduced to [2]

$$\frac{\partial^2 h_{ik}}{\partial x^j \partial x_j} = -\chi T_{ik}^* \tag{7}$$

with the definition

$$T_{ik}^* = T_{ik} - \frac{1}{2}\eta_{ik} T$$

and χ is the gravitational constant

$$\chi = \frac{8\pi G}{c^4}.$$

Note that in linearized theory the field source (represented by the energy-momentum tensor) can not depend on the metric tensor, because then the field would be a field source and the equations would not be linear, so we use η_{ik} and not g_{ik} for to lower and to raise indexes and therefore to obtain the trace of the energy-momentum tensor. That is, we eliminate the terms of type $h \cdot T$ by negligible.

The components of T_{ik}^* as a function of the components of T_{ik} are

$$\begin{gathered} T = \eta^{ik} T_{ik} = T_{00} - \sum_\alpha T^{\alpha\alpha} \\ T_{00}^* = \frac{1}{2} T^{00} + \frac{1}{2}\sum_\alpha T^{\alpha\alpha}; \quad T_{0\alpha}^* = -T^{0\alpha} \\ T_{\alpha\beta}^* = T^{\alpha\beta} \, (\alpha \neq \beta); \quad T_{\alpha\alpha}^* = T^{\alpha\alpha} + \frac{1}{2} T^{00} - \frac{1}{2}\sum_\alpha T^{\alpha\alpha} \end{gathered} \tag{8}$$

(7) is solved by the retarted potentials

$$h_{ik} = -\frac{4G}{c^4} \int \frac{\left[T_{ik}^*\right]}{r'} dV'$$

the bracket means retarted value. By (8) we find

$$\begin{gathered} h_{00} = -\frac{2G}{c^4} \int \frac{\left[T^{00}\right] + \sum_\alpha \left[T^{\alpha\alpha}\right]}{r'} dV'; \quad h_{\alpha\beta} = -\frac{4G}{c^4} \int \frac{\left[T^{\alpha\beta}\right]}{r'} dV' \, (\alpha \neq \beta) \\ h_{0\alpha} = \frac{4G}{c^4} \int \frac{\left[T^{0\alpha}\right]}{r'} dV'; \quad h_{\alpha\alpha} = -\frac{4G}{c^4} \int \frac{\left[T^{\alpha\alpha}\right] + \frac{1}{2}\left[T^{00}\right] - \frac{1}{2}\sum_\alpha \left[T^{\alpha\alpha}\right]}{r'} dV'. \end{gathered} \tag{9}$$

4 The equation of movement

The equation of motion of a free particle in a gravitational field is the geodesic equation (1). The symbols of Christoffel represent the gravitational action. For a weak gravitational field, of (1) we obtain the equation of motion of a particle subjected exclusively to the force of gravity, until the second order in the development with respect to the inverse of c [3]

$$\frac{d\mathbf{w}}{dt}=-\nabla\phi-4\frac{\partial\mathbf{A}}{\partial t}+4\mathbf{w}\wedge(\nabla\wedge\mathbf{A})-\nabla\left(\frac{2\phi^2}{c^2}+\frac{\psi}{c^2}\right)+3\frac{\mathbf{w}}{c^2}\frac{\partial\phi}{\partial t}+4\frac{\mathbf{w}}{c^2}(\mathbf{w}\cdot\nabla)\phi-\frac{w^2}{c^2}\nabla\phi$$

$\mathbf{w}$ is the velocity of the particle on which the force acts; the potential vector $\mathbf{A}$ and the scalar potentials ϕ and ψ are defined by

$$\phi=\frac{c^2}{2}\overset{(2)}{h}_{00}=-\frac{G}{c^2}\int\frac{\overset{(-2)}{\left[T^{00}\right]}+\sum_\alpha\overset{(-2)}{\left[T^{\alpha\alpha}\right]}}{r'}dV';\quad \psi=\frac{c^4}{2}\overset{(4)}{h}_{00}=-G\int\frac{\overset{(0)}{\left[T^{00}\right]}+\sum_\alpha\overset{(0)}{\left[T^{\alpha\alpha}\right]}}{r'}dV';$$

$$A^\alpha=-\frac{c}{4}\overset{(3)}{h}_{0\alpha}=-\frac{G}{c^3}\int\frac{\overset{(-1)}{\left[T^{0\alpha}\right]}}{r'}dV'$$

the number in brackets represents the order with respect to the inverse of c.

For the problem of determining the inductive force of the Universe on a test body in order to obtain the classic inertia force, it is sufficient to take the following terms

$$\frac{d\mathbf{w}}{dt}=-\nabla\phi-4\frac{\partial\mathbf{A}}{\partial t}+4\mathbf{w}\wedge(\nabla\wedge\mathbf{A}). \tag{10}$$

If the system is non-inertial, it will also be applicable (5), and both expressions (10) and (5) are equal. In the reasoning that we will do next we study the force that acts on a particle that has a velocity $-\mathbf{u}$ and an acceleration $-\mathbf{a}$ with respect to an inertial system in which the field sources are at rest. We will assume that the particle is permanently at rest at the origin of the non-inertial system, therefore

$$\mathbf{r}'=0;\quad \mathbf{w}=0;\quad \mathbf{V}=-\mathbf{u};\quad \dot{\mathbf{V}}=-\mathbf{a}$$

and from equations (5) and (10)

$$-\nabla\phi-4\frac{\partial\mathbf{A}}{\partial t}+4\mathbf{w}\wedge(\nabla\wedge\mathbf{A})=\mathbf{a}. \tag{11}$$

But the geodesic equation (1) is valid if the inertial mass is identical to the gravitational mass. But all we know is that both masses are proportional and that by choosing an adequate value of the universal gravitation constant, the two masses are numerically equal. But we do not have the guarantee that this equality will remain with the passage of time. Therefore, we are going to consider again the equation of motion (1) but without accepting that the inertial and gravitational mass are equal and leaving open the possibility that the constant of proportionality between both changes with time.

So the geodesic equation for a particle in a gravitational field and in Cartesian coordinates is

$$\frac{d^2x^r}{d\tau^2}+\frac{m_g}{m_i}\Gamma^r_{ik}\frac{dx^i}{d\tau}\frac{dx^k}{d\tau}=0$$

making the identification with (5)

$$-m_g\nabla\phi-4m_g\frac{\partial\mathbf{A}}{\partial t}+4m_g\mathbf{w}\wedge(\nabla\wedge\mathbf{A})=-m_i(-\mathbf{a})$$

the second member is the force of inertia, therefore we conclude that the force of inertia is the result of the gravitational action of the whole Universe and is given by

$$\mathbf{F}_i=-m_g\nabla\phi-4m_g\frac{\partial\mathbf{A}}{\partial t}+4m_g\mathbf{w}\wedge(\nabla\wedge\mathbf{A}). \tag{12}$$

This is the essence of Mach's principle: when a body moves with respect to the whole of the Universe, a force of gravitational induction acts on it, which must be identified with the force of inertia. In the later calculations we must check whether (12) actually reproduces the observed inertial force, that is: a force that is linear with respect to the acceleration of the body and that has opposite direction.

5 The cosmic momentum-energy tensor

To calculate the components of the energy-momentum tensor is necessary previously to relate the proper time τ with the coordinate time t. For this we start from

$$ds^2 = g_{00}c^2dt^2 + 2g_{0\alpha}cdtdx^\alpha + g_{\alpha\beta}dx^\alpha dx^\beta,$$

defining

$$\gamma_\alpha = \frac{g_{0\alpha}}{\sqrt{g_{00}}}; \quad \gamma_{\alpha\beta} = -g_{\alpha\beta} + \gamma_\alpha\gamma_\beta$$

therefore

$$ds^2 = -d\sigma^2 + \left(\gamma_\alpha dx^\alpha + \sqrt{g_{00}}cdt\right)^2 \tag{13}$$

the spatial distance is

$$d\sigma^2 = \gamma_{\alpha\beta}\, dx^\alpha dx^\beta.$$

From (13) we finally get

$$d\tau = \sqrt{\left(\gamma_\alpha \frac{u^\alpha}{c} + \sqrt{g_{00}}\right)^2 - \frac{u^2}{c^2}}\, dt, \tag{14}$$

it should be noted that u^α, like $u^2 = \sum\left(u^\alpha\right)^2$, is the velocity of the source with respect to the coordinate time t and not with respect to the proper time, however, in the order of approximation that we seek there is no difference between using one or the other. Note also that γ_α is of order 3 with respect to the inverse of c and therefore can be neglected. Limiting us to terms of order two with respect to the inverse of c we have

$$g_{00} = 1 + \overset{(2)}{h}_{00} = 1 + \frac{2\phi_0}{c^2}$$

where ϕ_0 is the gravitational potential of where the volume element that produces the field is located, since g_{00} is to determine the proper time of the source particle. But the metric tensor can not be included in the sources, since then the theory would not be linear, so in the following we will take $g_{00} \approx \eta_{00} = 1$; then the relation between the proper time and the coordinate is

$$d\tau = \sqrt{1 - u^2/c^2}\, dt.$$

We assume that the Universe is a perfect fluid and therefore its tensor energy-momentum is

$$T^{ik} = \left(\rho + p/c^2\right)u^i u^k - pg^{ik} \tag{15}$$

p is the cosmic pressure that is of order two with respect to c, then the component 0,0 is

$$T^{00} = \left(\rho + p/c^2\right)u^0u^0 - pg_{00} = \left(\rho c^2 + p\right)\left(\frac{dt}{d\tau}\right)^2 - p\eta_{00}$$

where we have used the Minkowski metric tensor to preserve the linear character of the equations. From the above equation we find

$$T^{00} = \frac{\rho c^2 + p}{1 - u^2/c^2} - p,$$

from here we find the order term -2

$$\overset{(-2)}{T}{}^{00} = \rho c^2, \tag{16}$$

the remaining terms are

$$T^{00} - \overset{(-2)}{T}{}^{00} = \frac{\rho c^2 + p}{1 - u^2/c^2} - p - \rho c^2 = \frac{\rho c^2 + p}{1 - u^2/c^2}\frac{u^2}{c^2}$$

from here we find

$$\overset{(0)}{T}{}^{00} = \left(\rho c^2 + p\right)\frac{u^2}{c^2}. \tag{17}$$

Note that in the previous calculation we have not assumed that u is small compared to c.

Now we calculate the components α,α

$$\sum_\alpha T^{\alpha\alpha} = \sum_\alpha \left(\rho + p/c^2\right)\frac{dx^\alpha}{d\tau}\frac{dx^\alpha}{d\tau} - \sum_\alpha p\eta_{\alpha\alpha} = \frac{\left(\rho + p/c^2\right)}{1-u^2/c^2}u^2 + 3p$$

from the above we deduce that

$$\sum_\alpha \overset{(-2)}{T}{}^{\alpha\alpha} = 3p. \tag{18}$$

To obtain the zero-order term we do the same as before and calculate

$$\sum_\alpha T^{\alpha\alpha} - \sum_\alpha \overset{(-2)}{T}{}^{\alpha\alpha} = \frac{\left(\rho + p/c^2\right)}{1-u^2/c^2}u^2$$

from here we see that

$$\sum_\alpha \overset{(0)}{T}{}^{\alpha\alpha} = \left(\rho + p/c^2\right)u^2, \tag{19}$$

we remember again that u is the calculated velocity with the time coordinate and not the proper time.

We calculate the components $0,\alpha$

$$T^{0\alpha} = \left(\rho + p/c^2\right)cu^\alpha\left(\frac{dt}{d\tau}\right)^2 = \frac{\left(\rho + p/c^2\right)cu^\alpha}{1-u^2/c^2}$$

from here we find

$$\overset{(-1)}{T}{}^{0\alpha} = \left(\rho + p/c^2\right)cu^\alpha .$$

which is the only term of the component $0, \alpha$ which we need for the later calculation.

Finally we calculate the trace of the energy-momentum tensor

$$T = \eta^{ik}T_{ik} = T_{00} - \sum_\alpha T_{\alpha\alpha} = T^{00} - \sum_\alpha T^{\alpha\alpha}. \tag{20}$$

By carrying (16), (17), (18), (19) and (20) to (8) we find

$$\overset{(-2)}{T^*_{00}} = \frac{1}{2}\overset{(-2)}{T}{}^{00} + \frac{1}{2}\sum_\alpha \overset{(-2)}{T}{}^{\alpha\alpha} = \frac{1}{2}\rho c^2 + \frac{3}{2}p$$

$$\overset{(0)}{T^*_{00}} = \frac{1}{2}\overset{(0)}{T}{}^{00} + \frac{1}{2}\sum_\alpha \overset{(0)}{T}{}^{\alpha\alpha} = \frac{1}{2}\left(\rho c^2 + p\right)\frac{u^2}{c^2} + \frac{1}{2}\left(\rho + p/c^2\right)u^2 = \left(\rho + p/c^2\right)u^2$$

$$\overset{(-1)}{T^*_{0\alpha}} = -\overset{(-1)}{T}{}^{0\alpha} = -\left(\rho + p/c^2\right)cu^\alpha .$$

With these results we find the two potential scalars and the potential vector

$$\phi = -G\int \frac{\left[\rho + 3p/c^2\right]}{r'}dV'; \quad \psi = -2G\int \frac{\left[\left(\rho + p/c^2\right)u^2\right]}{r'}dV';$$
$$A^\alpha = -\frac{G}{c^2}\int \frac{\left[\left(\rho + p/c^2\right)u^\alpha\right]}{r'}dV'. \tag{21}$$

6 Potentials of Liénard-Wiechert in General Relativity

The phenomenon of gravitational induction occurs when the source or the observer are in motion, which we call active and passive induction respectively. In our study we consider the second case, the gravitational induction is produced by the movement of the observer, while the source remains at rest [4].

The whole of the Universe is at rest with respect to the system K_0, which is an inertial reference system, this statement is the basis of Mach's principle, that is, the identification of the Universe as an inertial system. For an observer at rest in K_0 the gravitational scalar potential is

$$\phi = -G\int \frac{dm'}{r_0} \tag{22}$$

r_0 is the proper length from the observer to the source. But now the source of the field dm' is not only the mass but also the energy, that is the pressure. Therefore, the source of the potential is not only the 0,0 component of the energy-momentum tensor, but also the components α,α. But in the case of a perfect fluid, each of the three components α,α is equal to the pressure. This means that the source of the potential in the K_0 system is $T_{00} + \sum T_{\alpha\alpha}$. For an observer at rest at K_0 there are no inductive potentials **A** and ψ, since the velocity of the source is zero.Then from (22)

$$\phi = -G\int \frac{\rho + 3p/c^2}{r_0} dV_0 \quad \Rightarrow \quad h_{00} = -\frac{4G}{c^4}\int \frac{\overset{*}{T}_{00}}{r_0} dV_0, \tag{23}$$

dV_0 is the proper volume occupied by the source, therefore dV_0 is measured in the system K_0. If we consider the cosmological constant Λ, then the tensor energy-momentum (15) is modified

$$\tilde{T}_{ik} = \left(\tilde{\rho} + \tilde{p}/c^2\right)u_i u_k - \tilde{p} g_{ik}$$

$\tilde{\rho}$ is the sum of the density of matter and the density of the vacuum and is $\tilde{p}$ the sum of the electromagnetic pressure and the pressure of the vacuum

$$\tilde{\rho} = \rho + \frac{c^2\Lambda}{8\pi G}; \quad \tilde{p} = p - \frac{c^4\Lambda}{8\pi G},$$

which must be used in (23) if the cosmological constant is considered non-zero.

Let us consider an observer C at rest at the origin of the system K that moves with a velocity $-\mathbf{u}$ with respect to the system K_0, where the Universe as a whole is at rest.

To find the potential measured by C, we express h_{ik} in tensor notation, having to match its value in the system K with (23), then the expression found for h_{ik} is valid in the system K and in any other reference system (inertial or non-inertial).

Instead of considering the movement of K with respect to K_0, we now consider the movement of K_0 with respect to K. Then, the source dm' moves with velocity **u** with respect to C. For the observer K this is a retarded velocity, that is to say, it is the velocity that the mass dm' had when it emitted the gravitational action at the moment t' (retarded instant), which reaches observer C at the current time t. r' is the distance at which the mass dm' was at time t', that is to say, the distance at which C observes the mass dm' at the time t: $r' = c(t - t')$ and $\mathbf{r}'$ is the position of C with respect to the position retarded of dm' and therefore $-\mathbf{r}'$ is the vector of position of dm' with respect to C. Therefore the tetraposition of dm' and of the point C with respect to the system K are

$$x^k(dm') = (ct', -\mathbf{r}'); \quad x^k(C) = (ct, 0),$$

the tetravelocity of dm' with respect to K is

$$u^k = \frac{dx^k(dm')}{d\tau'} = \frac{dx^k(dm')}{dt'\sqrt{1-u^2/c^2}} = \left(\frac{c}{\sqrt{1-u^2/c^2}}, \frac{\mathbf{u}}{\sqrt{1-u^2/c^2}}\right)$$

$d\tau'$ is the proper time of the mass dm' at the time retarded and

$$\mathbf{u} = -\frac{d\mathbf{r}'}{dt'}.$$

We define the tetravector

$$R^k = x^k(dm') - x^k(C) = \left[c(t'-t), -\mathbf{r}'\right] = (-r', -\mathbf{r}')$$

therefore

$$u^i R_i = \frac{\mathbf{r}' \cdot \mathbf{u}}{\sqrt{1-u^2/c^2}} - \frac{r'c}{\sqrt{1-u^2/c^2}} = -\frac{cs}{\sqrt{1-u^2/c^2}}; \quad s = r' - \frac{\mathbf{r}' \cdot \mathbf{u}}{c}.$$

Equation (23) in tensor notation is

$$h_{ik}=\frac{4G}{c^3}\int\frac{\left[T^*_{ik}\right]}{u^jR_j}dV_0=-\frac{4G}{c^4}\int\frac{\left[T^*_{ik}\right]}{s}dV'$$

as it is a tensor relationship is valid in the system *K*. Then the potentials are

$$\begin{gathered}\phi=-G\int\frac{\left[\rho+3p/c^2\right]}{s}dV';\quad \psi=-2G\int\frac{\left[\left(\rho+p/c^2\right)u^2\right]}{s}dV';\\ \mathbf{A}=-\frac{G}{c^2}\int\frac{\left[\left(\rho+p/c^2\right)\mathbf{u}\right]}{s}dV'.\end{gathered}\tag{24}$$

where dV' is the element of volume measured in the system *K*, $dV'=dV_0\sqrt{1-u^2/c^2}$.

It is necessary to notice an essential difference between (21) and (24). In (21) the velocity that appears is the velocity of the source with respect to the inertial system K_0 . In equation (24) $-\mathbf{u}$ is the velocity of the observer with respect to the system K_0 .

The equations (24) are the generalization of the potentials of Liénard-Wiechert of the electromagnetic theory and we will use them to calculate the inductive action of the whole of the Universe.

7 Calculation of the induction force

We suppose a body C of gravitational mass m_g that has a velocity $-\mathbf{u}$ and an acceleration $-\mathbf{a}$ with respect to the Universe, that is to say, with respect to the inertial reference system K_0 . From equations (24) we obtain the potentials and by (12) we calculate the force of induction that acts on the body C.

To make the integration of the induction force acting on C we divide the Universe into spherical layers of negligible thickness dr' , of center in the body C and of radius r' . The position vector of an element of the spherical shell with respect to the system *K* in spherical coordinates is

$$-\mathbf{r}'=-r'\sin\theta\cos\varphi\mathbf{i}-r'\sin\theta\sin\varphi\mathbf{j}-r'\cos\theta\mathbf{k}.$$

To calculate the potentials of a spherical shell the terms that interest us are

$$\begin{gathered}\nabla\delta\phi=-G\int\left[\rho+3p/c^2\right]\nabla\left(\frac{1}{s}\right)dV';\quad \nabla\delta\psi=-2G\int\left[\left(\rho+p/c^2\right)u^2\right]\nabla\left(\frac{1}{s}\right)dV';\\ \frac{\partial\delta\mathbf{A}}{\partial t}=-\frac{G}{c^2}\int\left[\rho+p/c^2\right]\frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)dV'.\end{gathered}\tag{25}$$

To calculate the previous integrals we have to use relationships [5]

$$\begin{gathered}\nabla\left(\frac{1}{s}\right)=-\frac{\mathbf{r}'}{s^2r'}+\frac{\mathbf{u}}{cs^2}-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{u})}{cs^3r'}-\frac{\mathbf{r}'(\mathbf{r}'\cdot\mathbf{a})}{c^2s^3}+\frac{u^2\mathbf{r}'}{c^2s^3}\\ \frac{\partial}{\partial t}\left(\frac{\mathbf{u}}{s}\right)=\frac{r'}{s}\left[\frac{\mathbf{a}}{s}+\frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{u})}{s^2r'}+\frac{\mathbf{u}(\mathbf{r}'\cdot\mathbf{a})}{cs^2}-\frac{\mathbf{u}u^2}{cs^2}\right].\end{gathered}\tag{26}$$

We limit ourselves to non-relativistic velocities, because we intend is to identify the inertial force of classical mechanics with the force of gravitational induction (12). So $u\ll c$, therefore $s\approx r'$.

Since r' has cosmic dimensions, we neglect in the equations (26) the terms that depend on $1/r'^2$ versus the terms that depend on $1/r'$. With this simplification the only terms we consider are the fourth of the first equation (26) and the first and third of the second equation (26).

The direct calculation gives us

$$\nabla\delta\phi=\frac{4\pi}{3}\frac{G}{c^2}\left[\rho+3p/c^2\right]\mathbf{a}r'dr';\quad \frac{\partial\delta\mathbf{A}}{\partial t}=-4\pi\frac{G}{c^2}\left[\rho+p/c^2\right]\mathbf{a}r'dr'.$$

In these and following formulas is necessary to understand that both the density and the pressure are retardeds values, that is to say, those that the source had when it emitted a gravitational signal. By (12) the force of induction produced by the spherical shell located at distance r' is

$$\delta\mathbf{F}_i=-m_g\nabla\delta\phi-4m_g\frac{\partial\delta\mathbf{A}}{\partial t}=\frac{4\pi}{3}\frac{G}{c^2}\left[11\rho+9p/c^2\right]m_g\mathbf{a}r'dr'.\tag{27}$$

We add that, for reasons of symmetry, the cosmic expansion, which is isotropic, does not produce induction forces. We indicate that if we consider the cosmological constant, in (27) we must replace $\tilde{\rho}$ and $\tilde{p}$, which we define in section 6.

8 Cosmological parameters

The cosmic parameters of density of matter, radiation, vacuum and curvature are defined by the relations

$$\Omega_M = \frac{8\pi G}{3H^2}\rho_M; \quad \Omega_R = \frac{8\pi G}{3H^2}\rho_R; \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}; \quad \Omega_k = -\frac{kc^2}{H^2R^2}, \tag{28}$$

R is the cosmic scale factor; H is the Hubble «constant» defined by

$$H = \frac{\dot{R}}{R}$$

the point means derivation with respect to the coordinate time and not with respect x^0; k is the scalar curvature of the three-dimensional space, of value 1, -1 or 0; Λ is the cosmological constant; ρ_M, ρ_R and ρ_V are the energy densities of matter and radiation. Note that the cosmic parameters depend on time and therefore vary as the Universe evolves.

From (28) it follows

$$\rho_M = \Omega_M\rho_c; \quad \rho_R = \Omega_R\rho_c \quad \rho_V = \Omega_\Lambda\rho_c$$

ρ_c is the critical density defined by

$$\rho_c = \frac{3H^2}{8\pi G}$$

density that depends on time.

From the first of the Friedmann's equations

$$\dot{R}^2 + kc^2 = \frac{8\pi G}{3}\rho R^2 + \frac{1}{3}\Lambda c^2 R^2$$

$$H^2 = \frac{8\pi G}{3}(\rho_M + \rho_R) - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3},$$

therefore

$$\Omega_M + \Omega_R + \Omega_\Lambda + \Omega_k = 1,$$

expression that helps us to determine the cosmic scale factor in the present moment $R_0 = R(t_0)$. In fact, known the parameters of density of matter, radiation and vacuum at the current time, the parameter of curvature $\Omega_k^0 = \Omega_k(t_0)$ is calculated in the present moment t_0 and from here R_0 is determined, for which we need to know the Hubble constant H_0 at the current time t_0.

The densities of matter, radiation, and vacuum and the radiation and vacuum pressures are given by

$$\tilde{\rho} = \rho_M + \rho_V; \quad p_R = \frac{1}{3}\rho_R c^2; \quad p_V = -\rho_V c^2; \quad \rho_M \propto R^{-3}(t); \quad \rho_V \propto R^{-4}(t). \tag{29}$$

With these results, of the equation (27) we obtain

$$\delta\mathbf{F}_i = -\left(\frac{11}{2}\Omega_M + \frac{3}{2}\Omega_R + \Omega_\Lambda\right)\frac{H^2}{c^2}m_g\mathbf{a}\sigma d\sigma \tag{30}$$

we have identified r' with the proper distance σ.

9 Cosmic distance

We assume that the Robertson-Walker line element is applicable

$$ds^2 = c^2dt^2 - R^2(t)\left(dr + r^2\sin^2\theta d\varphi^2 + r^2 d\theta^2\right)$$

$R(t)$ is the cosmic scale factor and t is the cosmic time, that is, the coordinate time of the reference system with respect to which the Universe is at rest.

The movement of a gravitational signal that approaches to the center of the sphere is given by the expression

$$\frac{dr}{\sqrt{1-kr^2}} = -\frac{cdt}{R(t)} \tag{31}$$

integrating

$$r = \mathcal{S}\left[-ct_0 \int_{\hat{\zeta}}^{\zeta} \frac{d\zeta'}{R(\zeta')}\right] = \mathcal{S}\left[\frac{ct_0}{R_0} \int_{\zeta}^{\hat{\zeta}} \frac{d\zeta'}{a(\zeta')}\right],$$

we have defined

$$\zeta = \frac{t'}{t_0}; \quad a(\zeta) = \frac{R(\zeta)}{R_0}; \quad R_0 = R(t_0) = R(\zeta = 1),$$

we must notice that r does not depend on time, since the cosmic bodies have invariable coordinates. The function is defined as follows: if $k=1$ then $\mathcal{S}(x) = \sin x$; if $k=-1$, $\mathcal{S}(x) = \sinh x$ and if $k=0$, $\mathcal{S}(x) = x$.

From the Robertson-Walker metric it follows that the radial distance from the origin to a coordinate point r is

$$\sigma = R(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}}$$

and from (31) we find

$$\sigma(\zeta,\hat{\zeta}) = ct_0 a(\zeta) \int_{\zeta}^{\hat{\zeta}} \frac{d\zeta'}{a(\zeta')} \tag{32}$$

The derivative of the proper distance with respect to ζ is obtained from (32)

$$\sigma'(\zeta,\hat{\zeta}) = ct_0\left[a'(\zeta)\int_{\zeta}^{\hat{\zeta}} \frac{d\zeta'}{a(\zeta')} - 1\right]. \tag{33}$$

10 Inertia force expressed as a function of the cosmic parameters

The last step to determine the force induced on a particle due to the relative accelerated movement of the whole Universe, is the integration of (27) on all the spherical shells

$$\mathbf{F}_i = \int_0^{\hat{\zeta}} \left[\frac{11}{2}\Omega_M + \frac{3}{2}\Omega_R + \Omega_V\right]\frac{H^2}{c^2} m_g \mathbf{a}\,\sigma|\sigma'|d\zeta, \tag{34}$$

we have put

$$\sigma d\sigma = \sigma\frac{d\sigma}{d\zeta}d\zeta = \sigma|\sigma'|d\zeta$$

the absolute value appears because the proper distance (32) has a maximum, therefore its derivative is positive and then it becomes negative, but the negative value does not make sense because $d\sigma$ represents the thickness of the spherical shell, which is always positive.

It is easy to verify that

$$\Omega_M = \frac{\Omega_M^0}{a^3}\frac{H_0^2}{H^2}; \quad \Omega_R = \frac{\Omega_R^0}{a^4}\frac{H_0^2}{H^2}; \quad \Omega_\Lambda = \Omega_\Lambda^0 \frac{H_0^2}{H^2}$$

then for (27)

$$\mathbf{F}_i = \int_0^{\hat{\zeta}} \left[\frac{11}{2}\frac{\Omega_M^0}{a^3} + \frac{3}{2}\frac{\Omega_R^0}{a^4} + \Omega_\Lambda^0\right]\frac{H_0^2}{c^2} m_g \mathbf{a}\sigma|\sigma'|d\zeta. \tag{35}$$

To do this calculation we need the value of the density parameters at the current moment $\Omega_M^0, \Omega_R^0, \Omega_V^0$ and the Hubble constant H_0 also in the present moment. From the first equation of Friedman the current age of the Universe t_0 is determined

$$t_0 = \frac{1}{H_0}\int_0^1 \left[1 + \Omega_M^0\left(\frac{1}{a} - 1\right) + \Omega_R^0\left(\frac{1}{a^2} - 1\right) + \Omega_\Lambda^0\left(a^2 - 1\right)\right]^{-1/2} da,$$

the cosmic scale factor is determined by

$$\zeta = \frac{1}{H_0 t_0} \int_0^{R/R_0} \left[1 + \Omega_M^0 \left(\frac{1}{a} - 1 \right) + \Omega_R^0 \left(\frac{1}{a^2} - 1 \right) + \Omega_\Lambda^0 \left(a^2 - 1 \right) \right]^{-1/2} da.$$

Once function $a(\zeta)$ is known, $\sigma(\zeta)$ and $\sigma'(\zeta)$ can be determined by (32) and (33).

The derivative with respect to the coordinate time of the cosmic scalar factor is

$$\dot{a} = \frac{da}{dt} = H_0 \sqrt{\left[1 + \Omega_M^0 \left(\frac{1}{a} - 1 \right) + \Omega_R^0 \left(\frac{1}{a^2} - 1 \right) + \Omega_\Lambda^0 \left(a^2 - 1 \right) \right]},$$

the relationship between $\dot{a}$ and a' is

$$a' = \frac{da}{d\zeta} = \frac{da}{dt} t_0 = \frac{1}{H_0} (t_0 H_0) \dot{a}$$

therefore

$$a' = t_0 H_0 \sqrt{\left[1 + \Omega_M^0 \left(\frac{1}{a} - 1 \right) + \Omega_R^0 \left(\frac{1}{a^2} - 1 \right) + \Omega_\Lambda^0 \left(a^2 - 1 \right) \right]}. \tag{36}$$

The temporal variation of the density parameters can be known from the following expressions deduced from the definitions (28)

$$\Omega_M = \Omega_M^0 (t_0 H_0)^2 \frac{1}{a'^2 a}; \quad \Omega_R = \Omega_R^0 (t_0 H_0)^2 \frac{1}{a'^2 a^2}; \quad \Omega_\Lambda = \Omega_\Lambda^0 (t_0 H_0)^2 \frac{a^2}{a'^2},$$

where the apostrophe means derivation with respect to ζ. Finally, la constante de Hubble es

$$H = \frac{1}{t_0} \frac{a'}{a}.$$

11 Universe of density and constant pressure

In order to see the basic concepts without technical difficulties, we consider a simplified cosmic model. We assume that the Universe is large enough, of finite age, static and with a constant and uniform density and pressure. By integrating (27)

$$\mathbf{F}_i = \frac{2\pi}{3} G \left(11\tilde{\rho} + 9\,\tilde{p}/c^2 \right) t^2 m_g \mathbf{a} \tag{37}$$

the integration has been done on all the Universe connected casually with the observer. t is the age of the Universe. The force of inertia is $-m_i(-\mathbf{a})$ therefore

$$m_i = \frac{2\pi}{3} G \left(11\tilde{\rho} + 9\,\tilde{p}/c^2 \right) t^2 m_g \tag{38}$$

this equation shows us that the inertial mass of a body is the result of the inductive action of the Universe.

(37) shows that if the acceleration of the observer is zero, then there is no induction and therefore there is no force of inertia, as the law of inertia says. (37) also shows that the inertial force is proportional to the acceleration of the observer and in the opposite direction.

The equation (38) shows the proportionality between inertial and gravitational mass

$$m_i = \xi(t) m_g$$

to $\xi(t)$ we call it coefficient of inertia. (38) indicates that inertia is a phenomenon of a cosmic nature. Moreover, (38) informs us that the inertial mass varies with time, even in the case of a static Universe as we are considering.

The universal gravitation constant is chosen so that in the current epoch the inertial and gravitational mass coincide, therefore it must be fulfilled

$$\frac{2\pi}{3} G \left(11\tilde{\rho}_0 + 9\,\tilde{p}_0/c^2 \right) t_0^2 = 1 \tag{39}$$

$\tilde{\rho}_0$, $\tilde{p}_0$ y t_0 are the density, the pressure and the age of the Universe in the present moment.

We apply the results to the cosmic values

$$\Omega_M^0 = 0.27; \quad \Omega_R^0 \approx 0; \quad \Omega_\Lambda^0 = 0.73; \quad \rho_c^0 = 9.47 \cdot 10^{-27}\, Kg/m^3; \quad t_0 = 14 \cdot 10^9\, \textit{años}$$

then we find

$$m_i(t_0) = 4.5m_g$$

although it does not coincide with the expected result (39), it is a satisfactory result, given the simplified cosmic theory that we are considering.

12 Proportionality of the inertial and gravitational mass

According to equation (27) when a body moves with an acceleration with respect to the whole of the Universe, a gravitational induction force acts on it, proportional to the acceleration of the body and in the opposite direction, exactly like the force of inertia. Therefore we must identify both forces, concluding that the force of inertia acting on a body is the inductive force of the Universe.

(27) also shows us that the force of inertia does not depend on the velocity, at least at the classical level, and is proportional to the acceleration. Note that **a** is any type of acceleration, therefore (27) also explains the centrifugal force.

By the equation (38), applicable to a simplified cosmic model, we find that the inertial mass is proportional to the gravitational mass, although both magnitudes are conceptually independent. The universal gravitation constant is chosen so that the inertial and gravitational mass are equal in the current epoch.

The inertial mass varies as the square of the age of the Universe

$$m_i(t) = \left(\frac{t}{t_0}\right)^2 m_i(t_0).$$

In our simplified model this increase in inertial mass is explained because as time passes there are more masses of the Universe that are causally connected to the body that undergoes the force of induction.

Of (38) we deduce that gravity has to be an exclusively attractive force. In fact, if gravity were repulsive the scalar potential of equation (24) would be positive and the same will happen with the vector potential of (24). From these new equations we find again (24) but with a negative sign. But then the inertial mass would be negative, contrary to the observation that inertia is opposed to the change of movement of a body.

The previous reasoning is valid even in the case in which gravitational mass was negative. By equation (38) the gravitational mass can only have one sign, either positive or negative. If the gravitational mass had two signs (like the electric charge), there would be bodies with a negative inertial mass, which is absurd. Indeed, if the active gravitational mass (which produces the force) had a sign and the passive gravitational mass (on which the force acts) had the opposite sign, then its inertial mass would be negative, which is not observed in nature. Or in other words, if there were gravitational masses of the two signs, there would be bodies with a negative inertial mass.

The relation (38) produces effects that could be detectable, and which we will examine in a subsequent investigation. Among others, we point out that the variation of the inertial mass with time will affect to the emission frequency of the spectral lines, producing a shift of these lines, an effect that would overlap the shift caused by the cosmic expansion. The variation of the inertial mass will affect the orbital movements, so the rotation of distant galaxies will have a different law from the rotation of nearby galaxies. The equivalence between mass and the energy would also be affected by the variation of the inertial mass, in fact in the equation $E = mc^2$, m is the inertial mass, therefore its variation will affect the nuclear processes, which would have an impact on to stellar evolution.

13 Participation of the Universe in the formation of the inertial mass

According to Mach's principle, the inertial mass of a body is generated by the action of the whole Universe. On a body C act forces that are produced in all parts of the observable Universe. Some of these forces originate in very distant objects, that is, they were produced a long time ago; while the forces exerted by nearby objects were produced a short time ago.

Now we study the participation in the generation of the inertial mass of the different epochs of the Universe, applied to our simplified cosmic model. We consider the Universe formed by N

spherical shells in whose center is the body C. To the furthest shell we give the numeration $n = 0$ and in it are the objects that produce the force at the beginning of the Universe, whose effects reach the body C in the present moment, when the age of the Universe is t_0.

The thickness of the spherical shells is $c\tau$ where $\tau = t_0/N$. The inertial mass produced by shell *n* is by (38)

$$\delta m_i = \frac{4\pi}{3} G\left(11\tilde{\rho} + 9\,\tilde{p}/c^2\right) t\tau m_g \tag{40}$$

t is the time takes the gravitational interaction to travel from the spherical shell *n* to the body C, therefore

$$t = t_0 - n\tau$$

of (38) and (40) we get

$$\frac{\delta m_i}{m_i(t_0)} = \frac{2}{N}\left(1 - \frac{t'}{t_0}\right) \tag{41}$$

$t' = n\tau$ is the time retarded.

We must remember that (41) is applicable to the cosmic model that we are considering which has a constant and uniform density of matter. But independently of the cosmic model, (41) shows us that each epoch of the Universe contributes differently to the formation of the inertial mass of a body. (41) shows that the first moments of the Universe (when $t' = 0$) have a greater contribution to the inertial mass; while the present Universe ($t' = t_0$) the contribution is minimal. This result can be more pronounced in a realistic cosmic model.

14 The Universe of Einstein-De Sitter

We applied the previous concepts to the Einstein-de Sitter Universe, because it is a mathematically simple model, although it seems to be not adjusted to reality. This model is characterized by $\Omega_\Lambda = 0$, $k = 0$ and consists exclusively of matter. In this model the density of the Universe corresponds to its critical value and when applying (36) it is found

$$\left(\frac{da}{d\zeta}\right)^2 = t_0^2 H_0^2 \frac{1}{a}$$

integrating we find

$$H_0 t_0 \zeta = \frac{2}{3} a^{3/2} \quad \Rightarrow \quad \frac{R}{R_0} = \left(\frac{3}{2} H_0 t_0 \zeta\right)^{2/3},$$

when $t = t_0$ results $R = R_0$ and then $H_0 t_0 = 2/3$ therefore

$$R(\zeta) = R_0 \zeta^{2/3} \quad \Rightarrow \quad a = \zeta^{2/3}.$$

From the above equation we obtain that the Hubble constant has the following dependence with the cosmic time

$$t_0 H(\zeta) = \frac{2}{3}\frac{1}{\zeta}.$$

The density of this Universe is

$$\rho(\zeta) = \rho_c(\zeta) = \frac{3H^2(\zeta)}{8\pi G} = \frac{1}{6\pi G}\frac{1}{t_0^2}\zeta^{-2}.$$

The radial coordinate of where was a body that emitted the signal in *t* and that arrives in the moment $\hat{t}$ to the observer is

$$r = c\int_t^{\hat{t}} \frac{dt}{R(t)} = \frac{ct_0}{R_0}\int_\zeta^{\hat{\zeta}} \frac{d\zeta'}{a(\zeta')} = \frac{ct_0}{R_0}\int_\zeta^{\hat{\zeta}} \zeta'^{-2/3} d\zeta' = \frac{3ct_0}{R_0}\left(\hat{\zeta}^{1/3} - \zeta^{1/3}\right),$$

where ζ is the retarded moment.

The corresponding proper distance is calculated from

$$\sigma\left(\zeta,\hat{\zeta}\right) = R(\zeta) r\left(\zeta,\hat{\zeta}\right) = R_0 \zeta^{2/3} \frac{3ct_0}{R_0}\left(\hat{\zeta}^{1/3} - \zeta^{1/3}\right) = 3ct_0\left(\hat{\zeta}^{1/3}\zeta^{2/3} - \zeta\right),$$

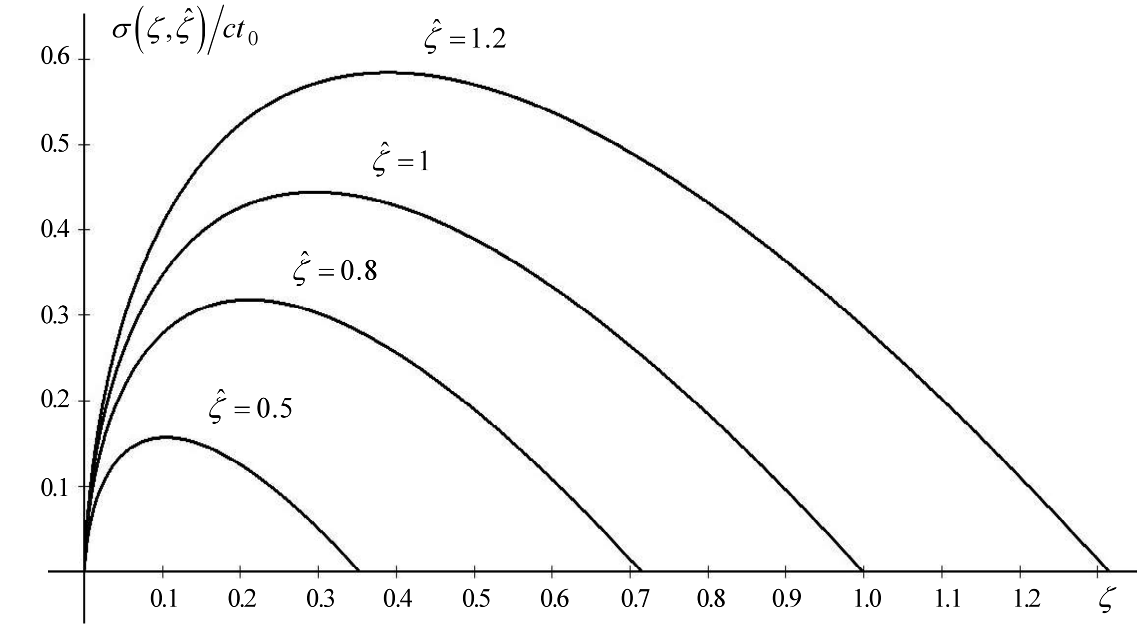

Illustration 1.- *Representation of proper distance σ with respect to time in the Universe of Einstein-De Sitter. The graph represents the value of the distance to which a source is located that emitting the gravitational interaction at the time ζ indicated on the horizontal axis, arrives at the field point at the moment $\hat{\zeta}$.*

therefore

$$\sigma d\sigma=\sigma\sigma' d\zeta=9c^2t_0^2\left(\frac{2}{3}\hat{\zeta}^{2/3}\zeta^{1/3}-\frac{5}{3}\hat{\zeta}^{1/3}\zeta^{2/3}+\zeta\right)d\zeta$$

$\zeta_{\max}$ is the moment when the farthest signal that reaches the observer at the moment $\hat{\zeta}$ was emitted, that is calculated with the condition $\sigma'=0$

$$\zeta_{\max}=\left(\frac{2}{3}\hat{\zeta}^{1/3}\right)^3=0.2963\hat{\zeta}$$

and the proper distance from which this interaction came out is

$$\sigma_{\max}=3ct_0\left[\left(\frac{2}{3}\right)^2\hat{\zeta}-\left(\frac{2}{3}\right)^3\hat{\zeta}\right]=\left(\frac{2}{3}\right)^2 ct_0 .$$

When applying the previous results to (27) it is found that the participation in the acceleration induced by a spherical shell is

$$\begin{aligned}\delta\mathbf{F}_i&=\left[\frac{11}{2}\Omega_M+\frac{3}{2}\Omega_R+\Omega_V\right]\frac{H^2}{c^2}m_g\mathbf{a}\sigma\left|\sigma'\right|d\zeta=\frac{11}{2}\Omega_M\frac{H^2}{c^2}m_g\mathbf{a}\sigma\left|\sigma'\right|d\zeta=\\&=22m_g\mathbf{a}\left|\frac{2}{3}\hat{\zeta}^{2/3}\zeta^{-5/3}-\frac{5}{3}\hat{\zeta}^{1/3}\zeta^{-4/3}+\zeta^{-1}\right|d\zeta .\end{aligned}\tag{42}$$

The effect caused by the whole Universe is the integration of the previous expression. This integral is divergent, but even without considering the precise moment of origin, the integral takes an excessively large value that can not correspond to reality. We must understand that the linearized theory does not apply to the Big Bang, because the field at that moment is excessively large and the non-linear terms can not be disregarded.

Even with this limitation we can find results that have physical interest. By making the integral (37) between the indicated limits, we find

$$\mathbf{F}_i=22m_g\mathbf{a}\int_{0.21231}^{1}\left|\frac{2}{3}\zeta'^{-5/3}-\frac{5}{3}\zeta'^{-4/3}+\zeta'^{-1}\right|d\zeta'=m_g\mathbf{a}\tag{43}$$

which would explain all the inertia of a body as a result of the gravitational induction of the rest of the Universe.

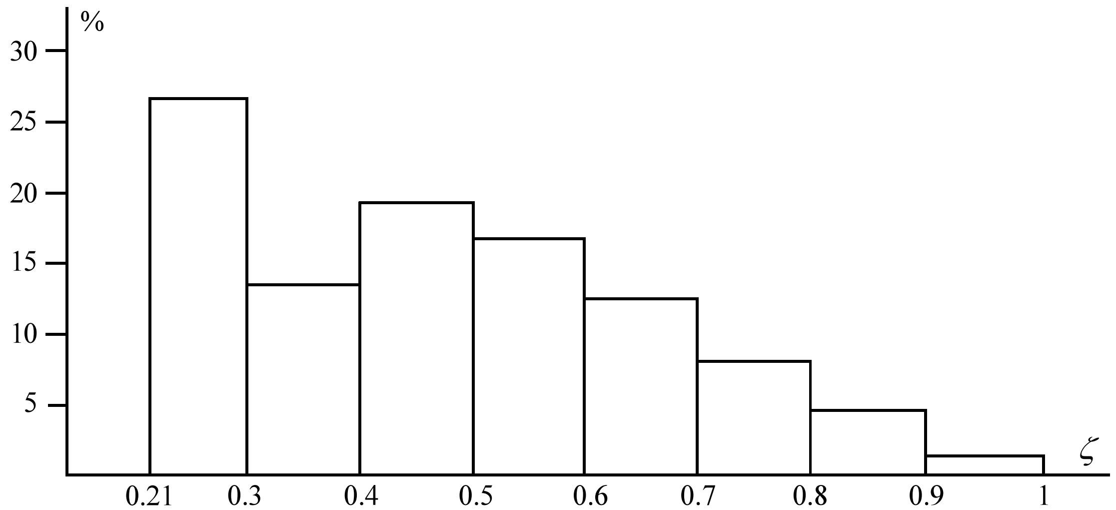

Illustration 2.- *The graph has been obtained from equation (43). It shows how the different cosmic epochs contribute to the formation of the inertial mass at the present time. It is observed that the first epochs of the Universe have a greater contribution and the recent epochs have a smaller contribution in the formation of the inertial mass. The period between the point we have taken as the beginning and the instant* $0.3t_0$ *is responsible for 26% of the inertial mass; however between* $0.9t_0$ *and* t_0 *the contribution is 1.3%.*

From equation (43) we see that the different epochs of the Universe contribute unequally to the inertial mass of a body. In the illustration number 2 we see the percentage in which each cosmic period contributes to the inertial mass of a body according to the model of Einstein-de Sitter that we are considering. From equation (43) we verify that the inertial mass varies with time $\hat{\zeta}$. In the illustration number 3 the numerical representation of this variation is shown. We verify that with the passage of time the inertial mass increases.

In a subsequent investigation we will analyze the applicability of the Mach's principle to several cosmological models.

14 Conclusions

We understand Mach's principle as the affirmation that the phenomenon of inertia (as well

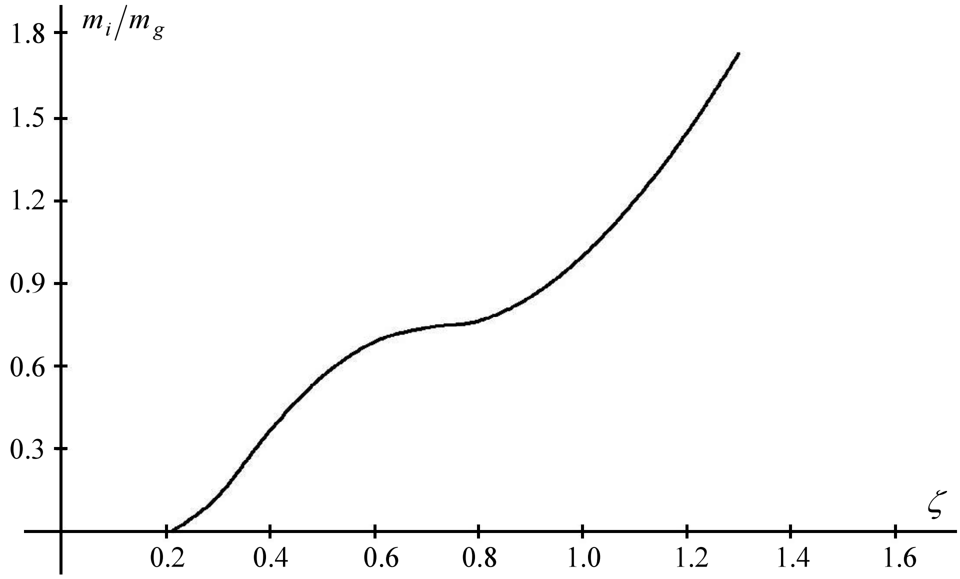

Illustration 3.- *Variation of the inertial mass with time. The inertial mass is proportional to the gravitational mass, but the constant of proportionality varies with time. In the Einstein-de Sitter model with the simplifications established in the text, the inertial mass increases with time.*

as the force of inertia and the inertial mass) are the result of the action of the Universe.

We define the inertial reference system as that system with respect to which the whole Universe on average is at rest or in uniform and rectilinear motion. This affirmation means to return to the concept of absolute Newtonian movement, as long as we identify absolute space with the Universe. Therefore, there is a preferred system of reference: that in which the masses of the Universe are at rest on average, a proposition that is supported by experience.

In any field theory the action takes a while to travel from the source to the observer. This circumstance makes induction phenomena exist, that is to say forces that arise by the movement either from the source of the field (active induction) or by the movement of the observer (passive induction).

When an observer moves with respect to the Universe, passive gravitational induction occurs. To calculate these forces we use the Liénard-Wiechert potentials, well known in electromagnetism and that we have applied to the linearized theory of General Relativity.

It must be emphasized that the two types of induction referred are different. If the source moves in relation to an inertial system, gravitational radiation can be generated, something that can not happen when it is passive induction. Then the relative motion of the Universe with respect to an observer does not produce gravitational radiation.

When we calculate the gravitational force induced on a body that moves with respect to the set of masses of the Universe, we verify that no forces arise if the movement is uniform and rectilinear, at least in the classical approximation. If the body carries an accelerated movement, then induction forces arise proportional to the acceleration and in the opposite direction. Therefore, these induction forces can be identified with the forces of inertia. Then we reproduce the first two laws of Newtonian mechanics, which turn out to be laws of cosmic origin.

The characteristic of the inertial reference systems is that the law of inertia is valid with respect to them. In effect, if a body is in uniform and rectilinear movement with respect to an inertial system, it will also have the same type of movement with respect to the Universe, and as we have said, in this situation no induction force is generated, that is, there is no force of inertia; something that does not happen in non-inertial systems, that is, those that are accelerated with respect to the Universe.

The coefficient of proportionality between the force of induction and acceleration is the inertial mass. From our calculation it is verified that this inertial mass is proportional to the gravitational mass and, in general, dependent on time. This last circumstance is what allows us to devise experiments to verify the correctness of the results we have obtained, as we will see in a later investigation.

We have applied our theory to a static and uniform cosmic model and when making the numerical application we have found results approximately coinciding with the observation that at present the inertial mass of a body coincides with its gravitational mass.

When applying the results to a simple cosmological model, such as that of Einstein-de Sitter, we find that they can not be maintained if the Big Bang is accepted, in effect the cosmic action on a body proceeds, not only from the whole Universe, but also of all the cosmic epochs, then the high densities in the Big Bang produce an extremely high induction force, something that is not observed.

In a future investigation we will study in more detail the different cosmological models and we will check the results that are obtained when the Mach's principle is applied. In a final investigation we will examine the consequences that derive from the results we have obtained.

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