On the nature of Dark Energy, Cosmological Constant and Dark Matter

Ramon Isasi

isasi.ramon@gmail.com

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Abstract

In the present essay, we consider the origin of the dark energy, cosmological constant and the dark matter. The dark energy, is consequence of the annihilation of the matter - antimatter at the very beginning of the big bang. This dark energy was the origin and cause of the cosmological expansion and the past and present creation of the whole space. We take into account the presence of the call “dark” matter as consequence of highly excited hydrogen and helium Rydberg’s atoms in perfect equilibrium with the CMB radiation. The cosmological constant is considered as an arbitrary *ad hoc* anti gravitational entity. Finally, we note that the Casimir effect, as a suitable and truly efficient physical method and a reliable resource in the experimental determination of the dark energy.

Keywords: Cosmology; Dark Matter; Dark Energy; Empty space, Casimir effect; Rydberg atoms; Hubble constant; CMB radiation.

1 Introduction

Nature always has the same design, which evolved from simple elementary units to produce more complex structures. In cosmology and astrophysics all its evolutionary processes was originated from a plasm of gas formed principally by ions, free electrons, neutrinos and radiation.

As the intensity of the electrical force is 10^{39} times stronger than the gravitational interactions, during the recombination the electrical charge are canceled and lather the gravitational collapse is produced in the formation of the massive structures as galaxies and stars. In consequence, the characteristics of all these structures - including all the minor unities - were imposed and established by the hegemonic arrangement of the whole. The Rydberg H and He highly excited atoms are the best candidates in the formation of the called “dark matter”. This Rydberg atoms, from the recombination they are in perfect equilibrium with the CMB radiation, and do no interacts with any other shorter forms of the electromagnetic radiation i.e. UV, IR, X, gamma. However, this Rydberg’s atoms together with the “ordinary” matter were fundamental in the formation of galaxies and stars. The evidence of his presence is noted in the anomalous rotation curves of spiral galaxies. Also, the observation of the phenomenon called “gravitational lensing” between galaxies and cumulus of galaxies can be only attributed to the “dark” matter (Zwicky, F. 1937). All this gravitational “anomalies” can be perfectly caused by the highly excited H and He Rydberg atoms.

The empty cosmological space is all “dark” energy. The source of this energy was originated from the initial conditions of the big bang process whose physical significance is related as the expansive evolution of the self space. This process was initiated from the annihilation of near the same matter - antimatter particles. At the present time development, the empty space energy density is the same as the registered by the Casimir effect.

2 Microspace vacuum and macrospace vacuum

2.1 Relativistic meaning of the spectral lines

For an electron pertaining to an isolated hydrogen atom, when $n = 1$ and $v = c/137$ ($v << c$), the Lorentz relativistic expression may by stated in the following form

$$m_0(1+\delta) = m_0\left(1+\frac{v^2}{c^2}\right)^{1/2} \simeq m_0\left(1+\frac{v^2}{2c^2}\right) \tag{1}$$

When this single electron evolves from a state (i) towards a state (j), there is within the empty contour of the atom a mass increase of Δm

$$m_i = m_e\left(1+\delta_i\right) \tag{2}$$

$$m_j = m_e\left(1+\delta_j\right) \tag{3}$$

$$\Delta m = m_i - m_j = m_e\left(\delta_i - \delta_j\right) \tag{4}$$

Taking into account $\delta = v^2/c^2 = \alpha^2/n^2$, being α the Sommerfeld constant, and as this relationship expresses the relativistic quantum ratio in electromagnetic emission (or absorption), we can insert it in (1) and (4)

$$\Delta m = m_e\left(\frac{v_i^2}{2c^2} - \frac{v_j^2}{2c^2}\right) = \frac{m_e\alpha^2}{2}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \tag{5}$$

Since $\alpha = e^2/\hbar c$, we have

$$\Delta m = \frac{m_e e^4}{2\hbar^2 c^2}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right); \Delta E = \frac{m_e e^4}{2\hbar^2}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \tag{6}$$

As with the expansion, the kinetic energy K diminishes and the potential energy U increases, this means that the potential energy is equivalent to the space[1] that has been created. Likewise, the spectral process of emission or absorption is an interaction between a microbubble of energy (atom) and the macrospace. In this process, both entities always exchange simultaneously the same quantum unit of energy. The absorption lines expresses the mass-energy quantity proceeding from the exterior space, and the emission lines are equivalent to the mass-energy transferred to the macrospace when the electron falls down to an inferior quantum level.

This correlativity of mass-energy and space-time reciprocal exchange is the foundation for establishing the equivalence between both vacuum contours: the microspace "vacuum" mass-energy and the macrospace "vacuum" mass-energy (Section 2.2).

From the Equations (1) and (6) it is evident that as the atomic radius increases, the electron energy decreases and Δm, which means the mass equivalent of the empty atomic space between n_i and n_j increases too.

Likewise, according to Bohr's formalism, the electron "velocity" is v_i for $n_i = 1$ and $r_i = a_0$ (Bohr radius). When the atom is excited or "expands", v decreases inversely and linearly in terms of the Bohr's radius: $v = \hbar/(m_e n a_0)$.

[1] The space is equivalent to mass or energy

As the velocity depression is also parametrized with respect to a scale unit (a_0), it allows us to establish the following formalism:

$$\frac{v_j}{v_i} = \frac{a_0}{r_j} = \frac{1}{n_j^2}; r_j = n_j^2 a_0 \tag{7}$$

2.2 Cosmological expansion and the hydrogen atom excitation

According to Eq. (6) and (7) it may be deduced that the relativistic mass of a photon emitted or absorbed for any transition is inversely proportional to the ratios of the corresponding stable orbits.

Consequently

$$\frac{m_i}{m_j} = \frac{r_j}{a_0}, \qquad r_j = 4a_0, 9a_0, \ldots, n^2 a_0 \tag{8}$$

Also the following proportions are valid:

$$\frac{m_i}{m_j} = \frac{E_i}{E_j} = \frac{T_i}{T_j} = \frac{r_j}{a_0} = \frac{n_j^2}{n_i^2} = \frac{\lambda_j}{\lambda_i} \tag{9}$$

This means, that the relativistic interactive photons mass-energy is directly proportional to the relativist temperature, and inversely proportional to the atomic radius. Even though it is certain that the expansion of the Universe operates in a continuous form, it is a particular congruence that the information of the event was provided by quantum transitions of atomic emissions systems. If for a practical convenience, we change the sign of these transitions, and instead of emission, it would correspond to the excitement of an electron of an isolated hydrogen atom. Here, its eigenvalues would have the same scale proportions in their spatial development as the cosmological values expressed as redshift or temperature of the CMB radiation.

This correspondence, is due to the equivalence between the energetic mechanism itself that originates the radiation (quantum states) associated to the size of the atoms, its emitted wavelength, and the size of the Universe in expansion.

It is because of that

$$z + 1 = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{R_U^0}{R_{em}} = \frac{n_j^2}{n_i^2} = \frac{T_{em}}{T_{obs}} \tag{10}$$

Where z is the measurement of the cosmological expansion; R_{em} and R_U^0 are the respective Universes radii when light was emitted, compared with the present radius.

For a transition $n_i = 1$ to $n_j = 2$, the atomic radius is 4 times greater, λ_{obs} for $n = 2$ is 4 times larger too. If $z = 3$, it means that the Universe was 4 times smaller: $R_{em} = R_u^0/4$, or what is the same, the wavelength emitted was a quarter of the observed $\lambda_{em} = \lambda_{obs}/4$.

As the monodimensional parameters of cosmological expansion have the same ratio as those of atomic excitation states, it means that the energetic implications of both vacuum contours are identical, despite of their scale magnitudes.

Thus, due to the cosmological expansion, the Lyman alpha line shift of the atomic hydrogen spectra, coincides with the ratio between the two atomic radii which generates this transition Eq. (10). E.g., for $z = 3$ the Lyman alpha line is shifted to a cosmological expansion with a factor 4. Likewise, when an isolated hydrogen atom originates a photon of $\lambda = 1,216\mathring{A}$, the electron has a quantum transition from $n_j = 2$ to $n_i = 1$, due to a spatial transition of $4a_0 \to a_0$. Although we have adopted the intense Lyman alpha line, the same properties for any other electronic transition can be verified.

2.3 Macrocosm and microcosm moments of interaction

In view of these preliminary antecedents of equivalence - which link the gravitational field with the Coulombic field - it is possible to relate the atomic and subatomic universe, to the cosmological Universe. In order to accomplish this, we will connect both scales through their corresponding moments of interaction (Law of the Lever). Then, the moment between the microscopic and macroscopic magnitudes can be made equal and be superimposed in the following way:

$$\begin{aligned} \text{Energy A} &= \text{Energy B} \\ F_G \times L_{max} &= F_{coul} \times r_e \end{aligned} \tag{11}$$

The "classical electron radius" r_e, physically has an entirely relativistic signification of equivalence between mass, radial distance and the unit of charge. This constant r_e, is the radial separation between two electrical charges when the Coulombic potential of energy is equivalent to the electron relativistic mass $m_e c^2 = e^2/r_e$. The gravitational interaction between a proton and an electron in the domain of r_e is $F_G = Gm_p m_e/(2\pi^2 r_e^2)$. As in this conditions $F_{coul} = e^2/r_e^2 = m_e c^2/r_e$ we can substitute it in the Eq. (11) and resolve for L_{max}.

$$L_{max} = \frac{2\pi^2 r_e e^2}{Gm_p m_e} = \frac{2\pi^2 r_e^2 c^2}{Gm_p} = 1.26 \times 10^{28} cm = 1.36 \times 10^{10} Ly \tag{12}$$

Being: L_{max} Universe critical radius; electron charge $e = 4.8 \times 10^{-10} cm^{3/2} g^{1/2} s^{-1}$; electron mass $m_e = 9.11 \times 10^{-28} g$; proton mass $m_p = 1.673 \times 10^{-24} g$; gravitational constant $G = 6.67 \times 10^{-8} cm^3 g^{-1} s^{-2}$; classical radius $r_e = 2.82 \times 10^{-13}$ cm.

It is evident the critical radius L_{max} established in this way is a constant, because it was originated by means of constants. Nevertheless, as L_{max} is a superior limit, it is accomplished when the expansive energy is the same as the potential energy. As the generalized state of expansion continues, this limit in the present has not been reached.

2.4 Critical number of protons

A microspace contour, and all the whole Universe contour are complementary when these opposite extremes are represented by means of a limited energy bubble with a well defined critical radius in its spherical surface. This makes it possible to determine another critical parameter, which can be found applying the Principle of Correspondence (Bohr, 1929) on Eq.(12). Its result lead to the obtainment of the number of baryons N_b and the critical mass m_U^0.

Multiplying and dividing Eq. (12) by N, being N a constant

$$L_{max} = \frac{2\pi^2 r_e^2 N c^2}{Gm_p N} = \frac{R_U^2 c^2}{2\, G m_U^0} \tag{13}$$

$$\text{as } m_U^0 = m_p N_b \text{ and } L_{max}^2 = 4\pi^2 r_e^2 N_b \tag{14}$$

from which we obtain the Schwartzchild Radius [2].

[2] The scale 4.10×10^{17}s : 1.23×10^{28}cm : 8.36×10^{55}g, is the same as Schwartzchild scale in Natural Units 1s : 3×10^{10}cm : 2×10^{38}g.

$$L_{max} = \frac{2GM}{c^2}; \quad M = m_U^0 \tag{15}$$

The equations (13) and (14) show that N_b represent the critical number of baryons. Therefore:

$$N_b = \left(\frac{L_{max}}{2\pi r_e}\right)^2 = \left(\frac{\pi\, r_e c^2}{G\, m_p}\right)^2 = \left(\frac{\pi\, e^2}{G\, m_p m_e}\right)^2 = 5.08 \times 10^{79} \tag{16}$$

The critical baryon density is:

$$\rho_b = \frac{3N_b m_p}{4\pi\, R_U^3} = \frac{3}{4\pi} N_b m_p \left(\frac{H_0}{c}\right)^3 = 1.09 \times 10^{-29} g \cdot cm^{-3} \tag{17}$$

The Principle of Correspondence is clearly expressed in the Eq.(16) when microphysics scale gets connected with cosmological scale through the N_b constant.

The ratio of interaction between two units of charges, with respect to the intensity of its gravitational field is 7.13×10^{39} times greater. It signifies that 7.13×10^{39} units of mass are required to produce a gravitational field of the same intensity as a single pair of units of charge. These assertion and principles can be applied to other very extensives or massive forms: galaxies formation, the structure of the Sun and stars.

3 The Hubble constant

3.1 Kinematics implications of H_0

Theorically, the Hubble constant is defined as a parameter established by the speed of the cosmological expansion within a scale unit (Misner *et al*; 1972).

$$H = \frac{\dot{R}}{R}$$

Starting from this constant, the following parameters can be deduced: a) Linear recession law: $v = dl/dt = \dot{l} = \dot{R}l/R$ being l, the mean distance between two referential physical points (*i.e.* galaxies). b) Hubble time $t_H = l/v = H^{-1}$ where t_H is the time from the present referential position, extrapolated to zero distance between galaxies moving at the recession rate observed today. c) Hubble length $L_H = c/H$, where L_H is the top distance, which is attained by use of the linear recession law when v is extrapolated to c.

3.2 The origin of H_0

One second of parallax (parsec), given by the diameter of the terrestrial orbit around the Sun, is an anthropic scale unit, and physically unmeaning by itself. The origin of H_0 must be marked by a physic unit of scale transcendent and significant as the radius of the gravitational collapse R_G, which it may allow us the acquisitions of physical implications which are comparative to the atomic referential radius as unit of scale (*i.e.* a_0 or r_e).

When the Universe radius was 6.20×10^{24} cm $= 2\,\mathrm{Mps} = R_G$ with a Planck's blackbody distribution curve corresponding to a temperature of $\sim 5,400°\mathrm{K} = T_G$, there still existed a fraction of photons in a thermic state equivalent to $\sim 166,000°\mathrm{K}$, whose number was the same as the whole population of baryons.

Starting from these conditions, the collapse of gravitation is produced; all the matter and radiation which up to that epoch was in an undiferenciated state of plasm, undergoes a 3-d globular packing condensation. The development of these clumps is a fundamental point of reference: the history of the cosmological expansion begins with the withdrawal of these formations, in order to mark the initial time of H_0^{-1}.

Since kilometer and megaparsec are units of distance, the dimensions km s^{-1}.Mps^{-1} means $second^{-1}$; then, as the cosmological space progresses, the Hubble expansion rate will decrease continuously, until it reaches the present time value of $H_0 = 75.4$ Km$\,s^{-1}$ Mps^{-1} = 151 Km$\,s^{-1}$ 2 Mps^{-1} (Table 1).

At the present the observational determination of H_0 shows acceptable precise values and by means of different methods converges within $H_0 = 73.8(\pm 3.1)$ Km·s^{-1}·Mps^{-1} (Riess, 2011); $H^0 = 73(\pm 2)(\pm 4)$ Km·s^{-1}·Mps^{-1} (Freedman - Madore, 2010); $H^0 = 73.9$ Km·s^{-1}·Mps^{-1} (Bonamente, 2006); $H_0 = 76.5$ Km·s^{-1}·Mps^{-1} (Holanda, 2011); $H_0 = 74.3$ Km·s^{-1}·$Mps^{-1}(\pm 2.1)$ (Freedman - Madore, 2012); $H^0 = 74.03(\pm 1.42)$ Km·s^{-1}·Mps^{-1} (Riess, 2019).

3.3 Non Inflationary Origin and evolution of the repulsive dark energy

All the 3.12×10^{87} photons of the CMB radiation Eq. (31) vibrate in all possible directions through a symmetric axis. But, as they have their origin in the annihilation of almost the same quantity of matter-antimatter, they are formed from $N_\gamma/2$ pairs of polarized waves. This polarization still remains after the inverse thermoionization (recombination) because the recombinant electron also collapses in atoms with two possible quantum states. Likewise, the electrons of the hydrogen atoms and He too, show two equal quantum states and emit polarized photons in both pairs.

Because the great supremacy of the $N_\gamma/2$ pairs of polarized waves, and despite of the perturbations provoked by the N_b baryons, this scheme remains invariant through all the cosmological evolution. Therefore, from the present conditions, if we fix an inverse sequential order towards a collapse on the space itself (gravitational implosion), it will show the following phases:

a - When the temperature is higher than $4,000°$K the electrons and the hydrogen and helium nucleus will still be at the plasma state. The N_γ photons of the CMB radiation keep their polarity, taking into account that they are $1/2N_\gamma(-)$ and $1/2N\gamma(+)$.

b - For the electrons' threshold temperature $T \sim 6 \times 10^{9}°$K and $R_U \sim 2.6 \times 10^{19}$ cm, 1/4 photons (-) and 1/4 photons (+) collapses as 1/4 electrons and 1/4 positrons.

c - When $R_U \sim 1.6 \times 10^{16}$ cm and $T > 2 \times 10^{12}°$K (neutron's threshold temperature) other 1/4 photons (+) plus 1/4 photons (-) collapse as 1/4 protons and 1/4 antiprotons, their final result being $N/2$ neutrons. $\left\{ \begin{array}{l} 1/4N(p^+ + e^- + \nu_{\text{neutrino}} \to n^0) \\ 1/4N(p^- + e^+ + \nu_{\text{antineutrino}} \to n^0) \end{array} \right\} N/2$ neutrons
Where $N/2 = 1.6 \times 10^{87}$ neutrons, and the constant $M_V = 2.6 \times 10^{63}$ g is the ***mass intake from the empty space or the space itself during the expansive phase.***

d - Finally, this $N/2$ neutrons coalesces to give $\sim 1.2 \times 10^{68}$ Planck's "particles" when $R_U = 6.4 \times 10^{-11}$ cm and $T = 1.62 \times 10^{32}$ °K.

In a cyclic, or periodic Universe, the Planck state was not an initial *ex-nihilo* big bang starting point, instead, it was a crossing point for a new cycle (Big Crunch). As at this point, there is not a preexistent surface, the Universe does not rebound on it, but it passes across itself, and expands toward any 3d points of space. Then, the former inward gravitational implosive falling energy was shifted to an opposite outward acceleration. This accelerative expansion, in fact, was not simultaneously canceled by an equal attractive gravitational field ($g = 0$). Consequently, this means **the origin of the cosmological expansion was produced exclusively from matter-antimatter annihilation (dark energy)**.

Both opposite interactions were still unchanged until the gravitational baryonic collapse, which started at 2Mps (Sect. 3.2).

3.4 Dynamical implications of H_0

Taking into account the Cosmological Principle, and considering H_0 for a simultaneous time (unobservable) for any enegypoint in all the extension of the cosmological space, we may establish the dynamic state of the Universe from the radiative transition, to baryonic up to present time. Thus, for the extremes $R_G = 2$ Mps (gravitational collapse) and R_U^0 (present time radius) we have

$$H_0 = \frac{c}{(z+1)R_G} = \frac{2\,v}{R_G} = 2.45 \times 10^{-18}\,\mathrm{s}^{-1} \tag{18}$$

being $v = 7.6 \times 10^6\,\mathrm{cm\,s}^{-1}$; $R_G = 6.20 \times 10^{24}$ cm

$$\text{Also: } H_0 = \frac{c}{R_U^0} = 2.45 \times 10^{-18}\ \mathrm{s}^{-1} \tag{19}$$

where $R_U^0 = 1.225 \times 10^{28}$cm

Equalizing (18) with (19) and reordering, we find the following dimensionless scale

$$\frac{2\,v}{c} = \frac{1}{z+1} = \frac{2\,H_0}{H} = \frac{R_G}{R_U^0} \tag{20}$$

Any value from the linear recession law is comparative to whatever intensive property of a system *i.e.* it is similar to the absolute temperature used universally as an indicator of the thermic state, or as a measure of energy for any system.

Then

$$\frac{2\,v}{c} = \frac{T}{T_G} \qquad T_G\text{: Gravitational collapse temperature} \tag{21}$$

Since H_0 defines the present Hubble expansion rate, and H defines a value of the Hubble constant at different epoch, we may extend (20) in the following way:

$$\frac{2\,v}{c} = \frac{1}{z+1} = \frac{2\,H_0}{H} = \frac{R_G}{R_U} = \frac{T}{T_G} = \left(\frac{8\,m_{de}}{M_V}\right)^{1/2} \tag{22}$$

$M_V = 2.64 \times 10^{63}$g is the mass of the cosmological vacuum (Sect. 3.3).

Raising to square all this dimensionless terms and reordering, we find

$$m_{de} = \frac{M_V v^2}{2\,c^2} = \frac{M_V}{8\,(z+1)^2} = \frac{M_V H_0^2}{2\,H^2} = \frac{M_V R_G^2}{8\,R_U^2} = \frac{M_V T^2}{8\,T_G^2} \tag{23}$$

These proportions express the dynamic state of the Universe unquestionably. The term $M_V v^2/2\,c^2$, the same as the other terms, represents **the relativistic vacuum m_{de} dark mass-energy equivalence of the space in expansion**.

Despite of the different methods for the determination of the Hubble constant, and the implications of the Universe age, for us, H_0^{-1} as well as t_0, are not independent quantities, since we consider for H_0 a clear point of arrival. Thus $R_G = 2$ Mps (6.20×10^{24} cm) referential point, is coincident with the initial withdrawal of the gas protogalaxies after gravitational collapses.

The Hubble time is an indicator of the cosmological age through the expansion rate in relation with the R_G referential point of departure. Hence, the present value of H_0^{-1} means the duration of the expansion from R_G until now. For this reason, t_0 = age, determined on the basis of the antiquity of the oldest objects, plus its time of formation, is the same as $t_0 \simeq H_0^{-1}$, as likewise $H_0 t_0 \simeq 1$.

4 The extension of the cosmological space

4.1 Energy constraint

All terms of the Eq.(23) determine the main implication of the Hubble parameter, because it establishes the dynamic index (scale factor) of the relativistic kinetic energy of the global expansion, prevailing for any point in space in a simultaneous time (Cosmological Principle).

In the initial evolutive process, because of $E_V > E_G$ and $m_V = m_{de} > m_M$, the Universe was hegemoneously expansive. Up to the $R_G \sim$ 2Mps, the gravitation collapses, and as it implies a force exerted, this makes a continuous decrease of $m_{de} = M_V v^2/2\,c^2$. Thus, the space expansion range can be established by means of the ratio between the expansive energy and the gravitational attractive force (Planck, 1926).

Slightly modifying the mechanical equivalent of heat, we have:

$$\text{Expansive energy} = F_{\text{grav}} L_{\max} \tag{24}$$

As $F_{\text{grav}} L_{\max}$ is the potential energy U, it determines that the expansive energy (kinetic K) be depressed continuously up to the equilibrium limit given by $\frac{1}{2}K = U$, which constraints the Universe extension ($L_{\max}$).

$$L_{\max} = \frac{M_V v^2}{2\,G {m_U^0}^2} R_U^2 \tag{25}$$

$$m_U^0 : \text{critical mass} = N_b m_p (\text{constant})$$

Being $N_b = 5 \times 10^{79}$; $m_U^0 = 8.35 \times 10^{55}$ g and M_V: mass of the vacuum 2.64×10^{63}g (Sect. 3.3), $K = m_V = m_{de} = M_V v^2/2\,c^2$ (dynamic mass of the vacuum or dark energy) and $U = m_M = 2\,G{m_U^0}^2/c^2 R_U$ (gravitational mass).

Considering R_G as the starting point, for any historic value of R_U and v, the results from Eq. (25) are always $L = 1.23 \times 10^{28}$cm $= 1.32 \times 10^{10}$ly.

The dark energy density is:

$$\rho_{darkenegy} = \frac{m_{de}}{Universe\,volume} = \frac{3 M_V v^2}{8\pi R_U^3 c^2} \tag{26}$$

4.2 Physical meaning of the cosmological term

The cosmological constant Λ, was designed exclusively for static model of universe. Its origin, was from an arbitrary *ad-hoc* constant of integration (Peebles P.J.E. 1993), and its negative sign, gives it the meaning of a repulsive "*antigravity*". Because of its static non-expansive nature, its implementation is ineffective for any previous stage in the evolution of the universe. Moreover, this repulsive term acts only on the space itself, but not acting on the matter. For this properties, it does not gravitates.

As a consequence of its static origin, the Eq.(25) is used with all terms as constants

$$L_{\max} = \frac{M_V v^2}{2G{m_U^0}^2} R_U^2$$

Multiplying both terms by $\frac{4\pi R_U c^2}{3}$ and reordering

$$\frac{3c^2}{8\pi R_U\, L_{\max}} = \frac{2\,G\rho_{m_U^0}\; m_U^0\, c^2}{M_V\, v^2} \qquad \text{Being } \rho_{m_U^0} = \frac{3m_U^0}{4\pi\, R_U^3} \simeq \rho_\Lambda = \rho_V$$

At present time $R_U^0 \simeq L_{\max}$, and $m_U^0 \simeq m_V \simeq m_\Lambda \simeq \frac{M_V v^2}{2\,c^2}$. Then

$$\Lambda = \frac{1}{{L_{\max}}^2} = \frac{-8\pi}{3\,c^2} G\,\rho_\Lambda \simeq \frac{8\pi}{3c^2} G\,\rho_{m_U^0} \tag{27}$$

As $c^2/{L_{max}}^2 \simeq {H_0}^2$

$${H_0}^2 = \frac{-8\pi\, G\,\rho_\Lambda}{3} \simeq \frac{8\pi\, G\,\rho_{m_U^0}}{3} \tag{28}$$

For $H_0 = 2.44 \times 10^{-18}\mathrm{s}^{-1}; \rho_\Lambda = 1.10 \times 10^{-29}\mathrm{g\,cm}^{-3}$

This means definitely, that the origin of the cosmological constant proceed from the referential energy of the vacuum $m_V = m_{de}$ (Eq. 23).

It is very evident that the cosmological constant Λ do no shows evolution; it not has pass or future: it is totally adjusted to the present time. Clearly it states that the cosmological constant is directly imposed by the present critical mass.

After the discovery of the expansive universe by Hubble, the cosmological constant was abandoned, and Einstein himself considered it as the worst *schlupfen* (blunder) of his life.

However, in the last 25 years, the cosmological constant was retaken and currently is considered as one of the greatest star than shines in the sky of the standard model of cosmology.

4.3 Photons number density of the CMB radiation

The energy spectrum of the CMB radiation registered at present, whose mean wavelength is 0.105 cm, represents a huge magnified copy of the photons produced by annihilation of $m_p^\pm$ and $m_e^\pm$ (matter - antimatter particles). The number of photons η_γ of the CMB radiation per cm^{-3} may be easily calculated from the Wien's Law which related the wavelength with temperature.

$$T \cdot \lambda_{CMB} = \frac{2\pi\,\hbar\,c}{5k} \quad \text{Reordering} \quad \frac{2\pi}{5\lambda_{CMB}}(cm^{-1}) = \frac{k\,T}{\hbar\,c}(cm^{-1}) \tag{29}$$

In order to obtain it, is only necessary to apply in both terms the sphere formula:

$$\eta_\gamma = \frac{6\pi^2\,\mathrm{cm}^{-3}}{(5\lambda_{CMB})^3} = 0.47374\,cm^{-3} \cdot \lambda_{CMB^{-3}}; \eta_\gamma = \frac{3}{4\pi}\left(\frac{k\,T_{CMB}}{\hbar\,c}\right)^3 cm^{-3} = 19.8^\circ\mathrm{K}^{-3} cm^{-3} T^3 \tag{30}$$

e.g. $T_{CMB} = 2.73^\circ$K and $\lambda_{CMB} = 0.105$cm; $\eta_\gamma = 404$ photons$\cdot$cm^{-3} and $\eta_\gamma = 408$ photons$\cdot$cm^{-2}.

Otherwise, the Universe photons number N_γ is a constant of nature since $T_{CMB} \cdot R_U$ and R_U/λ_{CMB} are constants:

$$k = 1.38 \times 10^{-16}\ \mathrm{erg} K^{-1}$$

$$N_\gamma = \left(\frac{2\pi R_U}{5\lambda_{CMB}}\right)^3 = \left(\frac{k\,T_{CMB} R_U}{\hbar\, c}\right)^3 = 3.12 \times 10^{87} \tag{31}$$

This equality shows the invariance of N_γ throughout the evolution of R_U. Multiplying the Eq. (30) by kT/c^2, we can arrive to the Universe density of the CMB radiation in g·cm^{-3}.

$$\rho_{CMB} = \frac{3}{4\pi}\frac{(kT_{CMB})^4}{\hbar^3 c^5} = 3.00 \times 10^{-36} g\, cm^{-3} {}^\circ\mathrm{K}^{-4} T^4 \tag{32}$$

Clearly in the expansion process from a temperature of $2 \times 10^9\,^\circ$K to $2.73\,^\circ$K, a huge transfer of energy to the own created space is produced. This means that all the cosmological empty space is entirely energy in itself (dark energy).

4.4 Dynamic repulsive density of the empty space

According to the foundations indicated in Sects. 3.3 and 4.3, it is possible to establish the density of the expanding empty space in terms of CMB rad. temperature evolution.

Because of its expansive origin (from annihilation of matter-antimatter particles) the cosmological vacuum is a repulsive "antigravitational" entity. Then, at the present temperature of the CMB radiation (2.73 °K), the vacuum density of dark energy is equivalent to

$$\rho_{de} = 102\left(m_p^+ + m_p^- + m_e^- + m_e^+\right) \cdot c^2 = 204 m_p^\pm \cdot c^2 = 0.31\ \mathrm{erg{\cdot}cm^{-3}}$$

$$\text{Since: } \left(m_p^+ + m_p^-\right) \simeq m_p^\pm \text{ and } \tfrac{1}{2} n_\gamma = \tfrac{1}{2}408 = 204$$

From Eq. (32) the static vacuum dark density at any temperature below $1 \times 10^{9\circ}$K is

$$\rho_{de} = \frac{1}{2}\eta_\gamma \cdot m_p^\pm \text{ and } \rho_{de} = \frac{3m_p^\pm}{8\pi}\left(\frac{k\,T}{\hbar\, c}\right)^3 = 1.66 \times 10^{-23} g \cdot \mathrm{cm}^{-3} \cdot {}^\circ\mathrm{K}^{-3} \cdot T^3 \tag{33}$$

Multiplying by $v^2/2c^2$ (see Eq. 1 and 22) we have the vacuum dark energy density in $g \cdot cm^{-3}$ for temperatures bellow to 5,400$^\circ K$.

$$\rho_{de} = \frac{3m_p^\pm \cdot v^2}{16\pi\, c^2}\left(\frac{k\,T}{\hbar\, c}\right)^3 \tag{34}$$

As the temperature $T_G = 5,400^\circ$K marks a point of departure from the gravitational collapse, by use of the Eq.(21), $v/c = T/2T_G$ we have:

$$\rho_{de} = \frac{3m_p^\pm \cdot T_{CMB}^5}{64\pi\, T_G^2}\left(\frac{k}{\hbar\, c}\right)^3 = 7.10 \times 10^{-32}\ \mathrm{g} \cdot \mathrm{cm}^{-3} \cdot {}^\circ\mathrm{K}^{-5} \cdot T^5 \tag{35}$$

At the present time $T = T_{CMB}$, $radiation = 2.73^\circ$K and $\rho_{de} = 1.08 \times 10^{-29} g\, cm^{-3}$.

Time seg.	Temp. °K	R_U cm	z	ρ_b g·cm^{-3}	ρ_{de} g·cm^{-3}	ρ_{CMB} g·cm^{-3}	H Km·s^{-1} R_G^{-1}	H Km·s^{-1} Mps^{-1}
2.13×10^{14}	5,400	6.20×10^{24}*	2,000	8.96×10^{-20}	3.25×10^{-13}	2.55×10^{-21}	3.0×10^{5}	1.50×10^{5}
3.34×10^{14}	3,350	1.01×10^{25}	1,280	1.97×10^{-20}	3.00×10^{-14}	3.82×10^{-22}	2.0×10^{5}	9.69×10^{4}
4.27×10^{14}	2,625	1.28×10^{25}	960	9.68×10^{-21}	8.85×10^{-15}	1.44×10^{-22}	3.0×10^{4}	1.45×10^{4}
—	—	—	—	—	—	—	—	—
5.33×10^{16}	21.00	1.60×10^{27}	7.68	4.96×10^{-27}	2.40×10^{-25}	5.89×10^{-31}	1,200	581
6.40×10^{16}	17.50	1.92×10^{27}	6.40	2.87×10^{-27}	1.16×10^{-25}	2.84×10^{-31}	1,000	484
7.47×10^{16}	15.00	2.24×10^{27}	5.55	1.88×10^{-27}	5.38×10^{-26}	1.53×10^{-31}	856	415
8.55×10^{16}	13.10	2.56×10^{27}	4.80	1.28×10^{-27}	2.73×10^{-26}	2.92×10^{-32}	750	363
—	—	—	—	—	—	—	—	—
4.10×10^{17}**	2.73	1.23×10^{28}	0	1.09×10^{-29}	1.07×10^{-29}	1.68×10^{-34}	156	75.5

* Gravitational collapse ** Present time

Table 1: The numerical result obtained from Eq.(23) show that the expansion rate is c when the radius of the universe is $R_G = 6.20 \times 10^{24} = 2$ Mps. Making use of this scale, the expansion rate progressively decreases up to the present value of 150 km·s^{-1}·(2 Mps)$^{-1}$ or 75.5 km·s^{-1}·Mps^{-1}. By use of the Eqts. (17) and (35), ρ_b and ρ_{de} are obtained. Using the Eq.(38) or $1/H$, time is accomplished, the ρ_{CMB} result from Eq. (32)

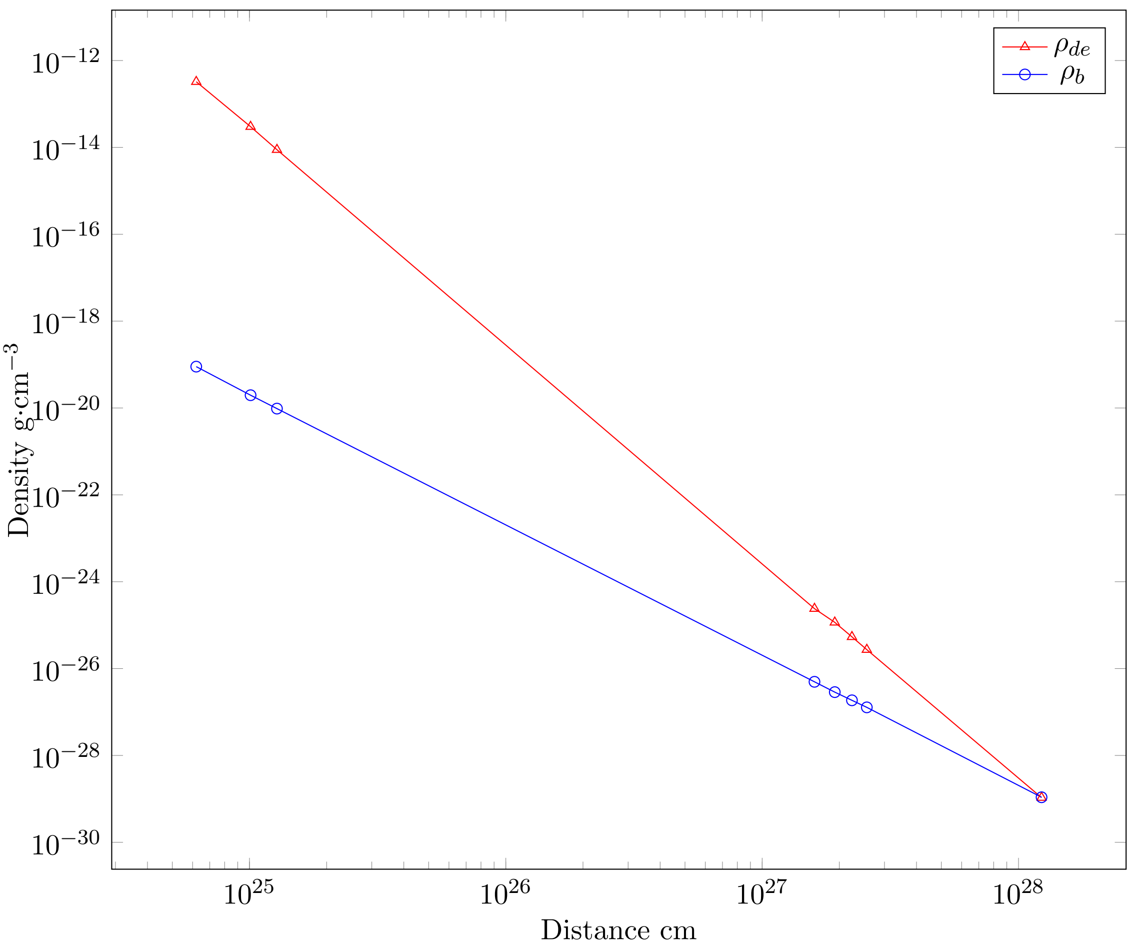

Figure 1: The red line represent the dynamical density of the vacuum ρ_V. The blue line represent the density of the baryonic matter ρ_b. The numerical values are from Table 1.

5 Cosmological time - temperature relationship

5.1 Expansive ideal gas of photons

Since the whole cosmological space is a black body contour full of radiation and a very repulsive-expansive bubble of energy, the ideal gas equation as a kinetic expression is a suitable tool to resolve this problem directly.

$$V = \frac{N_\gamma kT}{\rho_\gamma c^2} \tag{36}$$

As the relation volume to radius is

$$R = \left(\tfrac{3}{4\pi}V\right)^{1/3}$$

Since $t = \frac{R_U}{c}$ and $\rho_{CMB} = \rho_\gamma$, substituting ρ_γ by Eq. (32), and dividing by c, we can determine the universe age in terms of the radiation temperature

$$t = \frac{\hbar}{kT} N_\gamma^{1/3} \tag{37}$$

If $N_\gamma = 3.12 \times 10^{87}$ (Eq. 32), it means

$$T = 1.12 \times 10^{18}\mathrm{s} \cdot {}^\circ\mathrm{K} \cdot \mathrm{t}^{-1} \tag{38}$$

6 Formulation of the density of energy from the Casimir force

6.1 Conversion of force into energy density

In the plane - plane geometry, the areas of both planes are L^2, and the pressure is independent of the extension of the areas. But in the sphere-plane geometry, when the radius of curvature is relatively high, (10 - 15 cm), this surface can be considered approximately as a plane. However the spherical surface causes an optical dispersion, which geometrically can be corrected by the factor 2 [3]. Then, for the two system the Casimir pressure is expressed by

plane - plane	**plane - sphere**
$\rho = \frac{\pi^2 \hbar c}{240 d^4} = 1.30 \times 10^{-18}\,\mathrm{erg\,cm}\,d^{-4}$	$\rho = \frac{\pi^2 \hbar c}{120 d^4} = 2.60 \times 10^{-18}\,\mathrm{erg\,cm}\,d^{-4}$

(39)

At the point of closest separations, $d = 6.0 \times 10^{-5}$ cm, the static density of energy is $0.20\, erg\, cm^{-3}$. For the micro electromechanical MEMS devices, the Casimir force and the Coulombic force are expressed by the following equation.

$$F_{Cas} = \frac{\pi^3 \hbar c R}{360 d^3} = 2.73 \times 10^{-20}\,\mathrm{dyn\,cm^3}\,d^{-3} \tag{40}$$

$$F_{Coul} = \frac{\pi \varepsilon_0 R V^2}{d} = 5.15 \times 10^{-10}\,\mathrm{dyn\,cm}\,d^{-1} \tag{41}$$

[3]The ratio between hemisphere surface / circle surface= $\frac{2\pi R^2}{\pi R^2} = 2$

Being $R = 0.01\,\text{cm}$; $\varepsilon_0 = 7.97 \times 10^{-2}$ (dimensionless in c g s); $V = 0.136V$ ($1V = 3.333 \times 10^{-3}\,\text{cm}^{1/2}\,\text{g}^{1/2}\,\text{s}^{-1}$); $\hbar = 1.05457 \times 10^{-27} erg \cdot sec$
In all microelectromecanics systems (Chan and similar) at the "point of closest approach" (7.6×10^{-6}cm) the Casimir force ($\sim 7.0 \times 10^{-5}$ dyn) is equal to the Coulombic force at 136 mV. This point, marks a limit, due to the fact that in all experiments a constant is obtained when the different results are derived in the form of energy density. Then, we can infer that all these "coincident" results, **obeys the fact that are measurements of a constant of nature** (Table 2).
The Casimir energy density within the boundaries of any microscopic cavity appears from an initially attractive interaction, which then, at the point of closest separation begins to have a repulsive perturbation. Reaching this balance, allows us to register the *static* energy density.
If we consider in Eqts. (41) and (42) as $2R^2$ to be the effective area of the sphere, the static Casimir and the Coulombic pressure are:

$$\rho_{Cas} = \frac{\pi^3 \hbar c}{720 R d^3} = 1.36 \times 10^{-16}\,\text{erg}\; d^{-3} \tag{42}$$

$$\rho_{Coul} = \frac{\pi \varepsilon_0 V^2}{2Rd} = 2.57 \times 10^{-6}\,\text{dyn}\,\text{cm}^{-1}\; d^{-1} \tag{43}$$

e.g $d = 7.57 \times 10^{-6}$cm; $\rho_{Cas} = 0.314\,\text{erg}\,\text{cm}^{-3}$; $\rho_{Coul} = 0.337\,\text{erg}\,\text{cm}^{-3}$

Reference	Geometry	Sphere radius cm	Closest sep cm	Static Ener. Dens. ρ_{S_t} erg cm^{-3}	Dynamic dens. ρ_Λ g cm^{-3}
Sparnaay; 1958	plane - plane	—	$\sim 5.0 \times 10^{-5}$	∼0.21	$\sim 7.42 \times 10^{-30}$
Lamoreaux; 1997	plane - sphere	11.5	6.0×10^{-5}	0.20	7.06×10^{-30}
Roy - Mohideen; 1999	plane - sphere	0.01	6.5×10^{-6}	0.50	1.77×10^{-29}
Chan; 2001	plane - sphere	0.01	7.6×10^{-6}	0.31	1.10×10^{-29}
Bressi; 2002	plane - plane	—	5.0×10^{-5}	0.21	7.42×10^{-30}
Lisanti; 2005	plane - sphere	0.01	7.6×10^{-6}	0.31	1.10×10^{-29}
Iannuzzi; 2007	plane - sphere	0.01	7.0×10^{-6}	0.40	1.40×10^{-29}
Capasso; 2007	plane - sphere	0.01	7.5×10^{-6}	0.32	1.13×10^{-29}
Kim; 2009	plane - sphere	15.1	5.0×10^{-5}	0.41	1.48×10^{-29}
Sushkov; 2011	plane - sphere	15.6	7.0×10^{-5}	0.11	3.90×10^{-30}
Average				0.297	1.06×10^{-29}

Table 2: The results of static energy density ρ_{st} is obtained from Eqts. (40) and (43). The dynamic energy density ρ_Λ is obtained from Eqts. (49), (50) and (51)

The Casimir static experimental energy density is coincident with the obtained from the theoretical equation (34) $\rho_{vac} = 0.015\ \text{erg} \cdot \text{cm}^{-3} \cdot {}^\circ\text{K}^{-3} \cdot T^3$. Also the density is:
$\rho_V = \frac{0.296\ \text{erg}\cdot\text{cm}^{-3}}{C^2} = 3.30 \times 10^{-22} g \cdot \text{cm}^{-3}$
Which is the same:

$$\rho_V = \frac{3 M_{U\,vacuum}}{4\pi R_U^3} = 3.30 \times 10^{-22} g \cdot \text{cm}^{-3} \tag{44}$$

7 Cosmological H and HeI highly excited Rydberg atoms

7.1 Thermic equilibrium between the CMB radiation and the Rydberg atoms

The cosmological space, acts as an entity since it expands by itself as an active element (dark energy). This phenomenon becomes clear by stretching the N_γ photons of the CMB radiation which permeates the whole cosmological space in a homogeneous form.

In the halos of galaxies, the intergalactic medium and the medium between clusters and interclusters, the undetectable dark matter is formed by highly excited Rydberg hydrogen atoms at $n = 80 - 90$; $Z = 1$, and by helium Rydberg atoms $Z = 1.34$, at $n = 110 - 120$ which are in perfect thermic equilibrium with the CMB radiation. These Rydberg atoms can be detected by its gravitational effect as dark matter.

$$\text{From Eq. (6); } T = \frac{\Delta E}{k} = \frac{Z^2 m_e e^4}{2\hbar k}\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \tag{45}$$

$$\text{For } H_{n_i} \rightleftarrows H_{n_j} \frac{\Delta E_H}{k}; Z = 1 \text{ result } T = 157,100°K \left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \tag{46}$$

In the same way for $He_{n_i} \rightleftarrows He_{n_j}; Z = 1.34$

$$\text{We have: } T = 282,130°K \left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \tag{47}$$

As a representative example, in the Milky Way halo, there are 3,200 photons of the CMB radiation for one hydrogen Rydberg atom and 12,800 CMB radiation photons fo rone Rydberg He atom.

Hydrogen Rydberg atom				Helium Rydberg atom			
n_i	n_j	Temp. °K	$\lambda_{CMB} rad \cdot cm$	n_i	n_j	Temp. °K	$\lambda_{CMB} rad \cdot cm$
76	80	2.65	0.109	103	109	2.85	0.101
77	81	2.55	0.113	104	110	2.77	0.104
78	82	2.46	0.117	105	111	2.69	0.107
78	83	3.02	0.096	106	112	2.62	0.110
79	84	2.91	0.099	107	113	2.55	0.113
80	85	2.80	0.103	107	114	2.95	0.098
81	86	2.70	0.107	108	115	2.85	0.101
82	88	3.08	0.094	109	116	2.78	0.104
83	88	2.52	0.114	110	117	2.71	0.106
83	89	2.97	0.097	111	118	2.64	0.109

Table 3: According to Equations (46) and (47) we have some illustrative result for H and He Rydberg atoms. λ_{CMB} radiadion is obteined by use of the Eq. (30). $\lambda_{CMB} = 0.28804°K \cdot cm$

7.2 Dynamic expression form of the energy density

The transition from the radiative era, to the gravitational era, occurs after the baryonic mass surpasses the mass-energy of the radiation at $5,400$°K. At this epoch, the first clumps or clouds of gas were formed and this point marks the initial time of the Hubble parameter. From Eq. (23) the ratio between the Hubble constant at the present time, with respect its value at the gravitational collapse and other directly related parameters are equal to

$Y_D^2 = \frac{H_0^2}{2H^2} = \frac{v^2}{2c^2} = \frac{T_{CMB}^2}{8T_G^2} = \frac{R_G^2}{8R_U^2} = \frac{1}{8(z+1)^2}$

This index, defines the dynamic relativistic factor of the empty space in expansion. At present time ${Y_D}^2$ is

$$ {Y_D}^2 = \frac{1}{2}\left(\frac{75.4\text{km}\,\text{s}^{-1}\text{Mps}^{-1}}{c\ \text{km}\,\text{s}^{-1}\,\text{Mps}^{-1}}\right)^2 = \frac{1}{8}\left(\frac{2.73°\text{K}}{5,400°\text{K}}\right)^2 = \frac{1}{8}\left(\frac{6.4\times 10^{24}\text{cm}}{1.23\times 10^{28}\text{cm}}\right)^2 = \frac{1}{2}\frac{v^2}{c^2} = 3.18\times 10^{-8} \tag{48} $$

7.3 Expression of the dynamic density in the Casimir experimental results

The dynamic empty space density registered by the Casimir system is:

plane - plane

$$ \rho_{Cas} = \frac{\pi^2\,\hbar\,{Y_D}^2}{240\,c\,d^4} = 4.62\times 10^{-47}\,\text{g}\,\text{cm}\,d^{-4} \tag{49} $$

For $d = 5\times 10^{-5}$cm $\quad \rho_{Cas} = 7.40\times 10^{-30}$g cm^{-3}

plane - sphere

$$ \rho_{Cas} = \frac{\pi^2\,\hbar\,{Y_D}^2}{120\,c\,d^4} = 9.21\times 10^{-47}\,\text{g}\,\text{cm}\,d^{-4} \tag{50} $$

For $d = 5\times 10^{-5}$cm $\quad \rho_{Cas} = 1.50\times 10^{-29}$ g cm^{-3}

MEMS plane - microsphere

$$ \rho_{Cas} = \frac{\pi^3\,\hbar\,{Y_D}^2}{720\,R\,c\,d^3} = 4.82\times 10^{-45}\,\text{g}\,d^{-3} \tag{51} $$

For $d = 7.6\times 10^{-6}$cm $\quad \rho_{Cas} = 1.10\times 10^{-29}$ g cm^{-3}

7.4 CMB radiation anisotropies

Since the temperature of the CMB radiation and the density of the dark energy are directly related by Eq. 29 and 34, the anisotropies or fluctuations of the CMB radiation temperature must be caused by the density inhomogeneities which perveades between the cumulus and supercumulus of galaxies. According these equations 29 and 34, the fluctuations ratio can be expresses by the folowing form:

Fluctuation index $= \frac{\rho_{CMBradiation}}{\rho_{Darkenergy}} = \frac{3.00\times 10^{-36} g\cdot cm^{-3\circ}K^{-4}T_{CMB}^4}{5.29\times 10^{-31} g\cdot cm^{-3}T_{CMB}^3} = 5.67\times 10^{-6\circ}KT_{CMB}$
For $T = 2.725°K$

$$ \text{Fluctuation index} = \frac{\rho_{CMBradiation}}{\rho_{Darkenergy}} = 1.54\times 10^{-5} \tag{52} $$

This small ratio is in coincidence with the temperature fluctuation of the CMB radiation as well its expresses the spatial index in the anisotropies distribution of the dark matter and the dark energy. The same tiny fluctuations in the CMB radiation were registered by the Planck Mission which its experimental value is: $\sim \pm 1 \times 10^{-5 \circ} K$

In resume: the universe is formed by 0.05 of visible matter, 0.35 "dark" matter or Rydberg $H - H_e$ atoms and 0.60 of "dark" energy. The "dark" energy was originated at the very begining of the big bang process during the annhilation of near the same amount of matter - antimatter particles. This "dark" energy is the self energy of all the cosmological space in expansion (Sect. 3.3)

8 Conclusions

In natural science, particularly in any branch of the physics, it is extremely hard to construct a scientific framework on basis of generalized hypothetical and enigmatic unknown forms of matter energy. Hence, it is very unusual, as in the cosmological inflationary "standard model", it is considered as a natural fact, that ~95% of the matter-energy (near the whole) is in unknown hidden form.

Cosmology requires particles, radiation, space and energy; likewise, atomic physics and astrophysics require the same components. Since atomic physics and astrophysics do not make use of any hidden form of matter-energy, the theoretical cosmology must be free of these artifices. Given these failures, another physics, independent of the dogmas by the hegemonic standard model, would be essential and imperative. In this respect, a new interpretation of the experimental results of the Casimir effect, is a proof of this.

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