A classical explanation for the one-photon Mach-Zehnder experiment

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Abstract: In previous papers, we tried to show that the lack of an agreed-upon model of the electron may have contributed to an extraordinary convoluted explanation of the anomalous magnetic moment of an electron. We also suggested a classical electron model (the *Zitterbewegung* or the Dirac-Kerr-Newman model) may explain what is going on. The next logical step, of course, was to re-explore the classical idea of a photon to check if it can do what John Stewart Bell said cannot be done, and that is to explain interference at the level of a single photon. We think we have a classical explanation in this paper. If Mr. Bell was right, we must be wrong – we should be – but we don't see why.

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Introduction

The working title for this paper was a question: can a classical explanation explain quantum-mechanical interference? Bell's No-Go Theorem answers this question with a resounding: "No! Don't go there!" Pun intended. Now, we have not studied Bell's Theorem in detail but – after our discovery that one of Richard Feynman's famous thought experiments might be based on a flawed assumption[1] – we feel we are entitled to pursue the intuition and logic that we followed in previous papers – and that is to at least *try* to go there. We must admit we are very much encouraged by recent publications on the paradoxes that come out of weak measurement experiments – real as well as though experiments.[2] While these experiments are usually interpreted as a *confirmation* of orthodox quantum-mechanical theory, they make us think of Professor Ralston's evaluation of the current state of quantum mechanics:

> "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research."[3]

Let us recall the basics in regard to Feynman's thought experiment. Quantum physicists think of the elementary wavefunction as representing some theoretical spin-zero particle. Why? Spin-zero particles do not exist. All *real* particles have spin – electrons, photons, anything – and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. Hence, we can use the plus/minus sign of the imaginary unit in the $a{\cdot}e^{\pm i\theta}$ function to include spin in the mathematical description. Indeed, most introductory courses in quantum mechanics will show that both $a{\cdot}e^{-i\cdot\theta} = a{\cdot}e^{-i\cdot(\omega t - kx)}$ and $a{\cdot}e^{+i\cdot\theta} = a{\cdot}e^{+i\cdot(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction.[4] We would think physicists would then proceed to provide some argument why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[5]

Historical experience tells us theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things – think, for example, of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? Occam's Razor principle tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations, and the one obvious characteristic that would distinguish the two physical situations is the spin direction. Hence, we do *not* agree with the mainstream view that the choice is a matter of convention. Instead, we dare to suggest that the two mathematical possibilities represent identical particles with opposite spin. Combining this with the two possible

[1] Jean Louis Van Belle, *Euler's Wavefunction : The Double Life of –1*, 30 October 2018, http://vixra.org/pdf/1810.0339v2.pdf.
[2] See, for example: Aharonov, Cohen, and Elitzur, *Broadening the scope of weak quantum measurements II: Past and future measurement effects within a double Mach-Zehnder-interferometer setting*, https://arxiv.org/ftp/arxiv/papers/1207/1207.0667.pdf.
[3] John P. Ralston, *How to understand quantum mechanics* (2017), p. 1-10.
[4] These arguments usually show *other* waveforms – such as, for example, a real-valued sinusoid – are *not* acceptable.
[5] See, for example, the MIT's edX Course 8.04.1*x*, Lecture Notes, Chapter 4, Section 3.

directions of propagation (which are given by the +– or ++ signs in front of ω and k), we get the following table:

Table 1: Occam's Razor: mathematical possibilities versus physical realities

Spin and direction of travel	**Spin up (e.g. $J = +\hbar/2$)**	**Spin down (e.g. $J = -\hbar/2$)**
Positive *x*-direction	$\psi = a \cdot e^{-i\cdot(\omega t - kx)}$	$\psi^* = a \cdot e^{+i\cdot(\omega t - kx)}$
Negative *x*-direction	$\chi = a \cdot e^{-i\cdot(\omega t + kx)}$	$\chi^* = a \cdot e^{-i\cdot(\omega t + kx)}$

An added benefit of this interpretation is that we can now also associate some physical meaning with the complex conjugate of a wavefunction and – by extension – to various properties of quantum-mechanical operators, including hermiticity.[6] More generally speaking, we may say that we can finally offer a meaningful *physical* interpretation of the quantum-mechanical wavefunction.

What's wrong with Feynman's argument?

The above-mentioned redundancy in the quantum-mechanical mathematical framework – quantum physicists settling on a convention rather than exploiting the full power of Euler's function – is directly related to the logic leading to the rather uncomfortable conclusion that the wavefunction of spin-1/2 particles (read: electrons, practically speaking) has some weird 720-degree symmetry in space. This conclusion is uncomfortable because we cannot imagine such objects in space without invoking the idea of some kind of relation between the subject and the object (the reader should think of the Dirac belt trick here). It has, therefore, virtually halted all creative thinking on a physical interpretation of the wavefunction.

We have detailed Feynman's mistake in the above-mentioned paper and, hence, we do not want to repeat ourselves here. Let us just lift out the crucial logical error in the argument. It concludes a rather long-winded argument involving a thought experiment with three beam splitters (*S*, *T* and *U*) – placed at varying angles – in succession. We should probably note that the thought experiment involves spin-1/2 particles (the beam splitters are Stern-Gerlach apparatus). However, the analysis extends to the analysis of photons too because, while photons are spin-one particles, they do *not* have a zero-spin state. Hence, the Stern-Gerlach apparatuses in Feynman's thought experiment can be replaced by photon beam splitters.[7] Again, we refer the reader to our paper or, better, Feynman's original argument for what precedes this conclusion:

> "This result ($C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$) is just the original state all over again. Both amplitudes are just multiplied by −1 which gives back the original physical system. (It is again a case of a common phase change.) This means that if the angle between *T* and *S* in (b) is increased to 180°, the system (with respect to *T*) would be indistinguishable from the zero-degree situation, and the particles would again go through the (+) state of the *U* apparatus. At 180°, though, the (+) state of the *U* apparatus is the (−*x*) state of the original S apparatus. So a

[6] See the above reference: : Jean Louis Van Belle, 30 October 2018, *Euler's wavefunction: the double life of –1*, http://vixra.org/pdf/1810.0339v2.pdf.

[7] We find the fact that photons are spin-one particles without a zero-spin state (see, for example, Feynman's *Lectures*, III-11, footnote 1 as well as the context of this footnote) rather striking. It is weird that quantum theorists do not exploit it to try to be somewhat more creative in their thinking.

(+x) state would become a (−x) state. But we have done nothing to change the original state; the answer is wrong. We cannot have m = 1."[8]

This is where our physical interpretation (which, rather than making an arbitrary choice, maps all *mathematical* possibilities to all possible *physical* situations) makes the difference. The $C'_{up} = -C_{up}$ and $C'_{down} = -C_{down}$ do represent two different realities – two different *physical* states, that is. We do *not* a common phase change here. We write, somewhat enigmatically: $e^{+i\pi} \neq e^{-i\pi}$. The former ($e^{+i\pi}$) is a counterclockwise rotation. The latter is ($e^{-i\pi}$) is clockwise. In short, there are two different ways to get from +1 to −1 (and vice versa, of course), as illustrated below.[9]

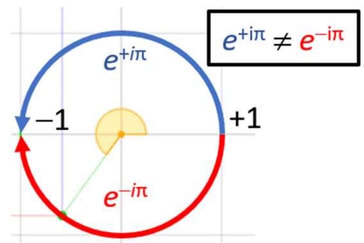

Figure 1: $e^{+i\pi} \neq e^{-i\pi}$

Hence, $a \cdot e^{+i\theta}$ and $a \cdot e^{-i\theta}$ represent opposite spin states, as illustrated below.

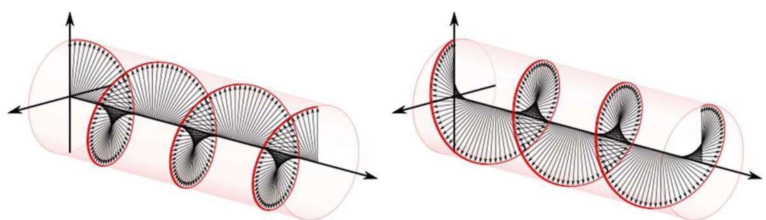

Figure 2: $a \cdot e^{+i\theta}$ and $a \cdot e^{-i\theta}$

Exploiting the full power of Euler's function opens up a whole new realm of interpretations. In the above-mentioned paper we argue, for example, that we can now interpret the Hermiticity condition as a *physical* reversibility condition – and we are not talking mere time symmetry here: reversing a physical process is like playing a movie backwards and, hence, we're talking CPT symmetry here. However, we do not have the time and space here to expand on that. We went off on a tangent and, hence, let us gradually move back to the topic of this paper. However, we need to make one more *detour* before we get there.

Physical interpretations of the wavefunction

The following series of diagrams and illustrations summarizes some of what we covered in our previous papers on a physical interpretation of the wavefunction. We refer to our previous papers for a detailed discussion of each of these.[10] Here we will just sum up the basics. We had a *Zitterbewegung* model, in which the elementary wavefunction represents a pointlike *charge* with zero rest mass and which,

[8] Feynman's *Lectures*, Vol. III, Chapter 6, Section 3.

[9] Mathematicians will cry wolf, of course, but our logic has no flaws as far as we can see: the +x state in Feynman's thought experiment effectively becomes a −x state when rotating the set-up over 180 degrees.

[10] See our series of viXra papers (http://vixra.org/author/jean_louis_van_belle). If we would have to choose one which sort of sums most, we would select our *Layered Motions: The Meaning of the Fine-Structure Constant* (http://vixra.org/pdf/1812.0273v3.pdf).

therefore, moves at the speed of light. This model explains Einstein's energy-mass equivalence relation in terms of a two-dimensional oscillation. The radius of the oscillation is the Compton radius of the electron. The *Zitterbewegung* electron – which combines the idea of a pointlike charge and Wheeler's idea of mass without mass[11] – can then be inserted into Bohr's quantum-mechanical model of an atom, which can also be represented using the elementary wavefunction. We have a different force configuration (because of the positively charged nucleus, we have a centripetal force now – as opposed to the tangential *zbw* force) but Euler's $a \cdot e^{\pm i\theta}$ function still represents an actual *position vector* of an electron which – because it acquired a rest mass from its *Zitterbewegung* – now moves at velocity $v = (\alpha/n) \cdot c$.[12] This should suffice to explain diagram 1, 2 and 3 below.

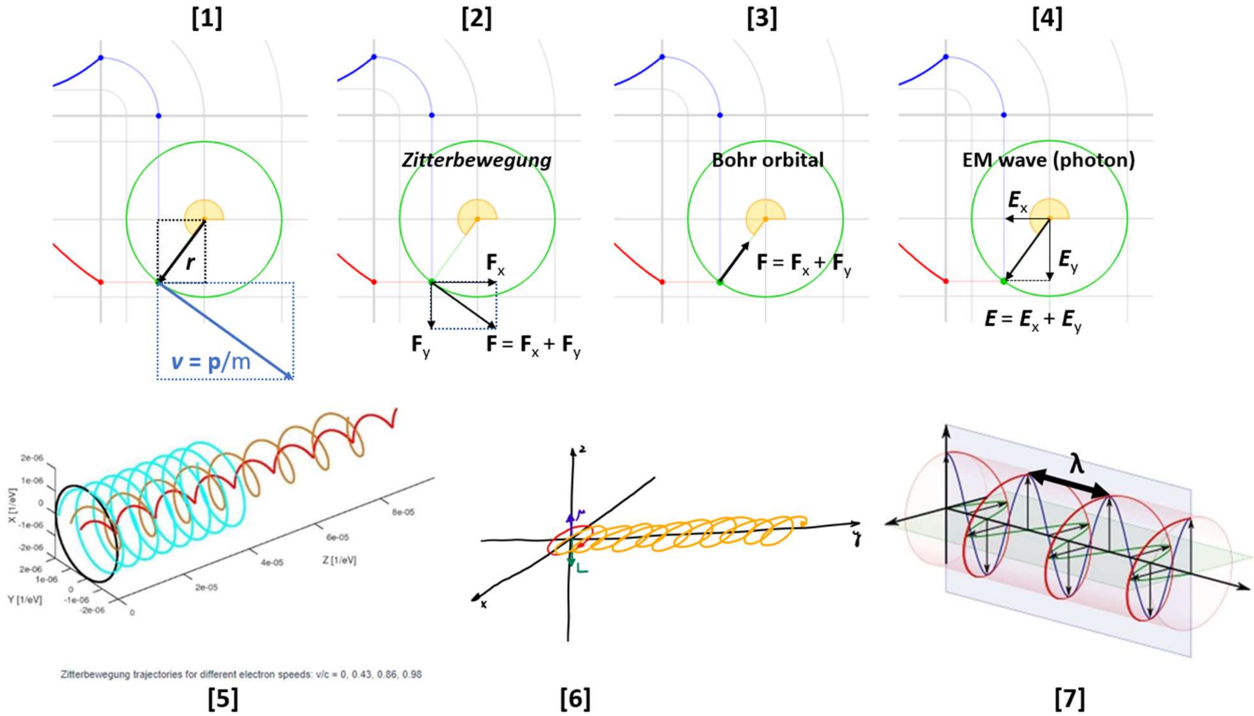

Figure 3: Physical interpretations of the wavefunction

Diagram 4 represents the idea of a photon that we get out of the Bohr model. We referred to it as the one-cycle photon model. The idea is the following. The Bohr orbitals are separated by a amount of (physical) action that is equal to h. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. Our photon will have to pack that, somehow. It will also have to pack the related energy, which is given by the *difference* of the energies of the two orbitals. This gives us not only the Rydberg formula – this we know since 1913 – but also a delightfully simple model of a photon and an intuitive interpretation of the Planck-Einstein relation ($f = 1/T = E/h$) for a photon. Indeed, we can do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

[11] The mass of the electron is the equivalent mass of the energy in the oscillation.

[12] The n is the number of the Bohr orbital (n = 1, 2, 3…). The α and c are the fine-structure constant and the speed of light. This formula comes out naturally of the Bohr model. See the referenced papers.

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the E = mc^2 mass-energy equivalence relation can be written as p = m*c* = E/*c* for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

We calculated the related force and field strength in our paper[13] so we won't repeat ourselves here. We would just like to point out something interesting – using diagram 5 above. Diagram 5 was copied from one of the many papers of Celani, Vassallo and Di Tommaso on the *Zitterbewegung* model, but we can use it to illustrate how and why we can associate a *radius* with the wavelength of a photon. Indeed, the diagram shows that, as an electron starts moving along some trajectory at a relativistic velocity – a velocity that becomes a more substantial fraction of *c*, that is – then the radius of the *Zitterbewegung* oscillation becomes smaller and smaller. In the limit ($v \to c$), it becomes zero ($r \to 0$), and the *circumference* of the oscillation becomes a simple (linear) wavelength in the process (this is illustrated in diagram 7, which provides a geometric interpretation of the *de Broglie* wavelength). Now, if we write this wavelength as λ_C (this is, of course, the Compton *wavelength*), then we get the usual relationship between a radius and a wavelength: $r_C = \lambda_C/2\pi$. This, then, provides an intuitive interpretation of the Eλ = *hc* equation for the photon and – more importantly – an intuitive explanation of the 2π factor in the formula for the fine-structure constant as a coupling constant. We write:

$$\alpha = \frac{2\pi \cdot \mathrm{q_e}^2}{h \cdot c} = \frac{k \cdot \mathrm{q_e}^2}{\hbar \cdot c} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F_\gamma} \cdot r_\gamma \cdot r_\gamma} = \frac{\mathrm{F_B} \cdot r_\mathrm{B}^2}{\mathrm{F_\gamma} \cdot r_\gamma^2} = \frac{\mathrm{E_B} \cdot r_\mathrm{B}}{\mathrm{E_\gamma} \cdot r_\gamma}$$

Needless to say, E_B, F_B, r_B and E_γ, F_γ, r_γ are the energies, forces and radii that are associated with the Bohr orbitals and our one-cycle photon.[14]

Finally – but this is a much finer and more philosophical point – diagram 5 gives us an intuitive geometric interpretation of one of the many ways in which Planck's quantum of action may express itself: the quantization of space. Indeed, at $v = 0$ (diagram 2), we have perfectly circular motion of a pointlike charge moving at the velocity of light, and we may associate Planck's quantum of action with the surface area of the circle. However, at $v = c$, the motion is purely linear – but we still think of the rotating field vector at the core (diagram 4). Planck's quantum of action now expresses itself space as a linear distance: the wavelength of the photon. We like to express this dual view as follows:

$$zbw\ electron\!: \mathrm{S} = h = \mathrm{p}_{Compton} \cdot \lambda_{Compton} = \mathrm{m_e} c \lambda_\mathrm{C} = \mathrm{m_e} c \cdot 2\pi r_\mathrm{C} = \mathrm{m_e} c \frac{h}{\mathrm{m_e} c} = h$$

$$photon\!: \mathrm{S} = h = \mathrm{p}_{photon} \cdot \lambda_{photon} = \frac{\mathrm{E_\gamma}}{c} \lambda_\gamma = \mathrm{m_\gamma} c \lambda_\gamma = \mathrm{m_\gamma} c \cdot 2\pi r_\gamma = \mathrm{m_\gamma} c \frac{hc}{\mathrm{E_\gamma}} = h$$

To be fully complete, we can add the same equation for the Bohr orbitals:

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[13] See the above-mentioned paper: Jean Louis Van Belle, *Layered Motions: The Meaning of the Fine-Structure Constant*, 23 December 2018, http://vixra.org/pdf/1812.0273v3.pdf.

[14] These formulas may appear as mind-boggling to the reader. If so, we advise the reader to first look at our other papers, whose pace is much more gradual.

$$n^{th}\ Bohr\ orbital\text{:}\ \mathrm{S} = n \cdot h = \mathrm{p}_n \cdot \lambda_n = \mathrm{m}_e v_n \lambda_n = \mathrm{m}_e \frac{\alpha c}{n} 2\pi \frac{n^2 \hbar}{\alpha \mathrm{m}_e c} = n \cdot h$$

We like these expressions because – in our humble view – there is no better way to express the idea that we should associate Planck's quantum of action (or any multiple of it) with the idea of a *cycle* in Nature.

We can imagine the reader is, by now, quite tired of these gymnastics and may question the relevance of this in light of the subject-matter of this paper. The answer is: we wanted to provide an introduction and, at the same time, refer to some history here. Prof. Dr. Alexander Burinskii – the author of the Dirac-Kerr-Newman electron model – told us he had started from the very same *Zitterbewegung* model in the year the author of this paper was born (1969). He published an article on this in the *Journal of Experimental and Theoretical Physics* (JETP)[15]. However, he told us he had always been puzzled about this one question: what keeps the pointlike charge in the *zbw* electron in its circular orbit? He, therefore, moved to exploring Kerr-Newman geometries – which has resulted in his Dirac-Kerr-Newman model of an electron.[16]

While the Dirac-Kerr-Newman model is a much more advanced model – it accommodates the theory of the supersymmetric Higgs field and string theory – we understand it does reduce to its classical limit, which is the *Zitterbewegung* model, if one limits the assumptions to general relativity and classical electromagnetism only. In our modest view, this validates our model. There is no mystery on the *zbw* force, we think: it is just the classical Lorentz force **F** = q**E** + q***v***×**B**. We, therefore, think that the *zbw* force results from the very same electric and magnetic field oscillation that makes up the photon. It is just the way that Planck's quantum of action expresses itself in space that is different here: we just get a different *form factor*, so to speak, when we look at the pointlike *zbw* charge. This, then, should solve Mr. Burinskii's puzzle – in our humble view, that is.

Finally, the attentive reader will have noticed that we did *not* discuss diagram 6. We inserted this diagram because when we considered the various *degrees of freedom* in interpreting Euler's wavefunction, we thought we should, perhaps, not necessarily assume that the plane of the circulatory motion – the *zbw* motion of the pointlike charge in the diagram – is perpendicular to the direction of propagation. In fact, the Stern-Gerlach experiment tells us the magnetic moment is literally *up* or *down*, which assumes the plane of the electric current should be parallel to the direction of motion. We like this alternative picture of the *zbw* electron because – intuitively – we feel it might provide us with some kind of *physical* explanation of relativistic length contraction: as velocities increase, the radius of the circular motion becomes smaller which, in this model, may be interpreted as a contraction of the size of the *zbw* electron.[17]

OK. This has been the longest introduction ever. It is time to have a closer look at the photon model now.

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[15] Burinskii, A.Y., *Microgeons with spin*, Sov. Phys. JETP 39 (1974) 193. One should note that Prof. dr. Burinskii refers to the *zbw* charge as an 'electron photon' or the 'electron EM wave'. However, its function in the model is basically the same. Prof. dr. Burinskii also told us that he was told *not* to refer to the *Zitterbewegung* model at the time, because it was seen as a classical model and, therefore, not in tune with the modern ideas of quantum mechanics.

[16] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008, https://arxiv.org/abs/hep-th/0507109. A more recent article of Mr. Burinskii (*New Path to Unification of Gravity with Particle Physics*, 2016, https://arxiv.org/abs/1701.01025, relates the model to more recent theories – most notably the "supersymmetric Higgs field" and the "Nielsen-Olesen model of dual string based on the Landau-Ginzburg (LG) field model." We admit we do not understand much of this – if anything at all.

[17] This is just a random thought at the moment. It needs further exploration.

The classical idea of a photon

Our analysis of Feynman's argument on the 720-degree of spin-1/2 particles should *not* be construed as a criticism of Feynman: it's not *his* argument – it's just orthodox QM. In general, we think Feynman's *Lectures* are still the best lectures on physics one can possibly get – if only because they make one *think* about what one is taught. We, therefore, borrow with very much pleasure two diagrams of his *Lectures* to complete the classical picture of a photon.

The first diagram (Feynman, I-34-9) brings in the oft-neglected magnetic field.[18] Feynman uses it to explain what he refers to as the 'pushing momentum' of light – which is more commonly referred to as radiation or *light pressure*. It is a bit of a strange term, because we are talking a *force*, really.

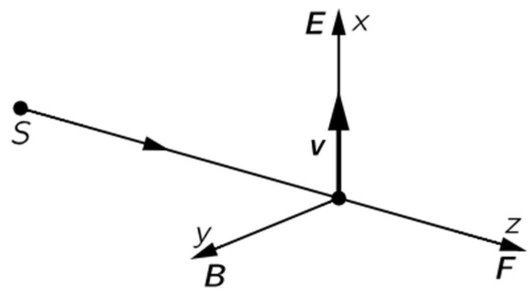

Figure 4: Feynman's explanation of the momentum of light

The basic idea is illustrated in another diagram, which is – unfortunately – separated from the diagram above by a full volume of lectures.[19] An electromagnetic wave – we take it to be a photon – will *drive* an electron, as shown below (Feynman, III-17-4). Hence, the magnetic force comes into play – as there is a charge and a velocity to play with now. 😊 The magnetic force – which is just denoted as ***F*** in the diagram above – will be equal to ***F*** = q***v***×**B**.

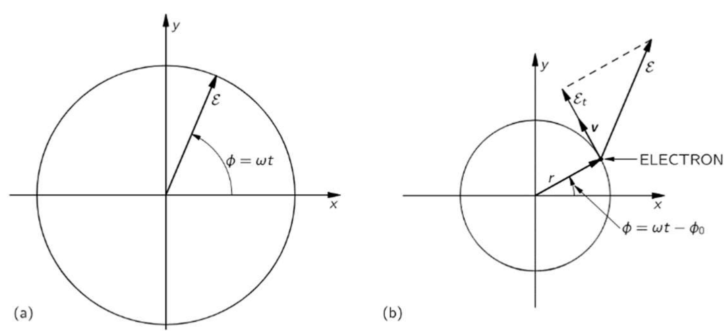

Figure 5: How the electric field of a photon might drive an orbital electron

Feynman then goes off on a bit of a tangent – analyzing the *average* force over time, which makes sense when one continues to take a classical view of an atom (or a Bohr (electron) orbital, practically speaking), and which gives some kind of meaning to the momentum of light.[20] The point is: his analysis fails to bridge classical mechanics with quantum mechanics because he fails to interpret Planck's quantum of action as a quantum: we're not only transferring energy here. We're also transferring angular momentum. In short: photon absorption and emission should respect *the integrity of a cycle*.

[18] Oft-neglected in the context of a *photon* model, that is.
[19] The first illustration comes from Feynman's volume on classical mechanics (Volume I), while the second comes from his lectures on quantum mechanics (Volume III). The volume in-between (Volume II) is on (classical) electromagnetism.
[20] Mr. Feynman gets some kind of explanation for the p = E/*c* relation out of his analysis.

What is this rule? Some new random interpretation of quantum mechanics? Yes. That is the one we offer here.

What happens when an electron jumps several Bohr orbitals? The angular momentum between the orbitals will then differ by several units of ħ. What happens to the photon picture in that case? It will pack the energy difference, but it will also pack several units of *ħ* (angular momentum) or – what amounts to the same – several units of *h* (physical action). In our humble opinion, we should still think of the photon a one-cycle oscillation. Hence, we do *not* think its energy will be spread over several cycles.[21] The two equations below need to make sense for *all* transitions[22]:

$$photon: \mathrm{S} = h = \mathrm{p}_\gamma \cdot \lambda_\gamma = \frac{\mathrm{E}_\gamma}{c}\lambda_\gamma = \frac{\mathrm{E}_\gamma}{f_\gamma} = \mathrm{E}_\gamma \cdot \mathrm{T}_\gamma$$

$$electron\ transition: \mathrm{S} = n \cdot h = \mathrm{p}_n \cdot \lambda_n = \mathrm{m}_e v_n \lambda_n = \mathrm{E}_n \cdot \mathrm{T}_n$$

The formulas above express the two most common expressions of what we referred to as the *Certainty Principle*. Pun intended.[23] We will leave it as an exercise for the reader to re-write these formulas in terms of a product of *force*, *distance*, and *time*.

So, what about Uncertainty, then? Nothing – absolutely *nothing* – of what we wrote above involves any uncertainty. It must be there somewhere, right? We would like to offer the following reflection. We have a few footnotes in previous papers, in which we suggest that Planck's quantum of action should be interpreted as a *vector*. The uncertainty – or the probabilistic nature of Nature, so to speak[24] – might, therefore, not be in its *magnitude*. We feel the uncertainty is in its *direction*. This may seem to be restrictive. However, because *h* is the product of a force (some *vector* in three-dimensional space), a distance (another three-dimensional concept) and time, we think we have the mathematical framework comes with sufficient degrees of freedom to describe any situation. Quantum-mechanical equations – such as Schrödinger's equation – should probably be written as vector equations.[25]

Linear and circularly polarized light

The photons above make for a circularly polarized beam. The spin direction may be left-handed or right-handed, as shown below.

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[21] When discussing the Mach-Zehnder experiment in the next version of our paper, we will bring a subtle but essential *nuance* to this point of view.

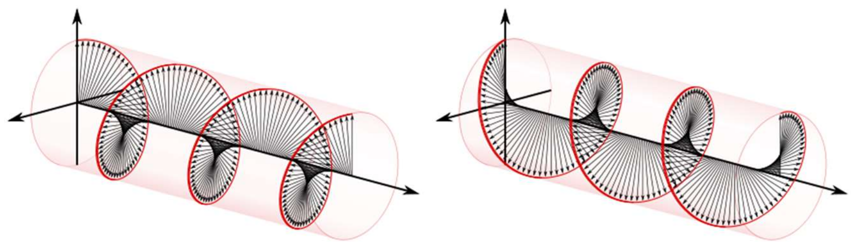
[22] The use of the same integer *n* for the *difference* in energy between Bohr orbitals might be confusing but we did not want to use another symbol – such as *m*, for example – because *m* would make one think of the fine-structure transitions (which we haven't discussed at all – not in this paper, not in previous one) and – more importantly – because we want to encourage the reader to think these things through for him- or herself. Symbols acquire meaning from the context in which they are used. We are tempted to go off on a tangent on Wittgenstein but we should restrain ourselves here. There is too much philosophy in this paper already. We advise the reader to critically cross-check the formula for electron transitions with what we wrote in previous papers. We warmly welcome comments.

[23] As we argued in previous papers, Planck's quantum of action should probably be interpreted as a *vector*. The uncertainty might not be in its *magnitude*. We feel the uncertainty is in its *direction*. Because *h* is the product of a force, a distance and time, we have a lot of dimensions to consider.

[24] A fair amount of so-called thought experiments in quantum mechanics – and I am not (only) talking the more popular accounts on what quantum mechanics is supposed to be all about – do *not* model the uncertainty *in Nature*, but on *our* uncertainty on what might actually be going on. Einstein was not worried about the conclusion that Nature was probabilistic (he fully agreed we cannot know everything): a quick analysis of the full transcriptions of his oft-quoted remarks reveal that he just wanted to see a theory that *explains* the probabilities. A theory that just *describes* them didn't satisfy him.

[25] We made a start with this in a previous paper: Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, http://vixra.org/pdf/1812.0202v1.pdf.

Figure 6: Left- and right-handed polarization[26]



We can think of these photons as the sum of two linearly polarized waves. We write:

$$cos\theta + i \cdot sin\theta = e^{i \cdot \theta} \text{ (RHC)}$$

$$cos(-\theta) + i \cdot sin(-\theta) = cos\theta - i \cdot sin\theta = e^{-i \cdot \theta} \text{ (LHC)}$$

Huh? *What is the geometry here?* It is quite simple. Let us spell it out so we have no issues of interpretation in the next section(s) of this paper. If ***x*** is the direction of propagation of the wave, then the ***z***-direction will be pointing upwards, and we get the ***y***-direction from the righthand rule for a Cartesian reference frame.[27] We may now think of the oscillation along the ***y***-axis as the cosine, and the oscillation along the ***z***-axis as the sine. If we then think of the imaginary unit *i* as a 90-degree counterclockwise rotation in the ***yz***-plane (and remembering the convention that angles (including the phase angle θ) are measured counterclockwise), then the right- and left-handed waves can effectively be represented by the wavefunctions above.

The point here is that easy visualizations like this strongly encourage us to think of a geometric representation of the wavefunction—if only because, conversely, one may also adopt the convention that the imaginary unit should be interpreted as a unit vector pointing in a direction that is perpendicular to the direction of propagation of the wave and one may then write the magnetic field vector as $\mathbf{B} = -i \cdot \mathbf{E}/c$.[28] The minus sign in the $\mathbf{B} = -i \cdot \mathbf{E}/c$. It is there because of consistency: we must combine a classical *physical* right-hand rule for **E** and **B** here as well as the *mathematical* convention that multiplication with the imaginary unit amounts to a *counter*clockwise rotation by 90 degrees. This allows us to re-write Maxwell's equations using complex numbers. We have done that in other papers, so if the reader is interested he can check there. [29] The point to note is that, while we will often sort of forget to show the magnetic field vector, the reader should always think of it – because it is an integral part of the electromagnetic wave: when we think of **E**, we should also think of **B**. Both oscillations carry energy.

The mention of energy brings me to another important point. As mentioned above, we think of a circularly polarized beam – and a photon – as a superposition of two linear waves. Now, these two linearly polarized waves will each pack *half* of the energy of the combined wave. It is a very important point to make because any classical explanation of interference – like the one we will offer in the next section – will need to respect the energy conservation law. Note that, while each wave packs *half* of the

[26] Credit: https://commons.wikimedia.org/wiki/User:Dave3457.

[27] Note the reference frame in the illustrations of the LHC and RHC wave – which we took from Wikipedia – is left-handed. Our argument will use a regular right-handed reference frame.

[28] As usual, we use **boldface** letters to represent geometric vectors – the electric (**E**) and magnetic field vectors (**B**), in this case. There is a risk of confusion between the energy E and the electric field E because we use the same symbols, but the context should make clear what is what.

[29] See, for example, Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, http://vixra.org/pdf/1812.0202v1.pdf.

energy of the combined wave, their (maximum) amplitude is the same: there is no change there. Let us briefly elaborate this point. The energy of any oscillation will always be proportional to (1) its amplitude (*a*) and (2) its frequency (*f*). Hence, if we write the proportionality coefficient as *k*, then the energy of our photon will be equal to:

$$\mathrm{E} = k \cdot a^2 \cdot \omega^2$$

What should we use for the amplitude of the oscillation here? It turns out we get a nice result using the *wavelength*[30]:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2 \frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

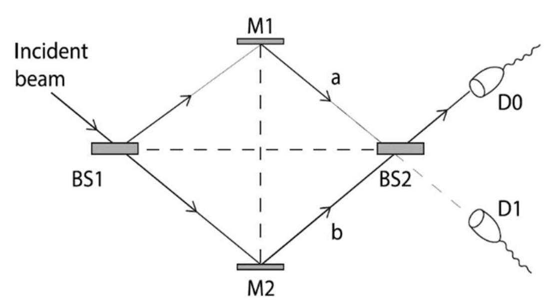
However, we should note this assumes a circularly polarized wave. Its linear components – the sine and cosine, that is – will only pack *half* of that energy. We can now offer the following classical explanation of the Mach-Zehnder experiment *for one photon only*.[31]

A classical explanation for the one-photon Mach-Zehnder experiment

We offered a geometric interpretation of the wavefunction. When analyzing interference in quantum mechanics, the wavefunction concept gives way to the concept of a probability amplitude which we associate with a possible *path* rather than a particle. The math looks somewhat similar but models very different ideas and concepts. Before the photon enters the beam splitter, we have one wavefunction: the photon. When it goes through, we have two probability amplitudes that – somehow – recombine and interfere with each other. What we want to do here is to explain this classically.

Let us look at the Mach-Zehnder interferometer once again. We have two beam splitters (BS1 and BS2) and two perfect mirrors (M1 and M2). An incident beam coming from the left is split at BS1 and recombines at BS2, which sends two outgoing beams to the photon detectors D0 and D1. More importantly, the interferometer can be set up to produce a precise interference effect which ensures all the light goes into D0, as shown below. Alternatively, the setup may be altered to ensure all the light goes into D1.

Figure 7: The Mach-Zehnder interferometer[32]



[30] We use the Eλ = h*c* ⇔ λ = h*c*/E identity. The reader might think we should use the amplitude of the electric and magnetic field. We could – the model is consistent – but it requires some extra calculations as we then need to think of the energy as some force over a distance. We refer to our papers for more details.
[31] We have written about this topic before (see: Jean Louis Van Belle, *Linear and circular polarization states in the Mach-Zehnder interference experiment*, 5 November 2018, http://vixra.org/pdf/1811.0056v1.pdf). Hence, we will only offer a summary of what we wrote there.
[32] Source of the illustration: MIT edX Course 8.04.1*x* (Quantum Physics), Lecture Notes, Chapter 1, Section 4 (*Quantum Superpositions*).

What is the classical explanation? The classical explanation is something like this: the first beam splitter (BS1) splits the beam into *two* beams. These two beams arrive in phase or, alternatively, out of phase and we, therefore, have constructive or destructive interference that recombines the original beam and makes it go towards D0 or, alternatively, towards D1.

When we analyze this in terms of a single photon, this classical picture becomes quite complicated – but we argue there is such classical picture. Our alternative theory of what happens in the Mach-Zehnder interferometer is the following:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

We will detail this in the next section, because what happens in a Mach-Zehnder interferometer is not all that straightforward. We should note, for example, that there are phase shifts along both paths: classical physics tells us that, on transmission, a wave does not pick up any phase shift, but it does so on reflection. To be precise, it will pick up a phase shift of π on reflection. We will refer to the standard textbook explanations of these subtleties and just integrate them in our more detailed explanation in the next section.[33] Before we do so, we will show the assumption that the two linear waves are orthogonal to each other is quite crucial. If they weren't, we would be in trouble with the energy conservation law. Let us show that before we proceed.

Suppose the beams would be polarized along the same direction. If *x* is the direction of propagation of the wave, then it may be the *y*- or *z*-direction of anything in-between. The magnitude of the electric field vector will then be given by a sinusoid. Now, we assume we have *two* linearly polarized beams, of course, which we will refer to as beam *a* and *b* respectively. These waves are likely to arrive with a phase difference – unless the apparatus has been set up to ensure the distances along both paths are exactly the same. Hence, the general case is that we would describe *a* by $cos(\omega \cdot t - k \cdot x) = cos(\theta)$ and *b* by $cos(\theta + \Delta)$ respectively. In the classical analysis, the difference in phase (Δ) will be there because of a difference of the path lengths[34] and the recombined wavefunction will be equal to the same cosine function, but with argument $\theta + \Delta/2$, multiplied by an *envelope* equal to $2 \cdot \cos(\Delta/2)$. We write[35]:

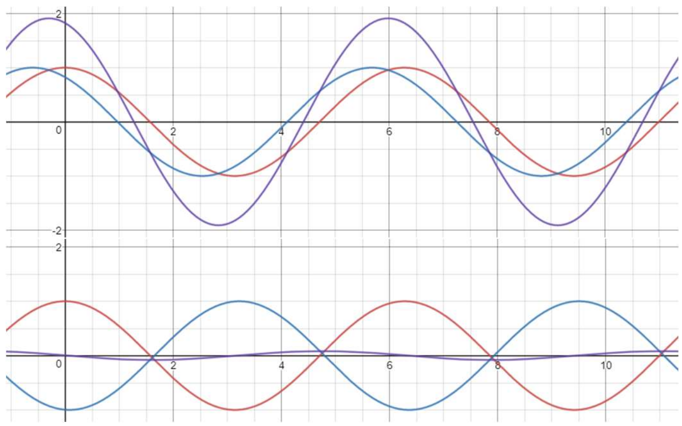
$$\cos(\theta) + \cos(\theta + \Delta) = 2 \cdot \cos(\theta + \Delta/2) \cdot \cos(\Delta/2)$$

We always get a recombined beam with the same frequency, but when the phase difference between the two incoming beams is small, its amplitude is going to be much larger. To be precise, it is going to be twice the amplitude of the incoming beams for $\Delta = 0$. In contrast, if the two beams are out of phase, the

[33] For a good *classical* explanation of the Mach-Zehnder interferometer, see: K.P. Zetie, S.F. Adams and R.M. Tocknell, January 2000, *How does a Mach–Zehnder interferometer work?* (https://www.cs.princeton.edu/courses/archive/fall06/cos576/papers/zetie_et_al_mach_zehnder00.pdf, accessed on 5 November 2018).
For a good quantum-mechanical explanation (interference of single photons), see – for example – the Mach-Zehnder tutorial from the PhysPort website (https://www.physport.org/curricula/QuILTs/, accessed on 5 November 2018).
[34] Feynman's path integral approach to quantum mechanics allows photons (or probability amplitudes, we should say) to travel somewhat slower or faster than *c*, but that should not bother us here.
[35] We are just applying the formula for the sum of two cosines here. If we would add sines, we would get $\sin(\theta) + \sin(\theta + \Delta) = 2 \cdot \sin(\theta + \Delta/2) \cdot \cos(\Delta/2)$. Hence, we get the same envelope: $2 \cdot \cos(\Delta/2)$.

amplitude is going to be much smaller, and it's going to be zero if the two waves are 180 degrees out of phase ($\Delta = \pi$), as shown below. That does not make sense because twice the amplitude means *four* times the energy, and zero amplitude means zero energy. The energy conservation law is being violated: photons are being multiplied or, conversely, are being destroyed.

Figure 8: Constructive and destructive interference for linearly polarized beams



Let us be explicit about the energy calculation. We assumed that, when the incoming beam splits up at BS1, that the energy of the *a* and *b* beam will be split in half too. We know the energy is given by (or, to be precise, proportional to) the square of the amplitude (let us denote this amplitude by *A*).[36] Hence, if we want the energy of the two individual beams to add up to $A^2 = 1^2 = 1$, then the (maximum) amplitude of the *a* and *b* beams must be $1/\sqrt{2}$ of the amplitude of the original beam, and our formula becomes:

$$(1/\sqrt{2})\cdot\cos(\theta) + (1/\sqrt{2})\cdot\cos(\theta + \Delta) = (2/\sqrt{2})\cdot\cos(\theta + \Delta/2)\cdot\cos(\Delta/2)$$

This reduces to $(2/\sqrt{2})\cdot\cos(\theta)$ for $\Delta = 0$. Hence, we still get *twice* the energy – $(2/\sqrt{2})^2$ equals 2 – when the beams are in phase and zero energy when the two beams are 180 degrees out of phase. This doesn't make sense.

Of course, the mistake in the argument is obvious. This is why our assumption that the two linear waves are orthogonal to each other comes in: we cannot just add the amplitudes of the *a* and *b* beams because they have different directions. If the *a* and *b* beams – after being split from the original beam – are linearly polarized, then the angle between the axes of polarization should be equal to 90 degrees to ensure that the two oscillations are independent. We can then add them like we would add the two parts of a complex number. Remembering the geometric interpretation of the imaginary unit as a counterclockwise rotation, we can then write the sum of our a and b beams as:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \Delta) = (1/\sqrt{2})\cdot[\cos(\theta) + i\cdot\cos(\theta + \Delta)]$$

What can we do with this? Not all that much, except noting that we can write the *cos*$(\theta + \Delta)$ as a sine for $\Delta = \pm\,\pi/2$. To be precise, we get:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \pi/2) = (1/\sqrt{2})\cdot(\cos\theta - i\cdot\sin\theta) = (1/\sqrt{2})\cdot e^{-i\cdot\theta}$$

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta - \pi/2) = (1/\sqrt{2})\cdot(\cos\theta + i\cdot\cos\theta) = (1/\sqrt{2})\cdot e^{i\cdot\theta}$$

[36] If we would reason in terms of average energies, we would have to apply a 1/2 factor because the average of the $\sin^2\theta$ and $\cos^2\theta$ over a cycle is equal to 1/2.

This gives us the classical explanation we were looking for:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

What about the 1/√2 factor? If the $e^{-i\cdot\theta}$ and $e^{i\cdot\theta}$ wavefunctions can, effectively, be interpreted geometrically as a *physical* oscillation in *two* dimensions – which is, effectively, our interpretation of the wavefunction[37] – then then each of the two (independent) oscillations will pack one half of the energy of the wave. Hence, if such *circularly* polarized wave splits into two *linearly* polarized waves, then the two linearly polarized waves will effectively, pack half of the energy without any need for us to think their (maximum) amplitude should be adjusted. If we now think of the ***x***-direction as the direction of the incident beam in the Mach-Zehnder experiment, and we would want to also think of rotations in the ***xz***-plane, then we need to need to introduce some new convention here. Let us introduce *another* imaginary unit, which we'll denote by *j*, and which will represent a 90-degree counterclockwise rotation in the ***xz***-plane.[38] We then get the following *classical* explanation for the results of the one-photon Mach-Zehnder experiment:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = cos\theta + i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta} = cos\theta - i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $-j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.

[37] We can assign the physical dimension of the electric field (force per unit charge, N/C) to the two perpendicular oscillations.

[38] This convention may make the reader think of the quaternion theory but we are thinking more of simple Euler angles here: *i* is a (counterclockwise) rotation around the *x*-axis, and *j* is a rotation around the *y*-axis.

Of course, we may also set up the apparatus with different path lengths, in which case the two linearly polarized beams will be out of phase when arriving at BS1. Let us assume the phase shift is equal to $\Delta = 180° = \pi$. This amounts to putting a minus sign in front of *either* the sine *or* the cosine function. Why? Because of the $cos(\theta \pm \pi) = -cos\theta$ and $sin(\theta \pm \pi) = -sin\theta$ identities. Let us assume the distance along the upper path is longer and, hence, that the phase shift affects the sine function.[39] In that case, the sequence of events might be like this:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = cos\theta + i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $j\cdot\sin(\theta + \pi) = -j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta} = cos\theta - i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta = = j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $-j\cdot\sin(\theta + \pi) = +j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.

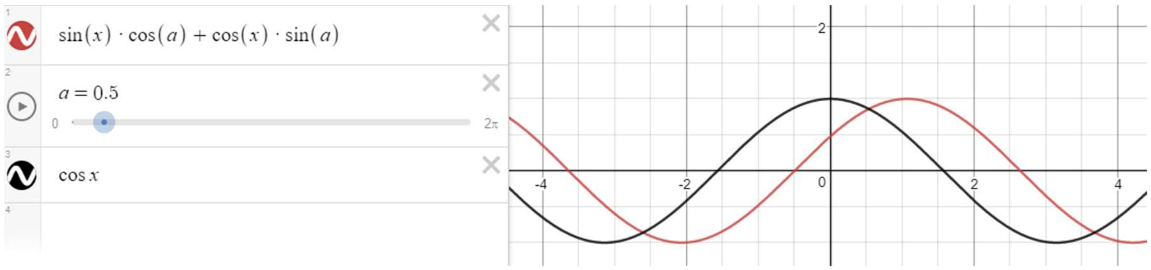
What happens when the difference between the phases of the two beams is not equal to 0 or 180 degrees? What if it is some random value in-between? Do we get an elliptically polarized wave or some other nice result? Denoting the phase shift as Δ, we can write:

$$cos\theta + j\cdot sin(\theta + \Delta) = cos\theta + j\cdot(sin\theta\cdot cos\Delta + cos\theta\cdot sin\Delta)$$

However, this is also just a circularly polarized wave, but with a random phase shift between the horizontal and vertical component of the wave, as shown below. Of course, for the special values $\Delta = 0$ and $\Delta = \pi$, we get $cos\theta + j\cdot sin\theta$ and $cos\theta - j\cdot sin\theta$ once more.

[39] The reader can easily work out the math for the opposite case (longer length of the lower path).

Figure 9: Random phase shift between two waves



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Mystery solved? Maybe. Maybe not. We just wanted to show that one should try to go everywhere. 😊

Jean Louis Van Belle, 29 December 2018

References

All references are in the text and/or footnotes.