

HOW $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ is generally, in the Galois sense solvable for a Kerr-Newman black hole affect questions on the opening and closing of a wormhole throat and the simplification of the problem, dramatically speaking, if $d=1$ (Kaluza Klein theory) and explaining the lack of overlap with the results when Applying the Gauss- Lucas theorem

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Abstract

First off, the term Δt is for the smallest unit of time step. Now, due to reasons we will discuss we state that. contrary to the wishes of a reviewer, the author asserts that a full Galois theory analysis of a quintic is mandatory for reasons which reflect about how the physics answers are all radically different for abbreviated lower math tech answers to this problem. I.e. if one turns the quintic to a quadratic, one gets answers materially different from when one applies the Gauss- Lucas theorem. So, despite the distaste of some in the physics community, this article pitches Galois theory for a restricted quintic. We begin our analysis of if a quintic equation for a shift in time, as for a Kerr Newman black hole affects possible temperature values, which may lead to opening or closing of a worm hole throat. Following Juan Maldacena, et. al, we evaluate the total energy of a worm hole, with the proviso that the energy of the worm hole, in four dimensions for a closed throat has energy of the worm hole, as proportional to negative value of (temperature times a fermionic number, q) which is if we view a worm hole as a connection between two black holes, a way to show if there is a connection between quantization of gravity, and if the worm hole throat is closed. Or open. For the quintic polynomial, we relate Δt to a $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ Quintic polynomial which has several combinations which Galois theoretical sense are generally solvable. We find that A_2 has a number, n of presumed produced gravitons, in the time interval Δt and that both A_1 and A_2 have an Ergosphere area, due to the induced Kerr- Newman black hole. If Gravitons and Gravitinos have the relationship the author purports in an article the author wrote years ago, as cited in this publication, then we have a way to discuss if quantization of gravity as affecting temperature T, in the worm hole tells us if a worm hole is open or closed. And a choice of the solvable constraints affects temperature, T, which in turn affects the sign of a worm hole throat. is far harder to solve. We explain the genesis of black hole physics negative temperature which is necessary for a positive black hole entropy, and then state our results have something very equivalent in terms of worm *ding* $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ we will be having $X = \Delta t$ assumed to be negligible, We then look at a quadratic version in the solution of $X = \Delta t$ so we are looking at four regimes for solving a quintic, with the infinitesimal value of Δt effectively reduced our quintic to a quadratic equation. Note that in the small Δt limit for $d = 1, 3, 5, 7$ we cleanly avoid any imaginary time no matter what the sign of T_{temp} is. In the case where we have $X = \Delta t$ assumed to be negligible, the connection in our text about coupling constants, if $d = 3$, may in itself for infinitesimal Δt lend toward supporting $d=3$. This is different from the more general case for general Galois solvability of $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$. $d \neq 1$ means we need to consider Galois theory. If $d=2, 4, 6$, need $T_{temp} < 0$ for coefficient A_1 to be greater than zero. If $d \neq 1$ and is instead $d=3, 5, 7$, there is an absence of general solutions in the Galois solution sense. This because if $d \neq 1$ $A_1 < 0$ whenever $d = 3, 5, 7$. And when $d=1$ in order to have any solvability one would need $X = \Delta t$ assumed to be infinitesimal in $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$.

Commented [AB1]: Reference to Appendix A deleted. 10 pages of Referees comments, also removed.

Keywords: Kerr Newman black hole, high-frequency gravitational waves (HGW), Solvable Quintic equations. Wormholes

I. Set up of the problem : precursor to answering innumerable issues

We assert that due to the fact that abbreviated lower math tech approximations to the derived quintic yield incommensurate very different physics answers to the delta t, Δt , problem, hence due to those very different answers, it is necessary to stop convenient approximations and to solve the problem via Galois theory. The godfather review of all solvable quintic problems is given here [1] and although a reviewer refused to learn the points raised, a solution to this specialized quintic is given in [2]. Whereas it will be the job of explaining in simple language why this is necessary. What we found is that if one changed the quintic to a quadratic, that the answers for the Δt problem look radically different from what we get when we take the derivative of the quintic, changing it, to understanding that golly gee, the following are not commensurate with each other. Note that the 2nd entry into Eq. (1) below comes from applying the Gauss- Lucas theorem [3] [4]. In the end the three different would be general solutions to Δt in these three equations look very different from each other. This is using manipulations of the original quintic as given by the author in [5]

$$\begin{aligned}
 (\Delta t)^3 + \frac{2A_1}{5} &= 0 \\
 \text{different } \Delta t \text{ answer from} & \\
 A_1 \cdot (\Delta t)^2 + A_2 &= 0 \qquad \qquad \qquad \{1\} \\
 \text{versus needing Galois solution to} & \\
 (\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 &= 0
 \end{aligned}$$

A reviewer did the assertion that a specialized solution to the third equation existed, whereas he was contravening several hundred years of Quintic polynomial research[6,7,8] . We will in the end answer that. And now to the physics of how the third equation the Quintic arose in the first place. [5]

This document will address the problem of a worm hole, as to the question of if its throat is opened or closed [9] , in doing so, the author references an earlier publication [5] which isolated a quintic polynomial in terms of delta t, i.e. Δt , and claims that a general solution in terms of what is called a restricted Quintic, with a fifth order term of \hbar helps determine the likelihood that a determination can be made as to if gravity is semi classical, or could be quantized. The quintic in question [5] is for a black hole[5] but if we make the assertion that a worm hole may connect two black holes, with information transmitted between them by quantum teleportation [10,11] we then assert that in a general sense the classical versus quantum nature of gravity of the worm hole may be ascertained. A subsidiary issue is, does the existence of a solution to Δt allow for a minimum uncertainty principle solution for gravitons via [12,13] $\Delta E \Delta t \doteq \hbar$, and if $\Delta E \Delta t \doteq \hbar$ is solved, do we have a criteria to state if gravity and gravitation is classical, semi classical or quantum? Note that the solution to the quintic, in [2] may have as noted by a reviewer, to have particular solutions which are trivial. We state for the record that such trivial solutions in no way contradict the complexity of the general solution and that the readers of this document should consult the Galois theory, and Abel's insolvability theorem [7,8]for general quintic solutions as a good reason as to disregard trivial solutions to the quintic as communicated to the author by a referee as given in [14] . i.e. one has to consider generalized solutions to the quintic according to problems, but if we go to higher dimensions, i.e. $d \neq 1$ gets very complicated fast. Hence this long article. And also, we will be

dealing with the reviewers [14] distaste for negative temperature, which is what started this inquiry in the first place due to comments raised by the reviewer in [14] is related to Kaluza Klein cosmology as given in reference [15] where we have an explanation as with respect to reference [16] and negative temperatures. As is noted in reference [16], negative temperatures when connected with the solution to the quintic as in [2] and [5] do, in certain cases which will be outlined connect solidly with negative temperatures. Contributing to positive entropy in black holes. This is relatable to the physics in [17,18] which will be in our article. [2] due to the range of values of A_1 and A_2 in [5]. This in turns of the additional dimensionality, d , for space times above four dimensions specifies T_{temp} . [5]. When $d=1$ we have Kaluza Klein type physics, and so it goes. The Kaluza Klein [15] situation with $d=1$ is by far and away the easiest situation to work with, and with the least .

Commented [AB2]: Here, reference [15] is included

Commented [AB3]: Appendix A removed, and so is the comment reference A itself

II. A reviewer’s complaints, and four cases to consider

The paper confused a reviewer who did not understand the references as to negative temperature. Hence, the first main part of the document is with regards to .negative temperature.[15]Then the idea of a general solution to a polynomial equation, the quintic. [2,5]

Before we do this temperature discussion , i.e. the necessary condition for picking the sign of T_{temp} is gone into, using results from [2], we can state then that (from the abstract) that, the following is what we adhere to.

There are here, though four cases to consider, and three of these arise if Δt is infinitesimally small, in which we have the following rules for the sign of T_{temp}

We are here, revising what is brought up in the discussion of Eq. (1) which is that we have three different would be equations to contend with which are linked to [5] and its results.

Case 1:

The first one, is for when we have an effective quadratic equation for Δt due to Δt being infinitesimally small. And we are avoiding at all costs having imaginary Δt

Note that for extra dimensions $d=1,3,5,7$, the coefficient A_1 is always less than zero, leading to no requirement for T_{temp} to be < 0 . . If $d=2,4,6$, need $T_{temp} < 0$ for coefficient A_1 to be less than zero . This will be shown to conflict with conditions for general Galois solvability of $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$.

Note, that special solutions for $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ are easy to obtain, as a reviewer noted, but that we are referring to completely general solutions, not specific special case solutions.

Now for the sign of T_{temp} , in terms of if we have $A_1 < 0$, and we claim this is also convenient as to obtain an easily determined value of , for $d = 1,2,3,4,5,6,7$, and a very small value of Δt

$$(\Delta t)^2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \cdot \left(\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \right)^{-1} \quad (2)$$

Note then that if $d=1$, as in Kaluza Klein theory, we have then that there are no questions of imaginary time, and also no T_{temp} restrictions. In answer to one of the reviewer's questions, we are avoiding having imaginary time, hence, this puts restrictions as to the choice of T_{temp} . Ironically, in the case of very small Δt , if $d = 1, 3, 5, 7$, we have Δt always real valued and setting $T_{\text{temp}} > 0$ is not necessary. I.e. negative temperature $T_{\text{temp}} < 0$ may occur. In doing so, if we do this, it means that there can be positive entropy, for black holes, as is discussed in [16]. Whereas for $d = 2, 4, 6$, and above we must have $T_{\text{temp}} > 0$ and then the case of if we have sufficiently small Δt an unavoidable situation for possible negative black hole entropy, no matter what which is discussed in [16], i.e. if we have small Δt and case 1 used, for $d = 3$ we may have a connection with quantized gravity for reasons we will discuss later on in this manuscript.

Case 2, infinitesimal Δt and $d=1$ the Kaluza Klein case.

We then always have Δt real valued, and no restrictions on T_{temp}

Case 3, infinitesimal Δt , and the possibility that Δt could be imaginary. If $d=2, 4, 6$, and $T_{\text{temp}} < 0$

The reviewer does not like imaginary time. Therefore, for the time being this is a mathematical demonstration only and will be only included in for completeness of this document. However, if we have $d=2, 4, 6$, and $T_{\text{temp}} < 0$ the following limiting behavior is noted, in Eq. (3)

This in all of what the reviewer has asked for is putting a very strong preference in for $d=1$ as the Kaluza Klein case avoids multiple pathologies, but again only in the case that Δt to the fifth power is neglected.

Can this dropping off of Δt to the fifth power be justified. A full comment on that issue will be in the final part of this manuscript.

For the record, this below is the case, and situation which the reviewer disliked the most.

Commented [AB4]: Nothing wrong with this.

Commented [AB5]: Removed the word also

$$\begin{aligned}
(\Delta t)^2 &= \frac{16\pi \cdot (\hbar)^2}{4\pi \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \cdot \left(\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \right)^{-1} \\
&\xrightarrow{d=2,4,6, \text{ and } T_{\text{temp}} < 0} \frac{16\pi \cdot (\hbar)^2}{4\pi \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \cdot \left(\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi |T_{\text{temp}}|}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \right)^{-1} \quad (3)
\end{aligned}$$

i.e. imaginary time, for $d=2,4,6,\dots$

Note this cannot happen, i.e. imaginary time, for $d=1,3,5,7$

If we can accept imaginary time, then in the case of $d=2,4,6$, we could have $T_{\text{temp}} < 0$. However, the reviewer of this manuscript has indicated that he does not favor the existence or acceptance of imaginary time. Needless to say though, for infinitesimal Δt if we wish to avoid imaginary times, it is best to consider dimensions $d = 1,3,5$, and above to have a situation for which Δt infinitesimal but real valued, no matter what the sign of T_{temp} is. And $d = 3$ ties in directly with the situation given in [17,18,19]; we have that there is a situation which favors $d = 3$ for reasons which are given on page 639 of [19] and which indicate a connection to coupling coefficients, of effective Yang Mills theory which will be commented upon in a reply to the referee in the later part of this document.

Note that in the small Δt limit for $d = 1,3,5,7$ we cleanly avoid any imaginary time no matter what the sign of T_{temp} is. But that for small Δt limit for $d = 2,4,6$, we can have imaginary time. And this, plus the connection to the discussion on page 639 about coupling constant, if $d = 3$, reference [19], page 639 may in itself for infinitesimal Δt lend toward supporting $d = 3$. This arises also because of the AdS/CFT correspondence brought up in [20, 21] which we use.

All this is well trod physics, and is not disturbing, but the problem becomes glaring if we have Δt not as infinitesimal, in which then we have some truly bizarre physics. i.e. in that case, we have to appeal to Galois theory and a quintic Galois solution [5][7][8]

Case 4, when we have a generalized solution for a Quintic polynomial, when Δt is not necessarily infinitesimal.

Note that for extra dimensions $d=1,3,5,7$, the coefficient A_1 is always less than zero, leading to no requirement for T_{temp} to be < 0 . The problem is though, that for $d=1,3,5,7$ and above, that if [5] is true, then there is no generalized Gauss theory solution to the restricted Quintic. As due to communication by the referee which we will discuss at length, due to [5] he very quickly came up with a specialized trivial example for solving this quintic, but in doing so he contravened not only [5] but also [7,8].

If we do not have an infinitesimal Δt and if $d=1,3,5,7$, the coefficient A_1 is always less than zero, then if the Galois solvability criteria is correct for the quintic as given in [5] as we will outline, we have a huge problem.

This for general Galois solvability of $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$. If $d=2,4,6$, need $T_{temp} < 0$ for coefficient A_1 to be greater than zero. This for general Galois solvability of $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$. Note, that special solutions for $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ are easy to obtain, as a reviewer noted, but that we are referring to completely general solutions, not specific special case solutions.

As has been noted by Galois, and others, there are trivial specific solutions as to the quintic, but what is referred to is a general polynomial solution to the quintic fifth order is not solvable in a general algebraic sense. i.e. there are noted fourth order general solutions to fourth order polynomial equations, but none in the sense of generalized solutions for fifth order polynomials [2,5,7,8] A reference to a Rocky mountain journal of mathematics is included for a general solution to a specific fifth order equation [2] [5], and as correctly noted by the reviewer, that in one sense the specialized general fifth order equation so derived by the author has a trivial special case solution Precisely because we do not have a physics reason for making the restriction to the specific special case solution suggested by the reviewer, we have to appeal to a general solution, and that involves a decomposition rooted in Galois theory, among others.

Finally, a comment as to the minimum uncertainty principle, as a way to imply quantization is included. Generally, as noted by the reviewer, the absence of a solution to a problem in terms of the minimum uncertainty principle, in this case $\Delta E \Delta t = \hbar$, written as $\Delta E \Delta t \doteq \hbar$ in itself is not evidence as to quantization. In this case, it actually does imply quantization [5] [6] for a reason given in this manuscript. The reviewer also is bothered by a discussion as to semi classicality versus alleged quantum solutions via an AdS/CFT [2] [16] correspondence discussion.

The main problem has been the Quintic polynomial, and this is taking up the lions share of this manuscript. i.e. it is famously noted by Galois and others that a generalized equation for completely general fifth order polynomials is not solvable. [5,7,8] The restricted general fifth order polynomial, the restricted quintic does have trivial specialized solutions, but it still is a very tough technical problem, for generalized solutions. Again, as noted, there is a reference as to solving the restricted fifth order general quintic polynomial. [5,7,8] And the author urges that people actually read it. And also review a bit of the literature as to Galois theory provided. [5,7,8]

In doing so, the author is not suggesting that there are not numerical solutions to the restricted fifth order quintic polynomial. Certainly they are, and the author actually has a PhD dissertation using Runge Kutta techniques [22,23,24,25] as to a condensed matter solution to a very tough condensed matter physics problem. [26] In a sense, this entire article is motivated by the author's PhD dissertation, as of 2001 which had to be numerically iterated, via Runge Kutta and also reviewed by quantum field theory to solve a similar extremely complicated nonlinear problem, [26] Due to the comments of the reviewer, the author hopes that readers take the time to review the Galois motivated manuscript, and realize that the author has a mathematics degree in numerical analysis, so the author is fully aware of the special case solution. The special case solution as alluded to by the reviewer is not a general equation solution [14], for reasons in Galois theory, and in other similar work by Abel and other mathematicians. [5,7,8] Having said that, we get to the first complaint area of the reviewer, as to the physical nature of assumed negative temperatures in black hole, and in our case, worm hole physics. Keep in mind that we will relate the closure of a worm hole throat to temperature, T_{temp} as given by Visser, [9]. So, all this is physically pertinent. The methods as to numerical interpolation were studied in [26, 27,28], whereas [27] and [28]

actually reflect some of the modeling issues which show up even today, and where the idea of gravitons, as information carriers, as given in [23] .

Before we proceed further, as a bridge to the negative temperature issue of black holes, we wish to address the most direct complaint raised by the reviewer, and that has to do with the problem of this formalism and its adherence to String theory

III. **How to reconcile String theory which is a quantum gravity regime, with results which seem to be inconsistent with quantum gravity.**

The reviewer, in [14] sent the following question which deserves an answer, i.e.

Quote

Another issue is that in all of this the author is working within a “stringy” framework, for instance the values of d are chosen such as to be compatible with string theory, AdS/CFT concepts are used throughout the work, and so on. However, string theory is a theory of quantum gravity. How can you make assumptions consistent with quantum gravity and then derive conditions which are inconsistent with quantum gravity at the same time? This is very inconsistent

End of quote

The author refers the readers to [19], specifically go to page 639 as to the coupling constants used in super Yang Mills theory. i.e. in the section labeled “the Coupling constants”, [24] write that

Quote, from [19], page 639

“The dimensional effective coupling of super Yang Mills theory in d+1 dimension is scale dependent. At an energy scale E, it is determined by dimensional analysis to be

$$g_{eff}^2(E) \sim g_{YM}^2 N E^{d-3} \quad (3)$$

This coupling is small, so that perturbation theory applies for large E (the UV) for d<3, and for small E (the IR) . The special case of d = 3 corresponds to $\mathbb{N} = 4$ super Yang Mills theory in four dimensions, which is known to be a UV finite, conformally invariant theory. In that case, $g_{eff}^2(E)$ is independent of the scale E and corresponds to the t’Hooft coupling constant

$$\lambda \sim g_{YM}^2 N \quad (4)$$

This is the constant which is held constant in the large – N expansion of the gauge theory discussed below

End of quote from page 639 of [19]

I.e. in our work, the question of d dependence will be crucial in the application of the T_{temp} to the question of if we have adherence to quantum gravity, via if we need a negative temperature, will show up as follows, namely

If we have from [2] the following decomposition of the quintic polynomial, and for this see Eq.(5) below, we will be able to go look at the dynamics of what may be occurring for d=3, i.e. what if we have independence of a coupling constant from energy, we have from d=3 in the situation where we have no dependence of the coefficient A_1 upon the sign of the T_{temp} . If say we have a typical dependence of system

energy, say $E_{\text{statistical}} = \frac{k_B T_{\text{applied-temperature}}}{2}$ we are saying, if we believe that this removes the necessity of

having a negative, or positive temperature, that then the possibility of, say a black hole having negative entropy (for positive temperature) as given by [15] is not important. But this would mean an effective statistically based negative energy, which would be for say energy flowing into a black hole . However, in our derivation of the quintic polynomial, in [2] we are dependent upon an entropy count based upon infinite statistics counting algorithm based upon entropy being based upon an admitted particle count, i.e. $S \sim$ particle count n , as given in [29]. The upshot is, that if we have $d = 3$ that we have a string theory-based removal of the sign of energy, and temperature in coupling which means that the coupling constant as given in Ea.(3) and Eq. (4) is also consistent with [30] and is also covered in [5] as we derived it. I.e. that the result we have, which uses [29] and [30] , for $d=3$ is fully consistent with the Eq. (3) and Eq. (4) removal of the centrality of how we evaluate energy, in terms of the sign of energy, if we in doing this regard our input energy, as say along the lines of $E_{\text{statistical}} = \frac{k_B T_{\text{applied-temperature}}}{2}$. In this sense, our results in

terms of removal of the importance of the sign of the temperature, and by extension statistical energy, given in Eq. (5) below may make a partial linkage between Eq. (5) below, and Eq. (4) if we can write

$E_{\text{statistical}} = \frac{k_B T_{\text{applied-temperature}}}{2} = E$, as an input into Eq. (4), with the applied temperature $T_{\text{applied-temperature}} = T_{\text{temp}}$

$$(\Delta t)^5 - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \equiv 0$$

$$\Rightarrow A_1 = - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3}$$

$$A_2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3}$$

$$\Rightarrow T_{\text{temp}} \text{ should - be - negative - if - } d = 2, 4, 6, \dots \text{ for } A_1 > 0 \quad (5)$$

$$\Rightarrow T_{\text{temp}} \text{ does - not - have - to - be - negative - if - } d = 1, 3, 5, 7, \dots \text{ for } A_1 < 0$$

but - the - solvability - requirement - for - a - Galois - solution, by [5]

is - impossible. And $A_1 < 0$ - all - the - time $A_1 > 0$

If the removal of the sign of the temperature, as given in T_{temp} , is similar to reducing the importance of the sign of energy, as an input using $E_{\text{statistical}} = \frac{k_B T_{\text{applied-temperature}}}{2} = E$, with E used in Eq. (4), we then have a connection with string theory which is in a sense answering the referees objections. This is different from when we have sensitivity as to the sign

In fact, as discussed earlier, using [2] and [5] we have that if we have this, that we can only use $d=2,4,6$, so as to have a preference for negative temperatures and if [16] is believed, then a situation for which all

black hole entropy is then positive. If we have positive entropy, and we model the worm hole as a connection between two black holes, then we may have a consistent physical model, indicating positive entropy.

i.e. for values of $d=2, d=4, d=6$, we have a situation where we are looking for where we have would be quantum behavior, i.e. a solution for this quintic, if we have negative temperatures. i.e. $A_1 > 0$. We claim then that we have a relationship to the situation given in Eq. (4) above. And thereby answer the reviewer's question.

When $d=1,3,5,\dots$ we claim then that Eq. (5) is in sync with Eq. (4) and that especially when $d=3$ we have the tie in with Eq. (5) and Eq. (4). And most telling the $d=3$ case appears to superimpose directly with Eq. (5) and the discussion as to what that implies given on page 639 but we rule out $d=3$, if we are looking at a generalized Galois solution given through Eq.(5)

IV. Negative temperatures.

One of the complaints of a reviewer has been about the idea of negative temperatures. Before we begin our discussion, we will briefly allude to the history of negative temperatures, and black hole physics, then allude as to what it may have to do with our problem.[16] is the starting reference, i.e. we will reference negative temperature as far as the history of black hole physics.

The executive summary of black hole physics, is that, indeed, as given by [16] and its additional references, as cited below that in order insure that the entropy of a black hole is non-negative, i.e. positive that we require having a negative Hawking temperature.

From [16] we will follow the following quote

II. NEW HAWKING TEMPERATURES FROM THERMODYNAMICS

In the spin systems the temperature can be negative, due to the upper bound of the energy spectrum [4]. Recently, a number of black hole solutions which have similar upper bounds of the black hole masses have been discovered [31,32,33,34,35,36] I have argued that the Hawking temperatures for these systems might not be given by the usual formula $T_+ = \hbar\kappa/2\pi$ [31,32,33,34] which is non-negative, but by new formulae which can be negative depending on the situations [35,36] The argument was based on the Hawking's area theorem and the second law. This has been found to agree completely with CF T analysis, being related to the AdS/CF T correspondence, as far as the CF T analysis is available [35,36].

End of quote

Admittedly, negative temperature appears to contravene the Hawking black hole temperature formula.

Quote, from [36], HERE WE ARE USING OUR APPENDIX ENTRIES TO COVER ENTRIES GIVEN IN [36]

But this seems to be physically nonsensical since the entropy is non-negative, "by its definition" as a measure of disorderliness [37]; the positiveness of the entropy is a "minimum" requirement that must be satisfied if the entropy has a statistical mechanical origin [33], [38], [39]. Moreover, without the guarantee of the second law, there would be no justification for identifying the entropies, even though they satisfy the first law [34] [40]. So, in this paper I consider a different approach which can resolve the two problems, simultaneously. The new resolution is to consider an entropy

$$S_{w'} = \left| \hat{\Omega} \right| \cdot \frac{2\pi r_+}{4G\hbar} \quad (6)$$

Commented [AB6]: ADDED [34]

Commented [AB7]: See this explanation as to the reasons for the numbering

which is non-negative manifestly and also satisfying the second law from the area theorem , as in the case of S_w in

$$S_w = \hat{\Omega} \cdot \frac{2\pi r_+}{4G\hbar} \quad (7)$$

for a positive Ω . But, in this case I must pay the price, by ^ considering a new temperature

$$T_+ ' \equiv -T_+ \quad (8)$$

End of quote

The tack of reference [9],[31][38] is that in order to have a positive black hole entropy, that we have to entertain negative temperature, which is given in Eq. (8) and which is elaborated on in page 5 of reference [9],[31][38] i.e. by the following adage, i.e. in order to have positive black hole entropy, the temperature has to be negative, i.e. Eq. (7) could give negative black hole entropy , and in order to obtain positive entropy for a black hole, as given by Eq. (6) we have to have Eq. (8) with negative temperature. To those whom still do not believe this summary? Go to reference [9],[31][38] and look it up. Now how does this connect worm holes ? i.e. a typical model of worm holes has in its formulation a worm hole bridge between two black holes. The complete Schwarzschild geometry consists of a black hole, a white hole, and two Universes connected at their horizons by a wormhole [41] . We have already discussed that negative temperature may exist in astrophysics, i.e. our next section is to link that to worm holes.[42]

Commented [AB8]: Ordering of references fixed

Commented [AB9]: Reference ordering fixed

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V. Negative Temperatures, and the total energy of worm holes

As we will argue accessing Juan Maldacena, et. al,[43] , the total energy of a worm hole reads as follows, h namely

$$\begin{aligned} E_{wormhole} &= -q / 8\ell \\ q &= 2j + 1 \\ \ell &= 1 / 2\pi T_{temperature} \end{aligned} \quad (9)$$

In short, if the total wormhole $T_{temperature}$, temperature is less than zero, we have, then that the $E_{wormhole}$ is greater than zero. So, what does this mean ? Negative energy appears in the speculative theory of wormholes, where it is needed to keep the wormhole open. A wormhole directly connects two locations which may be separated arbitrarily far apart in both space and time, and in principle allows near-instantaneous travel between them [44]

I.e. for a negative temperature, the worm hole throat is shut, and if the wormhole is open, the temperature has to be >0 , indeed $T_{temperature}$ is less than zero, we have a shut worm hole. But we observe in [44] in its figure 1, of page 1446 a figure 1, of [44] which has the likely interpretation of being a black hole, linked to a white hole, with a worm hole specifying entanglement between the two regimes. Of the two astrophysical objects. This is also part of [45]

VI. Wormholes and black holes, and possibly white holes

As mentioned before, we have in [44] in its figure 1, of page 1446 of [44] a possible linkage between a black hole, to a white hole via a worm hole. In any case, according to [3] there is a connection via quantum teleportation which may link two black holes, hence, this is akin to [44] [9][37] with some additional caveats/ i.e. as seen in [4]. And [4], the subject of a linkage of transversal worm holes is being revisited, and we claim also that we can add more specific structure to the analysis, as recently presented. Note that in [4] that the introduction to the abstract states, i.e. go to [4] and do not forget what is in [46][38] about quantum teleportation linkage between two black holes as to a worm hole bridge, Now, consider

Quote, from abstract of [4]

We study various aspects of wormholes that are made traversable by an interaction between the two of boundaries. We concentrate on the case of nearly AdS2 gravity and discuss a very simple mechanical picture for the gravitational dynamics.

End of quote

Our supposition goes beyond this, i.e. an analysis as to the physics of transversable worm holes is built upon gravitational physics as it affects the energy value, as given in Eq (9). i.e. we assume a set of given conditions which allow for if the temperature, $T_{temperature}$ is positive or negative. To do this though we will answer a complete mathematical mis understanding of quintic mathematics by the referee

VII. Answering a mis understanding by the referee as to the mathematical solution of a quintic polynomial, which is used to ascertain if $T_{temperature}$ is positive or negative

First of all, we ask the readers to review Eq. (9), and this will be to determine if $T_{temperature}$ **or positive and this comes from use of [2], i.e. we will look at the following equations (10) , Eq. (11) and then if Eq. (11) holds, Eq. (12) below which mandates having $A_1 > 0$ in Eq. (5) which then leads to what the reviewer incorrectly found a trivial solution for, i.e. the reviewer, and also readers are expected to look at Galois theory to come up with a generalized, as opposed to looking at Galois theory for general solvability.**

Note that for reasons which will be discussed in terms of its attendant physics in the later part of the manuscript, that for extremely small $(\Delta t)^5$ that in that situation where we have a simple quadratic, that instead of having $A_1 > 0$ we have, instead

$$A_1 = -\frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} < 0 \text{ when we have } (\Delta t)^5 \text{ about zero} \quad (10)$$

This is reflected in a simple general physics solution to

$$-\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \equiv 0 \quad (11)$$

If we have non-vanishing $(\Delta t)^5$ the situation changes, and we have then

$$(\Delta t)^5 - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \equiv 0 \quad (12)$$

We will, in spite of the protests of the reviewer, avoid the specialized solution, use a general solution, and then state

$$A_1 = -\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} > 0 \text{ when we have } (\Delta t)^5 \text{ still contributing} \quad (13)$$

If Eq. (11) no longer holds due to the fact we no longer have a quadratic equation due to $(\Delta t)^5$ not vanishing, we will have to go to what the reviewer found so distasteful, i.e. Eq. (13), and then the odd situation of what is given below. It is expected that the reviewer and also readers will take the time to go to this reference, which is in [2] and also [11] and then take the time to read some Galois theory. FTR we will then go back to Eq. (5) when setting up the usage of Eq. (14) below

Let a and b be nonzero rational numbers. We show that there are an **infinite number of essentially different**, irreducible, solvable, quintic trinomials $X^5 + ax + b$. On the other hand, **we** show that there are only five **essentially different, irreducible, solvable, quintic trinomials** x^5+ax^2+b , namely, by [2],[11]

$$\begin{aligned} &x^5 + 5x^2 + 3, \\ &X^5 + 5x^2 - 15, \\ &X^5 + 25x^2 + 300, \\ &X^5 + 100X^2 + 1000, \\ &\text{and } X^5 + 250X^2 + 625. \end{aligned} \quad (14)$$

Here, $X = \Delta t$, and we change the dimensional scaling of A_1 and A_2 , so as to be consistent with Eq. (14), and in addition, the d in Eq. (5) can range in size from d=2, 4,6 so as to keep our construction consistent with String theory.

If d=1,3,5,7 we have then that we could have then, with T_{temp} either greater than or less than zero, with the odd situation that at d=1, a situation where the sign, and the value of T_{temp} could even be zero itself, i.e. as an artifact of Kaluza Klein theory, but then all connection then to Eq. (14) would be lost and the following, at d=1 A_1 would always be negative. i.e.

If $d = 1$, then the following would always be true, (Kaluza Klein theory) and then we would be having

$$A_1 = - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} < 0 \quad (15)$$

The only way to avoid having all connections with Kaluza Klein theory removed is to say that in the case of $d=1$ that we would have to have $X = \Delta t$ infinitesimally small, hence we state the following

Theorem A

If $d = 1$ in order to come up with solvable conditions for Eq. (5) $X = \Delta t$ will be assumed to be negligible, i.e. we then look at a quadratic version in the solution of $X = \Delta t$ of Eq.(5), and that then only when $d=1$. I.e. $d=1$ will presumably be having use of Eq. (11), hence having a situation which involves no requirement on T_{temp} being less than zero. In fact, T_{temp} could be any value we wished including the positively weird situation that T_{temp} could go to zero itself. So long as $d=1$ that is allowed. Once d does not equal 1, we have then very T_{temp} dependent behavior.

If $d \neq 1$ we have then very T_{temp} dependent behavior. And then we have to go to the weirdness which the referee found so objectionable.

Now we will take the position of directly quoting the referee in [10] in full and to really answer him

Quote

Let me now come to the main problem of the paper. All the arguments of the paper rely on the fact that a given quintic polynomial of the form $X^5 + A_1X^2 + A_2 = 0$ is only solvable for certain choices of coefficients. In fact, the author says he shows there are only five essentially different, irreducible, solvable, quintic trinomials which are solvable. First of all, I don't understand what "essentially different" means. Does it mean polynomials which are not multiples of each other? I find it in any case very hard to believe that there are no other polynomials of that type which are solvable. For instance, the following equation: $X^5 + X^2 - 2 = 0$, (1) is trivially solved by $X = 1$, it is not a multiple of any of the other polynomials (assuming that's what is meant by essentially different) and is irreducible. And similarly, one can construct infinitely many other examples. So, the author should clarify this point,

End of quote

In the case of $d=1$, i.e. Kaluza Klein there is no problem, i.e. see Theorem 1 above. and it becomes a trivial general solution which is reflected in Eq. (15) at $d=1$ A_1 would always be negative. And the quintic would in $d=1$ reduce to solving Eq. (11), i.e. $d=1$ as being solvable would require $(\Delta t)^5$ not contributing, presumably due to being negligible in the full sense of the word and the only for

Eq. (11) , and $d=1$ we would then have (Kaluza Klein) a situation where the sign of T_{temp} , and its magnitude do not play any role in the determination of Δt

In the case of $d \neq 1$, we will then have to consider when $(\Delta t)^5$ intrudes, hence the following discussion below

I.e. the supposition given above is that there is a specific set of conditions for which the author specifically refutes this by the following statement. i.e. that this is verbatim. I.e. we are not using the specialized solution to the general solution for Eq. (16) . In particular we have that for a generalized quintic, even in trinomial form that one is not going to come up with a particular solution which fits the requirements of a general solution. i.e. what was done in [14] was to arbitrarily demand that Eq. (16) have $A_1 = -1$ and $A_2 = 1$, and then from there have a trivial solution made out which would simply satisfy the needed delta t value, which the referee set as equal to 1. We say without reservation that if we wish to have generalized inputs into A_1 and A_2 of the quintic equation that the following must be adhered to, and that without reservation we make, in the spirit of a generalized polynomial solution the following statement as to the values of the quintic equation. I.e..

$$\begin{aligned}
 (\Delta t)^5 - \frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} &\equiv 0 \\
 \Rightarrow A_1 = -\frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} &\neq 1 \\
 A_2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} &\neq -2
 \end{aligned} \tag{16}$$

There are no conceivable conditions for which one would have such a situation. We are referring to general solvability. Of quintics, by what is known as by radicals. See more on this as follows

In order to make this a bit more to the point, the author will go to Galois theory, temporarily, since the referee did not read the following i.e.[6,7,8] The next section of this paper will cite some of the foundational issues brought up in [8] which shows specifically the problem. First will be how the uncertainty principle is related to 5-dimensional physics, since this is one of the reasons why we actually bothered to have a quintic equation formed upon the minimum uncertainty given in the reference [5] which we will justify in our document.

VIII. How to relate and embed the uncertainty principle from five-dimensional physics

From [47] we have the following discussion which we find is very pertinent to d=1 Kaluza Klein physics and its relationship to the

i.e. consider first Let us now, briefly allude to the [48,49] reference, namely:
Start with the idea of an embedding of four-dimensional space-time in a 5-dimensional time interval. [47,48] and realize it inter connections with [50, 51, 52, 53] where L = length of canonical metric in 5-Dimensional theory

$$dS_{5\text{-dim}}^2 = \frac{L^2}{l^2} ds_{4\text{-dim}}^2 - \left(\frac{L^2}{l^2}\right)^2 dl^2$$

$$x_4 = l = h / mc \quad (17)$$

$$\Lambda = 3 / L^2$$

$$L = \text{scale} - \text{of} - \text{scale} - \text{of} - (\text{universe}) - \text{Potential} - \text{well}$$

And then we present, the five momenta as given by

$$P_\alpha = \frac{2L^2}{l^2} g^{\alpha\beta} \frac{dx^\beta}{dx} \quad (18)$$

$$P_l = -\frac{2L^4}{l^4} \frac{dl}{ds}$$

Then, if

$$P_\alpha = \frac{2L^2}{l^2} g^{\alpha\beta} \frac{dx^\beta}{dx}$$

$$P_l = -\frac{2L^4}{l^4} \frac{dl}{ds}$$

$$\int P_\alpha dx^\alpha = \int P_\alpha dx^\alpha + P_l dl = 0 \quad \text{iff} \quad dS_{5\text{-dim}}^2 = 0 \quad (19)$$

$$\Leftrightarrow l = l_0 e^{\pm s/L} \quad \& \quad (dl / ds) = \pm l / L$$

One eventually, as given by [48] obtains the Heisenberg type of relations that

$$|dp_\alpha dx^\alpha| = h \cdot \left\{ \frac{n}{c} \cdot \left(\frac{dl}{l}\right)^2 \right\} \quad (20)$$

Depending upon how we evaluate $\left\{ \frac{n}{c} \cdot \left(\frac{dl}{l}\right)^2 \right\}$, we can then say that if $n = L/l$, and if we have L as the length of the additional dimension, that we have from deterministic reasoning in 5 dimensions achieved

Eq. (20) which in four dimensions, depending upon how $\left\{ \frac{n}{c} \cdot \left(\frac{dl}{l}\right)^2 \right\}$ is evaluated is in common with $\Delta x \Delta p \geq \hbar$ [54]

To proceed with this further in [55] we have that $\Delta E \Delta t \geq \hbar$, and that the following holds, in cosmological physics, in a general sense, i.e. in cosmology we can depend upon the following assumptions, namely, as derived by the author in [56]. We use the approximation as presented in [56] which we reproduce below as also in [57,58]

$$\begin{aligned} (\Delta l)_{ij} &= \frac{\delta g_{ij} \cdot l}{g_{ij} \cdot 2} \\ (\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A \end{aligned} \quad (21)$$

If we use the following, from the Roberson-Walker metric [56]

$$\begin{aligned} g_{tt} &= 1 \\ g_{rr} &= \frac{-a^2(t)}{1-k \cdot r^2} \\ g_{\theta\theta} &= -a^2(t) \cdot r^2 \\ g_{\phi\phi} &= -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2 \end{aligned} \quad (22)$$

Following Unruh [57,58], write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters} \quad (23)$$

Then, if $\Delta T_{tt} \sim \Delta \rho$ [56,57,58]

$$\begin{aligned} V^{(4)} &= \delta t \cdot \Delta A \cdot r \\ \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\ \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}} \end{aligned} \quad (24)$$

This Eq(24) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time[56]

$$T_{tt} = \text{diag}(\rho, -p, -p, -p) \quad (25)$$

Then by [56]

$$\Delta T_{tt} \sim \Delta \rho \sim \frac{\Delta E}{V^{(3)}} \quad (26)$$

Then, by[56]

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (27)$$

Unless $\delta g_{tt} \sim O(1)$

In this case, looking at a rewrite of the Eq(20) to read, approximately as

$$|dp_\alpha dx^\alpha| \sim h \cdot \left\{ \frac{n}{c} \cdot \left(\frac{dl}{l} \right)^2 \right\}_\alpha \quad (28)$$

$$\text{With the } \alpha = 0 \Rightarrow |dp_0 dx^0| \sim h \cdot \left\{ \frac{n}{c} \cdot \left(\frac{dl}{l} \right)^2 \right\}_{\alpha=0} \Rightarrow \delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (29)$$

Unless $\delta g_{tt} \sim O(1)$

Having processed in how 5 dimensional geometry may allow for the HUP according to the above argument let us now see how, if we do not have $(\Delta t)^5$ not contributing, i.e. a quintic, in line with a simple reduction in complexity solution to the Eq. (16) problem, i.e. a quick and dirty solution [59, 60] [51, 52]

IX. Applying the Gauss- Lucas theorem to Eq. (16)

Gauss–Lucas theorem gives a [geometrical](#) relation between the [roots](#) of a [polynomial](#) P and the roots of its [derivative](#) P' , i.e... If P is a (nonconstant) polynomial with complex coefficients, all [zeros](#) of P' belong to the convex hull of the set of zeros of P . [52]

$$\begin{aligned} (\Delta t)^3 - \frac{2n_{\text{graviton-count}}}{5 \left(\frac{4\pi T_{\text{temp}}}{d} \right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar} \right)^3} (\Delta t)^0 &= 0 \\ \Rightarrow (\Delta t)^3 &= \frac{2n_{\text{graviton-count}}}{5 \left(\frac{4\pi T_{\text{temp}}}{d} \right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar} \right)^3} \end{aligned} \quad (30)$$

Superficially, this imposes the same sort of restrictions upon Δt for $d=1,3,5$, but then

$$(\Delta t)^2 \equiv \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \cdot \frac{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3}{n_{graviton-count}} \Rightarrow (\Delta t) \equiv \left(\frac{16\pi \cdot (\hbar)^2 \cdot \left(\frac{4\pi T_{temp}}{d}\right)^{d-1}}{n_{graviton-count}} \right)^{1/2} \quad (31)$$

$$(\Delta t)^3 = \frac{2n_{graviton-count}}{5 \left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \Rightarrow (\Delta t) = \left(\frac{2n_{graviton-count}}{5 \left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \right)^{1/3}$$

Hence, we have to do further root analysis.

X. Brief summary of reference [8] and the problem of a solution by radicals.

Readers are recommended to go to page 4 of [8] where the question of if a quintic polynomial is exactly solvable. Well it is not

The answer to why this is known as the Abel Ruffini theorem[53] i.e. to look at the following

The theorem does *not* assert that some higher-degree polynomial equations have *no* solution. In fact, the opposite is true: every non-constant polynomial equation in one unknown, with [real](#) or [complex](#) coefficients, has at least one complex number as a solution (and thus, by [polynomial division](#), as many complex roots as its degree, counting repeated roots); this is the [fundamental theorem of algebra](#). These solutions can be computed to any desired degree of accuracy using numerical methods such as the [Newton-Raphson method](#) or the [Laguerre method](#), and in this way they are no different from solutions to polynomial equations of the second, third, or fourth degrees. It also does *not* assert that *no* higher-degree polynomial equations can be solved in radicals: the equation $x^n - 1 = 0$ can be solved in radicals for every positive integer n , for example. The theorem only shows that there is no *general solution in radicals* that applies to *all* equations of a given degree greater than 4.

Also, see [61,62] , . i.e. what the referee does not understand is

quote

no general solution in radicals for degree five generalized quintic equations means the following cannot be done.

A general solution in radicals .An algebraic solution or solution in radicals is a closed form expression, and more specifically a closed-form algebraic expression, that is the solution of an algebraic equation in terms of the coefficients, relying only on addition, subtraction, multiplication, division, raising to integer powers, and the extraction of roots (square roots, cube roots, etc.).

As stated , we can also go to [63] i.e. page 54 where the definition of solvability by Radicals is done abstractly. See “ section 9, solvability of polynomials by radicals. Also [64]

The result of reference [11] which is mis understood here, is in determining if a radical solution of the given quintic exists. I.e.. In terms of Galois splitting field. The results of Eq. (31) ignored by the referee, is in obtaining a solution in terms of radicals is only achievable with regards to the five linear combinations of the sort given for coefficients given in Eq. (32). Now if we restrict the solution to the specialized quintic referred to in Eq. (11)

XI. Next objection by the referee. From [14], is the absence of being able to apply a minimum uncertainty principle , as a proof of lack of quantum gravity.

Quote from[14]

It is unclear to me how the author reaches certain conclusions about a possible quantum nature of gravity. For instance, the whole line of real solving in Eq. (11) is unclear. Why if T temperature > 0 then gravity must be semi-classical? Is it because then one cannot have a minimum uncertainty principle? If so, then it is unclear to me why the absence of a minimum uncertainty principle is in itself an indication that gravity cannot be quantum. Certainly, it hints in that direction, but it is not a solid indication.

End of quote

We will go to two cases, only since these are referred to in terms of first, very small Δt in the case of a definitely real value to the time interval, in which we will be looking at in terms of $d = 1,3,5,7\dots$

Case one, Tiny time step, temperature T can either be less than or greater than zero, and no imaginary time.

Again, as indicated by Eq. (1) we have that for a very small-time step, for a non-imaginary time, that no matter what the sign of Temperature, T, that

$$(\Delta t)^2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \cdot \left(\frac{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \left(\frac{Jc^2}{\hbar}\right)^3}{n_{graviton-count}} \right); d = 1,3,5,.. \quad (32)$$

In this case, the referee's question is pertinent. i.e. it is related to the minimum uncertainty principle. We do not, in the case of very small-time step, have a situation for which temperature T is required to be either positive or negative, hence we reduce this situation to being of the form $\Delta E \Delta t \doteq \hbar$

i.e.
$$(\Delta E)^2 = \frac{1}{4\pi} \cdot \left(\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1}} \right); d = 1, 3, 5, \dots \quad (33)$$

The sign T_{temp} plays no role in the determination of an energy value, other than that this conceivably be the minimum state of a graviton condensate.

Now let us consider what if $d=1$, i.e. Kaluza Klein, i.e. then we have

$$(\Delta E)^2 = \frac{n_{\text{graviton-count}}}{4\pi} \cdot \left(\frac{1}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1}} \right); d = 1, 3, 5, \dots \xrightarrow{d=1} \frac{n_{\text{graviton-count}}}{4\pi} \quad (34)$$

We are then leading to, if we have a distance, we call a_{graviton} .

$$\left[\left(\hbar / a_{\text{graviton}} \right) \cdot c \right]^2 \approx \frac{n_{\text{graviton-count}}}{4\pi} \quad (35)$$

If in this situation we have $a_{\text{graviton}} \approx \lambda_{\text{graviton}} \propto 1 / \omega_{\text{graviton}}$

$$\left[\left(\hbar / a_{\text{graviton}} \right) \cdot c \right]^2 \propto \hbar \omega_{\text{graviton}} \approx \frac{n_{\text{graviton-count}}}{4\pi} \cdot \left(\frac{1}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1}} \right); d = 1, 3, 5,$$

$$a_{\text{graviton}} \approx \lambda_{\text{graviton}} \propto 1 / \omega_{\text{graviton}} \quad (36)$$

if $d = 1$

$$\left[\left(\hbar / a_{\text{graviton}} \right) \cdot c \right]^2 \propto \hbar \omega_{\text{graviton}} \approx \frac{n_{\text{graviton-count}}}{4\pi}$$

We claim that in the case of $d=1$ in the situation for which $(\Delta t)^5 \rightarrow 0^+$, that indeed the ground state, as referred to in Eq. (36) is a strong indicator of quantum gravity. I.e.. The zero-point energy is dependent upon a graviton count, $n_{\text{graviton-count}}$

We see that in the case of minimum uncertainty in quantum mechanics, Quantum mechanically, the uncertainty principle forces the electron to have non-zero momentum and non-zero expectation value of position. If a is an average distance electron-proton distance, the uncertainty principle informs us that the minimum electron momentum is on the order of \hbar/a . I.e. if we have the same situation with a presumed graviton, and give it a mass of m_{graviton} infinitesimally small but not zero, and say we have a distance we call a_{graviton} . So, the minimum graviton momentum is

$$p(\text{momentum})_{\text{graviton}} \approx \hbar / a_{\text{graviton}} \quad (37)$$

Assume that gravitons are then endowed with mass, and then the mass vanishes

$$\begin{aligned} (p_{\text{graviton}} c)^2 &= E_{\text{graviton}}^2 - (m_{\text{graviton}} c^2)^2 \approx \left[\left(\hbar / a_{\text{graviton}} \right) \cdot c \right]^2 \\ \Rightarrow E_{\text{graviton}}^2 &\approx \left[\left(\hbar / a_{\text{graviton}} \right) \cdot c \right]^2 \text{ if } m_{\text{graviton}} \rightarrow 0 \end{aligned} \quad (38)$$

leads to a minimum energy equation looking like

$$\left[\left(\hbar / a_{\text{graviton}} \right) \cdot c \right]^2 \approx \frac{1}{4\pi} \cdot \left(\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d} \right)^{d-1}} \right); d = 1, 3, 5, \quad (39)$$

The HUP is central to the discussion of if a minimum uncertainty exists. In any stationary state $\langle p \rangle = 0$ or at least is a constant so any system in which there is a stationary state that has a gaussian wave function will have minimum position/momentum uncertainty. One case where this occurs is the ground state of the harmonic oscillator. In the case of a graviton we have that $\lambda (\equiv \lambda_{\text{graviton}}) = \frac{\hbar}{p} \left(\equiv \frac{\hbar}{p_{\text{graviton}}} \right)$ from the de Broglie hypothesis, we will answer in the last part of the question the final issues of if the quantum condition is due to a minimum uncertainty principle being satisfied.

Doing so means that we can, if $d = 1$, as in the case of Kaluza Klein theory, and 5-dimensional cosmology [5] still stick with $T_{\text{temperature}} < 0$. Other values of d will lead to different situations. I.e.. for $d = 0, d = 2, d = 4$, and $d = 6$ there is a chance for $T_{\text{temperature}} < 0$ leading to an exactly solvable value for Eq. (7) for the $X = \Delta t$ $X = \text{delta } t$ substitution

XII. Three theorems, so as to have a case by case rendition of the physics of our quintic polynomial. $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$

Theorem 1

For $d = 0, d = 2, d = 4,$ and $d = 6,$ Eq. (2) and Eq. (3) are solvable, in terms of $X = \Delta t,$ hence, then for the A_1 and A_2 terms, contributing to a value of $X = \Delta t$ we do not have an exactly solvable Quintic polynomial. Hence, then, $T_{temperature} < 0$ is not going to contribute to A_1 being changed from a negative value, as given in Eq. (2) to a positive value so it would be commensurate with Eq.(3). Hence, so that $T_{temperature} < 0$ changes $A_1 > 0.$

Hence, a necessary condition for exact solvability of the restricted quantic commensurate for Eq. (2) and Eq (3) and $A_1 > 0$ is that the dimensions, $d,$ as far as AdS/CFT correspondence have even values.

Theorem 2

For $d = 0, d = 2, d = 4,$ and $d = 6,$ Eq. (2) and Eq. (3) are solvable, hence we have that for these values of $d,$ that we have an exact solution for $X = \Delta t,$ hence then we do have a minimum uncertainty principle quantum gravity . We will then say that we DON'T have semi classical treatment of gravity.

Theorem 3

If we have $d = 1, d = 3, d = 5, d = 7$ set in AdS/CFT in dimensions, so that $T_{temperature} < 0$ changing $A_1 > 0$ is NO LONGER POSSIBLE . We have then no solvability of Eq. (2) and Eq. (3) hence, then ODD values of $d,$ as given above, lead to SEMI Classical gravity.

Corollary is that then, ODD values of $d,$ lead to SEMI classical treatment of gravity, and we can say then that the Kaluza Klein [5] 5-dimensional treatment is at best SEMI classical.

XIII. Analyzing when we have a very small $X = \Delta t$

changing

$$(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$$

to

$$(\Delta t)^2 + \frac{A_2}{A_1} = 0 \tag{40}$$

Theorem 4

$X = \Delta t$ very small, so that the first quintic polynomial term being ignorable, leads then to writing:

if $(\Delta t)^5 \approx 0^+$

$$(\Delta t)^2 \equiv +6\pi \cdot \hbar^2 \cdot \frac{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1}}{n_{graviton-count}} \quad (41)$$

We claim that this is rather than a case of semi classical, versus quantum a case of real and imaginary time, with a preference toward have $d = 1, d = 3, d = 5, d = 7$ set in AdS/CFT in dimensions, so that $T_{temperature} < 0$ is not necessary, and then we have the following $d = 1, d = 3, d = 5, d = 7$ to work with, so that we get

Theorem 5

Very small values of the sort with $(\Delta t)^5 \approx 0^+$ lead to , if $d = 1, d = 3, d = 5, d = 7$ then $T_{temperature} < 0$ is not necessary for real values of Δt , and then we have values of $\Delta E \Delta t \equiv \hbar$, so that ΔE is real valued. Also, then, ΔE is equivalent to H, with H a Hamiltonian system, i.e. a 1-1 and onto linkage then to the Hamiltonian being the same as the total energy of our system. This is in line with Abraham and Mardsen [6], Arnold [7], and Goldstein [8], as well as Spiegel [9] of a condition where the Hamiltonian is equal to the total energy of a system.

XIV. Conclusion, Relevance to the problem of the closed throat of a wormhole. And small to large delta t values

According to applying the criterial of [2] we have that if we look at a worm hole

Theorem 6

$$\begin{aligned} E_{wormhole} &= -q / 8\ell = -(2j+1) \cdot \pi \cdot T_{temperature} / 8 \\ E_{wormhole} < 0 &\Rightarrow \text{Open - wormhole - throat} \\ \Leftrightarrow T_{temperature} > 0 &\Rightarrow \text{Semi - Classical} \\ \Leftrightarrow \text{No - quantum - gravity - if } E_{wormhole} < 0 \end{aligned} \quad (42)$$

Keep in mind that this is making a connection with a Gravitino, of a very light mass, so as to be congruent with [2], we would have, say a gravitino of about .25 electron volts, i.e. see [10] whereas we make the connection to [11] as brought up by the author as a link between gravitons and gravitinos, and Mach's theorem. Should this be fleshed out in further generality, we will have the conundrum of addressing for very small delta t, Eq. (42) in conjunction with Eq.(43) below compared to Eq. (41) being usefully compared with connections to Eq. (41)

$$\text{if } (\Delta t)^5 \approx 0^+, (\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0 \Rightarrow (\Delta t)^2 + \frac{A_2}{A_1} = 0 \quad (43)$$

This would $d = 1, d = 3, d = 5, d = 7$ then $T_{temperature} < 0$ is not necessary for real values of Δt , and then we have values of $\Delta E \Delta t \equiv \hbar$, so that ΔE is real valued. And equal to the Hamiltonian.

Also, if Eq. (43) does not hold

Whereas for greater time step Δt , we have to consider the cases given in Theorems 1, 2, and 3 above. Where if $d = 1, d = 3, d = 5, d = 7$ then $T_{temperature} < 0$, and then the following summing up

Theorem 7

if Eq. (43) does not hold, i.e. for non-negligible Δt

if $d = 1, d = 3, d = 5, d = 7$ then $T_{temperature} < 0$, and then

- (i) $E_{wormhole} = -q/8\ell = -(2j+1) \cdot \pi \cdot T_{temperature} / 8 > 0$, HENCE the worm hole throat is closed
- (ii) We also do not have classical gravity if (i) is true. I.e. we can have quantum gravity
- (iii) Open throat worm hole means we assume semi classical gravity

Else

Theorem 8

if Eq. (43) does hold, i.e. for negligible Δt

if $d = 1, d = 3, d = 5, d = 7$ then $T_{temperature} < 0$ IS NOT NECESSARY, for real values of Δt , and then we have values of $\Delta E \Delta t \equiv \hbar$, so that ΔE is real valued. And equal to the Hamiltonian. Note then if $T_{temperature} < 0$ IS NOT NECESSARY for quantum gravity and then $E_{wormhole} = -q/8\ell = -(2j+1) \cdot \pi \cdot T_{temperature} / 8 < 0$ and we have an open worm hole throat

I.e. for very small Δt it is easy to come up with real values of Δt , and non-imaginary ΔE and it's easy to obtain $E_{wormhole} = -q/8\ell = -(2j+1) \cdot \pi \cdot T_{temperature} / 8 < 0$ for an OPEN worm hole throat.

Theorem 9

If Δt not so negligible, in order to obtain $E_{wormhole} = -q/8\ell = -(2j+1) \cdot \pi \cdot T_{temperature} / 8 < 0$ for an OPEN worm hole throat. We would then have to go to semi classical gravity. Due to the difficulty of obtaining $T_{temperature} > 0$

With regards to this problem, it is useful to make reference to [2], as its review of the fact that a general solution to Quintic 5th order polynomials does not exist. What we are doing is accessing instead results from Galois theory, as to Quintics, [6,7]

In a nutshell, we will be formally deriving $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ in our next section and from there ascertaining if the polynomial so derived, is explainable in terms of [5], in terms of exactly solvable solutions for Δt . For the sake of referencing the development of this article, we have as our motivating hypothesis, that if $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ is a polynomial in a form given in [5] that indeed, since n will be in terms of a graviton count from a black hole that then we have a NECESSARY condition for quantum gravity, at least in the framework of aligning $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ in terms of the polynomials given in [5] which are allegedly exactly solvable. If $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ does not meet the conditions given in [5], then we say that the criteria for exact solvability of an expression for Δt have not been met, and that indeed, then we have at best a semi classical treatment of gravity for reason which we will discuss at the end of our manuscript.

Finally, the reference [9] by C. A. Pickett and J. D. Zunda gives an area calculation which neatly fits into [10] and [11], whereas there is in [10] a precise calculation of entropy which also has an area to volume identification for black holes and entropy calculations. We close after all of this in stating that the energy, will be part of ΔE , as in the usual Heisenberg Uncertainty relationships, $\Delta E \Delta t \geq \hbar$, whereas we take the minimum condition of uncertainty by writing $\Delta E \Delta t \equiv \hbar$ [12], and [13] confirms that indeed we have that use of minimum uncertainty in terms of data analysis has a long history if done correctly. Keep in mind that we do an abbreviation of

$$\Delta E \equiv mc^2 = \hbar / \Delta t \Rightarrow m = \hbar / c \Delta t \quad (44)$$

This will allow us to obtain, in entropy a polynomial which we identify as $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$. The exact solution of this analysis, in terms of [2] will then form the basis of our analysis of if we have classical gravity, or quantum gravity, in terms of necessary conditions. If Eq. (44) and $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ is not exactly solvable, in terms of [5] we will the assert that this means gravity, in the case of the derived expression for Kerr – Newman black holes, is semi classical.

XV. Derivation of the polynomial $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$

We begin by looking at [20,21] for which we have that in terms of an AdS/CFT representation of entropy that we have, especially if we use [9] for Area, and S proportional to n for graviton count related to Entropy, as by [28], then

$$\Delta E \equiv mc^2 = \hbar / \Delta t \Rightarrow m = \hbar / c\Delta t$$

$$A_{area} = 16\pi m^2 + \frac{4\pi}{3} \cdot \left(\frac{J}{m}\right)^3 = 16\pi \cdot (\hbar / c\Delta t)^2 + \frac{4\pi}{3} \cdot \left(\frac{Jc \cdot \Delta t}{\hbar}\right)^3$$

$$dS^2 = \frac{L^2}{r^2} \cdot \left[-\left(1 - \left(\frac{r}{r_+}\right)^2\right) dt^2 + \frac{dr^2}{\left(1 - \left(\frac{r}{r_+}\right)^2\right)} + dx^i dx^j \right] \quad (45)$$

$$S_{entropy} = \frac{L^{d-1}}{4G_N} \cdot \left(\frac{r}{r_+}\right)^{d-1} \cdot \left(16\pi \cdot (\Delta E)^2 + \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\Delta E}\right)^3\right) \cdot \left(\frac{4\pi T_{temp}}{d}\right)^{d-1}$$

$$\propto n_{graviton-count}$$

We then have the following representation for a polynomial in Δt , namely if we have conflating of the material in Eq.(45) as far as a quantic treatment of delta t, as by [5] we have that

$$\frac{L^{d-1}}{4G_N} \cdot \left(\frac{r}{r_+}\right)^{d-1} \cdot \left(16\pi \cdot (\hbar / \Delta t)^2 + \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{(\hbar / \Delta t)}\right)^3\right) \cdot \left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \propto n_{graviton-count} \quad (46)$$

We will then, describe how to obtain from Eq. (46)(3) $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$

XVI. Obtaining $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ from Eq. (46)

In order to obtain this, we make the following substitutions below, and we will state specifically that in order to have a negative temperature in order to obtain the conditions as given in [5][65] for a Quintic polynomial which is solvable in the sense of what that article [5][65] is saying. We will later on describe this in detail. But below we put in the substitution needed so we can obtain the polynomial in delta t, which we will then subsequently modify. This also uses [20] and [21]

$$\begin{aligned}
& \frac{L^{d-1}}{4G_N} \cdot \left(\frac{r}{r_+}\right)^{d-1} \cdot \left(16\pi \cdot (\hbar / \Delta t)^2 + \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar / \Delta t}\right)^3\right) \cdot \left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \propto n_{graviton-count} \\
& \Rightarrow \left(16\pi \cdot (\hbar / \Delta t)^2 + \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar / \Delta t}\right)^3\right) \propto \frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1}} \\
& \Rightarrow \left(16\pi \cdot (\hbar)^2 + (\Delta t)^5 \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3\right) - \frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1}} (\Delta t)^2 \equiv 0 \tag{47} \\
& \Rightarrow (\Delta t)^5 - \frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \equiv 0
\end{aligned}$$

I.e. in order to obtain, in a sense a Quintic equation which can be solved, [2][5][65],

$$\begin{aligned}
& (\Delta t)^5 - \frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \equiv 0 \\
& \Rightarrow A_1 = -\frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \\
& A_2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \\
& \Rightarrow T_{temp} \text{ should - be - negative}
\end{aligned} \tag{48}$$

XVII. Can we have negative temperature?

This requires using [21, 22] and it is not clear that this is actually obtainable, in the experimental set up as given in our [21, 22] input into a black hole

What else do we need ?

According to the abstract of [2] and which is used in [5]

Quote

Let a and b be nonzero rational numbers. We show that there are an infinite number of essentially different, irreducible, solvable, quintic trinomials $X^5 + ax + b$. On the other hand, **we** show that there are only five essentially different, irreducible, solvable, quintic trinomials $x^5+ax^2+b=0$, namely,

$$\begin{aligned} &x^5 + 5x^2 + 3, \\ &X^5 + 5x^2 - 15, \\ &X^5 + 25x^2 + 300, \\ &X^5 + 100X^2 + 1000, \\ &\text{and } X^5 + 250X^2 + 625. \end{aligned} \tag{49}$$

End of quote

Aside from having a negative temperature, as for the reason given in Eq. (47) we have that if [21,22] is satisfied and still commensurate with reference [21,22] that we also need to have a polynomial in delta t, which is commensurate with Eq. (48)(6) which is also influenced by Eq. (49) which is taken from the abstract in [2],[5] and is linkable to Eq.(47).

XVIII. Conclusion. i.e. a necessary condition for quantization of induced Kerr Newman black hole

We first of all need to have a “negative “ temperature. I.e. is this doable ? This has to be rigorously explored experimentally and determined.

Secondly our Eq. (47) terms have to be consistently comparable to Eq. (48) (6). This requires rescaling of Eq. (47) but this is doable pending dimensional analysis, and perhaps Planckian physics units.

Both these conditions would be a NECESSARY condition for satisfying in terms of reference [5] $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ which we state would be due our construction a necessary condition for a complete quantum gravity analysis of gravitons being emitted from a Kerr- Newman black hole.

We state that these two points have to be determined and investigated, and also that an optimal value of d, for dimensions for a problem, involving Kerr Newman black holes would have to be ascertained in future research.

Finally, we refer the reader to references [65] [66] [67], [68] for additional ideas which may be used in future projects

Note also that Valev wrote [72]

$$\lambda_{graviton} \sim \frac{h}{m_{graviton} \cdot c} \tag{50}$$

, and Valev indicates in his article that this gives a light year, or more length GW of unimaginably low frequency. Obviously, in terms of experimental conditions, this breaks down, i.e. in the limit

of say a simulated worm hole in a laboratory, so it would be useful to find ways to experimentally test and vet Eq. (49) in our review of basics

Arguing further, the derivation done above, as for a HUP is likely doable and obtainable from higher dimensions. The referee asked that if a minimum uncertainty relation exists which is what I am asserting via [2] which is influenced by [5] that there are then several cases

In the situation of Kaluza Klein , $d = 1$ that we should assert the following

Going to the text, there are two equations which bear examination. i.e. see this in the text. Recall Eq. (35) and Eq. (36) of this text. We will summarize again what came in Eq. (35) and Eq. (36) as follows.

We are then leading to, if we have a distance, we call $a_{graviton}$. And Eq. (35) and Eq. (36) that if in this situation we have $a_{graviton} \approx \lambda_{graviton} \propto 1/\omega_{graviton}$ We go to Eq. (36) with the result that in the case of $d=1$ in the situation for which $(\Delta t)^5 \rightarrow 0^+$, that indeed the ground state , as referred to in Eq. (36) is a strong indicator of quantum gravity. I.e.. The zero-point energy is dependent upon a graviton count, $n_{graviton-count}$

End of my argument here.

I.e. my argument is that in the case of Eq. (36) due to the last line, that one is having a graviton count, as linked to lowest level uncertainty , for energy and that this, in itself is supporting a quantum interpretation of gravity based upon minimum time step

Keep in mind, too, what is in the answer to my answer to the reviewers first question. i.e. S (entropy) $\sim n$ (graviton count) is put in directly into the derivation of Eq. (5). There is no way to guarantee . S (entropy) $\sim n$ (graviton count) being positive as to two black holes at the two ends of a worm hole. i.e. that is one of the wormhole configurations. Unless one has NEGATIVE temperature. I.e. see the discussion of the text

on this, and that ties in directly with the sign of A_1 , as given in
$$A_1 = - \frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} < 0 \quad d \neq 1$$

The $d=1,3,5, \dots$ cases have a different behavior than what is in $d=2,4,6, \dots$ when we are looking at Eq. (5) it really hits home. And the sign of A_1 influences the solvability of finding Δt which in turn affects the likelihood of Eq.(36) above, and also, we have that we want a minimum energy to depend upon graviton count, with that process being inherently quantum nature of gravity.

The $d=1$ case, as with having $(\Delta E)^2 = \frac{1}{4\pi} \cdot \left(\frac{n_{graviton-count}}{\left(\frac{4\pi T_{temp}}{d}\right)^{d-1}}\right)$; $d=1,3,5, \dots$ i.e. if $d=1$, our minimum

uncertainty, which is solvable then will be giving us functional linkage to gravity and gravitons.

End of quote

The tack of reference [9] [31][38] is that in order to have a positive black hole entropy, that we have to entertain negative temperature, which is given in Eq. (8) and which is elaborated on in page 5 of reference [9] [31][38] . i.e. by the following adage, i.e. in order to have positive black hole entropy, the temperature has to be negative, i.e. Eq. (7) could give negative black hole entropy , and in order to obtain positive entropy for a black hole, as given by Eq. (6) we have to have Eq. (8) with negative temperature. To those whom still do not believe this summary? Go to reference [9] [31][38] and look it up. Now how does this connect worm holes ? i.e. a typical model of worm holes has in its formulation a worm hole bridge between two black holes. The complete Schwarzschild geometry consists of a black hole, a white hole, and two Universes connected at their horizons by a wormhole [41] . We have already discussed that negative temperature may exist in astrophysics, i.e. our next section is to link that to worm holes.[42]

4. Negative Temperatures, and the total energy of worm holes

As we will argue accessing Juan Maldacena, et. al,[43] , the total energy of a worm hole reads as follows, h namely given in Eq. (9) which has

$$\begin{aligned} E_{\text{wormhole}} &= -q / 8\ell \\ q &= 2j + 1 \\ \ell &= 1 / 2\pi T_{\text{temperature}} \end{aligned} \quad . \quad (9) \text{ reduplicated.}$$

End of quote

So far this is not stringy, or linked to AdS/CFT correspondence, but then observe the following

From the text. i.e.

IV. How to reconcile String theory which is a quantum gravity regime, with results which seem to be inconsistent with quantum gravity.

The reviewer, in [14] sent the following question which deserves an answer, i.e.

Quote

Another issue is that in all of this the author is working within a “stringy” framework, for instance the values of d are chosen such as to be compatible with string theory, AdS/CFT concepts are used throughout the work, and so on. However, string theory is a theory of quantum gravity. How can you make assumptions consistent with quantum gravity and then derive conditions which are inconsistent with quantum gravity at the same time? This is very inconsistent

End of quote

The author refers the readers to [19], specifically go to page 639 as to the coupling constants used in super Yang Mills theory. i.e. in the section labeled “the Coupling constants”, [24] write that

Quote, from [19], page 639

“The dimensional effective coupling of super Yang Mills theory in d+1 dimension is scale dependent. At an energy scale E, it is determined by dimensional analysis to be given by Eq. (3) we write as

$$g_{eff}^2(E) \sim g_{YM}^2 N E^{d-3} \quad (3) \text{ reduplicated}$$

This coupling is small, so that perturbation theory applies for large E (the UV) for d<3, and for small E (the IR) . The special case of d = 3 corresponds to $\mathbb{N} = 4$ super Yang Mills theory in four dimensions, which is known to be a UV finite, conformally invariant theory. In that case, $g_{eff}^2(E)$ is independent of the scale E and corresponds to the t’Hooft coupling constant which we use the results of Eq(4) we write as

$$\lambda \sim g_{YM}^2 N \quad (4) \text{ reduplicated.}$$

This is the constant which is held constant in the large – N expansion of the gauge theory discussed below

End of quote from page 639 of [19]

I.e. in our work, the question of d dependence will be crucial in the application of the T_{temp} to the question of if we have adherence to quantum gravity, via if we need a negative temperature, will show up as follows, namely

If we have from [2] the following decomposition of the quintic polynomial, and for this see Eq.(5) below, we will be able to go look at the dynamics of what may be occurring for d=3, i.e. what if we have independence of a coupling constant from energy, we have from d=3 in the situation where we have no dependence of the coefficient A_1 upon the sign of the T_{temp} . If say we have a typical dependence of system

energy, say $E_{statistical} = \frac{k_B T_{applied-temperature}}{2}$ we are saying, if we believe that this removes the necessity of having

a negative, or positive temperature, that then the possibility of, say a black hole having negative entropy (for positive temperature) as given by [15] is not important. But this would mean an effective statistically based negative energy, which would be for say energy flowing into a black hole . However, in our derivation of the quintic polynomial, in [2] we are dependent upon an entropy count based upon infinite statistics counting algorithm based upon entropy being based upon an admitted particle count, i.e. $S \sim$ particle count n, as given in [29]. The upshot is, that if we have d = 3 that we have a string theory-based removal of the sign of energy, and temperature in coupling which means that the coupling constant as given in Ea.(3) and Eq. (4) is also consistent with [30] and is also covered in [5] as we derived it. I.e. that the result we have, which uses [29] and [30] , for d=3 is fully consistent with the Eq. (3) and Eq. (4) removal of the centrality of how we evaluate energy, in terms of the sign of energy, if we in doing this

regard our input energy, as say along the lines of $E_{statistical} = \frac{k_B T_{applied-temperature}}{2}$. In this sense, our results in

terms of removal of the importance of the sign of the temperature, and by extension statistical energy, given in Eq. (5) below may make a partial linkage between Eq. (5) below, and Eq. (4) if we can write

$E_{statistical} = \frac{k_B T_{applied-temperature}}{2} = E$, as an input into Eq. (4), with the applied temperature $T_{applied-temperature} = T_{temp}$

XIX. Brief summary of reference [8]and the problem of a solution by radicals.

Readers are recommended to go to page 4 of [8] where the question of if a quintic polynomial is exactly solvable. Well it is not in general solvable. That is the point of reference [2], and the trinomial quintic. And Eq. (14)

The answer to why this is known as the Abel Ruffini theorem[53] i.e. to look at the following

The theorem does *not* assert that some higher-degree polynomial equations have *no* solution. In fact, the opposite is true: every non-constant polynomial equation in one unknown, with real or complex coefficients, has at least one complex number as a solution (and thus, by polynomial division, as many complex roots as its degree, counting repeated roots); this is the fundamental theorem of algebra. These solutions can be computed to any desired degree of accuracy using numerical methods such as the Newton–Raphson method or the Laguerre method, and in this way they are no different from solutions to polynomial equations of the second, third, or fourth degrees. It also does *not* assert that *no* higher-degree polynomial equations can be solved in radicals: the equation $x^n - 1 = 0$ can be solved in radicals for every positive integer n , for example. The theorem only shows that there is no *general solution in radicals* that applies to *all* equations of a given degree greater than 4.

Also, see [62] , . i.e. what the referee does not understand is

quote

no *general solution in radicals* for degree five generalized quintic equations means the following cannot be done.

A general solution in radicals an algebraic solution or solution in radicals is a closed form expression, and more specifically a closed-form algebraic expression, that is the solution of an algebraic equation in terms of the coefficients, relying only on addition, subtraction, multiplication, division, raising to integer powers, and the extraction of roots (square roots, cube roots, etc.).

As stated , we can also go to [63] i.e. page 54 where the definition of solvability by Radicals is done abstractly. See “ section 9, solvability of polynomials by radicals. Also [64]

The result of reference [11] which is mis understood here, is in determining if a radical solution of the given quintic exists. I.e.. In terms of Galois splitting field. The results of Eq. (31) ignored by the referee, is in obtaining a solution in terms of radicals is only achievable with regards to the five linear combinations of the sort given for coefficients given in Eq. (32). Now if we restrict the solution to the specialized quintic referred to in Eq. (11)

End of quote

It is important to review the issues brought up in [65],[66],[67],[68],[69],[70],[71],[72],[73] before going to the next point. I.e. what needs to be said is that we are looking at

- a. A minimum condition for quantization
- b. Looking at what happens to algebraic theory as to precise delineation as to roots
- c. Basic conditions as to black hole and worm hole physics.

Commented [YZ11]: Next one should be [65][66][67]...

Commented [AB12R11]: I just did put them in

Commented [AB13R11]: See the comments below

In order to parse this we should review the physics of why we are even going to review the application of [2].

This closed form solution is a direct result of the failure of the quadratic equation approximation and the application of **Gauss- Lucas theorem to have any commonality.**

We furthermore make the following observation, i.e.
Quote

There are tons of references to Galois theory in this paper. I.e. the readers should READ them. And the following is, in lieu of Eq. (31)

. We say without reservation that if we wish to have generalized inputs into A1 and A2 of the quintic equation that the following must be adhered to, and that without reservation we make, in the spirit of a generalized polynomial solution the following statement as to the values of the quintic equation. I.e. as given below we have a re duplication of Eq. (16) to consider

$$\begin{aligned}
 (\Delta t)^5 - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} (\Delta t)^2 + \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} &\equiv 0 \\
 \Rightarrow A_1 = -\frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} &\neq 1 \qquad (16). \text{ reduplicated} \\
 A_2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} &\neq -2
 \end{aligned}$$

There are no conceivable conditions for which one would have such a situation for a GENERAL solution . We are referring to general solvability. Of quintics, by what is known as by radicals. See more on this as follows

End of quote

The referee, and readers are enjoined to review this paper, and look at these details. Secondly, and I cannot stress this more than once, READ the following paper, i.e. [2] Spearman, B. and Williams, K. (1998) On Solvable Quintics $X^5 + ax + b$ and $X^5 + ax^2 + b$. Rocky Mountain Journal of Mathematics, 28.

Note that it is very important

5. Why was the Kerr Newman black hole chosen as a statement about quantum gravity ? What is special about it ? How can this be justified ?

Here, I urge people to read the following

Quote, from [73]

The Kerr-Newman metric describes a very special rotating, charged mass and is the most general of the asymptotically flat stationary 'black hole' solutions to the Einstein-Maxwell equations of general relativity. We review the derivation of this metric from the Reissner-Nordstrom solution by means of a complex transformation algorithm and provide a brief overview of its basic geometric properties. We also include some discussion of interpretive issues, related metrics, and higher-dimensional analogues

End of quote

It is the specific adage as to this black hole being the most GENERAL solution. I.e. this generality is why it was picked, as the most general, easily analyzed case.

We urge readers whom may not be satisfied by this to if they have to look at more extensions of this black hole business to look at [74] which is an encyclopedia of black holes in higher dimensions. It re enforces many of the same themes brought up here

Keep in mind that the next final section has essential details as to solvability of what is called the restricted trinomial quintic, which is the main focus of the second array of complaints by the reviewer. This is highly specialized and is algebraic field theory, and Galois theory. For your edification.

It is useful to include in the following

XX. FINAL SET OF COMMENTS as to the suitability of using Galois theory, i.e. [2], to solve the Quintic, due to additional questions raised.

See page 398, of [75] i.e. this came from subsequent questions in several additional rounds of inquiry by the referee, in [14]. Hence, to give a reality as to the restricted nature of the coefficients of Eq. (14) we first of all referred to the following theorem, as to what not to use in our problem. This primarily because the reviewer was so dead set against complex to imaginary time values. i.e. consider the following basic theorem

Theorem , the Fundamental theorem of algebra

The field of complex numbers is algebraically closed, that is, every polynomial in $\mathbb{C}[x]$ has a root in \mathbb{C}

i.e. in our case, as requested by the referee, we will be avoiding in analyzing a given polynomial $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$ having any A_1 and A_2 with complex coefficients; so as to avoid Δt be forced to be a root in \mathbb{C}

Commented [YZ14]: The order is not right.

Commented [AB15R14]: Fixed it. See the above.

Now, assume we are working with a real valued quintic equation. i.e.

In addition, we have Descartes' rule of sign [76] is used to determine the number of real zeros of a polynomial function, i.e. see this example. For the number of positive real roots, look at the polynomial, written in descending order, and count how many times the sign changes from term to term. This value represents the maximum number of positive roots in the polynomial. For example, in the polynomial $f(x) = 2x^4 - 9x^3 - 21x^2 + 88x + 48$, you see two changes in sign (don't forget to include the plus sign of the first term!) — from the first term ($+2x^4$) to the second ($-9x^3$) and from the third term ($-21x^2$) to the fourth term ($88x$). That means this equation can have up to two positive solutions.

Descartes's rule of signs says the number of positive roots is equal to changes in sign of $f(x)$ or is less than that by an even number (so you keep subtracting 2 until you get either 1 or 0, i.e. Negative real roots. For the number of negative real roots, find $f(-x)$ and count again. Because negative numbers raised to even powers are positive and negative numbers raised to odd powers are negative, this change affects only terms with odd powers. This step is the same as changing each term with an odd degree to its opposite sign and counting the sign changes again, which gives you the maximum number of negative roots. The example equation becomes $f(-x) = 2x^4 + 9x^3 - 21x^2 - 88x + 48$, which changes signs twice. There can be, at most, two negative roots. However, similar to the rule for positive roots, the number of negative roots is equal to the changes in sign for $f(-x)$ or must be less than that by an even number. Therefore, this example can have either 2 or 0 negative roots

This has been generalized in [77] in the following manner, i.e. In the 1970s Askold Georgevich Khovanskii developed the theory of fewnomials that generalises Descartes' rule. The rule of signs can be thought of as stating that the number of real roots of a polynomial is dependent on the polynomial's complexity, and that this complexity is proportional to the number of monomials it has, not its degree. Khovanskii showed that this holds true not just for polynomials but for algebraic combinations of many transcendental functions, the so-called Pfaffian functions.[78]

Here, a monomial is defined as [79], and in addition, note that If a polynomial doesn't factor, it's called prime because its only factors are 1 and itself. Having said that, let us now go to some other issues.

Note

Eq. (14) is a carbon copy of part of the abstract result from [2]

Observe

Here is the question. See eq. (14)

The referee in [14] questioned as to the following, i.e. these are desired combination of the given polynomial $(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$. This in itself is fair. But the allegation that Eq. (14) from the text below was constructed out of thin air is, actually from [2]. We use also, here that $X = \Delta t$ and that then we will review the math descriptions given in [2]

From [2] and also Eq.(14) of this manuscript. We observe that Eq. (14) is synchronized with the appendix entry of reference [2]

Let a and b be nonzero rational numbers. We show that there are an infinite number of essentially different, irreducible, solvable, quintic trinomials $X^5 + ax + b$. On the other hand, we show that there are only five essentially different, irreducible, solvable, quintic trinomials x^5+ax^2+b , namely, by [2], which is Eq.(14) of the text

The Descartes rule of signs would indicate that such combinations would allow for real valued $X = \Delta t$. Why is this important ? First, the referee has stated a preference for finding roots of $X = \Delta t$ being real valued. I.e. don't believe it ? Go to pages 28 and 29 of this manuscript where this preference is explicitly stated. Secondly, if say a worm hole is in its throat permitting negative time, say in conjunction that the time variable would become positive in the mouth of the worm hole. i.e. what we have been doing is to look at the conditions of the time dynamics in the throat of a worm hole. We shall go to the terms in reference [2] and begin to describe them, mathematically speaking. i.e. one of the first items is that the coefficients A_1 and A_2 are at least real valued. In fact, we have that from Eq. (5) of the text, that the break down of the equation is, given. In this case, go to Eq.(5) of the text.

If we have that $d=2,4,6$, the sign of temperature does not play a role, and we will have then that we will have no commensurate connection with Eq.(14) of the text. It also would indicate a positive time component, as to $X = \Delta t$ whereas we do wish to have the following convention

- A. For the throat, we would prefer to have negative time , which would transition to positive time, at the mouth of the worm hole. This so long as $d=2,4,6$
- B. If $d=1,3,5,7$, then we could have, by use of the Descartes sign convention negative time roots for time in the worm hole throat

Using [75], [76], [77],[78] , [79] we would have then a situation for which we would first of all avoid having imaginary time, if we use the conventions of Eq. (14) and also keep in mind the first part of Eq. (5) from the text we avoid imaginary, or complex time, which is what the referee would not stand for, and in addition, negative roots for $X = \Delta t$ as well as being real valued which is what we would prefer to have.

Commented [AB16]: Acceptable due to what was put in above, in the preceding page.

Note, that a possible problem, about using [2] is that the field as specified in Eq.(14) would require that A_1 and A_2 have rational coefficients. The restriction this would mean is that we would then say have to have, for the application of Eq. (14) the following, namely use this part of Eq.(5)

$$A_1 = - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \approx \text{Term - with - } \pi \text{ - canceled - out.}$$

{ Part of Eq.(5) from text }

$$A_2 = \frac{16\pi \cdot (\hbar)^2}{\frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \approx \text{Term - with - } \pi \text{ - canceled - out.}$$

In the case of A_2 this happens immediately. As for A_1 , it likely would mean defining $n_{\text{graviton-count}}$ or some other input variables in such a way as to lead to a canceling out of the π term. i.e. my preference would be to have T_{temp} and J defined in such a way as to effectively cancel out the π term from A_1 . Note if $n_{\text{graviton-count}}$ effectively vanished, we would then have a very easy to solve equation for $X = \Delta t$ i.e. no problem in terms of a defined $X = \Delta t$. However, in doing so we would have another problem in that the linkage to quantum gravity, i.e. a linkage to gravitons and quantum mechanics would be effectively demolished.

Next, in this is a question of the different terms in reference [2]. We will review them. First of all is the idea of irreducible polynomials. Let F be a finite field. As for general fields, a non-constant polynomial f in $F[x]$ is said to be **irreducible** over F if it is not the product of two polynomials of positive degree. A polynomial of positive degree that is not irreducible over F is called *reducible over F* . [81], [82], [83], [84], [85]

Now a polynomial of positive degree is such that the **degree** of a **polynomial** and the sign of its leading coefficient dictates its limiting behavior, and in our case, we have positive degrees with the term $(\Delta t)^5$.

Going back to [2] we have that the following shows up, i.e.

If the equation $f(x) = 0$ is solvable by radicals, the quintic polynomial $f(X)$ is said to be solvable. If $f(X)$ is solvable, its Galois group is solvable and is thus contained in the Frobenius group F_{20} of order 20, and hence is isomorphic to F_{20} . Here, polynomial $f(x) = 0$ is solvable by radicals, means that definitions as to solvability in [86] is satisfied in that we have operations given in the examples delineated by [87]

To re capitulate, what we choose in [2] was largely chosen due to the physical issues brought up in pages 48 to 51, as is conveniently brought up in Eq. (14) which was not arbitrarily chosen

Also, due to another issue once again, Eq. (16) of the text, as to what to avoid reads as $A_1 = 1$, $A_2 = -2$, and my objection is clearly rendered in Eq. *16 as reproduced below. As to avoiding $A_1 = 1$ and $A_2 = -2$ with these two values chosen not by me, And the equation below representing what we wish to avoid. I.e.. Particular solutions in the case where we want general solutions. Note the following as to what to avoid. i.e. see Eq. (16) In short, reference [2] was chosen as to its intersection with the Descartes result as of , once again

If we have that $d=2,4,6$, the sign of temperature does not play a role, and we will have then that we will have no commensurate connection with Eq.(14) of the text. It also would indicate a positive time component, as to $X = \Delta t$ whereas we do wish to have the following convention

- C. For the throat, we would prefer to have negative time , which would transition to positive time, at the mouth of the worm hole. This so long as $d=2,4,6$

D. If $d=1,3,5,7$, then we could have, by use of the Descartes sign convention negative time roots for time in the worm hole throat

Using [75], [76], [77],[78] , [79] we would have then a situation for which we would first of all avoid having imaginary time, if we use the conventions of Eq. (14) and also keep in mind the first part of Eq. (5) from the text we avoid imaginary, or complex time, which is what the referee would not stand for, and in addition, negative roots for $X = \Delta t$ as well as being real valued which is what we would prefer to have.

Both physics and mathematics is well served, and we used '[2]' also in addition to the above, due to Eq, (1) which we render again as the three cases, with the derivative of the polynomial having very different solution behavior for $X = \Delta t$, than what we would obtain for the quadratic approximation. Plus again, wishing to have by Descartes convention of signs the possibility of guaranteed access to non-imaginary, real valued roots, which could have, by Descartes convention of signs cases where not only could we have real valued $X = \Delta t$ but also negative time for $X = \Delta t$ in the throat of the wormhole.

See Eq. (1) reproduced below as to giving us this starting point.

$$(\Delta t)^3 + \frac{2A_1}{5} = 0$$

different Δt answer from

$$A_1 \cdot (\Delta t)^2 + A_2 = 0$$

(1) reduplicated again

versus needing Galois solution to

$$(\Delta t)^5 + A_1 \cdot (\Delta t)^2 + A_2 = 0$$

Note in addition that there are other wormhole issues, vitally important which will be brought up, extending these issues once review is commenced.

Keep in mind that we have one extension which will be stated here

As a parting remark, this business of choice of sign, for temperature and the behavior of a worm hole, and the question of if we have quantization behavior has similarities to some of the research work goals done by John Klauder [88] which we put in as the final reference as to our inquiry, especially if the worm hole construction is prevalent in the early phases of the expansion of the universe, as given in this document. In all we will seek connections with Dr. Klauder's work in future extensions of our inquiry.

Finally, and not to be minimized, we view that not only is Dr. Klauder's work important that we also have what is known as the Jones Polynomials to compare our polynomial idea with. i.e. see [89] , page 332.

Since we have referenced temperature, it would be expedient to go to page 332 where there is linkage to polynomials, and the idea of a partition function, and in page 328. Undoubtedly there

will be connections made to what is known as the Alexander – Conway polynomial of the Hopf link, as given in Figure 45 of page 328 of [89]

We close in stating also that there are more polynomial issues brought up in [90] which are linked to higher order curvature terms, which will be playing a role in our inquiries.

XXI. Final remarks to bring up for reference as to the next publication as a sequel to this document.

One of the issues which has been raised in conversations, has been about the dimensionality of d . ie. As Johnathan Dickau suggested [91] December 20 , it could be fractal or an irrational number . I.e. a fractal d may, with some caveats so that one would have Eq. (32) be consistent with the Galois theory of reference [2] so we could use directly the Rocky Mountain journal of mathematics as to having A_1 and A_2 with rational coefficients, which would make our results consistent with the choice of Eq. (14) and reference [2]. To do that we wish to have that the following equation , as given below avoid having irrational number character. As presented below.

$$A_1 = - \frac{n_{\text{graviton-count}}}{\left(\frac{4\pi T_{\text{temp}}}{d}\right)^{d-1} \cdot \frac{4\pi}{3} \cdot \left(\frac{Jc^2}{\hbar}\right)^3} \quad (51)$$

has no irrational character, but is a fraction

At the minimum, it would be also helpful to investigate if we could look at also, the role of additional dimensions, in terms of gravitational waves, as brought up in [92], as well as research done by Dr. Li, Dr. Wen Hao, and others in terms of [93] , as to how the character of gravitons which are in space time, as say in scalar-tensor gravitational theories influences polarizations.

A suggested update as to this research would be to investigate both the issues of references [92], and [93] in terms of the worm hole physics, as given in this document, as well as the extensions of worm hole physics brought up in [90].

Finally, [94], namely what Maggiore brought up in page 663 as to Thermal Tunneling theory, as to a first order phase transition material which may have very strong similarities as to the generation of GW as seen in our model, should be further developed and compared with our model, i.e. the Maggiore Tunneling and the bounce section of this manuscript, as of [94] may have GW characteristics similar to what we are bringing up in our problem. I.e. there is in page 668 of [94] a tunneling rate, as given by the physics of Eq. (33) below, which is for GW and gravitons emerging from the worm hole.

Commented [AB17]: This equation improperly labeled in my earlier submission to you Zoey and Christian. Fixed

$$\begin{aligned} \Gamma &= A \exp(-S_e) \\ A &= \text{proportinality} - \text{const} \\ S_e &= \text{Value} - \text{of} - \text{Euclidian} - \text{action} \end{aligned} \tag{52}$$

Commented [AB18]: Improper labeling of this Equation, Zoey and Christian. Fixed

Is such a construction even remotely feasible for the tunneling rate of gravitons say from a closed worm hole throat, to our present universe, and what is the counter part to the Euclidian action in our model ?

As of now, it is assuming a closed throat which appears to be consistent with our paper, but then say what is the value of the Euclidian action? A final issue to add, if this Eq. (52) is relevant, to graviton production and say if we restricted ourselves to $d=1$, i.e. the Kaluza Klein case, could we also look at an intermixture of gravitons with the electromagnetic field, which is given in [95], where from pages 295 to 299, the Kaluza Klein theory of electromagnetism is brought up, a purported linkage between the fine structure constant, and a nominal topological charge, i.e. if $d=1$, look at say a linkage between a topological charge, Q_n and a fine structure constant value. And possibly gravity itself as from the worm hole throat, via linkage between gravitons, eventually, and the $1/r^2$ gravitational potential. See this from [95] and its equations from 295 to 299 of [96] which gives an introduction to Kaluza Klein, and charges

Commented [AB19]: Very important future goal outlined as a future work in progress to consider

$$\begin{aligned} \alpha(\text{fine} - \text{structure} - \text{const}) &= \frac{4l_{\text{Planck}}^2}{\phi \cdot r^2} \\ \phi &= \text{scalar} - \text{field} \\ \& \\ Q_n &= \frac{\hbar \kappa}{r} \cdot \sqrt{\frac{2}{\phi}} \end{aligned} \tag{53}$$

Commented [AB20]: Eq.(53), is added, and is from reference [95]

The idea would be to make linkage between the production of Gravitons, and a gravitational potential energy system, i.e. in this case through the $1/r^2$ potential energy system. i.e. along the lines of a first order approximation of gravitational potential energy, as to a modification of the the $1/r^2$ potential energy system, and a linkage with that to gravitons, and then from there, using that, assuming some variant of Eq. (52) to then link graviton production behavior to the filling in of detail as to creating charge, Q_n , i.e. in this case creating a unification, via the cosmological constant with the idea of gravitational characteristics, and electromagnetics, in the $d=1$ case. Keep in mind that as given in [96] there are extensions of the electromagnetic field, beyond Maxwell's equation, as given by Terence Barrett, and that what we are asking about is in the same spirit. i.e. this is a long term project of linkage of electromagnetic field, with gravitation, in the case of the wormhole throat, and is a step beyond our present endeavor we should try for, i.e. for $d=1$ linkage of gravitons, with a $1/r^2$ potential and gravity and an open question of if this $1/r^2$ potential could be linked to the state of gravity emerging from a worm hole, and charge Q of electromagnetic fields.

Commented [AB21]: A long term project. i.e. linkage of gravitons, with a $1/r^2$ potential and gravity and an open question of if this $1/r^2$ potential could be linked to the state of gravity emerging from a worm hole, and charge Q of electromagnetic fields.

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