SunQM-3s7: Predict mass density r-distribution for gas/ice planets, and the superposition of {N,n//q} or |qnlm> QM states for planet/star

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Abstract

Using the same method developed in paper SunQM-3s6, the (close to the true) mass density r-distribution for gas/ice planets has been estimated based on {N,n} QM probability distribution. Based on this calculation (as well as on other scientists' calculation), the center core mass density for Jupiter, Saturn, Uranus, Neptune, and the undiscovered {3,2} planet (if it does formed) are estimated to be: ~ 26000 kg/m^3, ~ 23000 kg/m^3, ~ 16500 kg/m^3, ~ 17000 kg/m^3, and ~ 16000 kg/m^3 respectively. Although a celestial body's formation is primarily based on G-force, after passing a critical mass point (estimated between 1E+19 kg to 7E+22 kg), the {N,n} QM-force starts to affect the internal structure of this celestial body. For an in situ formed (large) celestial body, its {N,n} QM governed radial structure is always coupled with its gravity governed radial structure, although they may be de-coupled under certain situation. If we define the pFactor's quantum number as "q", then a p{N,n//q} QM state can be written as |qnlm>. The analysis suggests that q is also a superpositioned quantum number in |qnlm> QM state. The analysis reveals that Jupiter's current QM structure p{N,n//5} may not be at a Jupiter-massed celestial body's "global energy minimum" state. In other words, among all possible superpositional q(s), q=5 may not be the ground state for a Jupiter-massed celestial body's |qnlm> state. The analysis also suggests that q=6 is a Sun-massed celestial body's "global energy minimum" state, so Solar system's {N,n//6} QM structure is a |qnlm> ground state for the quantum number q.

Introduction

In a series of research articles, the quantum mechanics of Solar system has been established [1] ~ [11]. In previous paper SunQM-3s6 [10], from studying Earth's known internal structure and mass density distribution, I have developed a method which can be used to estimate any planet's internal structure and mass density. This method can be expressed as:

Planet mass = 4π ∫ (planet's QM probability density r-distribution) * W *D * r^2 dr

where mass density D = a * r + b, and W is a scaling factor. In paper SunQM-3s6, I applied this method to all four rocky planets, and obtained the internal structure and (close to the true) mass density r-distribution for these planets. In current paper, I will apply the same method to four gas/ice planets. Note: due to the size limitation, the same analysis for Sun has been spun-off from this paper, and moved to a new paper SunQM-3s8. Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see my paper SunQM-1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = 3.4×10^{12}, 5.6E-9 = 5.6×10^{-9}. Note: for all SunQM series papers, reader should check "SunQM-3s10: Updates and Q/A for SunQM series papers" for the most recent updates and corrections.

I. Predict Jupiter's internal structure and the mass density radial distribution by using p{N,n} QM probability function

So far, we do not have the experimental determined mass density radial distribution (like Earth's) for Jupiter. According to "https://sciencing.com/jupiters-core-vs-earths-core-21848.html", "*The core's density is estimated at 25,000 kg per cubic meter*". According to the method established from paper SunQM-3s6, let us first constitute the mass density linear equation D = a * r + b for Jupiter. After manual fitting, one good result is D = -0.000076 * r + 5310. It satisfies both conditions: 1) ∫ D dV = mass of Jupiter (see the integration equation below); 2) at surface r = 6.99E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 1a.

$$\int_0^{6.99\times 10^7} 4\pi(-0.000076\,x+5310)\,x^2\,dx = 1896554786375158606732460032$$

Table 1. Predict Jupiter's internal structure and the mass density r-distribution by using p{N,n} QM radial probability function.

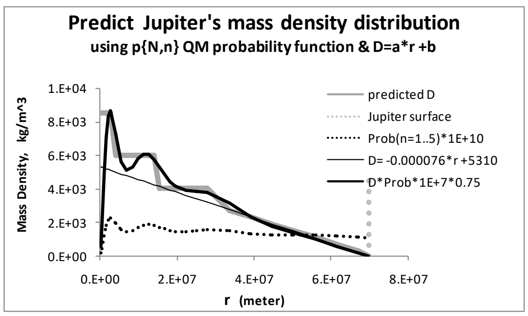
										A= 2.26E+05			
										B= 0.13			
r_1=2.80E+6 m			1.63E+27		1.00E+10	0.75	1.64E+27			C= 21600.0		1.63E+27	
r/r_1 =	r/r_1*r_1	D= -0.000076*r +5310	mass= D*ΔV	Prob(n=1..5)	Prob(n=1..5)*1E+10	D*Prob*1E+7*0.75	mass= (D*Prob*1E+7*0.75) *ΔV	r, p{-1,1}, p{0,1}	predicted D	D=A/r^B-C	(A/r^B-C)*Prob*1E+7	mass= ((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
unit	m	kg/m^3	kg			kg/m^3	kg	m	kg/m^3		kg/m^3	kg	kg/m^3
0.1	2.80E+05	5.29E+03	4.84E+20	1.386E-08	139	5.50E+02	5.03E+19	2.80E+05	8.50E+03	2.27E+04	3.14E+03	2.88E+20	2.60E+04
0.2	5.59E+05	5.27E+03	3.38E+21	4.537E-08	454	1.79E+03	1.15E+21	5.59E+05	8.50E+03	1.89E+04	8.55E+03	5.48E+21	2.60E+04
0.4	1.12E+06	5.23E+03	2.68E+22	1.213E-07	1213	4.75E+03	2.44E+22	1.12E+06	8.50E+03	1.54E+04	1.86E+04	9.55E+22	2.60E+04
0.6	1.68E+06	5.18E+03	7.21E+22	1.82E-07	1820	7.08E+03	9.85E+22	1.68E+06	8.50E+03	1.35E+04	2.45E+04	3.41E+23	2.60E+04
0.8	2.24E+06	5.14E+03	1.39E+23	2.159E-07	2159	8.32E+03	2.26E+23	2.24E+06	8.50E+03	1.22E+04	2.63E+04	7.13E+23	2.60E+04
1	2.80E+06	5.10E+03	2.28E+23	2.257E-07	2257	8.63E+03	3.85E+23	2.80E+06	8.50E+03	1.12E+04	2.53E+04	1.13E+24	2.60E+04
1.5	4.19E+06	4.99E+03	1.09E+24	1.936E-07	1936	7.25E+03	1.58E+24	4.19E+06	6.00E+03	9.53E+03	1.85E+04	4.01E+24	1.10E+04
2	5.59E+06	4.89E+03	2.07E+24	1.541E-07	1541	5.64E+03	2.39E+24	5.59E+06	6.00E+03	8.39E+03	1.29E+04	5.47E+24	1.10E+04
2.5	6.99E+06	4.78E+03	3.34E+24	1.418E-07	1418	5.08E+03	3.55E+24	6.99E+06	6.00E+03	7.53E+03	1.07E+04	7.45E+24	1.10E+04
3	8.39E+06	4.67E+03	4.87E+24	1.518E-07	1518	5.32E+03	5.54E+24	8.39E+06	6.00E+03	6.85E+03	1.04E+04	1.08E+25	1.10E+04
3.5	9.79E+06	4.57E+03	6.64E+24	1.689E-07	1689	5.78E+03	8.41E+24	9.79E+06	6.00E+03	6.28E+03	1.06E+04	1.54E+25	1.10E+04
4	1.12E+07	4.46E+03	8.63E+24	1.816E-07	1816	6.07E+03	1.17E+25	1.12E+07	6.00E+03	5.80E+03	1.05E+04	2.04E+25	1.10E+04
4.5	1.26E+07	4.35E+03	1.08E+25	1.852E-07	1852	6.05E+03	1.50E+25	1.26E+07	6.00E+03	5.39E+03	9.98E+03	2.48E+25	1.10E+04
5	1.40E+07	4.25E+03	1.32E+25	1.805E-07	1805	5.75E+03	1.78E+25	1.40E+07	6.00E+03	5.02E+03	9.06E+03	2.81E+25	5.00E+03
5.5	1.54E+07	4.14E+03	1.57E+25	1.708E-07	1708	5.30E+03	2.01E+25	1.54E+07	4.00E+03	4.69E+03	8.01E+03	3.03E+25	5.00E+03
6	1.68E+07	4.04E+03	1.83E+25	1.597E-07	1597	4.83E+03	2.20E+25	1.68E+07	4.00E+03	4.40E+03	7.02E+03	3.19E+25	5.00E+03
6.5	1.82E+07	3.93E+03	2.11E+25	1.504E-07	1504	4.43E+03	2.38E+25	1.82E+07	4.00E+03	4.13E+03	6.21E+03	3.33E+25	5.00E+03
7	1.96E+07	3.82E+03	2.39E+25	1.445E-07	1445	4.14E+03	2.59E+25	1.96E+07	4.00E+03	3.88E+03	5.60E+03	3.51E+25	5.00E+03
8	2.24E+07	3.61E+03	5.59E+25	1.434E-07	1434	3.88E+03	6.01E+25	2.24E+07	4.00E+03	3.44E+03	4.93E+03	7.64E+25	5.00E+03
9	2.52E+07	3.40E+03	6.75E+25	1.507E-07	1507	3.84E+03	7.63E+25	2.52E+07	4.00E+03	3.06E+03	4.61E+03	9.17E+25	5.00E+03
10	2.80E+07	3.19E+03	7.90E+25	1.573E-07	1573	3.76E+03	9.32E+25	2.80E+07	4.00E+03	2.73E+03	4.29E+03	1.06E+26	5.00E+03
12	3.36E+07	2.76E+03	1.84E+26	1.514E-07	1514	3.13E+03	2.09E+26	3.36E+07	2.70E+03	2.16E+03	3.26E+03	2.18E+26	2.40E+03
14	3.91E+07	2.34E+03	2.17E+26	1.325E-07	1325	2.32E+03	2.16E+26	3.91E+07	2.30E+03	1.68E+03	2.23E+03	2.08E+26	2.00E+03
16	4.47E+07	1.91E+03	2.36E+26	1.23E-07	1230	1.76E+03	2.18E+26	4.47E+07	1.90E+03	1.28E+03	1.58E+03	1.95E+26	1.60E+03
18	5.03E+07	1.49E+03	2.36E+26	1.247E-07	1247	1.39E+03	2.21E+26	5.03E+07	1.50E+03	9.36E+02	1.17E+03	1.86E+26	1.20E+03
20	5.59E+07	1.06E+03	2.10E+26	1.265E-07	1265	1.01E+03	2.00E+26	5.59E+07	1.10E+03	6.29E+02	7.96E+02	1.58E+26	8.00E+02
22	6.15E+07	6.35E+02	1.54E+26	1.213E-07	1213	5.78E+02	1.40E+26	6.15E+07	7.00E+02	3.56E+02	4.31E+02	1.05E+26	4.00E+02
24	6.71E+07	2.10E+02	6.11E+25	1.101E-07	1101	1.74E+02	5.05E+25	6.71E+07	3.00E+02	1.09E+02	1.20E+02	3.48E+25	1.00E+02
25	6.99E+07	-2.40E+00	-3.96E+23	1.039E-07	1039	-1.87E+00	-3.09E+23	6.99E+07	0.00E+00	-6.33E+00	-6.58E+00	-1.09E+24	0.00E+00

Note: if using r_1 = 0.28 (with unit of E+7 meter), then max Prob = 2.26. If using r_1 = 2.8E+6 (with unit of meter), then max Prob = 2.26E-7. The E-7 probability is due to that this radial wave function R(n*l*) is normalized for hydrogen atom's a_0 = 5.29E-11 m. So when using this probability, I need to scale it up to ~1E+7 times to make it around to 1. We can avoid this trouble by deducing out the radial wave function R(n*l*) that specifically normalized to Jupiter's r_1 = 2.8E+6 m. But I am only a citizen scientist of QM, it is too much work for me to do it.

From paper SunQM-1s3, we know that Jupiter (both its internal structure and its moon orbit system) can be described by a p{N,n//5} QM structure. If we define Jupiter's surface as p{0,1}, then p{0,1//5} = p{-1,5//5}. Let us set Jupiter's (Earth-sized) inner core as r_1, so $r_1 = r_{surface}$ / 5^2 = 6.99E+7 / 25 ≈ 2.80E+6 m. This means that when we set p{-1,1} as n=1 state, the radial probability density of Jupiter has function of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,*l*)|^2). Similar as that of Earth, Jupiter's mass radial distribution can also be (approximately, ignoring the p{-1,1] center region due to its small volume) described by a simple integration formula of QM probability

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 6.99E+7 m; θ=0, π; φ=0, 2π]
or
1.90E+27 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,*l*)|^2) *W *(-0.000076*r + 5310) * r^2 dr, [r=0, 6.99E+7 m]

For the probability of r^2 * |R(5,*l*)|^2, among *l* = 0, 1, 2, 3, and 4, only r^2 * |R(5,4)|^2, makes significant contribution within r/r_1 =25. So the r^2 * |R(5,*l*)|^2 is simplified as r^2 * |R(5,4)|^2. The calculation of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2 + |R(5,4)|^2) is the same as that in paper SunQM-3 Table 2, except now using r_1 = 2.80E+6 m. The table of r^2 *|R(n,l)|^2 calculation is not shown here. The resulted curve is shown in column 5 "Prob(n=1..5)" of Table 1, and it is plotted in Figure 1a as "Prob(n=1..5)*1E+10". Table 1 shows the calculation to predict Jupiter's internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 1a and Figure 1b shows the result.



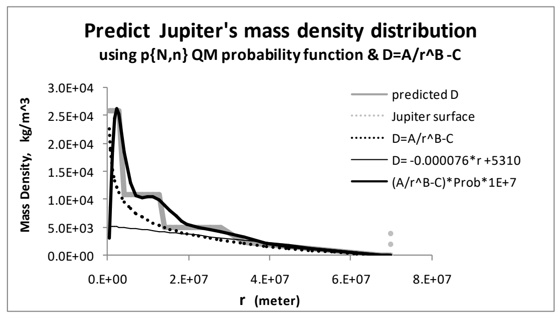


Figure 1a (left). Predict Jupiter's internal structure and the mass density r-distribution by using p{N,n} QM's radial probability function and a linear (D = a * r + b) scaling up.
Figure 1b (right). Predict Jupiter's internal structure and the mass density r-distribution by using p{N,n} QM's radial probability function and a curved (D = A / r^B - C) scaling up.

In Figure 1a, based on the D = -0.000076 * r +5310 linear curve, I manually adjust the factor W for curve of "D*Prob*1E+7*W", in a way that the "mass = (D*Prob*1E+7*W)*ΔV" value (in column 8 top line) equals to "mass = D*ΔV" value (in column 4 top line), the resulted W = 0.75. Then column 7 "D*Prob*1E+7*0.75" curve is plotted in Figure 1a. After that, I construct a stepped line (see the grey thick line in Figure 1a) according to the "D*Prob*1E+7*0.75" curve based on my eye judgment. According to this stepped line, I can predict that there are four (major) layers with three interfaces for Jupiter's internal structure: An obvious p{-1,1//5} core (or inner core, or Earth-sized core) with D ≈ 8500 kg/m^3, r ≈ 2.8E+6 m (same as p{-1,1} 's r = 6.99E+7 /25 = 2.8E+6 m), a p{-1,2//5} core with D ≈ 6000 kg/m^3, r ≈ 1.4E+7 m, (a little bit larger then p{-1,2//5} 's r =6.99E+7 /25*4 = 1.1E+7 m), and a unobvious p{-1,3//5} core (or a out core) with D ≈ 4000 kg/m^3, r ≈ 2.8E+7 m, (a little bit larger than p{-1,3//5} 's r =6.99E+7 /25*9 = 2.5E+7 m), and a most outer (atmosphere) layer with D decreasing from ≈2700 kg/m^3 at r ≈ 2.8E+7 m, to D ≈ 0 kg/m^3 at Jupiter surface.

This predication of internal structure closely matches the {N,n} QM analysis result for Jupiter in paper SunQM-1s3 section I-b. However, comparing to Jupiter's inner core mass density = 25000 kg/m^3 mentioned before, Figure 1a 's D = 8500 kg/m^3 is too low. It is obvious that this is caused by using the linear D = a * r + b for scaling up the probability curve. It is well known that due to the gravity compression, the radial distribution of mass density of a planet (or Sun) is curved line (not linear line). In paper SunQM-3, I used D = A / r^B to mimic this curved radial distribution of mass density for Sun. In current paper, after many tries, I find that I have to use D = A / r^B - C (instead of D = A / r^B) to mimic this curved radial distribution of mass density for Jupiter. The conditions for the right D curve are:
1) The total mass integration of "D*Porb*1E+7" from r = 0 to 6.99E+7 m has to equal to Jupiter's mass;
2) At Jupiter surface, "D*Porb*1E+7" ≈ 0 kg/m^3;
3) At Jupiter inner core (r < 2.8E+6 m), "D*Porb*1E+7" ≈ 25000 kg/m^3.

After fitting manually, one good result is D = 2.26E+5 / r^0.13 -21600 (see Table 1. columns 11-14). It satisfies all three conditions within reasonable error range.

Then I construct a stepped line (see column 15 of Table 1, and see the grey thick line in Figure 1b) according to the "(A / r^B - C)*Prob*1E+7" curve based on my eye judgment. According to this stepped line, I can predict that there are four (major) layers with three interfaces for Jupiter's internal structure: an obvious p{-1,1//5} core (or inner core, or Earth-sized core) with D ≈ 26000 kg/m^3, r ≈ 2.8E+6 m (same as p{-1,1//5} 's r = 6.99E+7 /25 = 2.8E+6 m); a p{-1,2//5} core with D ≈ 11000 kg/m^3, r ≈ 1.26E+7 m, (a little bit larger then p{-1,2//5} 's r = 6.99E+7 /25*4 = 1.1E+7 m); and a unobvious p{-1,3//5} core (or a out core) with D ≈ 5000 kg/m^3, r ≈ 2.8E+7 m, (a little bit larger than p{-1,3//5} 's r = 6.99E+7 /25*9 = 2.5E+7 m); and a most outer (atmosphere) layer with D decreasing from ≈ 2400 kg/m^3 at r ≈ 2.8E+7 m, to D ≈ 0 kg/m^3 at Jupiter surface. This is the internal structure and the mass density distribution I predicted for Jupiter. I believe it is very close to the true value of Jupiter's.

Comparing results between Figure 1b and Figure 1a, we see that they have the same internal (core) structure, but different mass density distribution. In this model, it is obvious that the Jupiter's {N,n} QM radial probability density curve determines Jupiter's internal core structure, and D curve (which is used to scale-up the QM probability distribution) determines the mass density of each core.

II. Predict Neptune's internal structure and the mass density r-distribution by using {N,n} QM probability function

So far no experimental determined mass density radial distribution (like Earth's) has been found for Neptune. Now let's constitute the mass density linear equation D = a * r + b for Neptune. After manual fitting, one good result is D = -0.000258 * r +6400. It satisfy both conditions 1) ∫ D dV = mass of Neptune (see the integration equation below); 2) at surface r = 2.48E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 2a.

$$\int_0^{2.48\times 10^7} 4\pi(-0.000258x + 6400)x^2\,dx = 102303203473436279213916160$$

From my analysis in paper SunQM-1s3 section IV and section X, the current Neptune has a p{N,n//2} QM structure. It includes a (Earth sized) core (let us define it as) p{0,1//2}, a p{-1,1//2} sized inner core, and a p{1,1//2} sized atmosphere. All of them have ~ 100% mass occupancy. A p{N,n//2} QM can be naturally described as a p{N,n//4} QM. If I choose to use p{N,n//2} QM to predict Neptune's internal structure, then I have to use the same method as that for Saturn (see section V). It is relatively complicated. If I choose to use p{N,n//4} QM to predict Neptune's internal structure, then I am able to use the same method as that for Jupiter (see section I), and it is relatively easier. So I choose to use p{N,n//4} QM to predict Neptune's internal structure. In p{N,n//4} QM, let's choose Neptune's inner core as p{0,1//4}, and its r as r_1 (= 2.48E+7 / 16 = 1.55E+6 m). Then its Earth-sized core is at p{0,2//4}, and its surface is at p{0,4//4}. The rest calculations are almost same as that for Jupiter's, except that Jupiter has a p{N,n//5} QM structure, while Neptune has a p{N,n//4} QM structure.

Similar as that of Earth and Jupiter, Neptune's mass radial distribution can also be described by a simple integration formula of QM probability:

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 2.48E+7 m; θ=0, π; φ=0, 2π]
or
1.02E+26 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W * (-0.000258*r +6400) * r^2 dr, [r=0, 2.48E+7 m]

Again, the table of r^2 *|R(n,*l*)|^2 calculation is not shown here. The resulted curve is shown in column 5 "Prob(n=1..4)" of Table 2, and it is plotted in Figure 2a as "Prob(n=1..4)*4E+9". Table 2 shows the calculation to predict Neptune's internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 2a and Figure 2b shows the result. In column 7 "D*Prob*1E+7*0.4" of Table 2, instead of integration, I manually scaled-up the Neptune's probability curve based on D = -0.000258 * r + 6400, with W = 1E+7 * 0.4, and this scaled-up curve is plotted in Figure 2a as "D*Prob*1E+7*0.4".

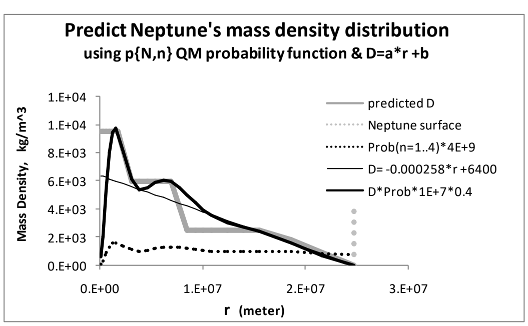
Again the linear D = a * r + b scaling up gives too low D (≈ 9500 kg/m^3) value at the inner core of Neptune. A curved scaling up with D = A / r^B - C has been tested (see columns 11-15 in Table 2). After many hours manual fitting, one possible curve is D = 169000 / r^0.014 - 133157. It gives:

1) The total mass integration of “D*Porb*1E+7” from r = 0 to 2.48E+7 m equals to Neptune's mass;
2) At Neptune surface, “D*Porb*1E+7” ≈ 0 kg/m^3 (see Figure 2b);
3) At Neptune's p{0,1//4} inner core (r < 1.55E+6 m), “D*Porb*1E+7” ≈ 21000 kg/m^3;

Both Figure 2a and 2b predict that Neptune has a p{0,1//4} inner core with r ≈ 1.55 ~ 1.8E+6 m, an Earth-sized core p{0,2//4} at r ≈ 6.98E+6 m, and an unobvious (liquid) atmosphere core p{0,3//4} at r ≈ 1.55E+7 m. However, the fittings in Figure 2a and 2b only give the range of the mass density for each core (9500~21000 kg/m^3, 6000~8000 kg/m^3, 2500~2500 kg/m^3, respectively). My best guess is 17000 kg/m^3 for p{0,1//4} inner core, 7000 kg/m^3 for p{0,2//4} core, 2500 kg/m^ for p{0,3//4} (liquid) atmosphere layer, and 2500 → 0 kg/m^3 for the outer atmosphere layer.

Table 2. Predict Neptune's mass density distribution by using p{N,n} QM probability function.

										A= 169000			
										B= 0.014			
r_1=	1.55E+06		8.15E+25			0.4	8.16E+25			C= 133157		8.15E+25	
r/r_1=	r/r_1*r_1	D= -0.000258*r +6400	mass= D*ΔV	Prob(n=1..4)	Prob(n=1..4)*4E+9	D*Prob*1E+7*0.4	mass= (D*Prob*1E+7*0.4)*ΔV	r, p{-1,1}, p{0,1}	predicted D	D=A/r^B-C	(A/r^B-C)*Prob*1E+7	mass= ((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
	m	kg/m^3				kg/m^3		m	kg/m^3	kg/m^3	kg/m^3	kg	kg/m^3
0.1	1.55E+05	6360	9.92E+19	2.488E-08	100	633	9.87E+18	1.55E+05	9500	9805	2439	3.80E+19	21500
0.2	3.10E+05	6320	6.90E+20	8.14E-08	326	2058	2.25E+20	3.10E+05	9500	8425	6858	7.49E+20	21500
0.4	6.20E+05	6240	5.45E+21	2.176E-07	870	5431	4.74E+21	6.20E+05	9500	7058	15358	1.34E+22	21500
0.6	9.30E+05	6160	1.46E+22	3.268E-07	1307	8052	1.91E+22	9.30E+05	9500	6264	20469	4.85E+22	21500
0.8	1.24E+06	6080	2.81E+22	3.877E-07	1551	9429	4.35E+22	1.24E+06	9500	5704	22112	1.02E+23	21500
1	1.55E+06	6000	4.57E+22	4.052E-07	1621	9725	7.40E+22	1.55E+06	9500	5270	21355	1.63E+23	21500
1.5	2.33E+06	5800	2.15E+23	3.471E-07	1388	8053	2.98E+23	1.80E+06	9500	4487	15575	5.77E+23	8000
2	3.10E+06	5600	4.04E+23	2.741E-07	1096	6140	4.43E+23	3.10E+06	6000	3934	10782	7.78E+23	8000
2.5	3.88E+06	5400	6.42E+23	2.493E-07	997	5385	6.40E+23	3.88E+06	6000	3506	8740	1.04E+24	8000
3	4.65E+06	5200	9.23E+23	2.647E-07	1059	5506	9.77E+23	4.65E+06	6000	3158	8359	1.48E+24	8000
3.5	5.43E+06	5000	1.24E+24	2.938E-07	1175	5877	1.46E+24	5.43E+06	6000	2864	8415	2.08E+24	8000
4	6.20E+06	4800	1.58E+24	3.162E-07	1265	6072	2.00E+24	6.20E+06	6000	2610	8252	2.72E+24	8000
4.5	6.98E+06	4600	1.95E+24	3.231E-07	1293	5946	2.52E+24	6.98E+06	6000	2386	7710	3.26E+24	8000
5	7.75E+06	4401	2.33E+24	3.151E-07	1261	5547	2.93E+24	8.50E+06	2500	2186	6889	3.64E+24	2500
5.5	8.53E+06	4201	2.71E+24	2.975E-07	1190	4998	3.23E+24	8.53E+06	2500	2006	5967	3.85E+24	2500
6	9.30E+06	4001	3.10E+24	2.766E-07	1106	4426	3.43E+24	9.30E+06	2500	1841	5093	3.94E+24	2500
6.5	1.01E+07	3801	3.48E+24	2.577E-07	1031	3918	3.58E+24	1.01E+07	2500	1690	4356	3.98E+24	2500
7	1.09E+07	3601	3.84E+24	2.441E-07	977	3516	3.75E+24	1.09E+07	2500	1550	3785	4.04E+24	2500
8	1.24E+07	3201	8.44E+24	2.356E-07	942	3016	7.95E+24	1.24E+07	2500	1299	3059	8.06E+24	2500
9	1.40E+07	2801	9.48E+24	2.439E-07	976	2732	9.25E+24	1.40E+07	2500	1077	2627	8.89E+24	2500
10	1.55E+07	2401	1.01E+25	2.543E-07	1017	2442	1.03E+25	1.55E+07	2500	879	2236	9.45E+24	2500
12	1.86E+07	1601	1.82E+25	2.472E-07	989	1583	1.80E+25	1.86E+07	1800	537	1329	1.51E+25	1800
14	2.17E+07	801	1.27E+25	2.101E-07	840	674	1.07E+25	2.17E+07	900	249	524	8.30E+24	900
16	2.48E+07	2	3.37E+22	1.793E-07	717	1	2.42E+22	2.48E+07	0	0	0	4.39E+21	0



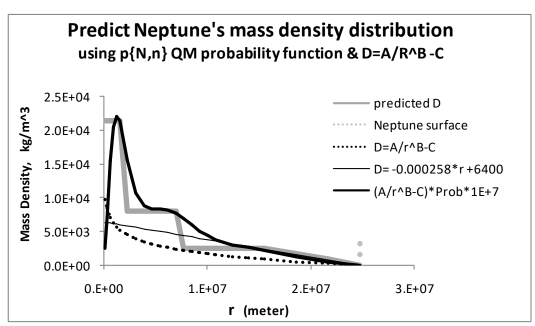


Figure 2a. Predict Neptune's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a linear (D=a*r +b) scaling up.
Figure 2b. Predict Neptune's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a curve (D=A/r^B -C) scaling up.

III. Predict Uranus' internal structure and the mass density r-distribution by using {N,n} QM probability function

So far no experimental determined mass density radial distribution (like Earth's) has been found for Uranus. The calculation for predicting Uranus internal structure and mass density is exactly the same as that for Neptune. First, let us constitute the mass density linear equation D= a * r + b for Uranus. After manual fitting, one possible result is D = -0.000193 * x + 4941. It satisfies both two conditions: 1) ∫ D dV = mass of Uranus (see the integration equation below); 2) at surface r = 2.56E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 3a.

$$\int_0^{2.56\times10^7} 4\pi(-0.000193\,x + 4941)\,x^2\,dx = 86\,819\,264\,480\,033\,892\,070\,326\,272$$

In p{N,n//4} QM, let's choose Uranus's inner core as p{0,1//4}, and its r as r_1 (= 2.56E+7 /16 = 1.6E+6 m). Then its Earth-sized core is at p{0,2//4}, and Uranus surface is at p{0,4//4}. Similar as that of Neptune, Uranus' mass radial distribution can also be described by a simple integration formula of QM probability:

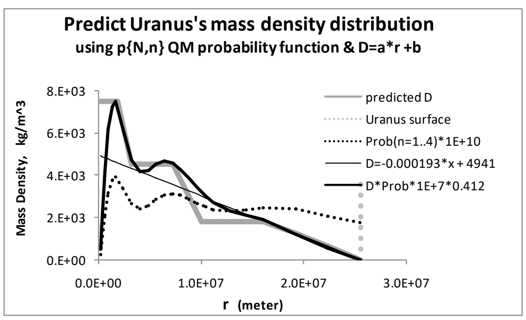
Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 2.56E+7 m; θ=0, π; φ=0, 2π]
or
8.68E+25 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2 + |R(4,*l*)|^2) *W * (-0.000193*r +4941) * r^2 dr, [r=0, 2.56E+7 m]

Again, the table of r^2 *|R(n,l)|^2 calculation is not shown here. The resulted curve is shown in column 5 "Prob(n=1..4)" of Table 3, and it is plotted in Figure 3a as "Prob(n=1..4)*1E+10". Table 3 shows the calculation to predict Uranus' internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 3a and Figure 3b shows the result. In column 7 "D*Prob*1E+7*0.412" of Table 3, instead of integration, I manually scaled-up the Uranus' probability curve based on D = -0.000193 * r + 4941, with W=1E+7*0.412, and this scaled-up curve is plotted in Figure 3a as "D*Prob*1E+7*0.412".

Again the linear D = a * r + b scaling up gives too low D (≈ 7500 kg/m^3) at the inner core of Uranus. A curved scaling up with D = A / r^B - C has been tested (see columns 11-15 in Table 3). After manual fitting, one possible curve is D = 143000 / r^0.013 -114559. It gives
1) The total mass integration of "D*Porb*1E+7" from r = 0 to 2.56E+7 m equals to Uranus' mass;
2) At Uranus surface, "D*Porb*1E+7" ≈ 0 kg/m^3;
3) At Uranus's p{0,1//4} inner core (r < 1.6E+6 m), "D*Porb*1E+7" ≈ 17000 kg/m^3;
Both Figure 3a and 3b predict that Uranus has a p{0,1//4} inner core with r ≈ 1.6 ~ 1.8E+6 m, an Earth-sized core p{0,2//4} at r ≈ 7.2E+6 m, and an unobvious (liquid) atmosphere core p{0,3//4} at r ≈ 1.6E+7 m. However, fittings in Figure 3a and 3b only give the range of the mass density for each core (7500~17000 kg/m^3, 4500~6500 kg/m^3, 1800~1800 kg/m^3, respectively). My best guess is 16500 kg/m^3 for p{0,1//4} inner core, 6000 kg/m^3 for p{0,2//4} core, 1800 kg/m^ for (liquid) atmosphere layer up to p{0,3//4}, and 1800 → 0 kg/m^3 for the outer atmosphere layer up to p{-1,4//4}.

Table 3. Predict Uranus' mass density r-distribution by using p{N,n} QM probability function.

r_1=	1.60E+06		6.91E+25			0.412	6.91E+25			A= 143000 B= 0.013 C= 114559		6.93E+25	
r/r_1=	r/r_1*r_1	D=-0.000193*x + 4941	mass= D*ΔV	Prob(n=1..4)	Prob(n=1..4)*1E+10	D*Prob*1E+7*0.412	mass= (D*Prob*1E+7*0.412)*ΔV	r, p{-1,1}, p{0,1}	predicted D	D=A/r^B-C	(A/r^B-C)*Prob*1E+7	mass= ((A/r^B-C)*Prob*1E+7)*ΔV	predicted D
	m	kg/m^3	kg			kg/m^3	kg	m	kg/m^3	kg/m^3	kg/m^3	kg	kg/m^3
0.1	1.60E+05	4910	8.42E+19	2.41E-08	241	488	8.36E+18	1.60E+05	7500	7813	1883	3.23E+19	17000
0.2	3.20E+05	4879	5.86E+20	7.886E-08	789	1585	1.90E+20	3.20E+05	7500	6715	5296	6.36E+20	17000
0.4	6.40E+05	4817	4.63E+21	2.108E-07	2108	4184	4.02E+21	6.40E+05	7500	5628	11863	1.14E+22	17000
0.6	9.60E+05	4756	1.24E+22	3.166E-07	3166	6203	1.62E+22	9.60E+05	7500	4996	15814	4.12E+22	17000
0.8	1.28E+06	4694	2.38E+22	3.756E-07	3756	7263	3.69E+22	1.28E+06	7500	4549	17086	8.68E+22	17000
1	1.60E+06	4632	3.88E+22	3.925E-07	3925	7491	6.27E+22	1.60E+06	7500	4204	16503	1.38E+23	17000
1.5	2.40E+06	4478	1.82E+23	3.363E-07	3363	6204	2.53E+23	1.80E+06	7500	3580	12039	4.91E+23	6500
2	3.20E+06	4323	3.43E+23	2.655E-07	2655	4730	3.75E+23	3.20E+06	4500	3139	8335	6.61E+23	6500
2.5	4.00E+06	4169	5.45E+23	2.415E-07	2415	4148	5.43E+23	4.00E+06	4500	2798	6757	8.84E+23	6500
3	4.80E+06	4015	7.84E+23	2.564E-07	2564	4242	8.28E+23	4.80E+06	4500	2520	6463	1.26E+24	6500
3.5	5.60E+06	3860	1.05E+24	2.846E-07	2846	4527	1.23E+24	5.60E+06	4500	2286	6507	1.77E+24	6500
4	6.40E+06	3706	1.34E+24	3.063E-07	3063	4677	1.70E+24	6.40E+06	4500	2083	6381	2.31E+24	6500
4.5	7.20E+06	3551	1.65E+24	3.13E-07	3130	4580	2.13E+24	7.20E+06	4500	1905	5963	2.77E+24	6500
5	8.00E+06	3397	1.97E+24	3.053E-07	3053	4273	2.48E+24	1.00E+07	1800	1745	5328	3.10E+24	1800
5.5	8.80E+06	3243	2.30E+24	2.882E-07	2882	3850	2.73E+24	1.00E+07	1800	1601	4615	3.28E+24	1800
6	9.60E+06	3088	2.63E+24	2.68E-07	2680	3409	2.90E+24	1.00E+07	1800	1470	3939	3.35E+24	1800
6.5	1.04E+07	2934	2.95E+24	2.497E-07	2497	3018	3.04E+24	1.04E+07	1800	1349	3369	3.39E+24	1800
7	1.12E+07	2779	3.26E+24	2.365E-07	2365	2708	3.18E+24	1.12E+07	1800	1238	2927	3.43E+24	1800
8	1.28E+07	2471	7.16E+24	2.282E-07	2282	2323	6.74E+24	1.28E+07	1800	1037	2366	6.86E+24	1800
9	1.44E+07	2162	8.05E+24	2.363E-07	2363	2104	7.83E+24	1.44E+07	1800	860	2032	7.57E+24	1800
10	1.60E+07	1853	8.62E+24	2.464E-07	2464	1881	8.74E+24	1.60E+07	1800	702	1729	8.04E+24	1800
12	1.92E+07	1235	1.54E+25	2.394E-07	2394	1219	1.52E+25	1.92E+07	1235	429	1028	1.28E+25	1235
14	2.24E+07	618	1.08E+25	2.035E-07	2035	518	9.03E+24	2.24E+07	618	199	405	7.06E+24	618
16	2.56E+07	0	4.64E+21	1.737E-07	1737	0	3.32E+21	2.56E+07	0	0	0	-3.15E+21	0



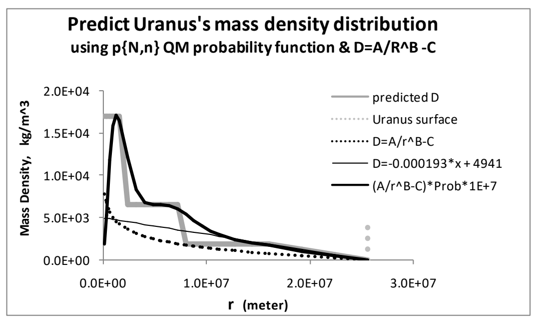


Figure 3a. Predict Uranus's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a linear (D = a * r + b) scaling up.
Figure 3b. Predict Uranus's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a curve (D = A / r^B - C) scaling up.

IV. Predict undiscovered planets {3,n=2..5}'s internal structure and the mass density r-distribution by using {N,n} QM probability function

In Table 2 of paper SunQM-3s6, we see that the undiscovered {3,2} planet (if the belt did have accreted into a planet) has mass of 7.12E+25 kg, and it is almost the same as that of Uranus (7.52E+25 kg). So I assume that it should have almost the same size and QM structure as that of Uranus. Therefore, based on the calculation of SunQM-3s6 Table 2, I predict that the undiscovered {3,2} planet has a p{0,1//4} inner core with r ≈ 1.36E+6 m, an Earth-sized core p{0,2//4} at r ≈ 5.45E+6 m, an unobvious (liquid) atmosphere core p{0,3//4} up to r = 2.18E+7 * (3/4)^2 ≈1.23E+7 m, and a surface at p{-

1,4//4} at r = 2.18E+7 m. Based on section III's result, my best guess is D ≈ 16000 kg/m^3 for p{0,1//4} inner core, 5500 kg/m^3 for p{0,2//4} core, 1750 kg/m^ for (liquid) atmosphere layer up to p{0,3//4}, and 1750 → 0 kg/m^3 for the outer atmosphere layer up to p{-1,4//4}.

For the rest three undiscovered planets {3,n=3..5} (if they did have accreted into planets), since their masses (from 3.99E+25 kg to 1.98E+25 kg) are close to {3,2}'s mass (7.12E+25 kg), I believe they all have the same {N,n//2} or {N,n//4} QM structure as that of Uranus'. The estimated sizes of atmosphere at p{-1,4//4}, the Earth-sized core at p{-1,2//4}, and the inner core at p{-1,1//4} for all {3,n=3..5} planets have been listed in Table 2 of paper SunQM-3s6. The mass density radial distribution should also be similar as that of Uranus too, only need to be scaled down a little bit. Therefore, I predict that their inner cores p{-1,1//4} 's mass density are between 15000 kg/m^3 to 12000 kg/m^3.

V. Predict Saturn's internal structure and the mass density radial distribution by using {N,n} QM probability function

So far, no experimental determined mass density radial distribution (like Earth's) for Saturn has been found. First, let us constitute the mass density linear equation D = a * r + b for Saturn. After manual fitting, one good result is D = -0.0000469 * r + 2730. It satisfies both conditions: 1) ∫ D dV = mass of Saturn (see the integration equation below); 2) at surface r = 5.82E+7 m, D ≈ 0 kg/m^3. It is plotted in Figure 4a.

$$\int_0^{5.82\times 10^7} 4\pi(-0.0000469\,x + 2730)\,x^2\,dx = 563\,846\,146\,050\,677\,159\,885\,799\,424$$

From my analysis in paper SunQM-1s3 section X, the current Saturn has an Earth-sized core (let us define it as) p{0,1//2}, and a inner core p{-1,1//2}, both with 100% mass occupancy, and both have p{N,n//2} QM structure. However, it has a p{0,3//2} sized atmosphere. The atmosphere from p{0,1//2} to p{0,2//2} has 100% mass occupancy. But the atmosphere from p{0,2//2} to p{0,3//2} has probably only ~50% mass occupancy. I believe that if Saturn has had 100% mass occupancy in atmosphere shell between p{0,2//2} and p{0,3//2}, it would have generated enough G-forced compression to transform its core from {N,1//2} QM to {N,1//3} QM, so that the whole Saturn would become a {N,n//3} QM structure. However, current Saturn's ~50% mass occupancy in n=3 shell is not enough to make this transformation to happen, so it is stuck at this hybridized (core is base-2, atmosphere is base-3) QM state. For this reason, it is better to choose Saturn's Earth-sized core (not the Saturn's surface) as the p{0,1}.

Based on the previous {N,n} QM structure model calculation of Earth and Jupiter, for Saturn I need to first analyze p{0,1//2} Earth-sized core, p{0,2//2} sized inner atmosphere, and p{0,3//2} sized outer atmosphere. This is equivalent to a p{N,n//3} QM, with p{0,1//3} Earth-sized core, p{0,2//3} sized inner atmosphere, and p{0,3//3} sized outer atmosphere. Since we choose the Earth-sized core as p{0,1//3}, and also choose it as r_1, so r_1 = 5.82E+7 /3^2 = 6.47E+6 m. Hence Saturn can be described by its QM radial probability density function of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2), as plotted in Figure 4a. Similar as that of Earth and Jupiter, Saturn's mass radial distribution can also be (approximately, ignoring the p{-1,1] center region due to its small volume) described by a simple integration formula of QM probability:

Mass (r, θ, φ) = ∭ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2) *W *D *sin(θ) * r^2 dr dθ dφ, [r=0, 5.82E+7 m; θ=0, π; φ=0, 2π]

or

5.68E+26 kg = 4π ∫ r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2 + |R(3,*l*)|^2) *W * (-0.0000469*r +2730) * r^2 dr, [r=0, 5.82E+7 m]

Table 4 shows the calculation to predict Saturn's internal structure and the mass density r-distribution using p{N,n} QM radial probability function, and Figure 4a and Figure 4b shows the result. The calculation of r^2 *|R(n,l)|^2 is shown in columns 1 through 8 in Table 4. The resulted curve is shown in column 8 "Prob(n=1..3)" of Table 4, and it is plotted in Figure 4a as "Prob(n=1..3)*E+10". In Table 4, instead of integration, I manually scaled-up the Saturn's probability curve based on D = -0.0000469 * r + 2730, with W = 1E+7*1.65 (see column 13 "D*Prob*1E+7*1.65" in Table 4), and this scaled-up curve is plotted in Figure 4a as "D*Prob*1E+7*1.65".

As mentioned before, Saturn has a inner core p{-1,1//2} inside the Earth-sized core p{0,1//2}. The probability density of this p[-1,1//2} core is not shown in the r^2 *(|R(1,0)|^2 + |R(2,l)|^2 + |R(3,l)|^2) curve with r_1 = 6.47E+6 m. To know the mass density of the inner core p{-1,1//2} inside the Earth-sized core p{0,1//2}, we need to use the probability density distribution of r^2 *(|R(1,0)|^2 + |R(2,*l*)|^2) with r_1 = 6.47E+6 /4 m, or r_1 = 1.62E+6 m (see dashed line in Figure 4a). This Prob(n=1..2) curve is (manually) scaled up to make its n=2 peak (at ~ 6.47E+6 m) to match Prob(n=1..3) curve n=1 peak (also at ~ 6.47E+6 m, see columns 15-19 in Table 4, and also see Figure 4a). So now this scaled up Prob(n=1..2) curve also reflects the (close to) true mass density of p{-1,1//2} core and p{0,1//2} core. Then I construct a stepped line (see column 20-21 of Table 4, and see the grey thick line in Figure 4a) according to the "D*Prob*1E+7*1.65" curve based on my eye judgment. According to this stepped line, I can predict that there are four (major) layers with three interfaces for Saturn's internal structure:

The p{-1,1} inner core (0 m < r < ~ 2.0E+6 m) with D ≈ 6300 kg/m^3.
The (Earth-sized) p{0,1} core (~2.0E+6 m < r < ~8.08E+6 m) with D ≈ 3800 kg/m^3.
The inner (liquid) atmosphere layer (~8.08E+6 m < r < ~3E+7 m) with D ≈ 1600 kg/m^3.
The outer atmosphere layer (~3E+7 m < r < ~5.82E+7 m) with D ≈ 1600 → 0 kg/m^3.

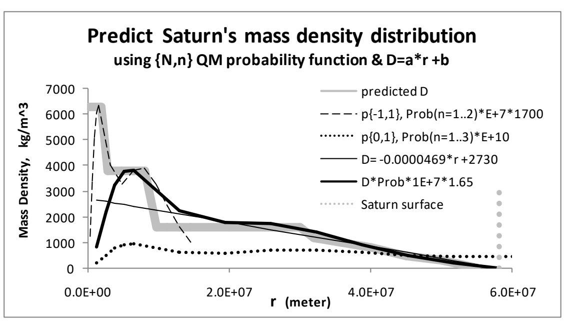
Table 4. Predict Saturn's internal structure and the mass density radial distribution by using p{N,n} QM probability function.

Saturn r	6.47E+06 meters						
r/r_1=	$r^2 * \|R_{1,0}\|^2$	$r^2 * \|R_{2,0}\|^2$	$r^2 * \|R_{2,1}\|^2$	$r^2 * \|R_{3,0}\|^2$	$r^2 * \|R_{3,1}\|^2$	$r^2 * \|R_{3,2}\|^2$	Prob(n=1..3)
0.2	1.66E-08	2.05E-09	8.44E-12	6.07E-10	2.96E-12	7.04E-16	1.93E-08
0.4	4.45E-08	5.31E-09	1.11E-10	1.56E-09	3.86E-11	3.94E-14	5.15E-08
0.6	6.71E-08	7.49E-09	4.58E-10	2.17E-09	1.59E-10	3.93E-13	7.73E-08
0.8	7.99E-08	8.00E-09	1.19E-09	2.27E-09	4.08E-10	1.93E-12	9.18E-08
1	8.37E-08	7.11E-09	2.37E-09	1.95E-09	8.07E-10	6.45E-12	9.60E-08
2	4.53E-08	0.00E+00	1.40E-08	3.31E-11	4.24E-09	2.12E-10	6.38E-08
3	1.38E-08	8.66E-09	2.60E-08	3.10E-09	6.20E-09	1.24E-09	5.90E-08
4	3.32E-09	2.27E-08	3.02E-08	5.90E-09	4.47E-09	3.58E-09	7.01E-08
5	7.02E-10	2.93E-08	2.71E-08	4.74E-09	1.40E-09	7.01E-09	7.03E-08
6	1.37E-10	2.76E-08	2.07E-08	1.68E-09	0.00E+00	1.07E-08	6.09E-08
7	2.52E-11	2.16E-08	1.41E-08	1.45E-11	1.42E-09	1.39E-08	5.11E-08
8	4.46E-12	1.49E-08	8.85E-09	1.17E-09	4.97E-09	1.59E-08	4.59E-08
9	7.63E-13	9.47E-09	5.22E-09	4.60E-09	9.20E-09	1.66E-08	4.50E-08
10	1.27E-13	5.62E-09	2.93E-09	8.83E-09	1.28E-08	1.60E-08	4.62E-08
12	3.36E-15	1.71E-09	8.21E-10	1.49E-08	1.57E-08	1.26E-08	4.57E-08
14	8.38E-17	4.54E-10	2.06E-10	1.52E-08	1.37E-08	8.37E-09	3.79E-08
16	2.01E-18	1.09E-10	4.75E-11	1.18E-08	9.60E-09	4.92E-09	2.65E-08
18	4.65E-20	2.44E-11	1.03E-11	7.71E-09	5.84E-09	2.63E-09	1.62E-08
20	1.05E-21	5.16E-12	2.12E-12	4.44E-09	3.19E-09	1.30E-09	8.94E-09
22	2.33E-23	1.04E-12	4.21E-13	2.33E-09	1.61E-09	6.08E-10	4.55E-09
24	5.08E-25	2.03E-13	8.07E-14	1.14E-09	7.60E-10	2.70E-10	2.17E-09
27	1.59E-27	1.66E-14	6.44E-15	3.48E-10	2.24E-10	7.42E-11	6.47E-10

r/r_1=	p{0,1}			4.46E+26	1.65	4.43E+26
	r/r_1*6.47E+6 (m)	p{0,1}, Prob(n=1..3)*E+10	D= -0.0000469*r +2730 (kg/m^3)	mass= D*ΔV (kg)	D*Prob*1E+7*1.65 (kg/m^3)	mass= (D*Prob*1E+7*1.65)*ΔV (kg)
0.2	1.29E+06	1.93E+02	2.67E+03	2.42E+22	848	7.68E+21
0.4	2.59E+06	5.15E+02	2.61E+03	1.65E+23	2216	1.41E+23
0.6	3.88E+06	7.73E+02	2.55E+03	4.39E+23	3252	5.60E+23
0.8	5.17E+06	9.18E+02	2.49E+03	8.34E+23	3768	1.26E+24
1	6.47E+06	9.60E+02	2.43E+03	1.34E+24	3842	2.12E+24
2	1.29E+07	6.38E+02	2.12E+03	1.68E+25	2234	1.77E+25
3	1.94E+07	5.90E+02	1.82E+03	3.92E+25	1771	3.81E+25
4	2.59E+07	7.01E+02	1.52E+03	6.36E+25	1756	7.36E+25
5	3.23E+07	7.03E+02	1.21E+03	8.39E+25	1407	9.72E+25
6	3.88E+07	6.09E+02	9.10E+02	9.38E+25	914	9.42E+25
7	4.53E+07	5.11E+02	6.07E+02	8.73E+25	511	7.36E+25
8	5.17E+07	4.59E+02	3.04E+02	5.81E+25	230	4.40E+25
9	5.82E+07	4.50E+02	4.20E-01	1.03E+23	0	7.67E+22
10	6.47E+07	4.62E+02				
12	7.76E+07	4.57E+02				
14	9.05E+07	3.79E+02				
16	1.03E+08	2.65E+02				
18	1.16E+08	1.62E+02				
20	1.29E+08	8.94E+01				
22	1.42E+08	4.55E+01				
24	1.55E+08	2.17E+01				
27	1.75E+08	6.47E+00				

r/r_1=	p{-1,1}	r1= 1.62E+06 meters			1700		linear
	r/r_1*1.62E+6 (m)	$r^2 * \|R_{1,0}\|^2$	$r^2 * \|R_{2,0}\|^2$	$r^2 * \|R_{2,1}\|^2$	p{-1,1}, Prob(n=1..2)*E+7*1700 (kg/m^3)	r, p{-1,1}, p{0,1} (m)	predicted D (kg/m^3)
0.2	3.23E+05	6.63E-08	8.20E-09	3.38E-11	1.27E+03	3.23E+05	6.30E+03
0.4	6.47E+05	1.78E-07	2.12E-08	4.42E-10	3.39E+03	6.47E+05	6.30E+03
0.6	9.70E+05	2.68E-07	2.99E-08	1.83E-09	5.10E+03	9.70E+05	6.30E+03
0.8	1.29E+06	3.20E-07	3.20E-08	4.74E-09	6.06E+03	1.29E+06	6.30E+03
1	1.62E+06	3.35E-07	2.84E-08	9.48E-09	6.34E+03	2.00E+06	6.30E+03
2	3.23E+06	1.81E-07	0.00E+00	5.58E-08	4.03E+03	2.80E+06	3.80E+03
3	4.85E+06	5.52E-08	3.46E-08	1.04E-07	3.29E+03	4.85E+06	3.80E+03
4	6.47E+06	1.33E-08	9.06E-08	1.21E-07	3.82E+03	6.47E+06	3.80E+03
5	8.08E+06	2.81E-09	1.17E-07	1.09E-07	3.89E+03	8.08E+06	3.80E+03
6	9.70E+06	5.47E-10	1.10E-07	8.28E-08	3.29E+03	9.70E+06	1.60E+03
7	1.13E+07	1.01E-10	8.64E-08	5.64E-08	2.43E+03	1.13E+07	1.60E+03
8	1.29E+07	1.78E-11	5.98E-08	3.54E-08	1.62E+03	1.29E+07	1.60E+03
9	1.46E+07	3.05E-12	3.79E-08	2.09E-08	9.99E+02	1.46E+07	1.60E+03
10						2.00E+07	1.60E+03
12						2.50E+07	1.60E+03
14						3.00E+07	1.60E+03
16						3.20E+07	1.20E+03
18						4.00E+07	8.00E+02
20						4.50E+07	5.00E+02
22						5.00E+07	3.00E+02
24						5.50E+07	3.00E+01
27						5.82E+07	0.00E+00

r/r_1=	A= 1.63E+05					
	B= 0.05		p{0,1}		p{-1,1}	
	C= 66672.0		4.42E+26	0.475	4.58E+24	curved
	D=A/r^B-C	(A/r^B-C)*Prob*1E+7 (kg/m^3)	((A/r^B-C)*Prob(1..3)*1E+7)*ΔV (kg)	(A/r^B-C)*Prob(1..2)*1E+7*0.475 (kg/m^3)	mass= ((A/r^B-C)*Prob*1E+7)*ΔV (kg)	predicted D (kg/m^3)
0.2	13978	2.69E+03	2.44E+22	7.00E+03	9.91E+20	2.30E+04
0.4	11230	5.78E+03	3.67E+23	1.59E+04	1.58E+22	2.30E+04
0.6	9667	7.48E+03	1.29E+24	2.16E+04	5.81E+22	2.30E+04
0.8	8577	7.87E+03	2.64E+24	2.37E+04	1.24E+23	2.30E+04
1	7742	7.43E+03	4.11E+24	2.32E+04	2.00E+23	2.30E+04
2	5207	3.32E+03	2.63E+25	1.17E+04	1.45E+24	7.80E+03
3	3764	2.22E+03	4.78E+25	8.12E+03	2.73E+24	7.80E+03
4	2759	1.93E+03	8.11E+25	8.27E+03		7.80E+03
5	1988	1.40E+03	9.66E+25	7.51E+03		7.80E+03
6	1365	8.31E+02	8.56E+25	5.75E+03		1.60E+03
7	843	4.30E+02	6.19E+25	3.86E+03		1.60E+03
8	393	1.80E+02	3.45E+25	2.35E+03		1.60E+03
9	0	-1.40E-01	-3.45E+22	1.34E+03		1.60E+03
10						1.60E+03
12						1.60E+03
14						1.60E+03
16						1.20E+03
18						8.00E+02
20						5.00E+02
22						3.00E+02
24						3.00E+01
27						0.00E+00



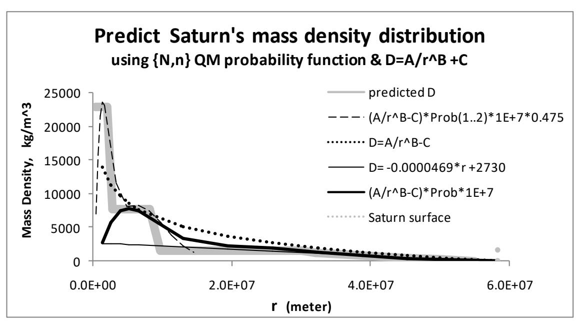


Figure 4a. Predict Saturn's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a linear (D = a * r + b) scaling up.
Figure 4b. Predict Saturn's internal structure and the mass density radial distribution by using p{N,n} QM probability function and a curve (D = A / r^B - C) scaling up.

Due to by using the linear D = a * r + b for scaling up, D ≈ 6300 kg/m^3 is too low for Saturn's inner core. According to Jupiter, Earth, and Sun's inner core mass density information (obtained from the wiki), after many fittings, I give the best estimated mass density for other planets' inner core (calculation is not shown here because the method is too rough). Saturn's inner core is estimated to have D ≈ 21000 kg/m^3. Then, same as that for other gas/ice planets, D = A / r^B - C model is used with following four conditions:

1) The total mass integration of "D*Porb*1E+7" from r = 0 to 5.82E+7 m has to equal to Saturn's mass;
2) At Saturn surface, "D*Porb*1E+7" ≈ 0 kg/m^3.
3) At surface r = 6.47E+6 m, p{-1,1//2} 's probability should match to p{0,1//3} 's probability.
4) The inner core has mass density not too different from 21000 kg/m^3.

After over 10 hours of manual fitting, the closest result is D = 1.63E+5 / r^0.05 - 66672 (see Table 4. columns 22-24, and Figure 4b). It satisfies conditions from 1) to 3), but has "D*Porb*1E+7" ≈ 23000 kg/m^3 at Saturn's p{-1,1} inner core (r < 2.0E+6 m). Both Figure 4a and 4b predict that Saturn has a p{-1,1//2} inner core with r ≈ 2.0E+6 m, an Earth-sized core p{0,1//2} at r ≈ 8.08E+6 m, and a (liquid) atmosphere core p{0,2//2} up to r ≈ 3E+7 m. However, fittings in Figure 4a and 4b only give the range of the mass density for each core (6300~23000 kg/m^3, 3800~7800 kg/m^3, 1600~1600 kg/m^3, respectively). My best guess is 23000 kg/m^3 for inner core p{-1,1//2}, 7800 kg/m^3 for p{0,1//2} core, 1600 kg/m^ for p{0,2//2} (liquid) atmosphere layer, and 1200 → 0 kg/m^3 for the outer atmosphere layer up to p{0,3//2}.

VI. What is the critical point of mass for a celestial body when the {N,n} QM structure effect starts to become effective?

From the analysis and results of SunQM-1s3, -3s6, -3s7 and -3s8, we now know that the formation of a celestial body (either planet or star)'s internal structure is mainly governed by a point-centered G-force (so its mass density r-distribution follows a smooth radial curve like D = (A / r^B - C) as mentioned above in the current paper), and then either modified by the chemical bond force (plus van der Waals force, etc.) if the total mass is lower than a critical point (see discussion in papers SunQM-1s3 section XI, and SunQM-5 section VII), or modified by the {N,n} QM force if the total mass above a critical point (so that it will generate the planetary differentiation, or follow the bumpy curve of r^2 *|R(n,l)|^2 probability r-distribution). So what is the value of the mass critical point that for a celestial body's internal structure to be modified by {N,n} QM force? We know that all eight (original) planets have their internal structures follow p{N,n} QM, so the critical point of mass must be smaller than that. Hence we need to check moons. We know our Earth's Moon has the internal structure following p{N,n} QM (see paper SunQM-1s3), so the critical point mass must smaller than our Moon (7.34E+22 kg). However, for most other moons, we don't know whether their internal structure follow p{N,n} QM or not. One thing we do know is that if a moon's internal structure follows the p{N,n} QM, then its shape must also follow the same p{N,n} QM structure, or it must be in spherical shape. Then we need to look for the smallest spherical shaped moon with rocky surface (not the ice/liquid surface because its spherical ice surface does not have to correlate to the internal structure). However, so far through online searching I am not able to find a rocky-surfaced and spherical shaped moon. From wiki "Moons of Saturn", the ice-surfaced moon Mimas (3.75E+19 kg) has nearly spherical shape but "*noticeably ovoid-shaped ... by the effects of Saturn's gravity*". The ice-surfaced moon Enceladus (1.08E+20 kg) has a perfect spherical shape. All smaller moons, like Phoebe (mass=8.29E+18 kg), Hyperion (mass=5.62E+18 kg), etc., do not have spherical shape, though still have ice surface. So by far, we only know that the high end of this critical point of mass is ~7E+22 kg, and low end could be ~1E+20 kg (or even lower, but must be higher than 1E+19 kg).

VII. The coupling (and de-coupling) of {N,n} QM governed radial structure to gravity governed radial structure

So now we know that as long as above the critical pint of mass, all originally (or naturally, or in situ) formed celestial bodies will have their internal structure governed primarily by G-force, and then modified by {N,n} QM force. For this situation, we can say that the {N,n} QM governed radial structure is coupled with gravity governed radial structure. Then how adding (or removing) some mass to (or from) planet/star will change planet/star's existing p{N,n//q} QM? Note: in the following discussion, for the gas/ice planet, let us use pCore{N,n//q} QM, where "pCore" means using gas/ice planet's Earth-sized core as p{0,1} and r_1, and q is the pFactor.

Saturn's current body size is the strongest evidence that how strong the {N,n} QM force may overcome the gravity force. If the Saturn were formed in situ with the current mass, I believe that it would have a perfect pCore{N,n//3} QM structure, with averaged mass density ~1600 kg/m^3 (like Neptune's mass density), with radius = (5.68E+26 / (4/3*π*1600))^(1/3) = 4.39E+7 meters, or ~25% smaller in comparison to Saturn's current radius 5.82E+7 m. The reason why current Saturn has such low mass density (568 kg/m^3) is that its original body (~20% of current mass) already determined its QM structure as pCore{N,n//2}, so the newly added ~80% mass has to follow this predetermined QM structure to fill all space of p{0,2//2}o orbit shell space. By doing this, it sacrificed gravity energy (because to make mass density extremely low or to make r larger than it should be does increased the G potential energy), but the favorable original pCore{0,3//2} QM state energy overcomes the unfavorable current (low mass density's) gravity potential energy, and sustains the current Saturn body as the original QM favorite but current gravity unfavorite size. So we can say that Saturn's pCore{N,n} QM governed radial structure is de-coupled from its gravity governed radial structure.

Another way to show that "the current Saturn's size is dominant by its QM structure which overcomes its gravity effect" is: according to the classical physics, Saturn should keep its body size by using the thermo pressure. As the heat keep escaping from the surface to the out space, Saturn's body should continuously shrink. According to {N,n} QM, each n shell will keep as one quanta to keep the (almost) same temperature and thermo pressure through the "photon thermos core effect" (see my paper SunQM-3s8 section II). So its size can only be quantum expand or collapse. As I mentioned before, the outmost shell of Saturn pCore{0,2//2}o is guessed to have only ~50% mass occupancy. The mass occupancy capacity of each n shell decreases as the temperature increase. So at high temperature, it appears that it has relative high mass occupancy, or the mass is more evenly distributed in both |210> and |211> states, therefore its shape turned to be more spherical. So when Saturn's pCore{0,3//2} QM structure was just formed, it had higher body temperature, and it must have almost perfect spherical shape. As it cooling down, its flattening value increases to today's 0.097. So from QM point of view, that "Saturn is more flatten than Jupiter" is not due to Saturn spins faster than Jupiter (actually it spins slower), it is because Saturn has too little mass to fill in |210> state, while Jupiter (due to it has close to 100% mass occupancy) has enough mass to fill in |210> state (if Jupiter is analyzed in pCore{N,n//3}, see the discussion in the next paragraph). As it further cooling down, the future Saturn will have even higher flattening value than that of today, simply because more mass in pCore{0,2//2}o orbit shell will shift from |210> state to the |211> state. But, whatever flatten it will be, it will not gradually decrease its radius (from current 5.82E+7 m to 27% smaller) because the {N,n} QM structure does not allow it to do that. (Notice that this rule is only for the n shell with mass occupancy > 30% (so it is still a ball-like shape). If n shell has mass occupancy less than a critical point (let's suppose = 10%), then the whole n shell's mass will quantum collapse into n-1 shell, and leave only < 0.1% of mass in n shell to become ring (e.g, Saturn's ABCD rings with pCore{0,4} QM structure or |4*lm*> state, see paper SunQM-3s4 section I). Then, as the ring mass further decreasing, the ring width (Δr) will decrease within the same n=4 shell space, and the ring's outer edge will quantum collapse from $r/r_1 \approx 25$ of |400> state, to $r/r_1 \approx 24$ of |411> state, then to $r/r_1 \approx 21$ of |422> state, finally to $r/r_1 \approx 16$ of |433> state, see paper SunQM-3s4 Figure 4's explanation).

In the case of Jupiter, if it were formed in situ with the current mass, it probably would have a perfect pCore{N,n//3}, or pCore{N,n//4}, or pCore{N,n//5} QM structure (actually, my best guess is pCore{N,n//3}), with averaged mass density ~1600 kg/m^3 (like Neptune's mass density), and with radius = (1.9E+27 / (4/3*π*1600))^(1/3) = 6.57E+7 meters, or ~6% smaller in comparison to the current radius 6.99E+7 m. However, its original mass (~10% of current mass, see SunQM-1s1) had determined it as pCore{N,n//2} QM structure. Comparing with Saturn where 4x of more mass added (20% original and 80% new), Jupiter added 9x of more mass (10% original, and 90% new). As the result, the huge unfavorable current gravity potential energy overcomes the original pCore{N,n//2} QM state energy, and forced Jupiter to adapt a complete new pCore{N,n//5} QM state which is the compromise of the current gravity formed pCore{N,n//q} QM state (which I believe is pCore{N,n//3}) and the original pCore{N,n//2} QM state. So Jupiter's current pCore{N,n//5} QM structure may or may not be the pCore{N,n//q} QM structure if Jupiter were formed in situ with the current mass.

Uranus lost ~50% mass after a catastrophic collision (see SunQM-1s1), but still keep its original pCore{N,n//2} QM structure. To do this, it has to decrease the mass density to keep the original size, so that Uranus' current averaged mass density is 1271 kg/m^3, significantly below Neptune's 1638 kg/m^3. It is obvious that the decreasing of mass density is mainly happened in Uranus atmosphere orbit shell pCore{0,2//2}o. From this, we can predict that the mass occupancy in

Uranus' atmosphere shell is ~ 50%. So we can say that Uranus' pCore{N,n} QM governed radial structure is de-coupled from its gravity governed radial structure also.

The above three example tells us that if an originally formed planet adds 4x more mass, its original pCore{N,n//2} QM format will not be changed. Under this situation, the planet's size and internal radial structure is not strictly based gravity's baseline curve, but based on the pre-existing pCore{N,n//q} QM structure. So we can say that the {N,n} QM governed radial structure is de-coupled with gravity governed radial structure. But if an originally formed planet adds 9x more mass, then its original pCore{N,n//2} QM format will be changed to accommodate the new gravity baseline curve. On the other side, if an originally formed planet lost 50% mass, its original pCore{N,n//2} QM format will not be changed.

The original Earth (25x of current Earth mass) lost 96% of mass and become today's Earth (1x mass, see paper SunQM-1s1), but it still keeps the old pCore{N,n//2} QM format, because the whole original atmosphere pCore{0,2//2}o orbit shell has been stripped off. Also for rocky planets, after stripping off the atmospheric mass, their gravity compressed core must have immediately expanded (see paper SunQM-3s6). However, their QM structure change might also lag-behind from their gravity governed radial structures change, so that these two structures might be de-coupled for certain period of time.

The reason for the de-coupling is the same as that has been explained in paper SunQM-1s3: the QM structure change happens between two different QM states, and there is a transition energy barrier between these two QM states (just like the chemical reaction's transition state theory). On one hand, the gravity structure change follows a smooth curve and does not have the (bumpy) transition energy barrier, so it does not have any time lagging. On the other hand, the transition between two QM states needs more time because it needs to climb over the bumpy transition state energy barrier. In the actual physical process of transition between two QM states for a celestial body, a pre-existing QM internal structure needs to be changed by:

1) melting down the interface (through friction and heating) between two n shells (to decrease the transition energy barrier),
2) re-adjusting mass density radial distribution according to both gravity force and QM force,
3) forming a new interface between the two newly formed n shells.

This is a very energy consuming process. Let's suppose that after the original Jupiter added 7x (rather than 9x) more mass, it had the 100% mass occupancy for the original p{0,2//2}o orbit space. To add the last 2x more mass (so total adding 9x more mass), Jupiter had to increase its volume by ~20%. This means that the radius had to increase by ~ 6% (due to 1.06^3 = 1.19). If using the original pCore{N,n} QM structure, Jupiter needed to increase every interface (pCore{-1,1//2}, pCore{0,1//2}, pCore{0,2//2}, pCore{0,3//2})'s r by ~6% one-by-one. The transition energy barrier was huge to do that. On the other hand, suppose if Jupiter could switch to a p{N,n//5} QM structure with ~30% volume increase with much lower transition energy barrier, then Jupiter would choose the second process, even though the final mass occupancy would be lower than 100% (or ~90% mass occupancy, see Table 6). So I believe, after capturing 9x new mass, between a "global energy minimum" state pCore{N,n//3} but very high transition energy barrier, and a "local energy minimum" state pCore{N,n//5} but much lower transition energy barrier, Jupiter chose the second process, to decrease the "total process energy", even though the end state was not at the "global energy minimum" state for a Jupiter-massed celestial body. Therefore if we could add external heat to completely melt down Jupiter, and then let it cool down slowly, I believe that Jupiter would form a pCore{N,n//3} QM structure instead of the current pCore{N,n//5} QM structure. I figured out this idea because as a previous biophysicist, I had the experience of protein crystallography (where the electron density map fitted protein structure needs to be cooked to minimize the global energy), as well as the experience of the molecular modeling for enzyme-inhibitor binding for drug discovery (where the binding energy needs to be minimized globally through varies methods).

VIII. The transitional energy barrier between QM superpositional states may affect how a celestial body change its {N,n} QM structure

This topic is originated from two questions:

1) Jupiter's four major moons (Io, Europa, Ganymede, and Callisto) can be equally well described by either pSurface{0,n=2..5//5} with Jupiter surface as p{0,1//5}, or by pCore{1,n=2..5//3} with Jupiter's original Earth-sized core as

p{0,1//3} (see paper Sun-3s4 Table 5 column 4 & 5 vs. 8 through 11). So Jupiter's moons can be in either p{N,n//5} or in p{N,n//3} QM structure. However, Jupiter's atmosphere band pattern clearly shows it has n=5 (not n=3) QM mode (see paper SunQM-3s3 Figures 3a, 3b, 3c). Therefore I choose pCore{N,n//5} to describe Jupiter's QM structure. Sun's mass is over 1000 times of Jupiter's, but Sun has a {N,n//6} QM structure with pFactor only =6, not much higher than Jupiter's pFactor =5. Saturn has almost same size and 1/3 of mass as Jupiter's, but Saturn is dominated by a pCore{N,n//3} QM structure with pFactor only =3, much smaller than Jupiter's pFactor =5. Neptune has ~1/19 of Jupiter's mass, and it has a pCore{N,n//2} QM structure with pFactor only =2, in comparison with Jupiter's pFactor =5. Furthermore, the original Earth and original Venus had the comparable mass and size as current Neptune's, and they all had p{N,n//2} QM structure (see paper SunQM-1s3). Let us name the pFactor quantum number as q, so {N,n//q} QM structure can be written as |qnlm> quantum state. I strongly believe that the q value in {N,n//q} QM structure or |qnlm> state of a celestial body is dependent on the mass of this celestial body. When we plot pFactor vs. planet/star's mass (see Table 5 and Figure 5), if we use pFactor =5 for Jupiter, it looks more like an outlier (see the red cross marker in Figure 5), However, if we use pFactor=3 for Jupiter, then all data can be fitted nicely with equation y = A log(x) -B, with R-square =0.98 (see the blue markers and fitting line in Figure 5). This analysis makes me to believe that the true (or the "global energy minimum") QM state of a Jupiter-massed planet is not pCore{N,n//5}, but pCore{N,n//3}. Our current Jupiter may be trapped in a "local energy minimum" QM state as pCore{N,n//5} QM structure. If the fitting equation pFactor = 0.4128 ln(mass) -22.794 (in Figure 5) is correct and meaningful, then a pFactor =5 celestial body will have mass around 1.75E+29 kg, or ~9% of Sun-mass. So some (small sized) red dwarf stars may have {N,n//5} QM structure.

Table 5. Correlate pFactor to planet/star's mass. The original Earth had mass ≈25x of Earth-mass, and the original Venus had mass ≈34x of Earth-mass (see paper SunQM-1s1)

Planets/Star	mass, kg	pFactor
Neptune	1.02E+26	2
ori-Earth, 25x	1.48E+26	2
ori-Venus, 34x	2.05E+26	2
Saturn	5.68E+26	3
Jupiter (pFactor=3)	1.90E+27	3
SUN	1.99E+30	6
Jupiter (pFactor=5)	1.90E+27	5

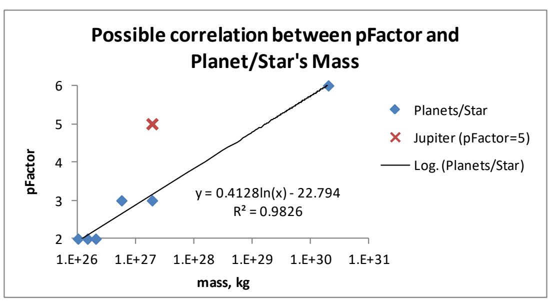


Figure 5. One possible correlation between pFactor and Planet/Star's mass.

2) The second question is, from QM point of view why Neptune's mass density D =1638 kg/m^3, and Jupiter, Uranus, and Saturn's mass densities are significantly less than ~1600 kg/m^3? What is the QM meaning? From wiki "Sun", Sun's averaged D = 1408 kg/m^3. From paper SunQM-1s1, we know that Sun's current r (hot-r) is 1.26x larger than the cold-r (or the G-forced r). So volume-wise, current Sun (or hot-r Sun)'s volume is 1.26^3 ≈ 2.0 of the cold-r (or the G-forced r) Sun's volume. If volume is decreased to 1/2, then D =1408*2 =2816 kg/m^3. Therefore if there is no H-fusion, a Sun-massed star will have mass density D = 2816 kg/m^3. Now we know the G-forced mass density for a Neptune-massed planet (1.02E+26 kg, 1638 kg/m^3) and for a Sun-massed star (1.99E+30 kg, 2816 kg/m^3, notice that both of them are gas-based celestial body), we can use a regression curve to calculate out G-forced mass density (D) for any gas planet within the mass range

between Neptune and Sun. But I don't know what formula we should use for D = function (mass). Like I (as a citizen scientist) did before in paper SunQM-1s1, I just pick a best function that I can think of (without knowing its physics meaning): log(D) = A log(mass) + B. After fitting the two known points (see Figure 6), log(D) =0.0549 log(mass) + 1.7877, then D = 10^1.7877 * r^0.0549, or, D = 61.3 * r^0.0549 (kg/m^3). Then we can use this fitted equation for the prediction (see Table 6 columns 6 through 8). If Jupiter, Saturn, Uranus follows log(D) =0.0549 log(mass) + 1.7877 relationship, then they will have mass density 1928, 1804, and 1627 kg/m^3, and their radius will be 12%, 28%, and 9% smaller than the current size. Note: If we use D =1638 kg/m^3 for all four gas planets, the calculated r change is still very similar as that in Table 6 column 8 (see paper SunQM-3s6 Table 2 columns 11 through 13), because r ∝ D^(1/3) decreases the change in D. According to these values, and combining with the information flatten (in column 9 of Table 6, obtained from wiki “Jupiter”, “Saturn”, “Uranus”, “Neptune” and “Sun”), the current mass density (in column 4 of Table 6), and the self-spin or day period (in column 5 of Table 6), I predicted that the current (G-forced) mass occupancy in the out atmosphere shell is ~ 90% for Jupiter, ~50% for Saturn, ~70% for Uranus, and ~100% for Neptune. For our Sun, the expected G-forced (cold) mass occupancy is ~50%, and the real (hot) mass occupancy is ~100% (see Table 6 columns 10 and 11).

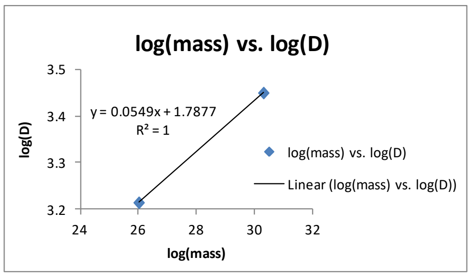


Figure 6. log(mass) vs. log(D) plot for linear regression fitting.

Table 6. Estimation of gas/ice planet's in situ formed size with its current mass, and the current % mass occupancy in the out atmosphere shell.

NASA's data of planets					predicted r from the current mass								atmosphere p{1,1//2}	Earth-sized core p{0,1//2}	inner core p{-1,1//2}
	current mass	planets body-r	current mass density	day period	predicted D from current mass = 61.3 *mass^0.0549	predicted r= [M/(D*4/3*π)]^(1/3)	% smaller than current r	Planet current flattening	predicted G-force mass occupancy	Heated mass occupancy	original planet mass	predicted D from original mass = 61.3 *mass^0.0549	original mass r= [M/(D*4/3*π)]^(1/3)		
unit	kg	m	kg/m^3	hour	kg/m^3	m					kg	kg/m^3	m	m	m
{1,3} Mercury	3.30E+23	2.44E+06		1407							3.04E+26	1743.3	3.47E+07	8.66E+06	2.17E+06
{1,4} Venus	4.87E+24	6.05E+06		-5832							2.05E+26	1706.0	3.06E+07	7.65E+06	1.91E+06
{1,5} Earth	5.97E+24	6.38E+06		23.9							1.48E+26	1675.7	2.76E+07	6.91E+06	1.73E+06
{1,6} Mars	6.42E+23	3.40E+06		24.6							1.16E+26	1653.5	2.56E+07	6.40E+06	1.60E+06
{2,2} Jupiter	1.90E+27	6.99E+07	1326	9.9	1928	6.17E+07	12%	0.065	~90%	-	1.92E+26	1699.9	3.00E+07	7.50E+06	1.87E+06
{2,3} Saturn	5.68E+26	5.82E+07	687	10.7	1804	4.22E+07	28%	0.098	~50%	-	1.11E+26	1649.5	2.52E+07	6.31E+06	1.58E+06
{2,4} Uranus	8.68E+25	2.56E+07	1271	-17.2	1627	2.34E+07	9%	0.023	~70%	-	7.52E+25	1614.6	2.23E+07	5.58E+06	1.39E+06
{2,5} Neptune	1.02E+26	2.48E+07	1368	16.1	1642	2.46E+07	1%	0.017	~100%	-	5.46E+25	1586.5	2.02E+07	5.04E+06	1.26E+06
SUN	1.99E+30	6.96E+08	1408		2824			9.00E-06	50%	100%	-	-	-	-	-

In Table 6 columns 12 through 16, using the original mass (obtained from Table 3b in paper SunQM-1s1), and the new mass densities predicted in column 6 of the current table, the radiuses of the original atmosphere pCore{1,1//2}, the Earth-sized core pCore{0,1//2}, and the inner core pCore{-1,1//2} are calculated for all eight planets. Since this calculation method is not significantly more accurate than that used in the Table 2 of paper SunQM-3s6, Table 2 of paper SunQM-3s6 will still be the standard radiuses for the original-massed planets’ p{N,n//2} QM structure for the SunQM series papers.

The analysis in question 2 reveals that among four gas/ice planets, only Neptune is at its “global energy minimum” QM state, or with ~ 100% mass occupancy in all n shells of its body. The other three are not in their “global energy minimum” QM state. We can use the well known quantum superposition state theory to explain this result. For example, Jupiter has a collection of superposition states of p{N,n//2}, p{N,n//3}, p{N,n//4}, p{N,n//5}, p{N,n//6}, etc. After (or during) adding 9x more mass, Jupiter encountered a much lower transitional energy barrier between the original pCore{N,n//2} to pCore{N,n//5} QM state than that between the original pCore{N,n//2} to pCore{N,n//3} QM state. So Jupiter transitioned to pCore{N,n//5} QM structure rather than to pCore{N,n//3}. After transitioned to pCore{N,n//5} QM state, the transitional energy barrier between the pCore{N,n//5} to pCore{N,n//3} is still too high to overcome, so currently Jupiter is trapped in pCore{N,n//5} QM state (which is not the "global energy minimum" QM state for its current total mass). So its mass density is ~ 1326 kg/m^3, not the lowest (from ~1638 to ~1928 kg/m^3), and its out atmosphere has around 90% mass occupancy instead of 100%. Then just like in the traditional |nlm> QM states, the quantum superposition happens within n quantum numbers, the same quantum superposition happens within q quantum numbers as well as within n quantum numbers in the {N,n//q} QM's |qnlm> states. So now Jupiter shows both pCore{N,n//5} and pCore{N,n//3} QM structural characters at the same time, because q is strongly superpositioned at both =5 and =3.

Saturn's q-superposition states also includes p{N,n//q=2..6}. As I mentioned before, beyond Earth-sized core, Saturn has pCore{0,3//2} QM structure, which is equivalent to a pCore{0,3//3} QM structure, and this is right for the Saturn-massed celestial body. However, within its Earth-sized core, it still keeps the original p{N,n//2} QM, not updated into pCore{N,n//3} yet. So the current Saturn is not in a complete pCore{N,n//3} QM, but in a hybridized (or superpositioned) QM of pCore{N,n//3} and p{N,n//2}.

Similarly, Uranus' q-superposition states also includes p{N,n//q=2..6}. As mentioned before, after a catastrophic collision, Uranus lost ~ 50% of mass, but it still keeps the original pCore{N,n//2} QM structure, which is still the right "global energy minimum" QM for Uranus' current mass. To do this, Uranus decreased its atmosphere shell mass occupancy to ~50% (or ~70% according to Table 6), so its current averaged mass density is ~ 1271 kg/m^3, significantly lower that the "global energy minimum" QM state value (~ 1600 kg/m^3).

So for a gas/ice planet with its mass equal or larger than that of Neptune, if it has D ≥ 1600 kg/m^3, then it is at the "global energy minimum" QM state (or the ground state of |qnlm>). If D is significantly lower than 1600 kg/m^3, then it is not at the "global energy minimum" QM state (or at least one of four quantum numbers q, n, l, m is not at ground state)! So far all these analyses are based on that Neptune is at the in situ formed "global energy minimum" QM state. After comparing all analyses, we are pretty sure that the current Neptune's mass should be within the range of 50% to 120% of the original Neptune's mass. Within this mass variation, Neptune should have kept the original size and the original pCore{N,n//2} QM structure.

After 100 million years, will Jupiter, or Saturn, or Uranus transit its QM structure from current "non-global energy minimum" state to the "global energy minimum" state? We don't know. The current {N,n} QM analysis is based on the time-independent theory, only provides the state energy information, not provide the transitional dynamics between two states. So we are waiting for somebody to develop a time-dependent {N,n//q} QM theory (like the time-dependent perturbation theory for atom) to answer this question.

Our Solar system's q-superposition states should also include {N,n//q=2..6}. What is the "global energy minimum" state of a Sun-massed Solar-system? Most likely it is {N,n//6}. Because the Sun-massed white dwarf is at {-1,1//6}, black hole is at {-3,1//6}, Kuiper belt's out edge is at {5,1//6}, and proton is at {-15,1//6}. These almost perfect fittings (especially the proton which is independent of Sun mass) strongly suggest that our Solar system is at its "global energy minimum" QM state {N,n//6}.

The identification of {N,n//6} QM structure for our Solar system may help us to trace back the size of the seed that trigged the formation of Solar system. The previous analysis showed that all eight planets in Solar system were originally formed in pCore{N,n//2} QM structure, so that {N,n//2} seems to be the most basic QM structure for the formation of celestial body (at least for the planets). There is a high probability that {N,n//2} is also the most basic QM structure for the formation of stars. If this is correct, and using current Sun surface {0,2//6} as r_1 of {0,1//2}, then Sun core {0,1//6} = {-1,1//2} is one "//2" level down, and {-1,3//6} = {-2,1//2} is two "//2" level down. This means that a {-1,3//6} sized QM structure may well be the seed for the formation of our Solar system (if the star formation follows {N,n//2} QM structure). We know Jupiter has size around {-1,3//6}, and that is why in paper SunQM-1s2 I mentioned that a wandering orphan Jupiter in the space may be the seed to trig the formation of our Solar system. It is always interesting to know what {N,n//q} QM structure will be if a star is formed by using a {-1,1//6} sized white dwarf, or a {-1,4//6} sized red dwarf as seed.

The result in paper SunQM-1s2 Table 1 (and paper SunQM-5 Table 1) illustrates that the {N,n//6} is our universe's dominated {N,n//q} QM structure, from {-17,1//6} quark to Virgo super cluster {10,1//6}. But for a 10x Sun-massed star, if Figure 5's prediction is right, then it will have pFactor ≈7. Will this contradict the result of "{N,n//6} is our universe's dominant {N,n//q} QM structure"? It should not. Because according to wiki "Stars", "*The more massive the star, the shorter its lifespan, primarily because massive stars have greater pressure on their cores, causing them to burn hydrogen more* *rapidly*". The lifetime of a star in the main sequence can be estimated by comparing it to solar evolutionary models according to the following formula T = 10^10(yrs) * [M / M_{sun}]^(-2.5), where T is the star's estimated main sequence lifetime, and M is the mass of the star (see wiki "Main sequence"). According to this formula, a 10x Sun-massed star will have a very short lifetime (only ~32 million years), in comparison with a ~1x Sun-massed star which has many billion years of lifetime. So a {N,n//6} QM structured star is a super-stable {N,n} QM structure in comparison with a {N,n//7} QM structured star, and our universe is dominated by {N,n//6} super stable QM structure, even other pFactor QM structures (like {N,n//7}, etc.) may exist.

This analysis also makes me to believe that our universe is made of a superpositioning {N,n//q} QM states with all values of pFactor (integer q), although the "global energy minimum" state is {N,n//6} QM structure. For each object (super cluster, galaxy, star, planet, moon, atom, proton, etc.) in our universe, it is also in a superpositioning {N,n//q} QM states with all values of pFactor, (or all four quantum numbers, q, n, l, and m in |qnlm> QM state, are superpositioned). Each object has a "global energy minimum" state based on this object's mass (or other character if it is not the G-force based QM), or has a |qnlm> ground state with a specific q, n, l, m quantum value. Most in situ formed objects stay in their "global energy minimum" {N,n//q} QM structure state (or |qnlm> ground state). So according to the quantum superposition, every QM state is possible!

The same idea has been presented in paper SunQM-2 section IV, but with the very different description. There our universe and each object (galaxy, star, atom, etc.) have been described with the matter wave that composed with all possible frequencies and RF(s). Meanwhile, our universe and each object (galaxy, star, atom, etc.) itself is a matter wave resonance chamber (MWRC). All possible matter waves with all possible frequencies and RF(s) are running in each of MWRC, but each MWRC only amplifies a specific set of matter wave frequencies and RF(s). So you can see that both |qnlm> superposition and matter wave resonance describe the same thing, but from a very different angle. This is similar as that Schrodinger's differential equation QM and Heisenberg's matrix QM, both describe the same QM, but with different angle. Actually the matter wave resonance description is more related to Schrodinger's differential equation QM, and |qnlm> superposition description may be more related to Heisenberg's matrix QM.

The evolution of our universe may be treated as a super gigantic quantum computer which is doing the quantum calculation with all possible quantum superpositional states. As the evolution (or quantum calculation) going, more and more quantum superpositional states collapse. Then the end of universe must be the final quantum calculated result of this super gigantic quantum computer, and it must be at the "global energy minimized" state.

Conclusions

Based on the hypothesis of "internal structure of all planets following their {N,n} QM radial probability density curve", the mass density for all gas/ice planets' internal structures are summarized in table below:

	Jupiter D (kg/m^3)	Saturn D (kg/m^3)	Uranus D (kg/m^3)	Neptune D (kg/m^3)	{3,2} planet D (kg/m^3)
inner core	26000	23000	~16500	~17000	~16000
	11000	7800	~6000	~7000	~5500
	5000	1600	1800	2500	~1750
	2400→0	1600→0	1800→0	2500→0	~1750→0

Although a celestial body's formation is primarily based on G-force, after passing a critical mass point (estimated to be between 1E+19 kg to 7E+22 kg), the {N,n} QM-force starts to affect the internal structure of this celestial body. For an in situ formed (large) celestial body, its {N,n} QM governed radial structure is always coupled with its gravity governed radial structure, although they may be de-coupled under certain situation. A p{N,n//q} QM state can be written as |qnlm>. The analysis suggests that q is also a superpositioned quantum number in |qnlm> QM state. The analysis reveals that Jupiter's current QM structure p{N,n//5} may not be at a Jupiter-massed celestial body's "global energy minimum" state. In other words, among all possible superpositional q(s), q=5 may not be the ground state for a Jupiter-massed celestial body's |qnlm> state.

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[11] Yi Cao, SunQM-3s8: Using {N,n} QM to study Sun's internal structure, convective zone formation, planetary differentiation and temperature r-distribution. http://vixra.org/pdf/1808.0637v1.pdf (submitted on 2018-08-29)

[12] A series of my papers that to be published (together with current paper):
SunQM-3s7: Predict mass density r-distribution for gas/ice planets, and the superposition of {N,n//q} or |qnlm> QM states for planet/star.
SunQM-3s9: Using {N,n} QM to explain the sunspot drift, the continental drift, and Sun's and Earth's magnetic dynamo.
SunQM-3s10: Updates and Q/A for SunQM series papers.
SunQM-3s4: Using {N,n} QM structure and multiplier n' to analyze Saturn's ring structure.
SunQM-3s5: Using {N,n} QM structure and n*l*0 effect to analyze the Bipolar outflow.
SunQM-5: A new version of QM based on interior {N,n}, multiplier n', |R(n,l)|^2 |Y(l,m)|^2 guided mass occupancy, and RF, and its application from string to universe.
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