Layered motions: the meaning
of the fine-structure constant

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23 December 2018

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Abstract: Following a series of papers on geometric interpretations of the wavefunction, this paper offers an overview of all of them. If anything, it shows that classical physics goes a long way in explaining so-called quantum-mechanical phenomena. It is suggested that the fine-structure constant can be interpreted as a scaling constant in a layered model of electron motion. Instead of one single wave equation explaining it all, we offer a theory of superposed motions based on the fine-structure constant, which we interpret as a scaling constant. The layers are the following:

1. To explain the electron's rest mass, we use the *Zitterbewegung* model. Here, we think of the electron as a pointlike charge (no internal structure or motion) with zero rest mass, and (1) its two-dimensional oscillation, (2) the $E/m = c^2 = a^2\omega^2$ elasticity of spacetime and (3) Planck's quantum of action (h) explain the rest mass: it is just the equivalent mass of the energy in the oscillation.

2. We then have the Bohr model, which shows orbitals pack the same amount of physical action (h) or a multiple of it ($S = n·h$). It just packs that amount in much larger loops which – of course – then also pack a different amount of energy. As it turns out, the equivalent energy ($E = h·f$) is equal to $\alpha^2 mc^2$. The fine-structure constant also acts as a scaling constant for all other dimensions (radii, velocities, and frequencies).

3. The difference between the energies of the Bohr orbitals is, of course, the energy of the photon when an electron makes a transition. Hence, we also offer an elegant one-cycle model of a photon and show the meaning of the fine-structure constant as a coupling constant in QED.

This leads to a more comprehensive interpretation of the fine-structure constant as a scaling constant. As an added bonus, we argue that the fine-structure constant also introduces a *form factor* (the electron is now viewed as a disk-like structure), which might explain the anomalous magnetic moment. We argue that the anomalous magnetic moment may, therefore, not be anomalous at all.

Keywords: Bohr model, Schrödinger's equation, rest matter oscillation, electron orbitals, wavefunction interpretations.

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I.   Introduction

This paper – like all the others we have written – is born out of curiosity and frustration. Curiosity – because we all want to understand the nature of matter and energy. But curiosity is not a sufficient reason to write yet another paper. There are so many out there. That is where frustration comes in: we have started to think one or two re-interpretations of quantum-mechanical concepts may be needed to make sense of it all – and we are surely *not* the only ones thinking that – while we always get numerically correct answers from quantum-mechanical models – their conceptual framework often appears as being mixed up or, worse, as being plain *ad hoc.*[1]

One absolutely inexcusable mistake is that quantum physicists think of the elementary wavefunction as representing some theoretical spin-zero particle. The mistake is inexcusable because the associated redundancy in the mathematical framework is directly related to the logic leading to the rather uncomfortable conclusion that the wavefunction of spin-1/2 particles (read: all matter-particles (fermions), practically speaking) has some weird 720-degree symmetry in space. This conclusion is uncomfortable because we cannot imagine such objects in space without invoking the idea of some kind of relation between the subject and the object (the reader should think of the Dirac belt trick here). It has, therefore, virtually halted all creative thinking on a physical interpretation of the wavefunction. We have written at length about this and other objections to a geometric interpretation of the wavefunction before[2], so we will just briefly summarize the point here. The two possibilities ($a·e^{-i·\theta}$ and $a·e^{+i·\theta}$) are visualized below.
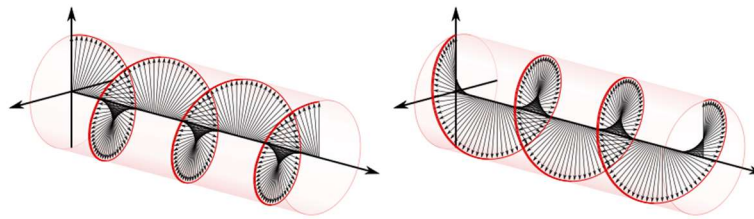

Figure 1: Left-handed and right-handed angular momentum

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[1] We love the self-criticism of John P. Ralston, Professor of Physics and Astronomy at the University of Texas: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (*How to understand quantum mechanics* (2017), p. 1-10) Having mentioned the comment, from our correspondence, we understand – and duly note – that he would surely *not* want to be associated with the classical ideas that are being presented in this paper.

[2] Such objections usually also include the idea that the coefficient (*a*) of the wavefunction $a·e^{i\theta}$ may be complex-valued, whereas in any *real* interpretation this (maximum) amplitude should be real-valued. This objection is also rejected. See: Jean Louis Van Belle, 30 October 2018, *Euler's wavefunction: the double life of −1*, http://vixra.org/pdf/1810.0339v2.pdf.

All *real* particles have spin – electrons, photons, anything – and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. Hence, why not use the *sign* of the imaginary unit in the $a \cdot e^{\pm i\theta}$ function to incorporate the spin direction in the description *from the start*? Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i\cdot\theta} = a \cdot e^{-i\cdot(\omega t - kx)}$ and $a \cdot e^{+i\cdot\theta} = a \cdot e^{+i\cdot(\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction – as opposed to, say, some real-valued sinusoid. We would think physicists would then proceed to provide some argument why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[3]

This is, frankly, quite shocking because we know, from historical experience, that theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things. Here we should think of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? Occam's Razor principle tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations, and the only characteristic that can make the difference is the spin direction.

Hence, we do *not* agree with the mainstream view that the choice is a matter of convention. Instead, we dare to suggest that the two mathematical possibilities represent identical particles with opposite spin. Combining this with the two possible directions of propagation (which are given by the +− or ++ signs in front of ω and k), we get the following table:

Table 1: Occam's Razor: mathematical possibilities versus physical realities

Spin and direction of travel	Spin up (e.g. $J = +\hbar/2$)	Spin down (e.g. $J = -\hbar/2$)
Positive *x*-direction	$\psi = a \cdot e^{-i\cdot(\omega t - kx)}$	$\psi^* = a \cdot e^{+i\cdot(\omega t - kx)}$
Negative *x*-direction	$\chi = a \cdot e^{-i\cdot(\omega t + kx)}$	$\chi^* = a \cdot e^{-i\cdot(\omega t + kx)}$

An added benefit of this interpretation is that we can now also associate some physical meaning with the complex conjugate of a wavefunction and – by extension – to various properties of quantum-mechanical operators, including their hermiticity (or not).[4] More generally speaking, we may say that we can finally offer a meaningful *physical* interpretation of the quantum-mechanical wavefunction. Let us get through the basics of it.

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[3] See, for example, the MIT's edX Course 8.04.1*x*, Lecture Notes, Chapter 4, Section 3.
[4] See the above reference: : Jean Louis Van Belle, 30 October 2018, *Euler's wavefunction: the double life of −1*, http://vixra.org/pdf/1810.0339v2.pdf.

II.   The quantum of action and the Bohr orbitals

To achieve some familiarity with the mathematical framework, we will use the quantum of action in the context of the Rutherford-Bohr model of an atom. This 105-year old model[5] was designed to explain the wavelength of a photon that is emitted or absorbed by a hydrogen atom – a one-electron atom, basically – and does a superb job of it. The idea is that the energy of such photon is equal to the *difference* in energy between the various orbitals. The energy of these orbitals is usually expressed in terms of the energy of the *first* Bohr orbital, which is usually referred to as the *ground state* of (the electron in) the hydrogen atom. The *Rydberg* energy E_R is just the combined kinetic and potential energy of the electron in the first Bohr orbital and it can be expressed in terms of the fine-structure constant (α) and the rest energy ($E_0 = mc^2$) of the electron[6]:

$$E_R = \frac{\alpha^2 mc^2}{2} = \frac{1}{2}(\frac{q_e^2}{2\varepsilon_0 hc})^2 mc^2 = \frac{q_e^4 m}{8\varepsilon_0^2 h^2} \approx 13.6 \text{ eV}$$

To be precise, the *difference* in energy between the various orbitals should be equal to:

$$\Delta E = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot E_R$$

The Rydberg formula then becomes self-evident. The idea of the wavelength of a wave (λ), its velocity of propagation (*c*) and its frequency (*f*)[7] are related through the λ = *c*/*f* relation, and the Planck-Einstein relation (E = *h·f*) tells us the energy and the wavelength of a photon are related through the frequency:

$$\lambda = \frac{c}{f} = \frac{hc}{E}$$

Hence, we can now write the Rydberg formula by combining the above:

$$\frac{1}{\lambda} = \frac{E}{hc} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot \frac{E_R}{hc} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \cdot \frac{\alpha^2 mc^2}{2hc}$$

The Rydberg formula uses the fine-structure constant, but actually describes the so-called *gross* structure of the hydrogen spectrum only (illustrated below). Indeed, when the spectral lines are examined at *very* high resolution, the spectral lines are split into finer lines. This is due to the intrinsic spin of the electron. This *intrinsic* spin of the electron is to be distinguished from its orbital motion. It shows we should not be thinking of the electron as a pointlike (infinitesimally small) particle: it has a

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[5] Around 1911, Rutherford had concluded that the nucleus had to be very small. Hence, Thomson's model – which assumed that electrons were held in place because they were, somehow, embedded in a uniform sphere of positive charge – was summarily dismissed. Bohr immediately used the Rutherford hypothesis to explain the emission spectrum of hydrogen atoms, which further confirmed Rutherford's conjecture, and Niels and Rutherford jointly presented the model in 1913. As Rydberg had published his formula in 1888, we have a gap of about 25 years between experiment and theory here.

[6] We should write m_0 instead of m – *everywhere*. But we are using non-relativistic formulas for the velocity and kinetic energy everywhere. Hence, we dropped the subscript.

[7] Our paper relates mathematical and physical concepts. Hence, we prefer to think of a wavelength as a mathematical *idea* right now, as opposed to some (physical) *reality*. Our ontological viewpoint is very simple: language describes reality. Hence, math describes physics. There is an intimate relation between both but – at the same time – we should not confuse the two.

radius.[8] Hence, we speak of *spin* angular momentum versus *orbital* angular momentum. However, as we will explain, there is some *coupling* between the two motions. We will come back to this later.
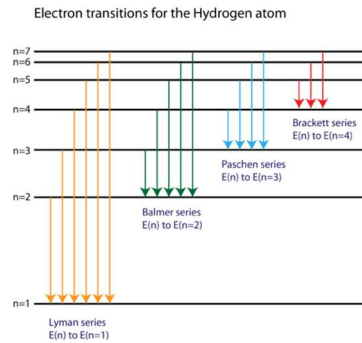

Electron transitions for the Hydrogen atom

Figure 2: The gross structure of the hydrogen spectrum

The Copenhagen interpretation of quantum mechanics – which, privately, we have started to think of as the *Heisenberg Diktatur*[9] – dismisses Bohr's model. However, it is actually a proper quantum-mechanical explanation and, as we have shown[10], Schrödinger's equation does not seem to add much in terms of a scientific explanation for the atomic electron orbitals. Feynman (*Lectures*, III-2-4) derives it from the momentum-space expression of the Uncertainty Principle which we may loosely state as follows: the product of the uncertainty in the momentum (Δp) and the uncertainty in the position (Δx) has an order of magnitude that is equal to Planck's quantum (h). His equation is the following:

$$\mathrm{p} \cdot a \approx \hbar \Leftrightarrow \mathrm{p} \approx \hbar/a$$

This allows him to write the kinetic energy of the electron as $mv^2/2 = p^2/2m = \hbar^2/2ma^2$. The potential energy is just the electrostatic energy $-e^2/a$.[11] The idea is then that the configuration must minimize the

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[8] We argue its radius is the Compton radius. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

[9] No one should take offense here. It is an opinion which is rooted in our experience trying to submit articles to scientific journals as well as interactions with academics. In fact, we should tone down and *not* specifically associate the Copenhagen interpretation with Heisenberg and other founding fathers of the quantum-mechanical framework, as they were part of the group of 'founding fathers' who actually became quite skeptical about the theory they had created because of the divergences in perturbative quantum electrodynamics (QED). Todorov (2018) specifically Heisenberg, Dirac, and Pauli in this regard, and mentions that QED, as a theory, only survived because of the efforts of the second generation of quantum physicists (Feynman, Schwinger, Dyson, etcetera). See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

[10] See: Jean Louis Van Belle, *A Geometric Interpretation of Schrödinger's Wave Equation*, 12 December 2018 (http://vixra.org/abs/1812.0202).

[11] The e^2 in this formula is the squared charge of an electron (q_e^2) divided by the electric constant ($4\pi\varepsilon_0$). The formula assumes the potential is zero when the distance between the positively charged nucleus and the electron is infinite, which explains the minus sign. We also get the minus sign, of course, by noting the two charges (electron and nucleus) have equal magnitude but opposite sign. One should note that the formulas are non-relativistic. This is justified by the fact that the velocities in this model are non-relativistic (the electron velocity in the Bohr orbit is given by $v_e = \alpha \cdot c \approx 0.0073 \cdot c$. This is an enormous speed but still less than 1% of the speed of light.

total energy E = $\hbar^2/2ma^2 - e^2/a$. The variable is the radius a and, hence, we get a by calculating the dE/da derivative and equating it to zero. We thus get the correct Bohr radius:

$$r_{\text{Bohr}} = \frac{\hbar^2}{\text{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\text{mq}_e{}^2} = \frac{1}{\alpha} \cdot r_{\text{Compton}} \approx 53 \times 10^{-12} \text{ m}$$

We find it useful to write the Bohr radius as the Compton radius divided by the fine-structure constant:

$r_B = r_C/\alpha = \hbar/\alpha mc \approx (386/0.0073) \times 10^{-15}$ m $\approx 53 \times 10^{-9}$ m.

We can now calculate the Rydberg energy – which is the *ionization* energy of hydrogen – by using the Bohr radius to calculate the energy E = $\hbar^2/2ma^2 - e^2/a$:

$$E_R = \frac{1}{2}\frac{\hbar^2}{m}\frac{m^2 e^4}{\hbar^4} - e^2\frac{me^2}{\hbar^2} = -\frac{1}{2}\frac{me^4}{\hbar^2} \approx -13.6 \text{ eV}$$

This amount equals the kinetic energy ($\hbar^2/2ma^2 = \alpha^2mc^2/2$). The electrostatic energy itself is *twice* that value ($-e^2/r_{\text{Bohr}} = -\alpha^2mc^2$).

Feynman's Uncertainty Principe is suspiciously certain. He basically equates the uncertainty in the momentum as the momentum itself (Δp = p) and the uncertainty in the position as a precise radius. We offer an alternative interpretation. If Planck's constant is, effectively, a physical constant ($h \approx$ 6.626×10⁻³⁴ N·m·s), then we should interpret it as such. If physical action – some force over some distance over some time – comes in units of h, then the relevant distance here is the loop, so that is $2\pi\cdot r_{\text{Bohr}}$. We would, therefore, like to re-write Feynman's p·$a \approx \hbar$ assumption as:

$S = h = p\cdot 2\pi\cdot r_{\text{Bohr}} = p\cdot\lambda$

The λ is, of course, the circumference of the loop. The equation resembles the *de Broglie* equation λ = h/p. How should we interpret this? We can associate Planck's quantum of action with a *cycle*: let us refer to it as a Bohr loop and, yes, we think of it as a circular orbit. As such, we can write h either as the energy times the cycle time or, else, as the (linear) momentum times the loop: h = p·2π·r_B. The latter expression not only reflects the second *de Broglie* relation but also the quantum-mechanical rule that angular momentum should come in units of $\hbar$ = h/2π. Indeed, the angular momentum can always be written in terms of the tangential velocity, the radius and the mass. As such, the two formulas below amount to the same:

$$L = m \cdot v \cdot r_B = p \cdot r_B = \hbar \Leftrightarrow S = p \cdot 2\pi \cdot r_B = p \cdot \lambda_B = h$$

Let us continue our calculations. We get the velocity out of the expression for the kinetic energy:

$$K.E. = \frac{mv^2}{2} = \frac{\alpha^2 mc^2}{2} \Leftrightarrow v = \alpha \cdot c \approx 0.0073 \cdot c$$

Of course, we should also be able to express the velocity as the product of the radius and an angular frequency, which we can do as follows:

$$v = \alpha \cdot c = r_B \cdot \omega_B = \frac{\hbar}{\alpha mc} \cdot \frac{\alpha^2 mc^2}{\hbar} = \alpha \cdot c \Leftrightarrow \omega_B = \frac{\alpha^2 mc^2}{\hbar}$$

We then calculate the cycle time T as T = $1/f_B$ = $2\pi/\omega_B$. Interestingly, the formula for f_B (or, thinking in terms of *angular* frequencies, for ω_B) reflects the first *de Broglie* relation: f_B = E/h = $\alpha^2 mc^2/h$. However, we should note that $\alpha^2 mc^2$ is *twice* the Rydberg energy – and, unlike some physicists[12], we do care about a 1/2 or π factor in our model of a Bohr electron. Hence, we should have a look at this energy concept. We will do so later. Let us – just for now – roll for a moment with this E = $\alpha^2 mc^2$ energy concept. It is, obviously, the energy that is associated with the loop. We wrote the quantum of action as the product of the (linear) momentum and the distance along the loop: h = p·λ_B = p·2π·λ_B. Likewise, we can write:

$$h = E \cdot T = \alpha^2 mc^2 \cdot \frac{2\pi \cdot r_B}{v} = \alpha^2 mc^2 \cdot \frac{2\pi \cdot r_C}{\alpha \cdot c \cdot \alpha} = mc^2 \frac{2\pi \cdot \hbar}{c \cdot m \cdot c} = h$$

Let us now generalize our formulas for all of the Bohr orbitals:

Table 2: Generalized formulas for the Bohr orbitals

Orbital electron (Bohr orbitals)
$S_n = n\text{h}$ for $n = 1, 2, \ldots$
$E_n = -\dfrac{1}{2}\dfrac{\alpha^2}{n^2}mc^2 = -\dfrac{1}{n^2}E_R$
$r_n = n^2 r_B = \dfrac{n^2 r_C}{\alpha} = \dfrac{n^2}{\alpha}\dfrac{\hbar}{mc}$
$v_n = \dfrac{1}{n}\alpha c$
$\omega_n = \dfrac{v_n}{r_n} = \dfrac{\alpha^2}{n^3 \hbar}mc^2 = \dfrac{\frac{1}{n^2}\alpha^2 mc^2}{n\hbar}$
$L_n = I \cdot \omega_n = n\hbar$
$\mu_n = I \cdot \pi r_n{}^2 = \dfrac{q_e}{2m}n\hbar$
$g_n = \dfrac{2m}{q_e}\dfrac{\mu}{L} = 1$

The reader can easily verify these formulas – by googling them, doing the calculations himself or, preferably, just doing some substitutions here and there. Let us substitute the equation for ω_n in the L_n formula, for example:

$$L_n = I \cdot \omega_n = \text{m} \cdot r_n{}^2 \cdot \frac{\alpha^2}{n^3 \hbar}mc^2 = \text{m} \cdot \frac{n^4}{\alpha^2}\frac{\hbar^2}{\text{m}^2 c^2} \cdot \frac{\alpha^2}{n^3 \hbar}mc^2 = n\hbar$$

The reader should note that these formulas are *not* so obvious as they seem. The table below shows what happens with radii, velocities, frequencies and cycle times as we move out. The velocities go down, all the way to zero for n → ∞, and the corresponding cycle times increases as the *cube* of *n*. Using totally

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[12] We are thinking of Richard Feynman's rather shocking *nonchalance* when introducing the quantum-mechanical idea of an electron orbital using the Uncertainty Principle (*Lectures*, Vol. III, Chapter 2, Section 4), noting from the outset that we should not "trust" the model "within factors like 2, π, etcetera."

non-scientific language, we might say the numbers suggest the electron starts to lose interest in the nucleus so as to get ready to just wander about as a free electron.

Table 3: Functional behavior of radius, velocity and frequency of the Bohr orbitals

n	1	2	3	4	5	6	7	8	9
$r_n \propto n^2$	1	4	9	16	25	36	49	64	81
$v_n \propto 1/n$	1	0.500	0.333	0.250	0.200	0.167	0.143	0.125	0.111
$\omega_n \propto 1/n^3$	1	0.125	0.037	0.016	0.008	0.005	0.003	0.002	0.001
$T_n \propto n^3$	1	8	27	64	125	216	343	512	729

The important thing is the energy formula, of course, because it should explain the Rydberg formula, and it does:

$$E_{n_2} - E_{n_1} = -\frac{1}{n_2{}^2}E_R + \frac{1}{n_1{}^2}E_R = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)\cdot E_R = \left(\frac{1}{n_1{}^2} - \frac{1}{n_2{}^2}\right)\cdot\frac{\alpha^2 m c^2}{2}$$

Let us know look at the energies once again and try to connect this model with the idea of a photon.

III.   The one-cycle photon

The Bohr orbitals are separated by a amount of *action* that is equal to h. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. Our photon will have to pack that, somehow. It will also have to pack the related energy, which is given by the Rydberg formula (see above). To focus our thinking, let us consider the transition from the second to the first level, for which the $1/1^2 - 1/2^2$ is equal 0.75. Hence, the photon energy should be equal to $(0.75)\cdot E_R \approx 10.2$ eV.[13] Now, if the total action is equal to h, then the cycle time T can be calculated as:

$$E \cdot T = h \Leftrightarrow T = \frac{h}{E} \approx \frac{4.135 \times 10^{-15} \text{eV} \cdot \text{s}}{10.2 \text{ eV}} \approx 0.4 \times 10^{-15} \text{ s}$$

This corresponds to a wave train with a length of $(3\times10^8 \text{ m/s})\cdot(0.4\times10^{-15} \text{ s}) = 122$ nm. That is the size of a large molecule and it is, therefore, much more reasonable than the length of the wave trains we get when thinking of transients using the supposed Q of an atomic oscillator.[14] In fact, this length is *exactly* equal to the wavelength $\lambda = c/f = c\cdot T = hc/E$.

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[13] This is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. The ozone layer of our atmosphere blocks most of it.

[14] In one of his famous *Lectures* (I-32-3), Feynman thinks about a sodium atom, which emits and absorbs sodium light, of course. Based on various assumptions – assumption that make sense in the context of the blackbody radiation model but *not* in the context of the Bohr model – he gets a Q of about 5×10^7. Now, the frequency of sodium light is about 500 THz (500×10^{12} oscillations per second). Hence, the *decay time* of the radiation is of the order of 10^{-8} seconds. So that means that, after 5×10^7 oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. That seems to be very short, but it still makes for 5 million oscillations and, because the wavelength of sodium light is about 600 nm (600×10^{-9} meter), we get a wave train with a considerable length: $(5\times10^6)\cdot(600\times10^{-9}$ meter$) = 3$ meter. *Surely you're joking, Mr. Feynman!* A photon with a length of 3 meter – or longer? While one

What picture of the photon are we getting here? Because of the angular momentum, we will probably want to think of it as a circularly polarized wave, which we may represent by the elementary wavefunction, as shown below.[15] We will call this interpretation of the wavefunction the *one-cycle photon*: the wavefunction represents the rotating electric field vector itself or, remembering the $F = q_eE$ equation, the force field.
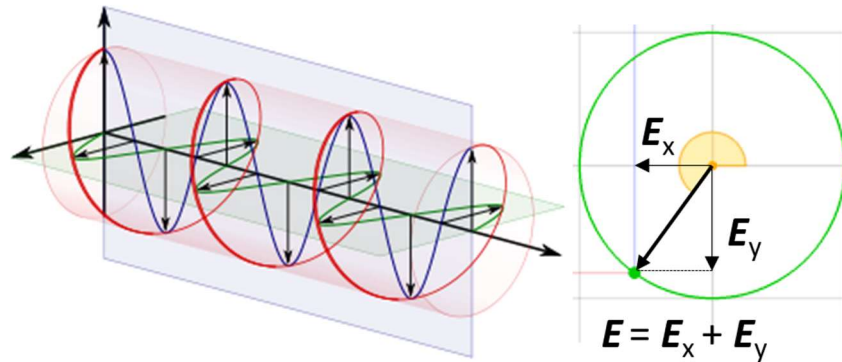

Figure 3: The one-cycle photon

It is a delightfully simple model: the photon is just one single cycle traveling through space and time, which packs one unit of angular momentum ($\hbar$) or – which amounts to the same, one unit of physical action (h). This gives us an equally delightful interpretation of the Planck-Einstein relation ($f = 1/T = E/h$) and we can, of course, do what we did for the electron, which is to express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the $E = mc^2$ mass-energy equivalence relation can be written as $p = mc = E/c$ for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

Let us now try something more adventurous: let us try to calculate the strength of the electric field. How can we do that? Energy is some force over a distance. What distance should we use? We could think of the wavelength, of course.

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might argue that relativity theory saves us here (relativistic length contraction should cause this length to reduce to zero as the wave train zips by at the speed of light), this just doesn't feel right – especially when one takes a closer look at the assumptions behind.

[15] Note that the wave could be either left- or right-handed.

However, the formulas above imply the following equation: E·λ = h·c. This suggest we should, perhaps, associate some *radius* with the wavelength of our photon. We write:

$$\mathrm{E} \cdot \frac{\lambda}{2\pi} = \mathrm{E} \cdot r = \hbar \cdot c \Longleftrightarrow r = \frac{\lambda}{2\pi} = \frac{\hbar \cdot c}{\mathrm{E}}$$

A strange formula? The reader can check the physical dimensions. They all work out: we do get a distance – something that is expressed in *meter*. But why the 2π factor? We do not want to confuse the reader too much but let us quickly insert a graph here from an article on the presumed *Zitterbewegung* of a free electron – which is interpreted as an oscillation of a pointlike *charge* (with *zero* rest mass) moving about a center at the speed of light. Now, as the electron starts moving along some trajectory at a relativistic velocity (i.e. a velocity that is a *substantial* fraction of *c*), the *radius* of the oscillation will have to diminish – because the tangential velocity remains what it is: *c*. The geometry of the situation (see below) shows the radius becomes a wavelength in this process. The radius is, of course, a circumference divided by 2π.
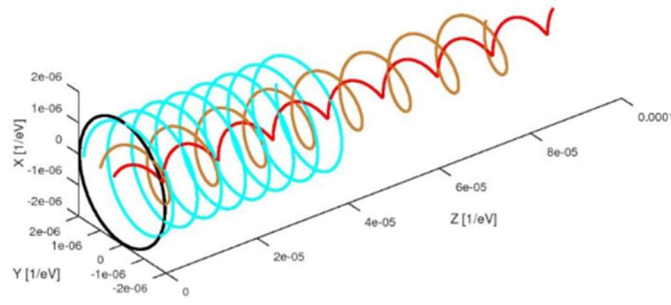
Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

We have probably confused the reader now, but he or she should just hang on for a while. Let us just jot down the following expression and then we can think about it:

$$\mathrm{E}_\gamma = \mathrm{F}_\gamma \cdot r_\gamma = \mathrm{F}_\gamma \cdot \frac{\lambda_\gamma}{2\pi}$$

We use the γ subscript to denote we're talking the energy, force and radius in the context of a photon because – in order to justify the formula above – we will remind ourselves of one of the many meanings of the fine-structure constant here: as a coupling constant, it is defined as the ratio between (1) $k \cdot q_e^2$ and (2) E·λ. We should probably interpret this as follows:

1. The $k \cdot q_e^2$ in this ratio is just the product of the electric potential between two elementary charges (we should think of the proton and the electron in our hydrogen atom here) and the distance between them:

$$\mathrm{U}(r) = \frac{k \cdot q_e^2}{r} = \frac{q_e^2}{4\pi\varepsilon_0 r} \Longleftrightarrow k \cdot q_e^2 = \mathrm{U}(r) \cdot r$$

2. The fine-structure constant can then effectively be written as:

$$\alpha = \frac{k \cdot q_e^2}{\hbar \cdot c} = \frac{k \cdot q_e^2}{\hbar \cdot c} = \frac{\mathrm{U}(r) \cdot r}{\mathrm{E}_{photo} \cdot r_{photon}}$$

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We can also write this in terms of forces times the *squared* distance:

$$\alpha = \frac{k \cdot q_e^{\ 2}}{\hbar \cdot c} = \frac{F_B \cdot r_B^{\ 2}}{F_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{F_B \cdot r_B^{\ 2}}{F_\gamma \cdot r_\gamma^{\ 2}} = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma}$$

This doesn't look too bad. We use B as a subscript in the denominator to remind ourselves we are talking the Bohr energies and radii. Let us write it all out – using the generalized formulas ($n = 1, 2,...$) above – to demonstrate the consistency of this formula:

$$\alpha = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 mc^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{mc}}{E_\gamma \cdot \frac{\hbar \cdot c}{E_\gamma}} = \alpha$$

Onwards! We think the following formula for the force may make sense now:

$$F_\gamma = \frac{E_\gamma}{r_\gamma} = \frac{2\pi \cdot E_\gamma}{\lambda_\gamma} = \frac{2\pi \cdot h \cdot f_\gamma}{\lambda_\gamma} = \frac{2\pi \cdot h \cdot c}{\lambda_\gamma^{\ 2}}$$

The electric field (E) is the force per unit charge which, we should remind the reader, is the *coulomb – not* the electron charge. Dropping the subscript, we get a delightfully simple formula for the strength of the electric field vector for a photon[16]:

$$E = \frac{\frac{2\pi hc}{\lambda^2}}{1} = \frac{2\pi hc}{\lambda^2} = \frac{2\pi E}{\lambda}\ (\frac{N}{C})$$

Let us calculate its value for our 10.2 eV photon. We should, of course, express the photon energy in SI units here:

$$E \approx \frac{2\pi \cdot 1.634 \times 10^{-1}\ J}{122 \times 10^{-9}\ m \cdot C} \approx 84 \times 10^{-12}\ \frac{N}{C}$$

This seems pretty reasonable![17] Let us make a final check on the logical consistency of this model. The energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Do we get any meaningful result when we apply that principle here? If we write the proportionality coefficient as k, we could write something like this:

$$E = k \cdot a^2 \cdot \omega^2$$

It would be wonderful if this would give some meaningful result – and even more so if we could interpret the proportionality coefficient k as the mass m. Why? Because we have used the $E = m \cdot a^2 \cdot \omega^2$ equation before: it gave us this wonderful interpretation of the *Zitterbewegung* as what we referred to as the *rest matter oscillation*. We will show, in the next section, that the idea of a two-dimensional

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[16] The *E* and E symbols should not be confused. *E* is the magnitude of the electric field vector and E is the energy of the photon. We hope the italics (*E*) – and the context of the formula, of course ! – will be sufficient to distinguish the electric field vector (*E*) from the energy (E).

[17] We got a rather non-sensical value in another paper (http://vixra.org/abs/1812.0028) but that's because we used the electron charge instead of the unit charge to calculate the field.

oscillation can also be applied to the Rutherford-Bohr model. Hence, can we repeat the trick here? We can, but the amplitude of the oscillation here is the *wavelength*. We can then write:

$$\mathrm{E} = ka^2\omega^2 = k\lambda^2\frac{\mathrm{E}^2}{h^2} = k\frac{h^2c^2}{\mathrm{E}^2}\frac{\mathrm{E}^2}{h^2} = kc^2 \Leftrightarrow k = \mathrm{m} \text{ and } \mathrm{E} = \mathrm{m}c^2$$

Sometimes physics can be *just nice*. I think we have a pretty good photon model here.

Before we move on, we need to answer an obvious question: what happens when an electron jumps several Bohr orbitals? The angular momentum between the orbitals will then differ by *several* units of $\hbar$. What happens to the photon picture in that case? It will pack the energy difference, but should it also pack several units of $\hbar$. In other words, should we still think of the photon as a one-cycle oscillation, or will the energy be spread over several cycles?

We will let the reader think about this, but our intuitive answer is: the photon is a spin-one particle and, hence, its energy should, therefore, be packed in one cycle only. This is also necessary for the consistency of the interpretation here: when everything is said and done, we do interpret the wavelength as a *physical* distance. To put it differently, the equation below needs to make sense:

$$h = \mathrm{p}\cdot\lambda = \frac{\mathrm{E}}{c}\cdot\lambda = \frac{\mathrm{E}}{f} = \mathrm{E}\cdot\mathrm{T}$$

IV.   Bohr's orbital as a two-dimensional oscillation

In previous papers, we explained the rest mass of the electron in terms of its *Zitterbewegung*. This interpretation of an electron, which goes back to Schrödinger and Dirac[18], combines the idea of motion with the idea of a pointlike *charge*, which has no inertia and can, therefore, move at the speed of light. The illustration below described the presumed circular oscillatory motion of the charge (the *Zitterbewegung*). We got wonderful results. The most spectacular result is the explanation for the *rest* mass of an electron: it is the equivalent mass of what we referred to as the *rest matter oscillation*.
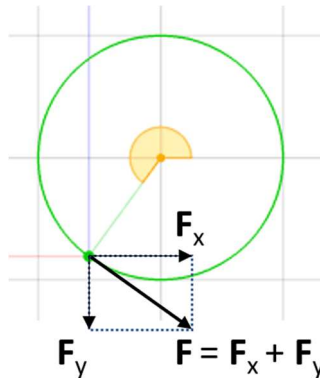

Figure 4: The *Zitterbewegung* model of an electron

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[18] Erwin Schrödinger derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very

The table summarizes the properties – angular momentum, magnetic moment, g-factor, etc. – we calculated:

Table 4: The properties of the free electron (spin-only)

Spin-only electron (*Zitterbewegung*)
$S = h$
$E = mc^2$
$r = r_C = \dfrac{\hbar}{mc}$
$v = c$
$L = I \cdot \omega = \dfrac{\hbar}{2}$
$\mu = I \cdot \pi r_C{}^2 = \dfrac{q_e}{2m}\hbar$
$g = \dfrac{2m}{q_e}\dfrac{\mu}{L} = 2$

The reader should keep his wits about him[19] here: the *Zitterbewegung* model should not be confused with our Bohr atom. We do not have any centripetal force here. There is no nucleus or other charge at the center of the *Zitterbewegung*. Instead of a tangential momentum vector, we have a tangential *force* vector (**F**), which we thought of as being the resultant force of two perpendicular oscillations.[20] This led us to boldly equate the E = mc^2, E = m·a^2·ω^2 and E = $\hbar$·ω equations – which gave us all the results we wanted. The *zbw* model – which, as we have mentioned in the footnote above, is inspired by the solution(s) for Dirac's wave equation for free electrons – tells us the velocity of the pointlike *charge* is equal to *c*. Hence, *if* the *zbw* frequency would be given by Planck's energy-frequency relation ($\omega = E/\hbar$), *then* we can easily combine Einstein's E = mc^2 formula with the radial velocity formula (*c* = *a*·ω) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$r_{\text{Compton}} = \frac{\hbar}{mc} = \frac{\lambda_e}{2\pi} \approx 0.386 \times 10^{-12} \text{ m}$$

The calculations relate the Bohr radius to the Compton radius through the fine-structure constant:

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high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

[19] The him could be a her, of course.

[20] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

$$r_{\text{Bohr}} = \frac{\hbar^2}{me^2} = \frac{4\pi\varepsilon_0\hbar^2}{mq_e{}^2} = \frac{1}{\alpha} \cdot r_{\text{Compton}} = \frac{\hbar}{\alpha mc} \approx 53 \times 10^{-12} \text{ m}$$

The fine-structure constant also relates the respective velocities, frequencies and energies of the two oscillations. We wrote:

$$v = \alpha \cdot c = r_{\text{B}} \cdot \omega_{\text{B}} = \frac{\hbar}{\alpha mc} \cdot \frac{\alpha^2 mc^2}{\hbar} = \alpha \cdot c \Leftrightarrow \omega_{\text{B}} = \frac{\alpha^2 mc^2}{\hbar}$$

As we mentioned before, the formula for the frequency of the motion of the electron in the Bohr orbitals reflects the first *de Broglie* relation: $f_B = E/h = \alpha^2 mc^2/h$. However, we noted that the $\alpha^2 mc^2$ is *twice* the Rydberg energy – and, unlike some physicists[21], we do care about a 1/2 or π factor in our model of a Bohr electron. Hence, we should have a look at this energy concept.

$$v = \alpha \cdot c = r_{\text{B}} \cdot \omega_{\text{B}} = \frac{r_{\text{B}}}{\alpha} \cdot \omega_{\text{B}} = \frac{\hbar}{\alpha mc} \cdot \frac{\alpha^2 mc^2}{\hbar} = \alpha \cdot c$$

The cycle time T is given as a function of the Bohr loop frequency by $T = 1/f_B = 2\pi/\omega_B$. Importantly, the formula for f_B *resembles* the first *de Broglie* relation: $f_B = E/h = \alpha^2 mc^2/h$.[22] Having said this, we noted that $\alpha^2 mc^2$ is *twice* the Rydberg energy – and, unlike some physicists[23], we do care about a 1/2 or π factor in our model of a Bohr electron. We said we would come back to this in a next section. This is where we are going to do this.

The $E = \alpha^2 mc^2$ energy concept is the energy that is associated with the loop. It is *twice* the kinetic energy, but it is a different energy concept altogether. In line with our interpretation of the elementary wavefunction in the context of our one-cycle photon and our free (spin-only) electron, we are thinking of the orbital motion as being driven by a two-dimensional oscillation, as illustrated below.
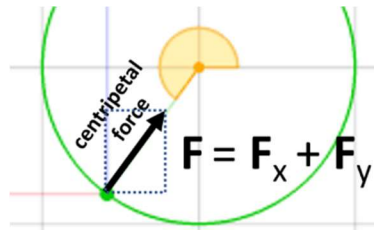

Figure 5: The oscillator model of the Bohr orbital

We look at the centripetal force as a *resultant* force here – a vector sum of two perpendicular components: **F** = **F**$_x$ + **F**$_y$. Needless to say, the **boldface** here indicates *vectors*: these force *components* have a magnitude as well as a direction. We can now develop yet another interpretation of the elementary wavefunction and think of a dual view of what is going on. Let us start with the description

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[21] We are thinking of Richard Feynman's rather shocking *nonchalance* when introducing the quantum-mechanical idea of an electron orbital using the Uncertainty Principle (*Lectures*, Vol. III, Chapter 2, Section 4), noting from the outset that we should not "trust" the model "within factors like 2, π, etcetera."
[22] In this section, we will just use the formulas for the first Bohr orbital ($n = 1$). It is easy generalize for $n = 2, 3, 4$, etc.
[23] We are thinking of Richard Feynman's rather shocking *nonchalance* when introducing the quantum-mechanical idea of an electron orbital using the Uncertainty Principle (*Lectures*, Vol. III, Chapter 2, Section 4), noting from the outset that we should not "trust" the model "within factors like 2, π, etcetera."

of the physical position (i.e. the *x*- and *y*-coordinates) of the electron. This is the green dot in the illustration of Euler's function above. Its motion is described by:

$$r = a \cdot e^{i\theta} = x + i \cdot y = a \cdot cos(\theta) + i \cdot a \cdot sin(\theta) = (x, y)$$

We can now think of this motion being driven by two perpendicular oscillations. These oscillations are associated with a kinetic and a potential energy. We illustrate this below for *one* oscillator only.
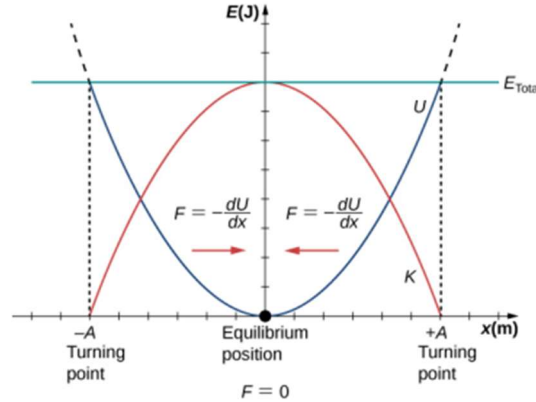

Figure 6: Kinetic (K) and potential energy (U) of an oscillator

Now, if the amplitude of the oscillation is equal to *a*, then we know that the sum of the kinetic and potential energy of the oscillator will be equal to $(1/2) \cdot m \cdot a^2 \cdot \omega^2$. In this case (the Bohr orbital), we have two oscillators, and we can add their kinetic and potential energies because of the 90-degree phase difference. Indeed, it is easy to see that the *total* kinetic energy – added over the *two* oscillators – will effectively be constant *over the cycle* and will be equal to:

$$\text{K. E.} = \frac{1}{2} m \cdot r_B{}^2 \cdot \omega^2 = \frac{1}{2} m \cdot v^2 = \frac{1}{2} \alpha^2 \cdot m \cdot c^2$$

The potential energy will be equal to the kinetic energy and we, therefore, get the desired result: the *total* energy of the loop is equal to $E = \alpha^2 mc^2$. We can now re-write the quantum of action as the product of the energy and the cycle time:

$$h = \text{E} \cdot \text{T} = \alpha^2 mc^2 \cdot \frac{2\pi \cdot r_B}{v} = \alpha^2 mc^2 \cdot \frac{2\pi \cdot r_C}{\alpha \cdot c \cdot \alpha} = mc^2 \frac{2\pi \cdot \hbar}{c \cdot m \cdot c} = h$$

Of course, we can also write it as the product of the (linear) momentum and the distance along the loop:

$$h = \text{p} \cdot \lambda_B = \text{m} \cdot v \cdot 2\pi \cdot r_B = \text{m} \cdot \alpha \cdot c \cdot 2\pi \cdot \frac{\hbar}{\alpha mc} = h$$

All makes sense. Now, we said we have a *dual* view of the meaning of the wavefunction here. What is the dual view? It is that of the force vector: we will want to write the energy as the product of a force over a distance. Hence, what is the force and what is the distance here? The Bohr model implies the circular motion of the *electron* is driven by (1) its inertia[24] and (2) a centripetal force (because of the

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[24] Note the difference with the *Zitterbewegung* model, which assumes a pointlike *charge* with *no inertia to motion*. Its orbital velocity is, therefore, effectively equal to the speed of light (*c*). This is *very* different from the Bohr

presence of a nucleus with the opposite charge). The geometry of the situation shows we can write **F** = **F**$_x$ + **F**$_y$ as:

$$\mathbf{F} = -F\cdot\cos(\omega t) - i\cdot F\cdot\sin(\omega t) = -F\cdot e^{i\theta}$$

The nature of this force is electric, of course. Hence, we should write in in terms of the electric field vector E[25]: **F** = q$_e$**E**. The electric field is, of course, the force on the unit charge which, in this case, is a force between q$_e$ (the electron) and −q$_e$ (the proton or hydrogen nucleus).[26] Let us calculate the magnitude of the force by using the fine-structure constant to check the consistency of the model:

$$F = \mathrm{q}_e E = \frac{\mathrm{q}_e{}^2}{4\pi\varepsilon_0 r_B{}^2} = \frac{\alpha\hbar c}{r_B{}^2} = \frac{\hbar v}{r_B{}^2} = \frac{\hbar r_B \omega_B}{r_B{}^2} = \frac{\hbar}{r_B}\frac{\alpha^2 mc^2}{\hbar} = \frac{\mathrm{E}}{r_B}$$

This $F = \mathrm{q}_e E = \mathrm{E}/r_B$ is confusing (E is the electric field, but E is the *energy*) but very interesting because it allows us to write the quantum of action in its usual dimensions – which is the product of a force, a distance (the radius of the oscillation, in this particular case), and a time:

$$h = F \cdot r \cdot \mathrm{T} = \frac{\mathrm{E}}{r_B}\cdot r_B \cdot \frac{1}{f} = \frac{\mathrm{E}}{r_B}\cdot r_B \cdot \frac{h}{\mathrm{E}} = h$$

Hence, we have a bunch of equivalent expressions for Planck's quantum of action – all of which help us to understand the *complementarity* of the various viewpoints:

$$h = \mathrm{p}\cdot 2\pi\cdot r = \mathrm{p}\cdot\lambda$$

$$h = \mathrm{E}\cdot\mathrm{T} = \mathrm{E}/f$$

$$h = r\cdot\mathrm{T}\cdot F = r\cdot\mathrm{T}\cdot\mathrm{q}_e E = r\cdot\mathrm{T}\cdot\mathrm{E}/r = \mathrm{E}\cdot\mathrm{T}$$

We could also combine these formulas with the classical formulas for a centripetal force – think of the F = m·r·ω² and F = m·v²/r = p·v/r formulas here – but we will let the reader play with that.

The point is: there is an *energy* in this oscillation, and the energy makes sense if we think of it as a two-dimensional oscillation. We can write this two-dimensional oscillation – using Euler's formula - in various but complementary ways. We can use the position vector, the force vector, or the electric field vector:

$$\mathbf{F} = -F_x\cdot\cos(\omega t) - i\cdot F_x\cdot\sin(\omega t) = -F\cdot e^{i\theta}$$

$$\mathbf{E} = -(\mathrm{E}/\mathrm{q}_e)\cdot\cos(\omega t) - i\cdot(\mathrm{E}/\mathrm{q}_e)\cdot\sin(\omega t) = -F\cdot e^{i\theta}$$

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model, in which the electron moves at a non-relativistic speed $v = \alpha c$ with $\alpha \approx 0.0073$. However, the two models are obviously complementary: the *Zitterbewegung* model – Dirac's electron, we might say – effectively explains the (rest) mass of the Bohr electron.

[25] Symbols may be confusing. We use E for the energy, but E for the electric field vector. Likewise, I is a moment of inertia, and I is an electric current. The context is usually clear enough to make out what is what.

[26] The concepts of potential, potential energy and the electric field can be quite confusing. The potential and the potential energy of a charge in a field vary with $1/r$. The electric field is the electric force – generally defined as the *Lorentz* force $\mathbf{F} = \mathrm{q}\mathbf{E} + \mathrm{q}(\mathbf{v}\times\mathbf{B})$ – on the unit charge. Hence, the $\mathbf{F} = \mathrm{q}\mathbf{E}$ formula here is nothing but the $\mathbf{E} = \mathbf{F}/\mathrm{q}$ formula. The electric field varies with $1/r^2$ and is, therefore, associated with the inverse-*square* law. It is also quite confusing that q$_e$ is actually the (supposedly negative) electron charge and that we have to, therefore, use a minus sign for the charge of the (supposedly positive) proton charge – but then the signs always work out, of course.

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot cos(\theta) + i \cdot a \cdot sin(\theta) = (x, y)$$

The various viewpoints of the oscillation are complementary. They pack the same energy ($E = \alpha^2 mc^2$), and they pack one unit of physical action (h). We will leave it to the reader to generalize for the n = 2, 3, etc. orbitals. It is an easy exercise: the energy for the higher loops is equal to $E_n = \alpha^2 mc^2/n^2$ and the associated action is equal to $S = n \cdot h$. One obvious way to relate both is through the frequency of the loop. We write:

$$f_n = \frac{E_n}{S_n} = \frac{\frac{1}{n^2}\alpha^2 mc^2}{nh} = \frac{\alpha^2}{n^3 h} mc^2$$

V.   The fine-structure constant as a scaling constant

The fine-structure constant pops up as a scaling constant in all of the calculations above. It relates the Bohr radius to the Compton radius, for example:

$$r_{\text{Bohr}} = \frac{\hbar^2}{me^2} = \frac{4\pi\varepsilon_0 \hbar^2}{mq_e{}^2} = \frac{1}{\alpha} \cdot r_{\text{Compton}} = \frac{\hbar}{\alpha mc} \approx 53 \times 10^{-12} \text{ m}$$

But it also relates the respective velocities, frequencies and energies of the two oscillations. We may summarize these relations in the following equations:

$$v = \alpha \cdot c = r_{\text{B}} \cdot \omega_{\text{B}} = \frac{\hbar}{\alpha mc} \cdot \frac{\alpha^2 mc^2}{\hbar} = \alpha \cdot c$$

But this is not the only meaning of the fine-structure constant. We know it pops up in many other formulas as well. To name just a few:

1. It is the mysterious quantum-mechanical *coupling* constant.
2. It explains the so-called anomalous magnetic moment – which, as we will explain in a moment, might *not* be anomalous at all!
3. Last but not least, it explains the *fine structure* of the hydrogen spectrum – which is where it got its name from, of course!

Can we make some more sense of this as a result of the interpretations we have offered above? Let us start with the coupling constant because there is a lot of nonsensical writing on that.[27] We basically showed that, as a coupling constant, the fine-structure continues to act as a dimensional scaling constant. We wrote:

$$\alpha = \frac{k \cdot q_e{}^2}{\hbar \cdot c} = \frac{F_{\text{B}} \cdot r_{\text{B}}{}^2}{F_\gamma \cdot r_\gamma \cdot r_\gamma} = \frac{F_{\text{B}} \cdot r_{\text{B}}{}^2}{F_\gamma \cdot r_\gamma{}^2} = \frac{E_{\text{B}} \cdot r_{\text{B}}}{E_\gamma \cdot r_\gamma}$$

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[27] Feynman's *QED: The Strange Theory of Light and Matter* (1985) refers to its (negative) *square root* as the coupling constant, and states that is "the amplitude for a real electron to emit or absorb a real photon." We take it to be just one example of an ambiguous remark by a famous physicist that is being explained by an amateur physicist. The book was not written by Richard Feynman: it is a transcription of a short series of lectures by Feynman for a popular audience. We are not impressed by the transcription.

We use B as a subscript in the denominator to remind ourselves we are talking the Bohr energies and radii. Let us use the generalized formulas (n = 1, 2,…) for the Bohr orbitals once again and write it all out:

$$\alpha = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma} = \frac{\frac{1}{n^2}\alpha^2 mc^2 \cdot \frac{n^2}{\alpha}\frac{\hbar}{mc}}{E_\gamma \cdot \frac{\hbar \cdot c}{E_\gamma}} = \alpha$$

While the formula is obvious, its interpretation is not necessarily as obvious: what *is* this product of an energy and a radius? How should we interpret this? The physical dimension of this product (in the denominator and the numerator, of course) is J·m = N·m·m = N·m². We get the same physical dimension if we multiply action or angular momentum with a velocity, so let us try this to check if it makes us any wiser:

$$\alpha = \frac{E_B \cdot r_B}{E_\gamma \cdot r_\gamma} = \frac{L_n \cdot v_n}{L_\gamma \cdot v_\gamma} = \frac{n\hbar \cdot \frac{1}{n}\alpha c}{\hbar \cdot c} = \alpha = \frac{S_n \cdot v_n}{S_\gamma \cdot v_\gamma} = \frac{nh \cdot \frac{1}{n}\alpha c}{h \cdot c} = \alpha$$

The formulas show we should, most probably, just think of them as yet another expression of the idea of a scaling constant.

Let us think of the fine-structure constant in yet one more way. We know the Compton and Bohr radius are related through the fine-structure constant. We used this formula many times already:

$$r_C = \alpha \cdot r_B$$

Let us write it out:

$$r_C = \frac{\hbar}{mc} = \frac{\hbar c}{mc^2} = \alpha \cdot \frac{\hbar c}{\alpha mc^2} = \alpha \cdot r_B$$

The numerator in these ratios is an energy. We write:

$$r_C = \frac{\hbar c}{m_e c^2} = \frac{\hbar c}{E_e}$$

$$r_B = \frac{\hbar c}{\alpha mc^2} = \frac{\hbar c}{E_B}$$

The E_e is just the (rest) energy of the electron, and E_B is the energy in the (first) Bohr orbital. Hence, we can also write the fine-structure constant as the ratio between these two energies:

$$\alpha = \frac{r_C}{r_B} = \frac{\frac{\hbar c}{E_e}}{\frac{\hbar c}{E_B}} = \frac{E_B}{E_e}$$

Because $r_n = n^2 r_B$ and $E_n = \frac{E_B}{n^2}$, we know that $r_n = \frac{n^2 \hbar c}{\alpha mc^2} = \frac{\hbar c}{E_n}$ and, hence, we can easily generalize for the n = 2, 3,… orbitals:

$$\alpha = \frac{r_C}{r_n} = \frac{\frac{\hbar c}{E_e}}{\frac{\hbar c}{E_n}} = \frac{E_n}{E_e}$$

The explorations above – and the interpretation of the fine-structure constant as a scaling constant – raise an interesting question. We know there is also the idea of a classical electron radius, which is related to the Compton radius in the same way as the Compton radius to the Bohr radius:

$$r_e = \alpha \cdot r_C = \alpha^2 \cdot r_B$$

We have already explained the second identity ($\alpha r_C = \alpha^2 r_B \Leftrightarrow r_C = \alpha r_B$) but what about $r_e = \alpha r_C$? Let us think about that in a separate section.

VI.   The fine-structure constant and the classical electron radius

Let us write it all out and see if there is something triggering some idea:

$$r_e = \frac{e^2}{mc^2} = \alpha \cdot \frac{\hbar c}{mc^2}$$

We, once again, have two energies in the numerator – but they are exactly the same! Hence, when writing the fine-structure constant as the ratio between the two radii, we get:

$$\alpha = \frac{r_e}{r_C} = \frac{\frac{e^2}{mc^2}}{\frac{\hbar c}{mc^2}} = \frac{e^2}{\hbar c} = k q_e^{\;2} = \frac{1}{4\pi\varepsilon_0} \frac{q_e^{\;2}}{\hbar c}$$

We just get the usual formula for the fine-structure constant here. What does it mean in terms of interpretation? Here we should probably think of the meaning of e^2. There is something interesting here: the elementary charge e^2 has the same physical dimension – the *joule-meter* (J·m) – as the h·c = E·λ product:

$$[e^2] = \left[\frac{1}{4\pi\varepsilon_0} q_e^{\;2}\right] = \frac{N \cdot m^2}{C^2} \cdot C^2 = N \cdot m^2 = J \cdot m$$

What was that h·c = E·λ product again? We got it in the context of our photon model. To be precise, we got it by applying the *second* de Broglie equation to a photon:

$$h = p \cdot \lambda = \frac{E}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{E}$$

In fact, it appears we may apply this relation to any particle that is traveling at the speed of light. *Huh? What other particle do we have here?* It is our pointlike *charge*: this charge has, effectively, no rest mass in the *Zitterbewegung* model and, therefore, does make us think of a photon. But we should be precise here: it is the *square* of the elementary charge that has that *joule-meter* dimension. We write:

$$[e^2] = [E]\cdot[\lambda] = [h]\cdot[c]$$

This is strange: what energy and what wavelength would we associate with this pointlike charge. I am not sure – but if we try the energy and circumference of the loop of the *Zitterbewegung*, we get a

sensible relation on the right-hand side:

$$\mathrm{E} \cdot \lambda = \mathrm{m}c^2 \cdot \frac{h}{\mathrm{m}c} = h \cdot c$$

Obvious, you'll say. But, *no*, this is not obvious: we are not talking the energy and mass of a photon here, but the energy and mass of… Well… Our pointlike charge? Yes: our pointlike charge in its *Zitter* motion, that is. But what about the suggestion that we should be able to write something like $e^2 = E \cdot \lambda$? Well… We can start by re-writing the formula for the classical electron radius so it gives a product of an energy and a distance:

$$\mathrm{e}^2 = r_\mathrm{e} \mathrm{m}c^2 = r_\mathrm{e} \mathrm{E}$$

Does this make sense? Yes. It gives us the formula for the fine-structure constant:

$$\mathrm{e}^2 = r_\mathrm{e} \mathrm{m}c^2 = \alpha r_\mathrm{C} \mathrm{E} = \alpha \frac{\hbar c}{\mathrm{m}c^2} \mathrm{E} = \alpha \frac{\hbar c}{\mathrm{m}c^2} \mathrm{m}c^2 = \alpha \hbar c \Leftrightarrow \alpha = \frac{\mathrm{e}^2}{\hbar c}$$

By now, the reader is probably tired of these gymnastics and, hence, we will stop here. What was the use? Interpretation. The formulas are not presenting anything new: we have just been substituting and re-arranging equations but we have, hopefully, succeeded in presenting a coherent picture while doing so.

VII.   The fine-structure constant and the anomalous magnetic moment

We mentioned other meanings of the fine-structure constant. It pops up in the explanation of the electron's anomalous magnetic moment and, naturally, in the context of where physicists noted it first: the fine structure of the hydrogen spectrum. So, what can we say about these? Let us first discuss the so-called anomalous magnetic moment. Such discussion is simple and complicated at the same time. We have, therefore, opted to write a separate paper on that.[28] Hence, we refer the interested reader to that paper and will just summarize the basic ideas here.

The reader will know that most of the anomalous magnetic moment – about 99.85% of the so-called anomaly, to be precise – is explained by Schwinger's $\alpha/2\pi$ factor: he got a Nobel Prize for that, and he has the factor engraved on his tomb. Hence, the suggestion that the anomalous magnetic moment might have a classical explanation may sound somewhat disrespectful. We ensure no disrespect is intended here. Having said that, in the mentioned paper, we argue we would expect to see an $\alpha/2\pi$ factor – or a similar factor – in a classical analysis *too*! Why is that so?

The *Zitterbewegung* model or – if the reader would prefer a much more sophisticated model, the Dirac-Kerr-Newman electron[29] – comes with a *form factor*: we should not think of the electron as a pointlike particle. In other words, we should *not* think of it as an infinitesimally small but perfect sphere. The models above show that we should think of the electron as a disk-like structure, and the fine-structure

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[28] See: Jean Louis Van Belle, *The Not-So-Anomalous Magnetic Moment*, 21 December 2018 (http://vixra.org/pdf/1812.0233v3.pdf).
[29] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008 (https://arxiv.org/abs/hep-th/0507109). For the (much simpler) *Zitterbewegung* model, see: Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, 12 December 2018 (http://vixra.org/pdf/1812.0202v1.pdf).

constant relates the radius of the disk (the Compton radius) to its thickness (the classical electron radius – aka Thomson or Lorentz radius). To be precise, Dirac-Kerr-Newman geometries which – as mentioned, are far more sophisticated than our simple *Zitterbewegung* model – yield a "highly oblate ellipsoid" (Burinskii, 2008), with a thickness that is effectively of the order of the classical electron radius, and a radius of the disk itself that is of the order of the Compton radius. The *ratio* between these two is, obviously, the fine-structure constant and the fine-structure constant is, therefore, a form factor that should pop up when calculating the various angular momenta that we can associate with the various layers in the motion.

We, therefore, claim that this form factor – and the equations of motion which, as shown below, are rather complicated – should explain why the measured gyromagnetic ratio of the system as a while is *not* exactly equal to 2. Indeed, the complicated motions in the Penning traps[30] that are used to measure the magnetic moment are illustrated below, and the reader will now understand why, in the mentioned paper, we argue that what is surprising is *not* that the measured g-factor is *not* equal to 2 but that it is *so nearly* equal to 2.
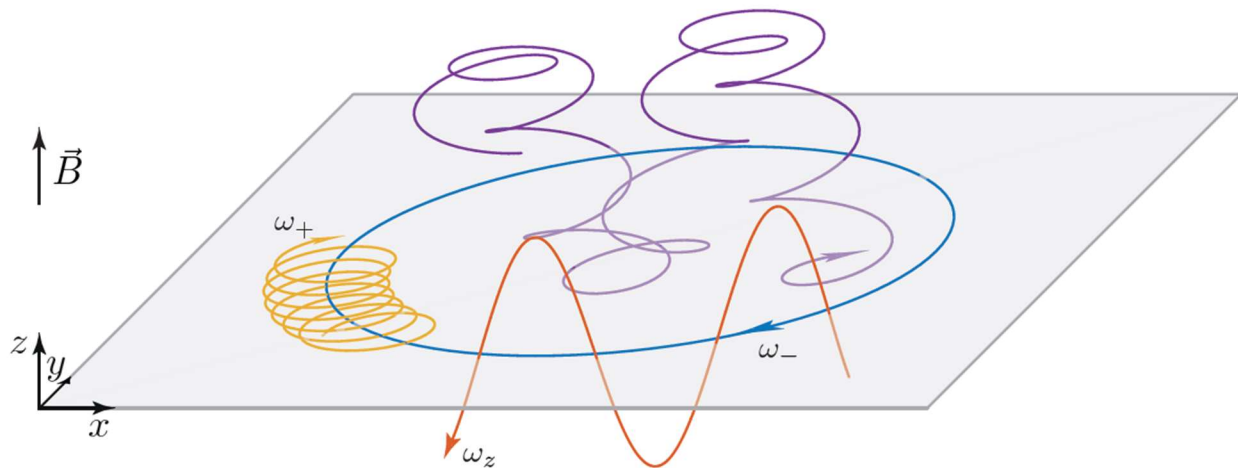

Figure 7: The three principal motions and frequencies in a Penning trap (**purple** is combined motion)

Hence, while our models give us a perfect number for the gyromagnetic ratio for the spin and orbital angular momentum (2 and 1 respectively), we would expect a rather classical coupling to take place when combining these motions and, hence, the form factor will have an impact on the system as a whole. The idea of a coupling between these motions is illustrated below. In a magnetic field, we will see a precession: the orbital plane is wobbling around the central axis. This is the axial oscillation. It is the ω_z oscillation in the illustration above. It is defined by (1) a frequency () but also by (2) a precession angle (θ). This precession angle is the angle between the original orbital momentum vector **L** now and the *actual* angular momentum vector **J**, as shown below.

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[30] These Penning traps combine a cyclotron and a magnetron. Indeed, to trap the electron, there is a magnetic *and* an electric field. We refer to the above-mentioned paper for a more detailed description of the geometry of the situation and, importantly, a more accurate description of what is actually being measured in those experiments.
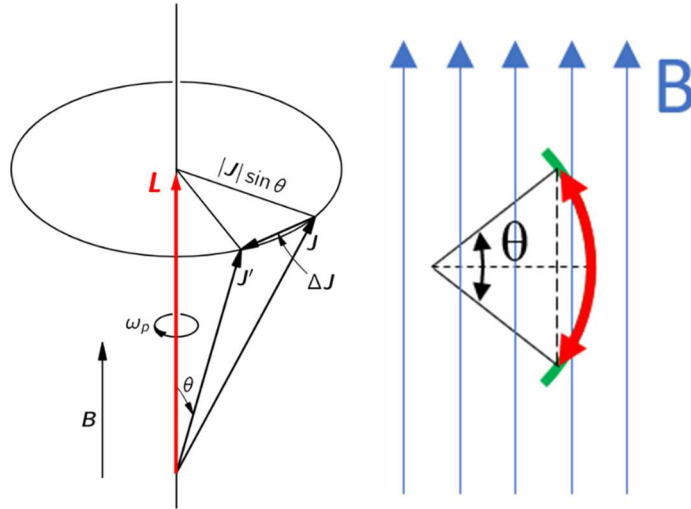
Figure 8: The classical coupling between the precession and the orbital angular momentum

To be precise, we can write the orbital angular momentum vector **L** as a vector dot product of the angular precession vector $\boldsymbol{\omega_p}$ and the new angular momentum vector **J**:

$$\mathbf{L} = \boldsymbol{\omega_p} \cdot \mathbf{J} = \left|\boldsymbol{\omega_p}\right| \cdot |\mathbf{J}| \cdot cos\theta = \omega_p \cdot \mathbf{J} \cdot cos\theta$$

The illustration on the right-hand side takes a sideway view of the rotation and shows that, as a result of the precession, we now have a smaller *effective* radius of the rotational motion. Hence, this reduces the electric current of the loop (I) and, therefore, it also reduces the magnetic moment. The reader can easily verify this for himself by checking the usual formulas here:

$$I = \frac{q}{T} = q \cdot f = q\frac{v}{2\pi \cdot r}$$

$$\mu = I \cdot \pi \cdot r^2 = q\frac{v}{2\pi \cdot r} \cdot \pi \cdot r^2 = \frac{q \cdot v \cdot r}{2}$$

Of course, the reader will note that the magnetic moment and the angular momentum have the same direction and that, therefore, their magnitudes may change but that this should not have any impact on the gyromagnetic ratio. However, the formula for the angular momentum is quite different: it does *not* depend on the current, but on the moment of inertia *I*.[31]

$$\mathbf{J} = I \cdot \boldsymbol{\omega}$$

To make a long story short: we should analyze the motions in the system, and we should analyze them in terms of vector dot and cross products. We *think*[32] the sine and cosine factors, combined with the form factor (the α-ratio between the Compton and Thomson radius), may yield Schwinger's α/2π factor.

What about the second- and higher-order corrections? As mentioned, while the velocities are non-

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[31] We apologize for the possible confusion: I is a current, *I* (italicized) is the moment of inertia of the system. Note that we should write the angular momentum as a *vector* (dot) product.

[32] We admit we have not come up with a proof, *yet*. The reason is quite simple: lack of time and lack of mathematical skills. We hope more advanced readers of this paper will be encouraged to go through the motions – literally – themselves. If this would yield any result, the author of this paper would appreciate feedback.

relativistic (*v* is a relatively *small* fraction of *c*, that is), we do think that relativistic corrections might go a long way in explaining the remaining differences. For example, if we would equate *v* with the typical Bohr velocity (*v* = α·*c*), we get a Lorentz factor that is equal to 1.000026627, and the table below shows this explains most of the remaining difference after applying Schwinger's α/2π factor.

Table 5: Example of a possible relativistic correction to Schwinger's α/2π factor

α	0.00729735256640
α/2π	0.00116140973243
a_e	0.00115965218073
First-order difference	-0.00000175755170
%	-0.152%
Lorentz factor (γ)	1.00002662674068
γα/2π	0.00116144065698
Remaining difference	-0.00000003092538
%	-0.003%

To sum it all up, we dare argue that the so-called anomaly in the anomalous magnetic moment may not be an anomaly: it might be a simple form factor. The basic idea here is to *not* think of the electron as a perfect sphere. In fact, the Dirac-Kerr-Newman models suggest we should probably also *not* think of it as a perfect disk!

VIII.   The fine-structure constant and the fine structure

We should now explain the final and last meaning of the fine-structure constant – the one that gave it its name! Why is that the fine-structure constant explains the fine structure of the hydrogen spectrum? However, as this paper has exceeded the usual length of a paper – and because this is actually a topic that is well covered in standard physics textbooks – we will, effectively, refer the reader to such physics textbooks – and conclude here.[33]

Jean Louis Van Belle, 23 December 2018

References

All references are given in the footnotes.

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[33] We would warmly recommend Feynman's *Lectures* here, because Feynman treats the topic in these lectures both from a classical as well as quantum-mechanical perspective (see the lectures in Volume II versus the lectures in Volume III here).