The Not-So Anomalous Magnetic Moment

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Abstract: This paper is a didactic exploration of the geometry of the experiments measuring the anomalous magnetic moment. It is argued that there may be nothing anomalous about it. We argue that Schwinger's $\alpha/2\pi$ factor and the other quantum-mechanical corrections might be explained by a form factor: the electron should, perhaps, *not* be thought of as a perfect sphere or a perfect disk. If this possibility is allowed for, the anomalous magnetic moment might possibly be explained in terms of a classical explanation.

Contents:

The Not-So-Anomalous Magnetic Moment

I. Classical Theory

Basic concepts and equations

The theory around the anomalous magnetic moment is straightforward but complicated at the same time. We should note, from the outset, that what is actually being measured in the experiments is a ratio of two *frequencies*. There is, therefore, no *direct* measurement of the electron's magnetic moment. So, what frequencies are being measured then? We will get into that in a moment. We will first *define* the anomalous magnetic moment because the confusion starts right there – with its definition, that is. The *Physics Today* article on the 2006 experiments[1] *defines* the electron's anomalous magnetic moment (denoted by a_e) as the (half-)difference between (1) a supposedly *real* gyromagnetic ratio (a *measured* g_e) and (2) Dirac's theoretical value for the gyromagnetic ratio of a spin-only electron ($g = 2$):

$$a_e = \frac{g_e - g}{2} = \frac{g_e - 2}{2} = \frac{g_e}{2} - 1$$

That looks like a non-starter to us: it is plain weird to use the (theoretical) g-factor for the intrinsic spin of an electron ($g = 2$) because the electron in the magnetron (a Penning trap) is *not* a spin-only electron. It follows an orbital motion – in fact, there is a superposition of motions here, as we will explain shortly – and, hence, *if* some theoretical value for the g-factor has to be used here, then it should probably the g-factor that is associated with the orbital motion of an electron, which is that of the Bohr orbitals (g = 1).

The attentive reader may wonder why we *italicized* the *if* in the phrase above. The answer is that we do so because we think the concept of the gyromagnetic ratio may not be all that useful: in our humble view, the introduction of separate g-factors does not clarify but *obscure* the classical coupling between orbital and spin angular momentum. However, we are not there yet. Let us stay clear from the more popular accounts of the actual experiments and explore the journal articles themselves.[2]

The original experiment was done by a group of researchers from the Physics Department from Harvard University and we will use their 2009 article because that is freely available online.[3] While this article does provide some theory as well, it does *not* invoke the theoretical g-factor for a spin-only electron. It simply writes a_e as:

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[1] See: *Physics Today*, 1 August 2006, p. 15 (https://physicstoday.scitation.org/doi/10.1063/1.2349714). *Physics Today* is the flagship publication of the American Institute for Physics. While one would not expect top-notch journal articles in *Physics Today* (the aim is more to *promote* such articles), one would also *not* expect any misrepresentation of research. However, that seems to be the case here.

[2] We will continue to refer to the above-mentioned article a couple of times for the sake of correcting its mistakes.

[3] See: D. Hanneke, S. Fogwell, N. Guise, J. Dorr and G. Gabrielse, *More Accurate Measurement of the Electron Magnetic Moment and the Fine-Structure Constant*, in: Proceedings of the XXI International Conference on Atomic Physics. As mentioned, we prefer this article over the 2006 article (G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *New Determination of the Fine Structure Constant from the Electron g Value and QED*, Phys. Rev. Lett. 97, 030802, 2006) because it can be freely consulted online: http://gabrielse.physics.harvard.edu/gabrielse/papers/2009/PushingTheFrontiersOfAtomicPhysics.pdf.

$$a_e = \frac{g}{2} - 1$$

The 2009 article states that the *measured* value of $g/2$ is equal to 1.00115965218073(28). The 28 (between brackets) is the (un)certainty: it is equal to 1.00000000000028, i.e. 28 *parts per trillion* (ppt) and it is measured as a standard deviation. In 2006, its measured value was 1.00115965218085(76), so that's an improvement in the accuracy of a factor of about 73/28 ≈ 2.7. More recent experiments have come up with even more precise numbers. However, let us repeat that these experiments do *not* directly measure a_e. What is being measured in the *Penning traps* that are used in these experiments are frequencies. What frequencies? According to the mentioned *Physics Today* article, only two frequencies matter: a cyclotron frequency – which is defined as the frequency of the electron orbitals in the cyclotron (a Penning trap[4]) – and a precession frequency – which is said to be caused by the electron's intrinsic spin. According to the same article, the anomalous magnetic moment a_e is then defined as the fractional difference between the two:

$$a_e = \frac{f_s}{f_c} - 1$$

The two frequencies can be easily calculated – we will do so in a moment[5] - but we should first note that the *Physics Today* formula is a simplification which may or may not make sense. The motion in these Penning traps is very complicated and one needs to think about three motions and, therefore, three frequencies – for starters, that is. The three frequencies are illustrated below.[6]

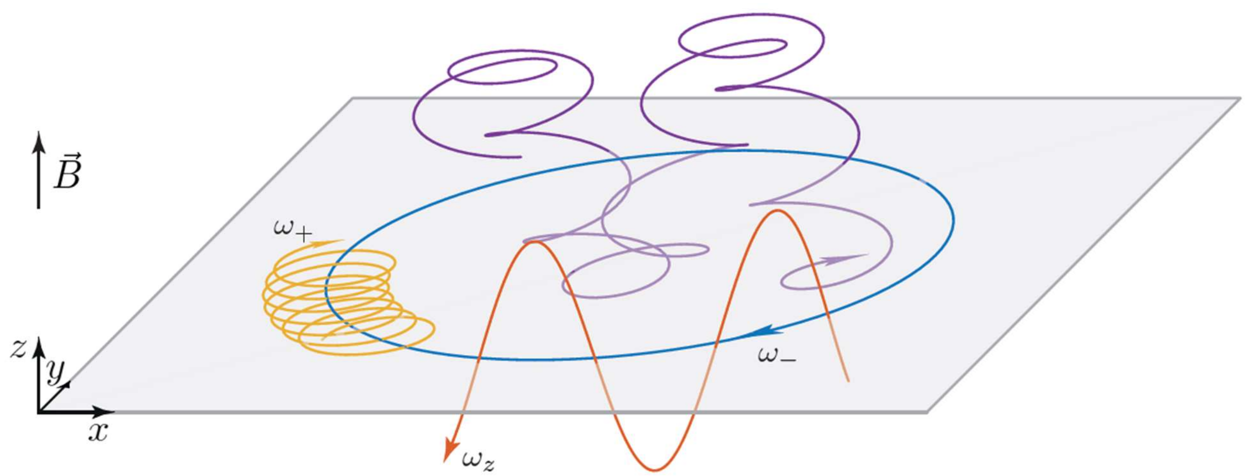

Figure 1: The three principal motions and frequencies in a Penning trap

[4] The terms cyclotron and magnetron are, de facto, used interchangeably. This is a bit regrettable, but it is what it is. In any case, the Penning trap combines features of both: there is a magnetic field *and* an electric field.

[5] An easy-to-follow derivation for the cyclotron frequency is: https://www.didaktik.physik.uni-muenchen.de/elektronenbahnen/en/b-feld/anwendung/zyklotron2.php. As for the precession frequency, we refer to Feynman's *Lectures* (II-34-3, *The Precession of Atomic Magnets*): http://www.feynmanlectures.caltech.edu/II_34.html#Ch34-S3.

[6] We took this illustration from an excellent article on the complexities of a Penning trap: *Cylotron frequency in a Penning trap*, Blaum Group, 28 September 2015 (https://www.physi.uni-heidelberg.de/Einrichtungen/FP/anleitungen/F47.pdf).

Let us go through the motions – literally! We first have the cyclotron frequency, which is written as ω_- here and which – confusingly – should actually be referred to as the *magnetron* frequency: in the context of a Penning trap, one will, effectively refer to the ω_+ motion as the *cyclotron* frequency, while the circular motion (associated with the ω_- frequency) is the magnetron frequency. The reason is simple: the regular circular motion ω_- is caused by the magnetic field (**B**) – and the magnetic field *only.* In contrast, the ω_+ is caused by an additional *electric* field (**E**). We will come back to this. As we are now used to the subscript *c*, we should probably think of *c* for *circular* then.

It is the orbital motion – the blue loop – and the related frequency is easily calculated from a simple analysis of the Lorentz force, which is just the magnetic force here: $\mathbf{F} = \mathrm{q}(\boldsymbol{v} \times \mathbf{B})$. This force equals the *centripetal* force here. We can therefore write:

$$\mathrm{q} \cdot v \cdot \mathrm{B} = \mathrm{F}_c = m \cdot \mathrm{F}_c = \frac{mv^2}{r}$$

The v^2/r factor is the centripetal acceleration, and the mass factor is just the equivalent mass of the rest energy of the electron. Hence, the $F = m \cdot v^2/r$ does effectively represent Newton's force law. The equation above yields the following formula for *v* and the *v*/*r* ratio:

$$v = \frac{\mathrm{q} \cdot r \cdot \mathrm{B}}{\mathrm{m}} \Rightarrow \frac{v}{r} = \frac{\mathrm{q} \cdot \mathrm{B}}{\mathrm{m}}$$

Note that the velocity will usually be non-relativistic ($v \ll c$). One should, therefore, not be confused by the subscript: the *c* in f_c stands for *c*ircular, *c*entripetal, or – preferably *not* for the reasons mentioned above – *c*yclotron. Hence, *c* stands for everything but the speed of light ! We can now derive the orbital frequency f_c from the following equation:

$$v = \omega \cdot r = 2\pi \cdot f_c \cdot r \Leftrightarrow f_c = \frac{v}{2\pi \cdot r}$$

Re-arranging and substituting *v* for q·*r*·b/m yields the formula we expected to find:

$$f_c = \frac{\mathrm{q} \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}$$

Note that the frequency does *not* depend on the velocity or the radius of the circular motion. This is actually the whole idea of the trap: the electron can be inserted into the trap with a precise kinetic energy and will follow a circular trajectory if the magnetic field can be kept constant. Let us now calculate the associated magnetic moment and the angular momentum. To calculate the magnetic moment, we can calculate the associated *current*, which is equal to:

$$\mathrm{I} = \mathrm{q} \cdot f = \mathrm{q}\frac{v}{2\pi \cdot r} = \frac{\mathrm{q}^2 \cdot \mathrm{B}}{2\pi \cdot \mathrm{m}}$$

The magnetic moment (μ) is equal to the current times (I) times the area of the loop (πr^2). We get:

$$\mu = \mathrm{I} \cdot \pi \cdot r^2 = \mathrm{q}\frac{v}{2\pi \cdot r} \cdot \pi \cdot r^2 = \frac{\mathrm{q} \cdot v \cdot r}{2}$$

For the angular momentum (L), we need to calculate the moment of inertia (I).[7] For regular shapes, we always have an easy formula for the moment of inertia and, hence, for the angular momentum: it is just mr^2 times a ***form factor***. The form factor for a disk, for example, is 1/2. We write: I = (1/2)·mr^2. However, here we just have a rotating point mass. Hence, the form factor is 1 and we can simply write:

$$\mathrm{L} = I \cdot \omega = \mathrm{m}r^2 \cdot \frac{v}{r} = \mathrm{m} \cdot r \cdot v$$

Hence, we can write the g-factor for the orbital motion as:

$$g_c = \frac{2\mathrm{m}}{\mathrm{q}}\frac{\mu}{\mathrm{L}} = \frac{2\mathrm{m}}{\mathrm{q}} \cdot \frac{\mathrm{q} \cdot v \cdot r}{2\mathrm{m} \cdot r \cdot v} = 1$$

It is what we would expect it to be: it is the gyromagnetic ratio for the *orbital* angular momentum of the electron. It is *one*, not 2.

We should now note a very common mistake. Because the g-factor is equal to 1 for the *orbital*, it is tempting to write the following:

$$\frac{2\mathrm{m}}{\mathrm{q}}\frac{\mu}{\mathrm{J}} = 1 \Leftrightarrow \frac{\mu}{\mathrm{J}} = \frac{\mathrm{q}}{2\mathrm{m}}$$

However, this is an identity which should *not* be used in subsequent calculations when analyzing the other motions. One first needs to examine what the impact of the two other oscillatory motions will be on the current, the magnetic moment and the angular momentum. In other word, *it would, effectively, be a logical mistake to conclude – from the analysis above only, which is the analysis of just one layer in the motion of our electron – that the g-factor for the whole system should be equal to 1.*

Such mistake is, in effect, of the same order as the mistake made in the mentioned *Physics Today* article, where the author – for some obscure reason (probably lack of familiarity with the topic) – assumes the g-factor for the whole system should, somehow, be equal to 2.

The form factor

We mentioned what we refer to as a *form factor* in the context of the angular momentum because it explains the g = 2 result for the intrinsic spin moment of an electron in the *Zitterbewegung* model or – if the reader prefers a more sophisticated model – the Dirac-Kerr-Newman electron.[8] The formulas below illustrate the profound difference between the concept of intrinsic spin and the concept of orbital angular momentum.

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[7] J is the symbol which Feynman uses. In many articles and textbooks, one will read L instead of J. Note that the symbols may be confusing: I is a current, but I is the moment of inertia. It is equal to m·r^2 for a rotating mass.

[8] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008 (https://arxiv.org/abs/hep-th/0507109). For the (much simpler) *Zitterbewegung* model, see: Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, 12 December 2018 (http://vixra.org/pdf/1812.0202v1.pdf).

Table 1: Intrinsic spin versus orbital angular momentum

Spin-only electron (*Zitterbewegung*)	Orbital electron (Bohr orbitals)
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = -\frac{1}{2}\frac{\alpha^2}{n^2}\mathrm{m}c^2 = -\frac{1}{n^2}\mathrm{E}_R$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\omega = \frac{v}{r} = c \cdot \frac{\mathrm{m}c}{\hbar} = \frac{\mathrm{E}}{\hbar}$	$\omega_n = \frac{v_n}{r_n} = \frac{\alpha^2}{n^3\hbar}\mathrm{m}c^2 = \frac{\frac{1}{n^2}\alpha^2\mathrm{m}c^2}{n\hbar}$
$\mathrm{L} = I \cdot \omega = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega_n = n\hbar$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

Note that the formulas in the right column are the formulas for the properties of the Bohr orbitals. These *resemble* the cyclotron orbitals – to some extent – but one should not confuse them: the cyclotron orbitals have no nucleus at their center. In fact, the oft-quoted description of these magnetron orbitals – or of the Penning trap itself – as an artificial atom[9] is quite confusing and, therefore, not very useful: the radius and kinetic energy of the electron in a magnetron is of an entirely different order of magnitude! As such, we should – perhaps – not have mentioned the Bohr orbitals. We did so only to point to the popular misconception that, somehow, these electrons in Penning traps would resemble an electron in an atom. They do *not*. There are *similarities*, but these may actually be misleading – as opposed to helpful – in our understanding of what's going on.

We have been quite verbose here. Let us, therefore, move on and have a look at the other frequencies now. Before we do so, we should point out that the formulas above do show us the natural *unit* for measuring a magnetic moment. Indeed, one recognizes the Bohr magneton in the formulas, which is defined as:

$$\mu_\mathrm{B} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \approx 9.274 \cdot 10^{-24}\,\frac{\mathrm{J}}{\mathrm{T}}$$

The J/T unit is joule (J) per *tesla* (T). Needless to say, the tesla is the SI unit for the magnitude of a magnetic field. We can also write it as [**B**] = N/(m·A), using the SI unit for current, i.e. the ampere (A). Now, 1 C = 1 A·s and, hence, 1 N/(m·A) = 1 (N/C)/(m/s). Hence, the physical dimension of the magnetic field is the physical dimension of the electric field (N/C) divided by m/s. We like the [**E**] = [**B**]·m/s

[9] See, for example, this story: https://physics.aps.org/story/v4/st11, which variously describes the Penning trap as a 'quantum cyclotron' and 'an example of an artificial atom.' We think such language is exaggerated or, worse, misleading and, therefore, inappropriate.

expression because it reflects the *geometry* of the electric and magnetic field vectors. *Onwards!*

Precession

We will first deal with the precession, because that is easiest to explain. The f_s frequency in the *Physics Today* article is the *Larmor* or precession frequency. It is a classical thing: if we think of the electron as a tiny magnet with a magnetic moment that is proportional to its angular momentum, then it should, effectively, *precess* in a magnetic field. In the illustration above, it is written as the angular frequency ω_z. That notation makes a lot of sense because it is, effectively, an oscillation along the *z*-direction (assuming the *xy*-plane is the plane of the orbital rotation). It is, therefore, also referred to as an *axial frequency*.

However, we will argue it is not *exactly* along the *z*-direction: there is a classical *coupling* between the two motions. Let us analyze it. The geometry of the situation is shown below.

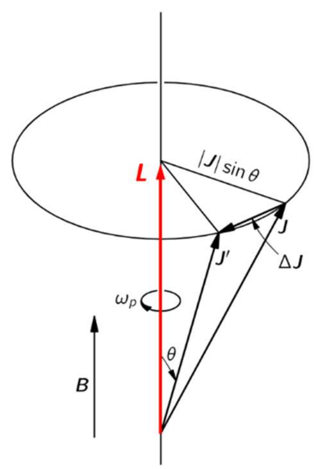

Figure 2: The precession of an orbital electron

Let us go through the derivation of the formula for the precession frequency[10]. The angular momentum change from **J** to **J'** in some small time interval Δt. The **boldfaced J** to **J'** makes it clear these are *vector* quantities: their magnitude is the same, but their direction is *not*. In fact, that is the whole point of the analysis. The angle with the *z*-direction – which is the direction of the magnetic field ***B*** – is equal to the angle of precession, which we write as θ. Now, we wrote **J** to **J'** as vectors, but the ΔJ in the illustration is an actual *distance* – not a vector. The geometry of the situation shows us that we can write it as:

$$\Delta J=(J\cdot\sin\theta)\cdot(\omega_p\cdot\Delta t)$$

Hence, the *rate of change* of the angular momentum is equal to:

$$\frac{dJ}{dt} = \omega_p \cdot J \cdot sin\theta$$

This must equal the torque[11]:

[10] This derivation is taken from Feynman's *Lectures* (II-34-3), from which we also copied the illustration – to which we added the angular momentum vector ***L***. We like it because it is precise but intuitive at the same time.

[11] We are sorry to have to bother the reader with the basics of circular motion, but it is necessary.

$$\tau = \mu \cdot B \cdot sin\theta$$

We, therefore, can write the following:

$$\omega_p \cdot J = \frac{dJ}{dt} \cdot \frac{1}{sin\theta} = \frac{\tau}{sin\theta} = \frac{\mu \cdot B \cdot sin\theta}{sin\theta} = \mu \cdot B \Leftrightarrow \omega_p = \frac{\mu}{J} \cdot B$$

What g-factor would we associate with this? It is tempting to think it is just the same one and – technically speaking – it is. We're just measuring in another plane. Indeed, the plane of rotation is wobbling around now but we can still define a *g*-factor for it. The velocity *v* and radius *r* remain the same, right? They do. We can, effectively, write something like this:

$$g_p = \frac{2m}{q}\frac{\mu}{J} = \frac{2m}{q} \cdot \frac{q \cdot v \cdot r}{2m \cdot r \cdot v} = 1$$

However, this does not make much sense, because we have a *new* angular momentum **J** and **μ** here. Their *magnitude* is the same as that of the **L** and **μ** vectors we were looking at before – when we were considering orbital motion only, but their direction has changed. To be precise, we can write the orbital angular momentum vector **L** as a vector dot product of the angular precession vector $\boldsymbol{\omega_p}$ and the new angular momentum vector **J**:

$$\mathbf{L} = \boldsymbol{\omega_p} \cdot \mathbf{J} = |\boldsymbol{\omega_p}| \cdot |\mathbf{J}| \cdot sin\theta = \omega_p \cdot J \cdot sin\theta$$

This equation gives us the *coupling* between the orbital and precessional motion. The precession effectively causes the electron to wobble: its plane of rotation – and, hence, the axis of the angular momentum (and the magnetic moment) – is no longer fixed. This wobbling motion changes the orbital plane constantly and we can, therefore, we can no longer trust the values we have used in our formulas for the angular momentum and the magnetic moment. At first, we may think there will no impact on the current. Taking a radial view, the motion along the *z*-direction is going to look like this:

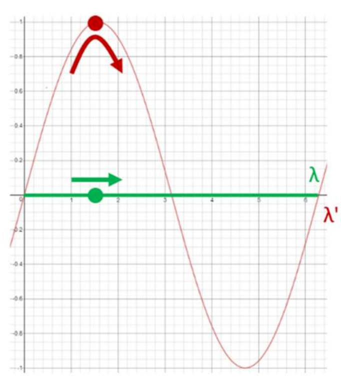

Figure 3: A radial view of the precessional rotation

Hence, we may think there will be no change in the *amount* of current that is going around, because electric charge is relativistically invariant: the up-and-down motion comes with a higher (linear) velocity, but that does not cause any change in the amount of charge over the distance, which we wrote as λ = *r* in the illustration above. However, it is easy to see that the up-and-down motion will follow an arc-like trajectory – as illustrated below – which is going to reduce the *effective radius* of the orbital loop. As such, the effective current will also diminish. That is only logical because the energy – and the physical action – in the loop remains the same. It is now just distributed over *two* superposed motions, rather than just one.

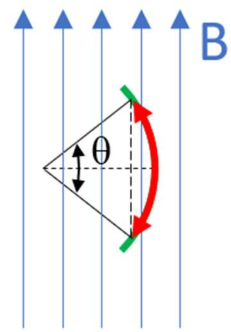

Figure 4: A *sideway* view of the precessional rotation

Now, the formulas above give us the angle that is to be associated with this arc: it is just the angle of the precession: we just turned it by 90 degrees. Hence, we may be tempted to just substitute the new values into the equations for the orbital magnetic moment (and the orbital angular momentum) and consider the problem solved but, *no!* We would, once again, make another logical mistake: we need to look at the system as a whole and, therefore, we need to solve a *set of equations*. At this point, we would like to quickly clarify why we wrote – somewhat disrespectfully, perhaps – that the introduction of separate *g*-factors does not clarify but *obscure* the classical coupling between orbital and spin angular momentum.

Indeed, as we have pointed above, it is a mistake to equate the orbital angular momentum vector **L** with the combined angular momentum vector **J**. By now, the reader of this paper will say: yes, of course! However, this is what is, effectively, being done when calculating those ratios of g-factors. Let us have another look at this logical error that is routinely made in the more popular presentations of the experiment – and, more in particular, in the mentioned *Physics Today* article. The author associates a gyromagnetic ratio with the precessional motion. Logically, it should be equal to 1. We then get the following formula:

$$a_e = \frac{g_p - g_c}{g_c} = \frac{g_p - 1}{1} = \frac{1 - 1}{1} = 0$$

Note that he or she made not one but *two* mistakes. First, it was totally random to equate g_c with 2. Second, it makes no sense whatsoever to calculate a *numerical* ratio of these two quantities: the underlying *vector* quantities don't have the same direction. The equation, therefore, becomes meaningless. In fact, if the equation would make sense – which it doesn't – we should not wonder why the anomalous magnetic moment is not equal to one, but why it's not equal to *zero*!

Let us move to third layer of motion in the diagram above.

The cyclotron frequency

A Penning trap combines features of a magnetron and a cyclotron. There is a magnetic field – which explains the magnetron frequency (mistakenly referred to as the *cyclotron* frequency in the mentioned *Physics Today* article) – and there is an electric field, which explains what we're going to explain now: the cyclotron frequency. It is the ω_+ oscillation: a circular motion within the orbital motion, so to speak.

As mentioned, it is there because of the *electric* field (**E**) which – together with the magnetic field – keeps the electron effectively trapped in the… Well… The Penning trap. 😊

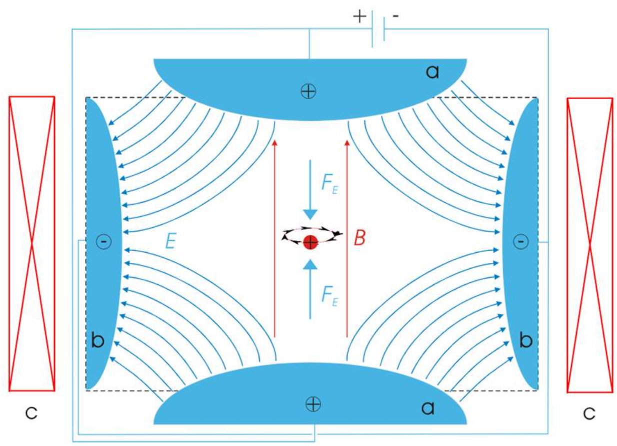

Figure 5: The electric and magnetic fields in a Penning trap[12]

We will not say too much about this, except that the velocities involved are (also) non-relativistic. All in all, the motion resembles Ptolemean physics – circular motion within circular motion, as illustrated below.[13]

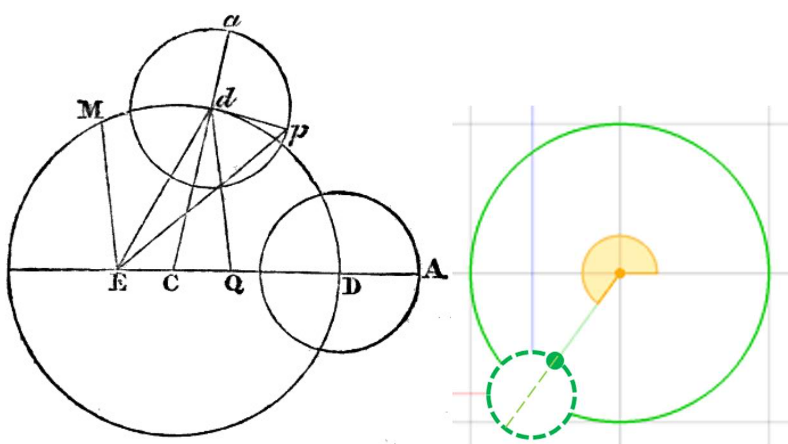

Figure 6: Ptolemean loops

The large circular motion is referred to as the deferent, while the smaller circular motion is known as the epicycle. If we denote the (angular) frequency of the deferent and epicyclical motion as ω_- and ω_+ respectively, then the illustrations below show what we get from combining the ω_- and ω_+ motions.

[12] The diagram is from Wikipedia (https://upload.wikimedia.org/wikipedia/commons/b/b6/Penning_Trap.svg).
[13] The first illustration is taken from the *Project Gutenberg* e-publications: *The Life of Galileo*, written by John Elliot Drinkwater Bethune (https://www.gutenberg.org/files/43877/43877-h/43877-h.htm). It is supposed to represent the motion of planets in the Ptolemean system. The second is the author's simpler rendition of the pretty much the same thing – but in the context of presumed electron orbitals.

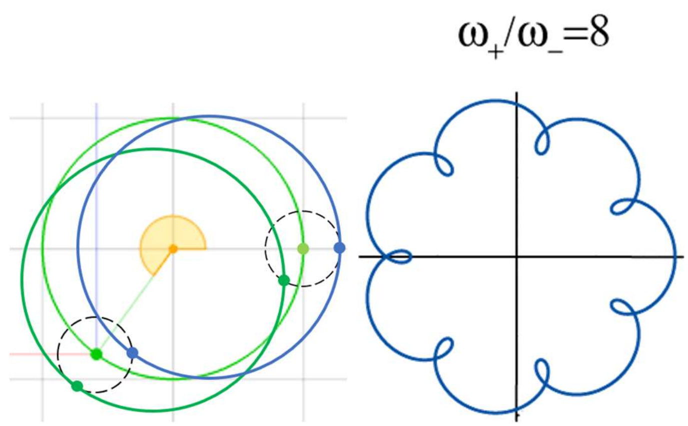

Figure 7: Ptolemean physics?

The combined motion is referred to as an *epitrochoid*, and the illustration on the left shows the motion for $\omega_+ = 0$. Note that, if we are rotating a disk which can freely rotate itself, its inertia will *not* cause any rotation of the disk itself. Any point on that disk will, therefore, just cover the same distance as any point on the deferent. Hence, the epitrochoid will just describe the same circle as the deferent, but its center will move about. This leads to something interesting. Indeed, carefully look at the illustration on the right-hand side: the ratio of the two frequencies is equal to 8, but we do *not* have 8 loops within the larger loop. There are only 7.

OK. This is all very interesting, but the question is: what is the impact on the (electric) current and, therefore, on the magnetic moment? We repeat the formulas here:

$$\mathrm{I} = \frac{\mathrm{q}}{\mathrm{T}} = \mathrm{q} \cdot f = \mathrm{q}\frac{v}{2\pi \cdot r}$$

$$\mu = \mathrm{I} \cdot \pi \cdot r^2 = \mathrm{q}\frac{v}{2\pi \cdot r} \cdot \pi \cdot r^2 = \frac{\mathrm{q} \cdot v \cdot r}{2}$$

We are not very well versed in the math of epitrochoids but, intuitively, it would seem the superposition of the two motions would *not* change anything in regard to the current: the velocity (*v*) and the distance (*r*) will constantly change, of course, but the charge that goes round and round is the same and, hence, there will be some *effective* velocity and radius that will give us the same current we get from simple orbital motion.[14] Hence, Ptolemean physics are probably *not* going to help to explain the anomalous frequency. However, they do need to be considered when calculating what is being calculated in these experiments. Indeed, it is now time to answer the burning question: what is actually being measured in those experiments?

What is being measured, *exactly*?

Let us give you the formula from the mentioned 2009 article of the Harvard group:

[14] We may also note that charge – unlike mass or energy – is relativistically invariant.

$$\frac{g}{2} \simeq 1 + \frac{\bar{\nu}_a - \bar{\nu}_z^2/(2\bar{f}_c)}{\bar{f}_c + 3\delta/2 + \bar{\nu}_z^2/(2\bar{f}_c)} + \frac{\Delta g_{cav}}{2}$$

Figure 8: The formula for the *measured* anomalous magnetic moment

Does this make your headache any worse? If not, it should. We will refer to the mentioned article[15] for the methodology. However, you can make sense of this equation by noting we do have three frequencies here, plus the so-called *anomalous* frequency, which is… Well… The supposed anomaly. Think of the other factors as relativistic and other corrections – which is what they are, really.

The point is: when you see this formula – and considering the complexities as explained above – one really starts to wonder why the anomalous magnetic moment is, effectively, so *nearly* one. Indeed, what we find is that all these complicated motions, taken altogether, give us a (theoretical) gyromagnetic ratio that is very nearly equal to two.

Yes, *two*. Not one. Because a_e = g/2. Hence, we write, somewhat disrespectfully:

$$g_e \approx 2.00232$$

We should, of course, give you the 14 digits but… Well… You can find them, right?

[15] D. Hanneke, S. Fogwell, N. Guise, J. Dorr and G. Gabrielse, *More Accurate Measurement of the Electron Magnetic Moment and the Fine-Structure Constant*, in: Proceedings of the XXI International Conference on Atomic Physics. We prefer this article over the 2006 article (G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. Odom, *New Determination of the Fine Structure Constant from the Electron g Value and QED*, Phys. Rev. Lett. 97, 030802, 2006) because it can be freely consulted online: http://gabrielse.physics.harvard.edu/gabrielse/papers/2009/PushingTheFrontiersOfAtomicPhysics.pdf.

II. The quantum-mechanical explanation

Quantum physicists explain the anomaly in the magnetic moment as a series of first-, second-, third-, n^{th}-order *corrections*, which are written as follows:

$$a_e = \sum_n a_n \left(\frac{\alpha}{\pi}\right)^n$$

The first coefficient (a_1) is equal to 1/2 and the associated first-order correction is, therefore, equal to:

$$\alpha/2\pi \approx 0.00116141$$

Using "his renormalized QED theory", Julian Schwinger had already obtained this value back in 1947. He got it from calculating the "one loop electron vertex function in an external magnetic field." I am just quoting here from a well-informed article (Todorov, 2018[16]). Indeed, Todorov's article is an article that beautifully describes the *math* behind this "tennis match between experiment and theory" – as Brian Hayes referred to it.[17]

Julian Schwinger is, of course, one of the most prominent representatives of the second generation of quantum physicists, and he has this number on this tombstone. Hence, we surely do not want to question the depth of his understanding of this phenomenon. However, the difference that needs to be explained by the 2nd, 3rd, etc. corrections is only 0.15%, and Todorov's work shows all of these corrections can be written in terms of a sort of exponential series of α/2π and a *phi*-function ϕ(*n*) which had intrigued Euler for all of his life. We copy the formula for the (the sum of) the first-, second- and third-order term of the theoretical value of a_e as calculated in 1995-1996 (*th* : 1996).[18]

$$\begin{aligned} a_e(th:1996) &= \frac{1}{2}\frac{\alpha}{\pi} + \left[\phi(3) - 6\,\phi(1)\,\phi(2) + \phi(2) + \frac{197}{2^4\,3^2}\right]\left(\frac{\alpha}{\pi}\right)^2 \\ &+ \left[\frac{2}{3^2}\left(83\,\phi(2)\,\phi(3) - 43\,\phi(5)\right) - \frac{50}{3}\,\phi(1,3) + \frac{13}{5}\,\phi(2)^2\right. \\ &+ \left.\frac{278}{3}\left(\frac{\phi(3)}{3^2} - 12\,\phi(1)\,\phi(2)\right) + \frac{34202}{3^3\,5}\,\phi(2) + \frac{28259}{2^5\,3^4}\right]\left(\frac{\alpha}{\pi}\right)^3 + \dots \\ &= 1.159652201(27)\times 10^{-3} \end{aligned} \tag{19}$$

We also quote Todorov's succinct summary of how this result was obtained: "Toichiro Kinoshita of Cornell University evaluated the 72 [third-order loop Feynman] diagrams numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. A year later, the

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[16] See: Ivan Todorov, *From Euler's play with infinite series to the anomalous magnetic moment*, 12 October 2018 (https://arxiv.org/pdf/1804.09553.pdf).

[17] See: Brian Hayes, *Computing Science: g-ology*, in: *American Scientist*, Vol. 92, No. 3, May-June 2004, pages 212-216. The subtitle says it all: it is an article 'on the long campaign to refine measurements and theoretical calculations of a physical constant called the *g* factor of the electron.' (https://pdfs.semanticscholar.org/4c12/50f66fc1fb799610d58f25b9c1e1c2d9854c.pdf).

[18] It is worth quoting Todorov's succinct summary of how this result was obtained: Toichiro Kinoshita of Cornell University evaluated the 72 [Feynman] diagrams [corresponding to the third-order loop] numerically, comparing and combining his results with analytic values that were then known for 67 of the diagrams. later the last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna.

last few diagrams were calculated analytically by Stefano Laporta and Ettore Remiddi of the University of Bologna."

Apparently, the calculations are even more detailed now: the mentioned Laporta claims to have calculated 891 *four*-loop contributions to the anomalous magnetic moment.[19]

Hence, what is going on here? One gets an uncanny feeling here: if one has to calculate a zillion integrals all over space using 72 third-order diagrams to calculate the 12^{th} digit in the anomalous magnetic moment, or 891 fourth-order diagrams to get the next level of precision, then there might something wrong with the theory. Is there an alternative? We think there is, and the idea is surprisingly simple. We explore a possible classical solution to the problem in the next and final section of our paper.

[19] See: Stefano Laporta, *High-precision calculation of the 4-loop contribution to the electron g-2 in QED*, Stefano (https://www.sciencedirect.com/science/article/pii/S0370269317305324).

III. The elements for a classical explanation

In light of what we wrote above, it is obvious that our suggestion that there might be some rather simple classical explanation for the anomalous magnetic moment is quite disrespectful. However, that is what we are going to do: we are going to think of a very simple classical explanation: the *form factor* in the moment of inertia.

As mentioned, for regular shapes, we always have an easy formula for the moment of inertia: it is just mr^2 times a ***form factor***. The idea is the following. The form factor for a disk is 1/2: $I = (1/2)\cdot mr^2$. It is this form factor which explains the $g = 2$ result for the intrinsic spin moment of an electron in the *Zitterbewegung* model or – if the reader prefers a more sophisticated model – the Dirac-Kerr-Newman electron.[20] Now, we have detailed the model elsewhere and, hence, we will not go into too much detail here.[21] It is an interpretation of an electron which goes back to Schrödinger and Dirac[22], and which combines the idea of motion with the idea of a pointlike *charge*, which has no inertia and can, therefore, move at the speed of light. The most spectacular result of the model is the explanation for the *rest* mass of an electron: it is the equivalent mass of what we referred to as the *rest matter oscillation*. The model also gives the right formulas for all the measured properties of a free electron, such as angular momentum, magnetic moment, g-factor, etcetera:

Table 2: The properties of the free electron (spin-only)

Spin-only electron (*Zitterbewegung*)
$\mathrm{S} = \mathrm{h}$
$\mathrm{E} = \mathrm{m}c^2$
$r = r_{\mathrm{C}} = \dfrac{\hbar}{\mathrm{m}c}$
$v = c$
$\omega = \dfrac{v}{r} = c \cdot \dfrac{\mathrm{m}c}{\hbar} = \dfrac{\mathrm{E}}{\hbar}$

[20] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008 (https://arxiv.org/abs/hep-th/0507109). For the (much simpler) *Zitterbewegung* model, see: Jean Louis Van Belle, *A geometric interpretation of Schrödinger's equation*, 12 December 2018 (http://vixra.org/pdf/1812.0202v1.pdf).

[21] See, for example, Jean Louis Van Belle, *Einstein's Mass-Energy Equivalence Relation: an Explanation in Terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/abs/1811.0364).

[22] Erwin Schrödinger derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

$\mathrm{L} = I \cdot \omega = \frac{\hbar}{2}$
$\mu = \mathrm{I} \cdot \pi r_\mathrm{C}{}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$

The reader should keep his wits about him[23] here: the *Zitterbewegung* model should not be confused with the model for the Bohr orbitals. We do not have any centripetal force here. There is no nucleus or other charge at the center of the *Zitterbewegung*. Instead of a tangential momentum vector, we have a tangential *force* vector (**F**), which we thought of as being the resultant force of two perpendicular oscillations.[24] This led us to boldly equate the $E = mc^2$, $E = m \cdot a^2 \cdot \omega^2$ and $E = \hbar \cdot \omega$ equations – which gave us all the results we wanted. The *zbw* model – which, as we have mentioned in the footnote above, is inspired by the solution(s) for Dirac's wave equation for free electrons – tells us the velocity of the pointlike *charge* is equal to *c*. Hence, *if* the *zbw* frequency would be given by Planck's energy-frequency relation ($\omega = E/\hbar$), *then* we can easily combine Einstein's $E = mc^2$ formula with the radial velocity formula ($c = a \cdot \omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$r_{\mathrm{Compton}} = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_\mathrm{e}}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

By now, the reader will probably wonder: what is the point here? What is the relation with the anomalous magnetic moment? The point is that the calculations also relate the Bohr radius to the Compton radius through the fine-structure constant:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{mq_e}^2} = \frac{1}{\alpha} \cdot r_{\mathrm{Compton}} = \frac{\hbar}{\alpha \mathrm{m}c} \approx 53 \times 10^{-12}\ \mathrm{m}$$

The same fine-structure constant also relates the respective velocities, frequencies and energies of the Bohr and Compton oscillations. Indeed, one easily show the following:

$$v = \alpha \cdot c = r_\mathrm{B} \cdot \omega_\mathrm{B} = \frac{\hbar}{\alpha \mathrm{m}c} \cdot \frac{\alpha^2 \mathrm{m}c^2}{\hbar} = \alpha \cdot c \Leftrightarrow \omega_\mathrm{B} = \frac{\alpha^2 \mathrm{m}c^2}{\hbar}$$

The fact that the fine-structure constant pops up naturally here – as a dimensional constant, so to speak – makes us feel that this might explain two things in one movement:

1. Why the measured g-factor is so close to *two* – i.e. the g-factor that is related to the *spin* angular momentum – as opposed to one, which is the g-factor that is related to the *orbital* angular momentum.
2. Why the difference between the measured g-factor and 2 is equal to Schwinger's $\alpha/2\pi$ factor.

[23] The him could be a her, of course.

[24] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

The latter idea is related to the fact that we also have a classical electron radius, which is equal to[25]:

$$r_{\text{Thomson}} = r_{\text{e}} = \frac{\text{e}^2}{\text{m}c^2} = \alpha \frac{\hbar c}{\text{m}c^2} \approx 2.818 \times 10^{-15} \text{ m}$$

Hence, we have a trio of radii here, all related by the same constant (α):

$$r_{\text{e}} = \alpha \cdot r_{\text{C}} = \alpha^2 \cdot r_{\text{B}}$$

If the fine-structure constant acts as a dimensional constant here, would it be any surprise it pops up as a *form factor* when calculating angular momenta? For us, this is an obvious intuition. Having said that, an intuition is, obviously, something else than a full-blown proof, of course. Of course, the second- and third-order corrections will also need explain. However, relativistic corrections may go a long here.

Indeed, the Lorentz factor for v = α·*c* is equal to 1.000026627, and the table below shows we can also *very* rapidly explain a second-order difference when combining this factor with Schwinger's α/2π factor.

Table 3: Successive corrections using α/2π and the Lorentz factor

α	0.00729735256640
α/2π	0.00116140973243
a_e	0.00115965218073
First-order difference	-0.00000175755170
%	-0.152%
Lorentz factor (γ)	1.00002662674068
γα/2π	0.00116144065698
Remaining difference	-0.00000003092538
%	-0.003%

Hence, the suggestion is that the so-called anomaly in the anomalous magnetic moment is just a simple form factor. In other words, the idea here is to *not* think of the electron as a perfect sphere or a perfect disk.

Jean Louis Van Belle, 21 December 2018

References

All references are given in the footnotes.

[25] The e^2 is the squared electron charge but expressed in its natural unit: itself. Expressed in SI units, it is written as $k \cdot q_e^2 = q_e^2/4\pi\varepsilon_0$.