A geometric interpretation of Schrödinger's wave equation

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Abstract: Following a series of papers on a geometric interpretation of the wavefunction, this paper offers a geometric interpretation of the wave *equation* itself. It interprets Schrödinger's equation as a differential equation for elliptical orbitals. As such, it complements a revised Rutherford-Bohr model which is also based on the assumption that – if electron orbitals would be actual orbitals – they would be elliptical rather than circular.

Keywords: Bohr model, Schrödinger's equation, rest matter oscillation, electron orbitals, wavefunction interpretations.

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A geometric interpretation of Schrödinger's wave equation

The electron model

In previous papers[1], we combined a *Zitterbewegung* (*zbw*) model of an electron with the classical Rutherford-Bohr model of electron orbitals to explain the *combined* atomic magnetic moment as measured in a Stern-Gerlach experiment. Hence, we looked at a superposition of *motions*. It is worth quoting Dirac's summary of Schrödinger's discovery of the presumed *Zitterbewegung* of an electron in this regard:

> "The variables [in Dirac's wave equation] give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."[2]

The formulas that come out of the two models are summarized below.

Table 1: The formulas for the *zbw* electron and the Bohr orbitals

Spin-only electron (*Zitterbewegung*)	**Orbital electron (Bohr atom)**
$\mathrm{S} = \mathrm{h}$	$\mathrm{S}_n = n\mathrm{h}$ for $n = 1, 2, \ldots$
$\mathrm{E} = \mathrm{m}c^2$	$\mathrm{E}_n = \frac{\alpha^2}{n^2}\mathrm{m}c^2$
$r = r_\mathrm{C} = \frac{\hbar}{\mathrm{m}c}$	$r_n = n^2 r_\mathrm{B} = \frac{n^2 r_\mathrm{C}}{\alpha} = \frac{n^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$
$v = c$	$v_n = \frac{1}{n}\alpha c$
$\mathrm{L} = I \cdot \omega = \frac{\hbar}{2}$	$\mathrm{L}_n = I \cdot \omega = n\hbar$
$\mu = \mathrm{I} \cdot \pi {r_\mathrm{C}}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$	$\mu_n = \mathrm{I} \cdot \pi {r_n}^2 = \frac{\mathrm{q_e}}{2\mathrm{m}}n\hbar$
$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 2$	$\mathrm{g}_n = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = 1$

[1] Jean Louis Van Belle, *Bohr's Atom, the Photon, and the [Un]Certainty Principle*, 3 December 2018 (http://vixra.org/pdf/1812.0028v1.pdf).

[2] See: Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf.

The velocity is a tangential velocity, and α is the fine-structure constant. The mass m is the electron *rest* mass $m_0 = E_0/c^2$. Hence, we could write the *zbw* action, energy, radius, etc. as S_0, E_0, r_0 etc. Let us recap the basics of the two models to make sure there is no confusion.

1. The Zitterbewegung

The *Zitterbewegung* concept of an electron combines the idea of a very high-frequency circulatory motion with the idea of a pointlike charge which – importantly – has no inertia and can, therefore, move at the speed of light ($v = c$). The center of the *Zitterbewegung* is plain nothingness and we assume some two-dimensional oscillation makes the pointlike (and massless) electric *charge* go round and round, as shown below.

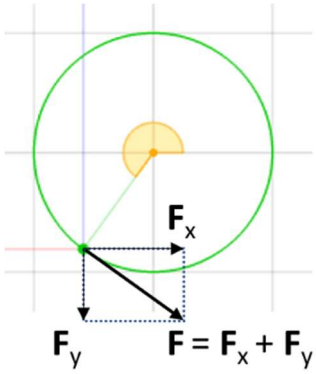

Figure 1: The *Zitterbewegung* model of a (stationary) free electron

The *x*- and *y*-component of the tangential force are given by the same periodic function but with a phase difference of 90 degrees: $\cos(\theta) = \sin(\theta + \pi/2)$. We think of the force as an electric force, of course.[3] If the position vector is given by $\boldsymbol{r} = r_C \cdot \cos(\theta) + i \cdot r_C \cdot \sin(\theta)$, then the force will be equal to:

$$\mathbf{F} = q_e \cdot \boldsymbol{E}_0 = q_e \cdot E_0 \cdot \cos(\omega t + \pi/2) + i \cdot q_e \cdot E_0 \cdot \sin(\omega t + \pi/2)$$

The angular frequency of the *Zitterbewegung* rotation is given by the Planck-Einstein relation ($\omega = E/\hbar = mc^2/\hbar$) and we get the *Zitterbewegung* radius ($r_{Compton} = \hbar/mc$) by equating the $E = mc^2$ and $E = ma^2\omega^2$ equation. We note the correspondence between the angular momentum ($\hbar/2$) and the (physical) *action* that is packed in one cycle. We write it as the product of (1) the force ($F = mc^2/\lambda_C$), (2) the distance traveled ($\lambda_C = 2\pi \cdot r_C$) and (3) the cycle time ($T = 2\pi \cdot r_C/c = h/mc^2 = h/E = 1/f$)[4]:

$$S = F \cdot \lambda_C \cdot T = \frac{mc^2}{\lambda_C} \lambda_C \frac{h}{E} = h$$

The energy – and, therefore, the (equivalent) mass of the electron – is in the oscillation and we, therefore, should also associate the momentum $p = E/c = mc^2/c$ with the oscillation as a whole[5] or, if we

[3] The pointlike charge has no (rest) mass. Hence, the force can only grab onto the charge and must, therefore, be electric.

[4] For the detail of the calculations and, more importantly, the *rationale* of these formulas, see: Jean Louis Van Belle, *The Metaphysics of Physics*, 30 November 2018 (http://vixra.org/pdf/1811.0399v3.pdf).

[5] As mentioned above, the idea of the electron here combines the idea of a very high-frequency circulatory motion with the idea of a pointlike *charge* which has no inertia (and, thus, no rest mass) and which can, therefore, move at

would have to associate it with a single point in space, with the center of the oscillation (as opposed to the changing position of the rotating massless *charge*). This explains the 1/2 factor in the angular momentum (L), which is due to this *form factor*, which allows us to use the $I = m \cdot r^2/2$ formula for the moment of inertia.[6]

Needless to say, mankind will never be able to *prove* this model because of the extreme frequency (f_e = 1/T = E/h ≈ 0.123×10^{-21} Hz) and the sub-atomic scale ($r_C = \hbar/mc \approx 386\times10^{-15}$ m). It is, therefore, a *logical* model only: it gives us the right values for the angular momentum ($L = \hbar/2$), the magnetic moment ($\mu = (q_e/2m)\cdot\hbar$ and the gyromagnetic factor (g = 2). More importantly, it explains the *rest* mass of a free (stationary) electron in terms of the *Zitterbewegung*, which we can now refer to as the *rest matter oscillation*.

We should, of course, note an important *logical* issue with this model: the charge should go off on a tangent. We will not dwell on this – as we did so in previous papers.[7] We can only note two possible explanations. One involves gravity. Indeed, we can calculate the force as follows:

$$F = q_e E_0 = \frac{\mathrm{E}}{\lambda_\mathrm{e}} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-1}\ \ \mathrm{m}} \approx 3.3743 \times 10^{-2}\mathrm{N}$$

This force is equivalent to a force that gives a mass of about 37.5 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second. This is a *huge* force in light of the tiny distance scale. We should, therefore, think through the implications in terms of the distortion of spacetime caused by the presence of such energy in such tiny volume. Suggestions that the Schwarzschild radius of an electron may be equal to its Compton radius, such as those stemming from Kerr-Newman models of the electron[8], are very encouraging in this regard.

Secondly, we should not forget about the magnetic field, which is perpendicular to the electric field. The *magnitude* of the magnetic field is E/*c*, but we should multiply it with the velocity of the charge to get the Lorentz force, which is *c* in this case. Hence, the magnitude of the electric and the magnetic force are the same here, and the assumption of two perpendicular oscillations with the same magnitude makes, therefore, a lot of sense.[9]

2. The Bohr orbitals

The electron in the Bohr orbitals does *not* revolve around emptiness: the orbitals have the positively charged nucleus at their center, and its electron has an effective rest mass. The tangential velocity $v_n = r_n \cdot \omega$ of the electron is, therefore, only a *fraction* of the speed of light ($v_n = (\alpha/n)\cdot c$). We can, therefore,

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the speed of light. As such, we cannot say anything reasonable about the momentum of the (massless) charge itself: one should associate the p = E/*c* momentum with the (two-dimensional) *oscillation* of the charge.

[6] The moment of inertia (aka angular mass or rotational inertia) $I = mr^2/2$ is needed to calculate the angular momentum. It is to be distinguished from the current $I = q_e/T$, which is needed to calculate the magnetic moment.

[7] See the reference above (http://vixra.org/pdf/1811.0399v3.pdf).

[8] See: Alexander Buriinski, *The Dirac-Kerr-Newman Electron*, 19 March 2008, https://arxiv.org/pdf/hep-th/0507109.pdf. Also see the above-mentioned paper for a discussion (http://vixra.org/pdf/1811.0399v3.pdf).

[9] There is, of course, the issue of the 90-degree phase difference, but this may be explained by the fact that the influence must travel at the speed of light. However, these intuitions need more reflection and detailing, which is outside of the scope of this paper.

also associate some non-zero momentum $p_n = m \cdot v_n$ with the electron, which we can relate to the electric force using the classical $F_n = p_n \cdot \omega_n = m v_n^2 / r_n$ formula.

The Rutherford-Bohr model assumes circular orbits (illustrated below) that are associated with an angular momentum equal to ħ, 2ħ, 3ħ, etcetera. Needless to say, the *n* is the orbital number (1, 2,…) and we assume the Bohr atom has one electron only.[10]

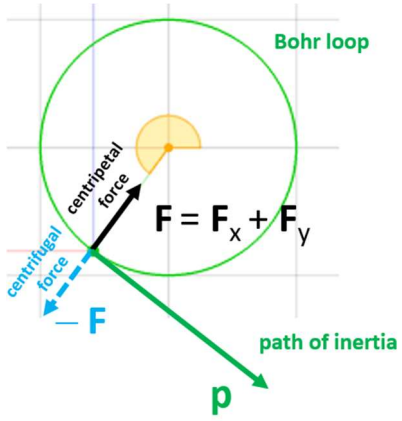

Figure 2: The position, force and momentum vector in a Bohr loop

The force is different: it is an electric force too, of course, but it is centripetal. To facilitate the discussion and the calculations, we will only consider the *first* Bohr orbital (*n* = 1) because the analysis can easily be generalized for *n* = 2, 3,…. Hence, we can calculate the force as the electrostatic force between the charge and the nucleus:

$$\mathrm{F} = \frac{{q_e}^2}{4\pi\varepsilon_0 {r_B}^2} = \alpha \cdot \frac{\hbar c}{{r_B}^2}$$

We can calculate the physical action over a cycle as:

$$\mathrm{S} = \mathrm{F} \cdot r_B \cdot \mathrm{T} = \alpha \cdot \frac{\hbar c}{{r_B}^2} \cdot r_B \cdot \frac{2\pi r_B}{v} = \alpha \cdot \frac{\mathrm{h} c}{\alpha c} = \mathrm{h}$$

All is consistent. However, we should note the implied energy concept is somewhat surprising:

$$\mathrm{S} = \mathrm{h} = \mathrm{E} \cdot \mathrm{T} = \mathrm{E} \cdot \frac{2\pi r_B}{v} = \mathrm{E} \cdot \frac{\frac{h}{\alpha \mathrm{m} c}}{\alpha c} \Leftrightarrow \mathrm{E} = \alpha^2 \mathrm{m} c^2$$

Surprising but not inconsistent. It is consistent with the calculated velocity and frequency:

$$v = r_B \cdot \omega = \frac{r_C}{\alpha} \frac{\mathrm{E}}{\hbar} = \frac{\hbar}{\alpha \mathrm{m} c} \frac{\alpha^2 \mathrm{m} c^2}{\hbar} = \alpha c$$

But how can we *explain* this $\mathrm{E} = \alpha^2 \mathrm{m} c^2$ value? It is, once again, the model of a two-dimensional oscillator that helps us out here. At this point, we should think about the energy concepts (kinetic and potential)

[10] We will want to establish a mathematical equivalence between an augmented Rutherford-Bohr model and (the solutions to) Schrödinger's wave equation, which describes a hydrogen atom. Hence, Schrödinger's model also describes an atom with one electron only.

that are associated with a harmonic oscillator. We know the energy in such oscillator is constant. To be precise, the *sum* of the kinetic and potential energy will be constant. To be very precise, we know that, *over one cycle*, the kinetic energy (K) will go from 0 to its maximum and then back to zero, while the potential energy (U) will go from its maximum value to zero, and the back to its maximum value, as shown below.

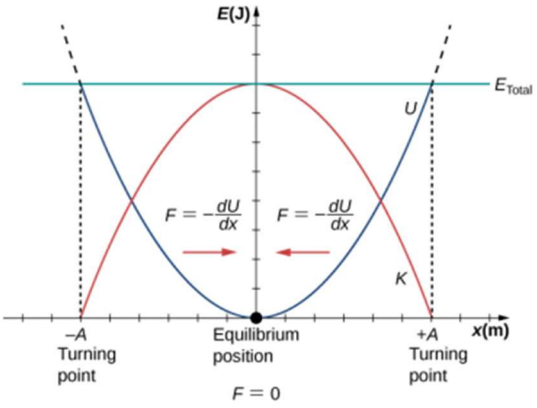

Figure 3: Kinetic (K) and potential energy (U) of an oscillator

If the amplitude of the oscillation is equal to *a*, then we know that the sum of the kinetic and potential energy of the oscillator will be equal to $(1/2)\cdot m\cdot a^2\cdot\omega^2$. In this case (the Bohr orbital), we have two oscillators, and we can add their kinetic and potential energies because of the 90-degree phase difference. The *total* kinetic energy – added over the *two* oscillators – will effectively be constant *over the cycle* and will be equal to:

$$K = \frac{1}{2} m \cdot r_B{}^2 \cdot \omega^2 = \frac{1}{2} m \cdot v^2 = \frac{1}{2} \alpha^2 \cdot m \cdot c^2$$

The potential energy will match this and we get the desired result: the *total* energy is equal to $E = \alpha^2 mc^2$. In one of the next sections, we will try to interpret Schrödinger's equation as a *vector* equation in three-dimensional space. To prepare the reader for that analysis, we should probably make some notes and clarify some relations here.

It is useful to note that we interpret the imaginary unit as an *operator*: when combined with a vector, it will rotate that vector – counterclockwise – over 90 degrees. The *plane* of rotation is established by the two-dimensional oscillation itself. The $\cos(\theta) = \sin(\theta + \pi/2)$ identity is now written as:

$$i\cdot\cos(\theta) = i\cdot\sin(\theta + \pi/2)$$

It is a significant formula: the introduction of the imaginary unit in an equation gives numbers a *vector* quality: it gives them a direction – albeit a direction that is defined in terms of each other. The identify above can be verified by squaring both sides:

$$i^2\cdot\cos^2(\theta) = i^2\cdot\sin^2(\theta + \pi/2) \Leftrightarrow -\cos^2(\theta) = -\sin^2(\theta + \pi/2) = -\cos^2(\theta)$$

It is easy to see that the position vector $\boldsymbol{r} = \boldsymbol{r}_B$ can be represented as an elementary wavefunction:

$$\boldsymbol{r} = r\cdot e^{i\theta}$$

It is equally easy to see that the *direction* of the momentum and velocity vectors (**p** and ***v***) is perpendicular to the direction of the position vector – in the *clockwise* direction, as illustrated below.

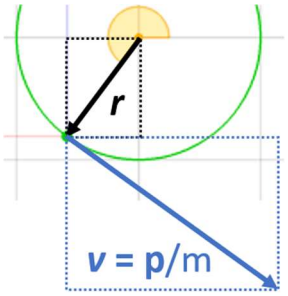

Figure 4: Kinetic (K) and potential energy (U) of an oscillator

This allows us to write the velocity vector as:

$$\boldsymbol{v} = i{\cdot}v{\cdot}e^{i\theta} = i{\cdot}v{\cdot}\cos(\theta) + i{\cdot}v{\cdot}i{\cdot}\sin(\theta) = v{\cdot}\cos(\theta + \pi/2) + i{\cdot}v{\cdot}\sin(\theta + \pi/2) = v{\cdot}e^{i(\theta+\pi/2)} = r_B{\cdot}\omega{\cdot}e^{i(\theta+\pi/2)} = \alpha{\cdot}c{\cdot}e^{i(\theta+\pi/2)}$$

Now, to show the formulas are consistent but not obvious, we can define ***v*** as the sum of an *x*- and a *y*-component. Hence, the following formulas should make sense[11]:

$$\boldsymbol{v} = \boldsymbol{v}_x + \boldsymbol{v}_y = v_x + i{\cdot}v_y$$

$$\mathrm{K} = \mathrm{K_x} + \mathrm{K_y} = \frac{1}{2}\mathrm{m}\cdot {v_x}^2 + \frac{1}{2}\mathrm{m}\cdot {v_y}^2 = \frac{1}{2}\mathrm{m}\cdot\left({v_x}^2 + {v_y}^2\right) = \frac{1}{2}\mathrm{m}v^2 = \frac{1}{2}\mathrm{m}\alpha^2c^2$$

We are now ready to look at Schrödinger's so-called wave equation again.

Schrödinger's equation and elliptical orbitals

We do not want to refer to Schrödinger's equation as a wave equation. We think it is just a differential equation for orbitals. We think the elementary wavefunction represents a position vector. To be precise, we think it represents the *exact* position of the center of the electron's rest matter oscillation. Hence, we think the elementary wavefunction $\psi = r{\cdot}e^{i\theta}$ represents the *true* position of the *zbw* electron. Hence, we write this true position of the *zbw* electron (with rest mass E ≈ 0.511 MeV/c^2) as:

$$\boldsymbol{r} = \psi = r{\cdot}e^{i\theta} = r{\cdot}\cos(\theta) + i{\cdot}r{\cdot}\sin(\theta)$$

However, we should not assume that the circular orbitals are the true orbitals: we should allow for elliptical orbitals. In fact, because of the pure spin moment of our *zbw* electron, we think the circular orbital is the exception: it cannot be the rule. Here we need to re-examine the argument of the wavefunction:

$$\theta = \frac{\mathrm{E}\cdot t - \mathbf{p}\cdot\boldsymbol{x}}{\hbar} = 2\pi\cdot\frac{\mathrm{E}\cdot t - \mathbf{p}\cdot\boldsymbol{x}}{\mathrm{h}}$$

[11] There is a very subtle thing in notation here: we could write $\boldsymbol{v}_x$ or $\boldsymbol{v}_x$. Should we *boldface* the subscript or not? We think we should: velocity has a direction which is defined with respect to the direction of the *x*- (or *y*-) axis. We therefore prefer to *boldface* the subscript too.

As mentioned above, we should think of h as the amount of (physical) *action* that is packed in one cycle. If the orbital is circular, then the **p**·**x** dot product vanishes because of the right angle between the two vectors: **p**·**x** = p·x·cos(ϕ) = p·x·cos(π/2) = p·x·0 = 0, and the argument of the wavefunction reduces to:

$$\theta = \omega \cdot t = \frac{\mathrm{E} \cdot t}{\hbar} = 2\pi \cdot \frac{\mathrm{E} \cdot t}{\mathrm{h}} = 2\pi \cdot \frac{t}{\mathrm{T}} = 2\pi \cdot \frac{t}{\mathrm{T}}$$

This equation makes it clear that we should think of measuring the time in units of T, and that each cycle corresponds to 2π radians. The cycle time T is equal to

$$\mathrm{T} = \frac{2\pi r_{\mathrm{B}}}{v} = \frac{2\pi r_{\mathrm{C}}}{\alpha v} = \frac{2\pi\hbar}{\alpha v \mathrm{m}c} = \frac{\mathrm{h}}{\alpha^2 \mathrm{m}c^2} = \frac{\mathrm{h}}{\mathrm{E}} = \frac{1}{f}$$

However, we should now think of elliptical orbits. Hence, it is just like moving from the Copernicus model (circular orbits) to Kepler's model (elliptical orbits): the radius *r* and velocity *v* are no longer constant, and Kepler's laws should apply. For example, the radius line between the nucleus and the electron should sweep out equal areas during equal intervals of time, as shown below.

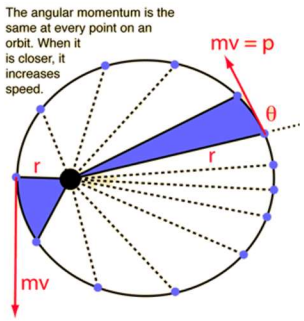

Figure 5: Kepler's second law

This *augmented* Rutherford-Bohr model – which allows for elliptical orbitals – should enable us to interpret Schrödinger's wave equation as a differential equation for the orbitals. Indeed, if we think of ψ = ***r*** = $r \cdot e^{i\theta}$ as a position vector, then we should not think of it as a wave equation. Schrödinger's equation is given by:

$$i\hbar \frac{\partial \psi}{\partial \mathrm{t}} = -\frac{\hbar^2}{2\mathrm{m}} \nabla^2 \psi + \mathrm{V}\psi$$

Replacing ψ by ***r*** = $r \cdot e^{i\theta} = r \cdot e^{i \cdot (\mathrm{E}/\hbar) \cdot \mathrm{t} - (\mathbf{p}/\hbar) \cdot \boldsymbol{r})}$ makes it look somewhat less mysterious:

$$i\hbar \frac{\partial \boldsymbol{r}}{\partial \mathrm{t}} = -\frac{\hbar^2}{2\mathrm{m}} \nabla^2 \boldsymbol{r} + \mathrm{V}\boldsymbol{r}$$

Let us carefully analyze each term of the equation. Let us start with the left-hand side. The derivative of the position vector with respect to time is the velocity vector ***v***. Hence, the left-hand side can be written as *i*·ħ·***v***. We should probably think of *i*·ħ as a vector now and, hence, rewrite it as the following vector *cross*-product[12]:

[12] We could have used boldface for ħ but we prefer the traditional arrow for a vector here so as to emphasize what we are thinking of. We should also note that *i* is *not* a rotation in the plane of oscillation here: it is a rotation in a

$$i \cdot \hbar = \vec{\hbar} = \boldsymbol{r} \times \mathrm{m} \cdot \boldsymbol{v}$$

The magnitude of this vector is given by:

$$\left|\vec{\hbar}\right| = \hbar = r \cdot \mathrm{m} \cdot v \cdot \sin(\phi)$$

Hence, the left-hand side of Schrödinger's equation may be written as:

$$i \cdot \hbar \cdot \boldsymbol{v} = \vec{\hbar} \cdot \boldsymbol{v} = \boldsymbol{r} \times \mathrm{m} \cdot \boldsymbol{v} \cdot \boldsymbol{v} = \mathrm{m}v^2\boldsymbol{r} = (\alpha^2 \mathrm{m}v^2)\boldsymbol{r} = \mathrm{E} \cdot \boldsymbol{r}$$

Schrödinger's equation can now be rewritten as:

$$i\hbar \frac{\partial \boldsymbol{r}}{\partial \mathrm{t}} = \mathrm{E} \cdot \boldsymbol{r} = -\frac{\hbar^2}{2\mathrm{m}} \nabla^2 \boldsymbol{r} + \mathrm{V} \cdot \boldsymbol{r}$$

We now need to interpret the right-hand side. We would like to show the following:

$$\mathrm{E} \cdot \boldsymbol{r} = (\mathrm{K} + \mathrm{U}) \cdot \boldsymbol{r} = \mathrm{K} \cdot \boldsymbol{r} + \mathrm{U} \cdot \boldsymbol{r} = -\frac{\hbar^2}{2\mathrm{m}} \nabla^2 \boldsymbol{r} + \mathrm{V} \cdot \boldsymbol{r}$$

We know that U is equal to V = V(*r*), because V is effectively defined as the potential energy of the electron in the electrostatic field of the proton.[13] Hence, all that is left to prove now is:

$$-\frac{\hbar^2}{2\mathrm{m}} \nabla^2 \boldsymbol{r} = \mathrm{K} \cdot \boldsymbol{r} = \frac{\mathrm{p}^2}{2\mathrm{m}} \boldsymbol{r}$$

This is not so easy as it seems. ∇^2 is the *vector Laplace operator* acting on the position vector ***r***. If we write ***r*** as ***r*** = (*x*, *y*, *z*), then we get the following expression:

$$\nabla^2 \boldsymbol{r} = (\nabla^2 x, \nabla^2 y, \nabla^2 z)$$

The components of this vector are:

$$\nabla^2 x = \frac{\partial^2 x}{\partial x^2} + \frac{\partial^2 x}{\partial y^2} + \frac{\partial^2 x}{\partial z^2} = \frac{\partial^2 x}{\partial y^2} + \frac{\partial^2 x}{\partial z^2}$$

$$\nabla^2 y = \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} + \frac{\partial^2 y}{\partial z^2} = \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial z^2}$$

$$\nabla^2 z = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial z^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$$

We must assume the $-\frac{\hbar^2}{2\mathrm{m}} \nabla^2 \boldsymbol{r} = \frac{\mathrm{p}^2}{2\mathrm{m}} \boldsymbol{r}$ identity can be established, but we have not managed to prove it. In any case, we did achieve the objective of this paper, and that is to provide the reader with a *geometric* interpretation of Schrödinger's wave equation. This geometric interpretation is quite simple: Schrödinger's wave equation is not a wave equation but a differential equation for non-circular orbitals.

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plane that is perpendicular to the plane of oscillation. Hence, one might want to use a different symbol, such as *j*, for example.

[13] See: Feynman, III-19-1. We should note that this is, perhaps, not so obvious as it seems. The symbol V is often used to denote the electric potential, which is defined not in terms of the *electron* charge but in terms of the *Coulomb* (C), i.e. the SI unit charge.

As such, it is mathematically equivalent to what we refer to as an *augmented* Rutherford-Bohr model of the atom.

Occam's Razor

We should wrap up this paper by noting that the wavefunction we started out with does *not* respect the usual convention: physicists usually write ψ as $a{\cdot}e^{-i\theta}$. We will make a small but necessary digression here. Most introductory courses in quantum mechanics will show that both $\psi = \exp(i\theta) = \exp[i(kx-\omega t)]$ and $\psi = \exp(-i\theta) = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$ are acceptable waveforms for a particle that is propagating in the *x*-direction – as opposed to, say, some real-valued sinusoid. We would think physicists would then proceed to provide some argument why one would be better than the other, or some discussion on why they might be different, but that is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[14]

This is, frankly, quite shocking because we *know*, from *experience*, that theoretical or mathematical possibilities in quantum mechanics often turn out to represent real things. Here we should think of the experimental verification of the existence of the positron (or of anti-matter in general) after Dirac had predicted its existence based on the mathematical possibility only. So why would that *not* be the case here? *Occam's Razor* tells us that we should not have any redundancy in the description. Hence, if there is a physical interpretation of the wavefunction, then we should not have to choose between the two mathematical possibilities: they would represent two different physical situations. Of course, the only characteristic that can make the difference here would be spin. Hence, we would *not* agree with the mainstream view that "the choice is a matter of convention" and that "happily, most physicists use the same convention"[15] but, instead, dare to suggest that the two mathematical possibilities may represent identical particles with opposite spin (i.e. *real* spin-1/2 particles as opposed to non-existing spin-zero particles), in which case we get the following table.

Table 2: Occam's Razor: mathematical possibilities versus physical realities

Spin and direction of travel	**Spin up ($J = +\hbar/2$)**	**Spin down ($J = -\hbar/2$)**
Positive *x*-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative *x*-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

The reader of this paper may wonder why this point should matter. The answer is that the redundancy in the description is directly related to the logic which leads us to the rather uncomfortable conclusion that the wavefunction of spin-1/2 particles have a 720-degree symmetry in space. This conclusion is uncomfortable because we cannot imagine such objects in space without invoking the idea of some kind of relation between the subject and the object (the reader should think of the Dirac belt trick here),

[14] See, for example, the MIT's edX Course 8.04.1*x*, Lecture Notes, Chapter 4, Section 3.
[15] See the reference above.

which we want to avoid. We have written at length about this and other objections to a geometric interpretation of the wavefunction before, so we will just refer the reader there.[16]

Conclusions

In one of his introductory *Lectures* on electrodynamics (*Lectures*, Volume II, Chapter 5), Feynman briefly discusses the Rutherford-Bohr model of an atom. He duly notes the model's key advantage over the preceding static models (the electrons are kept from falling in toward the nucleus by their orbital motion), but then dismisses it based on the usual objection: "With such motion, the electrons would be accelerating (because of the circular motion) and would, therefore, be radiating energy. They would lose the kinetic energy required to stay in orbit and would spiral in toward the nucleus." He then sums up the quantum-mechanical model of an atom as follows:

> "The electrostatic forces pull the electron as close to the nucleus as possible, but the electron is compelled to stay spread out in space over a distance given by the Uncertainty Principle. If it were confined in too small a space, it would have a great uncertainty in momentum. But that means it would have a high expected energy—which it would use to escape from the electrical attraction. The net result is an electrical equilibrium not too different from the idea of Thompson—only is it the negative charge that is spread out, because the mass of the electron is so much smaller than the mass of the proton."

This explanation is a bit sloppy, and one has to patiently wait for Feynman to introduce Schrödinger's equation and the related derivation of the electron orbitals to get the following clarification:

> "The wave function $\psi(\boldsymbol{r})$ for an electron in an atom does not describe a smeared-out electron with a smooth charge density. The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge." (Feynman's *Lectures*, III-21-4)

This leaves us bewildered, because it is not clear at all how this quantum-mechanical picture is supposed to solve the radiation problem! Indeed, if the pointlike charge is sometimes here, and sometimes there, then it must – logically – also go from here to there once in a while, and then it should generate some electromagnetic radiation too! We, therefore, wanted to re-examine the Rutherford-Bohr model and prove that, with some modifications, it explains all what needs to be explained. We have, therefore, demonstrated that classical mechanics goes quite a long way in explaining quantum mechanics – further than most would think, that is !

A final note concerns the Uncertainty Principle. We assume the uncertainty is in the direction of h (and, hence, $\hbar$). It is not in its magnitude. The uncertainty in the direction of h should explain why atoms are actually spherical – as opposed to this *flat* orbital model that we have offered.

Jean Louis Van Belle, 12 December 2018

[16] Such objections usually also include the idea that the coefficient (a) of the wavefunction $a\cdot e^{i\theta}$ may be complex-valued, whereas in any *real* interpretation this (maximum) amplitude should be real-valued. This objection is also rejected. See: Jean Louis Van Belle, 30 October 2018, *Euler's wavefunction: the double life of −1*, http://vixra.org/abs/1810.0339.

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to:

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.

4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.

5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

Most illustrations in this paper were made by the author. The others are are open source. Finally, we should also mention another critical appraisal of the quantum-mechanical framework:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

It is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Last but not least, we would like to mention the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html). However, we have not gone into the nitty-gritty of it and, therefore, do not want to pretend we have.