Rotation Operators in Headpiece Fashion

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1 Introduction

Ever since caps were worn, fashion deemed it necessary that everyone eventually migrate to new styles as the older styles became "worn" and "uncool." Unfortunately, as of yet, there have been no mathematical explanations for how these new styles have come about. We henceforth set about creating a mathematical foundation for the styles people use for wearing their hats, caps, and other headpieces of choice.

Figure 1: A Cap

2 Rotation Matrix Formalism

Rotation Matrices in 3 dimensions are defined by a rotation by an angle θ counterclockwise about an axis. For example, the rotations by θ in the x, y, and z axes respectively are:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}, R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3 Applying the Matrices

First, we a consider a human wearing a hat.

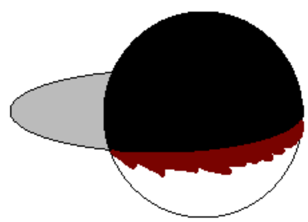

Figure 2: A brown haired man facing away (the hair color is arbitrary).

We then define the axes of the Cartesian coordinate plane, with x pointing out from the face of the man, y point out from the left side, and z pointing above the man.

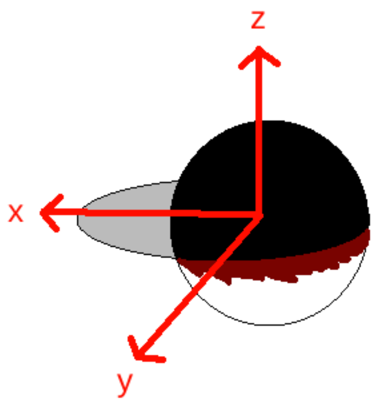

Figure 3: Cartesian Coordinate Axes with the center of the head (sphere) as the origin

From here it is trivial to note that not rotating the cap, i.e. the θ=0 case, is the familiar Identity Matrix:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Now, after everyone wore their caps in the default orientation, deviants began to wear their caps backwards instead [4].

Figure 4: Deviant

This is the result of rotation about the z-axis of angle π:

$$R_z(\pi) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Eventually this style too grew stale and a rotation of $\pi/2$ became fashionable[5],

Figure 5: The deviant Deviant

with

$$R_z(\pi/2) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This hatstyle however, was degenerate (the human head is symmetric about the z axis) and could be reproduced by applying $R_z(\pi/2)$ and $R_z(\pi)$ consecutively.

Afterward, this style too was replace in favor of the $\pi/4$ approach [6],

Figure 6: *Deviant*[3]

with the resulting matrix:

$$R_z(\pi/4) = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

This too is degenerate as you can apply $R_z(\pi/4)$, then $R_z(\pi/2)$ one, two, or three times.

Finally, the xy plane was abandoned for brighter pastures and a rotation about the y-axis was performed as part of the counterculture:

$$R_y(\pi/4) = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ -\sqrt{2}/2 & 0 & \sqrt{2}/2 \end{bmatrix}$$

with the following result [7].

Figure 7: Transcendent Deviant

4 Future Fashions

As of yet, there remain many more possibilities for fashionistas and researchers to discover. Some of the possibilities include mirror reflections (turning the hat inside out), $R_y(\pi/2)$ rotations (the hat covers your face and you bump into things) and time dependent rotations (your hat spins at an angular frequency ω but you also lose all your hair due to friction).