Electroweak physics reconstrued using null cone integrals

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Abstract

We present a novel formulation of particle physics that dispenses with space-time derivative operators in favour of null cone integrations. It is shown that the loss of locality incurred is compensated by gains in conceptual and mathematical simplicity, the absence of non-physical gauge degrees of freedom and the concomittant complications of ghosts etc..
Central to the formulation is a dimensionless homologue of the Lagrangian density, formed from integrals of scalar product terms over null cones. Instead of covariant derivatives, the gauge fields are represented by rotations over the simple product of the internal and Lorentz symmetry groups. We demonstrate that application of a variational principle to this quasi-action functional yields essentially the same equations of motion as the SM. As a consequence of the enlarged symmetry group, the primordial electroweak Higgs field is shown to be the origin of *all* bosonic degrees of freedom, not just the Goldstone modes, prior to the symmetry breaking that reduces it to an isospin carrying scalar.
Although this paper is restricted to considerations of leptons and the electroweak $SU(2)_L \times U(1)_Y$ symmetry group, the extension of the method to quarks and $SU(3)_C \subset SO(10)$ would appear to be straightforward and will be the subject of a subsequent paper.

1 Motivation

Its magnificient successes notwithstanding, the Lagrangian of the Standard Model is clearly not the final word in mathematical representation of the physical world. There is however currently no empirical evidence in favour of any of the (mostly SUSY-based) models that still remain standing after the complete lack of evidence for any BSM particles at the LHC and elsewhere. It therefore behoves us to subject the *a priori* assumptions of the lagrangian model to the minutest scrutiny. One such assumption, sacrosant since at least the 1930s, is that the space-time derivatives of particle fields are as fundamental as the fields themselves. For example, QFT treats fermionic ψ and $\partial_\mu \psi$ as conjugate variables. If however one looks at the Minkowskian geometry of space-time with an unprejudiced eye, this equality of status appears to rest on a somewhat shaky foundation. Specifically, it could be argued that, in order to synthesize $\partial_\mu \psi(x)$, nature needs to have knowledge of $\psi(x)$ at 8 infinitesimally separated field points $x_\mu \pm \delta x_\mu$. But of these 8 points, only one lies within the past cone of the causally connected past. The derivative field value $\partial_\mu \psi$ therefore contains contributions that are causally unrelated to one another. Thus $\partial_\mu \psi$ would appear to be inherently less fundamental object than ψ.
This misgiving concerning derivative fields has led me to look for a formulation of particle physics that is (initially) innocent of space-time derivatives but from which the familiar differential equations of motion emerge naturally. The obvious candidate tool was the null cone, on account of its intrinsically Minkowskian geometry and nice fourier properties, which we summarize in the next section.

2 Fourier properties of null cones

Consider a complex spinor $\Lambda = \begin{pmatrix}\lambda_1 + i\lambda_2, & \lambda_3 + i\lambda_4\end{pmatrix}$ of metric dimension $+\frac{1}{2}$, with a $U(1)$ degenerate mapping onto the past null cone with vertex at the origin of x:

$$x_\mu = \Lambda^\dagger \sigma_\mu \Lambda$$

$$d\Lambda = \prod_{i=1}^{4} d\lambda_i = 2\pi\delta(t^2 - r^2)d^4x = 2\pi\frac{d^3\mathbf{r}}{r}$$

It can be shown that, for all $k^2 \neq 0$

$$\int_{-\infty}^{0} e^{-ik_\nu x^\nu} d\Lambda = \frac{1}{k^2} \tag{1}$$

and

$$\int_{-\infty}^{0} x_\mu e^{-ik_\nu x^\nu} d\Lambda = \frac{2ik_\mu}{k^4} \tag{2}$$

The product of two Λ cones maps onto $(x_\mu x^\mu > 0)$ space-time with a $U(1)_L \times SU(2) \times U(1)_R$ degeneracy:

$$x_\mu = x_{1,\mu} + x_{2,\mu} = \Lambda_1^\dagger \sigma_\mu \Lambda_1 + \Lambda_2^\dagger \sigma_\mu \Lambda_2$$

$$\iint e^{-ik_\nu x^\nu} d\Lambda_1 d\Lambda_2 = \frac{1}{k^4} \tag{3}$$

$$d\Lambda_1 d\Lambda_2 = 4\pi^2 d^4x$$

So, for an arbitrary field f:

$$f = \partial_\nu\partial^\nu \int f d\Lambda = \frac{1}{2}\partial^\mu\partial_\nu\partial^\nu \int f x_\mu d\Lambda = (\partial_\nu\partial^\nu)^2 \iint f d\Lambda_1 d\Lambda_2 \tag{4}$$

3 Fermions on cones

Consider the following dimensionless analogue of the lagrangian density for a multiplet of a freely propagating Dirac fermions with a diagonal mass matrix m:

$$\mathcal{Q}_f = \int ix^\nu \left[\psi_L^\dagger \sigma_\nu \psi_L + \psi_R^\dagger \tilde{\sigma}_\nu \psi_R\right] d\Lambda - \iint \left[\psi_L^\dagger m \psi_R + \psi_R^\dagger m \psi_L\right] d\Lambda_1 d\Lambda_2 \tag{5}$$

If we suppose that the "quasi-action" $\mathcal{Q}_f$ is invariant w.r.t. variations in ψ and $\psi^\dagger$ then:

$$\frac{\partial \mathcal{Q}_f}{\partial \psi_L^\dagger} = 0 \implies \int ix^\nu \sigma_\nu \psi_L \, d\Lambda - \iint m\psi_R \, d\Lambda_1 d\Lambda_2 = 0 \tag{6}$$

$$\frac{\partial \mathcal{Q}_f}{\partial \psi_R^\dagger} = 0 \implies \int ix^\nu \tilde{\sigma}_\nu \psi_R \, d\Lambda - \iint m\psi_L \, d\Lambda_1 d\Lambda_2 = 0 \tag{7}$$

$$\implies \begin{cases} \iint [i\sigma_\nu \partial_\nu \psi_L - m\psi_R] \, d\Lambda_1 d\Lambda_2 = 0 \\ \iint [i\tilde{\sigma}_\nu \partial_\nu \psi_R - m\psi_L] \, d\Lambda_1 d\Lambda_2 = 0 \end{cases} \tag{8}$$

..where we have used (4).
For typographical reasons, we will formally omit the implicit integration measures $d\Lambda$ and $d\Lambda_1 d\Lambda_2$ for the rest of this paper.
Although the fermion masses m are constants in the present epoch, let us consider a primitive earlier state in which they are in the form of a Higgs-type field Φ that couples fermions of opposite chirality. Dimensional and renormalizability considerations lead us to consider the following general form for the quasi-action of the combined lepton + electroweak Higgs fields system:

$$\begin{aligned}\mathcal{Q}_{l\phi} = &\int \left[ix^\nu[\psi_L^\dagger \sigma^\nu \psi_L + \psi_R^\dagger \tilde{\sigma}^\nu \psi_R] + \langle 0|\, \Phi^\dagger \Phi \,|0\rangle\right] \\ &- \iint \psi_L^\dagger \Phi \mathbf{Y} \psi_R + \psi_R^\dagger \Phi \mathbf{Y} \psi_L + \langle 0|\left[\mu^2 \Phi^* \Phi - \lambda(\Phi^*\Phi)^2\right]|0\rangle \end{aligned} \tag{9}$$

where μ has the dimensions of mass and λ is a dimensionless self-coupling constant and $\mathbf{Y}$ is a (diagonal) matrix of Yukawa coupling constants.

4 SO(6) $\times$ SO(3,1)

The fermions of one family can be assigned to the $\mathbf{16} \oplus \overline{\mathbf{16}}$ (complex spinor representation) of $SO(10)$ and the leptons of one family to the $\mathbf{4} \oplus \overline{\mathbf{4}}$ of an $SO(6)$ subgroup[1]. By including spin as an extra rank, the fundamental adjoint representation becomes the product of Lorentz and internal charge symmetry operations. Representing the $SO(8)$ vacuum by $|0\rangle$ ($\equiv |0,0,0,0\rangle$) with the orthonormality property:

$$\langle 0|\, \tau_a \times \tau_b \times \tau_c \times \sigma_d \,|0\rangle = \delta_{0a}\delta_{0b}\delta_{0c}\delta_{0d}$$

..where we have used σ_ν rather than $\frac{1}{2}\tau_\nu$ in the 4th rank so as make a clarifying distinction between charge and spin spaces.
We have fermions:

$$\begin{cases} \hat{\nu}_{\mathbf{L}\uparrow}\,|0\rangle = |-1,+1,-1,+1\rangle \\ \hat{\mathbf{e}}_{\mathbf{L}\uparrow}\,|0\rangle = |+1,-1,-1,+1\rangle \\ \hat{\nu}_{\mathbf{R}\uparrow}\,|0\rangle = |-1,-1,-1,+1\rangle \\ \hat{\mathbf{e}}_{\mathbf{R}\uparrow}\,|0\rangle = |+1,+1,-1,+1\rangle \end{cases} \tag{10}$$

The conserved hypercharge, isospin and electric charge operators are:

$$\begin{cases} Y = [\tau_0 \times \tau_0 \times \tau_3 \times \tau_0] - \frac{1}{2}[\tau_0 \times \tau_3 \times \tau_0 \times \tau_0] - \frac{1}{2}[\tau_3 \times \tau_0 \times \tau_0 \times \tau_0] \\ T_3 = \frac{1}{4}[\tau_0 \times \tau_3 \times \tau_0 \times \tau_0] - \frac{1}{4}[\tau_3 \times \tau_0 \times \tau_0 \times \tau_0] \\ Q \equiv \frac{1}{2}Y + T_3 = \frac{1}{2}[\tau_0 \times \tau_0 \times \tau_3 \times \tau_0] - \frac{1}{2}[\tau_3 \times \tau_0 \times \tau_0 \times \tau_0] \end{cases}$$

In the chiral representation of the first two ranks of this algebra:

$$\psi \equiv \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \equiv \begin{pmatrix} \nu_L \\ e_L \\ \nu_R \\ e_R \end{pmatrix}$$

and the field introduced in (9) has the following 16 complex components:

$$\Phi = \phi^{\mu\nu}[\tau_1 \times \tau_\mu \times \tau_0 \times \sigma_\nu] \tag{11}$$

where $\mu, \nu = \{0,1,2,3\}$.
Invoking the invariance of $\mathcal{Q}_{l\Phi}$ w.r.t. each component of Φ, we have:

$$\frac{\partial \mathcal{Q}_{l\phi}}{\partial \phi^*_{\mu\nu}} = 0 \implies \phi_{\mu\nu} = \int [\mu^2 - 2\lambda(\phi^{*\nu}_\mu \phi_{\mu\nu})]\phi_{\mu\nu}\, d\Lambda \tag{12}$$

The only solutions of (12) are the trivial $\phi_{\mu\nu} = 0$ or[1] $\mu^2 \approx 2\lambda\phi^{*\nu}_\mu \phi_{\mu\nu}$.
We suppose that at some point in the early universe, the underlying symmetry of $\mathcal{Q}_{l\phi}$ was broken by ϕ_{00} acquiring a real v.e.v. $v = \sqrt{\mu^2/2\lambda}$.
With $\phi_{00} \equiv v + h$ we have:

$$\partial_\mu \partial^\mu h = -2\lambda[v^2 h + 3\lambda v h^* h + \lambda h^* h^2] \tag{13}$$

...which describes a scalar particle with effective mass $M_H = \sqrt{2\lambda}\, v$ and tri- and quartic self-interactions.
The v.e.v. of the Higgs field in the same representation as (11) is:

$$\Phi_0 = (v+h)[\tau_1 \times \tau_0 \times \tau_0 \times \sigma_0]$$

In a generalization of the Goldstone mechanism, the remaining 15 components

[1] The cosmological $\Sigma\overline{\psi}Y\psi$ charge term is negligible compared to each of the static LHS terms, so the approximation holds to a very high order of accuracy

can be expressed as non-unitary rotations of Φ_0 in the $SU(2)\times U(1)\times SO(3,1)$ space. In the representation space of the leptons:

$$\Phi = e^{-\Theta^*}\Phi_0 e^{\Theta} \equiv (v+h)\begin{pmatrix} e^{-\Theta_L^*} & 0 \\ 0 & e^{-\Theta_R^*}\end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix} e^{\Theta_L} & 0 \\ 0 & e^{\Theta_R}\end{pmatrix}$$
$$= (v+h)\begin{pmatrix} 0 & e^{\Theta_R-\Theta_L^*} \\ e^{\Theta_L-\Theta_R^*} & 0\end{pmatrix} \tag{14}$$

$$\Phi^\dagger = (v+h^*)\begin{pmatrix} 0 & e^{\Theta_L-\Theta_R^*} \\ e^{\Theta_R-\Theta_L^*} & 0\end{pmatrix}$$

The Yukawa terms in (9) are formally independent of the proto-Higgs field components when expressed in terms of transformed fermions ψ_θ defined by the counter-transformation:

$$\begin{cases} \begin{pmatrix} \psi_{L\theta} \\ \psi_{R\theta}\end{pmatrix} \equiv \begin{pmatrix} e^{\Theta_L} & 0 \\ 0 & e^{\Theta_R}\end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R\end{pmatrix} \\ \begin{pmatrix} \psi_{L\theta}^\dagger & \psi_{R\theta}^\dagger\end{pmatrix} \equiv \begin{pmatrix} \psi_L^\dagger & \psi_R^\dagger\end{pmatrix}\begin{pmatrix} e^{-\Theta_L^*} & 0 \\ 0 & e^{-\Theta_R^*}\end{pmatrix}\end{cases} \tag{15}$$

$$\Longrightarrow\ \psi_L^\dagger \Phi \mathbf{Y}\psi_R + \psi_R^\dagger\Phi\mathbf{Y}\psi_L \equiv \psi_{L\theta}^\dagger\Phi_0\mathbf{Y}\psi_{R\theta} + \psi_{R\theta}^\dagger\Phi_0\mathbf{Y}\psi_{L\theta}$$

The effective low energy chiral (L,R) couplings for the $SU(2)$ and $U(1)$ subgroups are respectively $(g,0)$ and (g',g') so we define:

$$\begin{cases} \Theta_L \equiv \frac{i}{2}\begin{pmatrix} gw_\nu^3 - g'b_\nu & gw_\nu^+ \\ gw_\nu^- & -gw_\nu^3 - g'b_\nu\end{pmatrix}\sigma_\nu \\ \Theta_R \equiv i\begin{pmatrix} 0 & 0 \\ 0 & -g'b_\nu^*\end{pmatrix}\tilde{\sigma}_\nu\end{cases} \tag{16}$$

with $\nu = \{0,1,2,3\}$.
As will be shown below, the w_0^3 and b_0 components correspond to the number of electroweak charge quanta and the $w_0^\pm$ components correspond to the number of isospin-changing events. Assuming that the universe as a whole is uncharged, the said components will all be zero, allowing us to replace the ν index by $j=\{1,2,3\}$ in the following expansion:

$$\langle 0|\,\Phi^\dagger\Phi\,|0\rangle = |v+h|^2\Big[4 + \mathrm{Tr}\big[(\Theta_R^*-\Theta_L)^2 + (\Theta_L^*-\Theta_R)^2\big] + \mathcal{O}(g^4) + \mathcal{O}(g'^4)\Big]$$
$$\approx |v+h|^2\left[4 - \mathrm{Tr}\left[\begin{matrix} -gw_j^3+g'b_j & gw_j^+ \\ gw_j^- & gw_j^3-g'b_j\end{matrix}\right]^2 + \left[\begin{matrix} gw_j^{3*}-g'b_j^* & -gw_j^{+*} \\ -gw_j^{-*} & -gw_j^{3*}+g'b_j^*\end{matrix}\right]^2\right]$$
$$= 4|v+h|^2\Big[1 - \big[(g^2+g'^2)(z_j^2+z_j^{*2}) + g^2(w_j^-w_j^+ + w_j^{-*}w_j^{+*})\big]\Big] \tag{17}$$

where a_ν, z_ν are related to w_ν^3, b_ν by the Weinberg rotation:

$$\begin{pmatrix} z_\nu \\ a_\nu\end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix} w_\nu^3 \\ b_\nu\end{pmatrix}, \qquad \tan\theta_W = \frac{g'}{g}$$

In terms of these new variables, and using for convenience:

$$\begin{cases} q \equiv g\sin\theta_W \\ g_z^2 \equiv g^2 + g'^2 \\ g'_z = g\cos\theta_W - g'\sin\theta_W\end{cases} \tag{18}$$

... (16) becomes:

$$\begin{cases} \Theta_L = \frac{i}{2}\begin{pmatrix} g_z z_\nu & gw_\nu^+ \\ gw_\nu^- & -2qa_\nu - g'_z z_\nu\end{pmatrix}\sigma_\nu \\ \Theta_R = i\begin{pmatrix} 0 & 0 \\ 0 & -qa_\nu^* + g'\sin\theta_W z_\nu^*\end{pmatrix}\tilde{\sigma}_v\end{cases} \tag{19}$$

Rewriting (9) in terms of these transformed fields, together with additional bilinear boson propagator terms $\mathcal{Q}_b$

$$
\begin{aligned}
\mathcal{Q} \equiv \mathcal{Q}_b + \mathcal{Q}_{l\phi} \equiv \frac{1}{2}\big[a_\nu a^\nu + a^*_\nu a^{\nu *} + z_\nu z^\nu + z^*_\nu z^{\nu *} + w^{+*}_\nu w^{-\nu *} + w^{-*}_\nu w^{+\nu *}\big] \\
+ \int \big[ix^\nu[\psi^\dagger_{L\theta} e^{\Theta^*_L}\sigma_\nu e^{-\Theta_L}\psi_{L\theta} + \psi^\dagger_{R\theta} e^{\Theta^*_R}\tilde{\sigma}_\nu e^{-\Theta_R}\psi_{R\theta}] \\
+ \int |v+h|^2 \Big[1 - \big[(g^2 + g'^2)(z^2_j + z^{*2}_j) + g^2(w^-_j w^+_j + w^{-*}_j w^{+*}_j)\big]\Big] \\
- \iint \Big[\psi^\dagger_{L\theta}[v+h]\mathbf{Y}\psi_{R\theta} + \psi^\dagger_{R\theta}[v+h^*]\mathbf{Y}\psi_{L\theta} \\
+ \lambda \iint 2v^3(h+h^*) + 5v^2|h|^2 + 2v|h|^2(h+h^*) + |h|^4
\end{aligned}
\tag{20}
$$

We suppose that (20) expresses the complete quasi-action for the leptonic electroweak sector and that the independent fields (for the purposes of the variational principle) are $\psi_\theta, \psi^*_\theta, a_\nu, w_\nu, z_\nu$ and h.

5 Electrons and photons

With $z_\nu = w_\nu = 0$, (19) reduces to:

$$
\begin{cases}
\theta_L \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & -iqa_\nu\sigma_\nu \end{pmatrix} \\
\theta_R \rightarrow \begin{pmatrix} 0 & 0 \\ 0 & -iqa^*_\nu\tilde{\sigma}_v \end{pmatrix}
\end{cases}
\tag{21}
$$

electrons are decoupled from neutrinos:

$$
\begin{cases}
\begin{pmatrix} e_{L\theta} \\ e_{R\theta} \end{pmatrix} \rightarrow \begin{pmatrix} e^{-iqa\cdot\sigma} & 0 \\ 0 & e^{-iqa^*\cdot\tilde{\sigma}} \end{pmatrix} \begin{pmatrix} e_L \\ e_R \end{pmatrix} \\
\begin{pmatrix} e^\dagger_{L\theta} & e^\dagger_{R\theta} \end{pmatrix} \rightarrow \begin{pmatrix} e^\dagger_L & e^\dagger_R \end{pmatrix} \begin{pmatrix} e^{-iqa^*\cdot\sigma} & 0 \\ 0 & e^{-iqa\cdot\tilde{\sigma}} \end{pmatrix}
\end{cases}
\tag{22}
$$

and (14) reduces to $\Phi \rightarrow \Phi_0$
The part of $\mathcal{Q}$ that is dependent upon a_ν, e_L and e_R is just:

$$
\begin{aligned}
\mathcal{Q}_a = \frac{1}{2}[a^\nu a_\nu + a^{\nu *} a^*_\nu] + \int ix^\nu[e^\dagger_{L\theta} e^{iqa^*\cdot\sigma}\sigma_\nu e^{iqa\cdot\sigma} e_{L\theta} + e^\dagger_{R\theta} e^{iqa\cdot\tilde{\sigma}}\tilde{\sigma}_\nu e^{iqa^*\cdot\tilde{\sigma}} e_{R\theta}] \\
- y_e \iint e^\dagger_{L\theta}\frac{v+h}{\sqrt{2}} e_{R\theta} + e^\dagger_{R\theta}\frac{v+h^*}{\sqrt{2}} e_{L\theta}
\end{aligned}
\tag{23}
$$

$$
\frac{\partial \mathcal{Q}_a}{\partial a_\nu} = 0 \implies a_\nu = q\int x^\nu \left[e^\dagger_L \sigma_\nu\sigma_\mu e_L + e^\dagger_R \tilde{\sigma}_\mu\tilde{\sigma}_\nu e_R\right]
$$

$$
\frac{\partial \mathcal{Q}_a}{\partial a^*_\nu} = 0 \implies a^*_\nu = q\int x^\nu \left[e^\dagger_L \sigma_\mu\sigma_\nu e_L + e^\dagger_R \tilde{\sigma}_\nu\tilde{\sigma}_\mu e_R\right]
$$

In terms of the current density $J_\nu \equiv e^\dagger_L\sigma_\nu e_L + e^\dagger_R\tilde{\sigma}_\nu e_R$ we have:

$$
\begin{cases}
a_0 = a^*_0 = q\int x^\nu J_\nu = 0 \\
a_j = q\int x_j J_0 - x_0 J_j + i\epsilon_{jki} x_i J_k \\
a^*_j = q\int x_j J_0 - x_0 J_j - i\epsilon_{ijk} x_i J_k
\end{cases}
\tag{24}
$$

This set of equations expresses the unification of quantized charge with the electric and magnetic field that it sources.
Using (4):

$$
a_j = q\iint \partial_j J_0 - \partial_0 J_j + i\epsilon_{jki}\partial_i J_k
\tag{25}
$$

$$\implies \begin{cases} \partial^j a_j = q \iint \partial^j \partial_j J_0 - \partial_0 \partial^j J_j = -q \int J_0 \\ \partial^0 a_j + i\epsilon_{jkl}\partial^k a_l = q \iint \partial_j \partial^0 J_0 - \partial^0 \partial_0 J_j - \epsilon_{jkl}\partial^k \epsilon_{lcd}\partial_c J_d = -q \int J_j \end{cases} \tag{26}$$

..where we have used $\partial^\nu J_\nu = 0$. Similarly:

$$a_j^* = q \iint \partial_j J_0 - \partial_0 J_j - i\epsilon_{jki}\partial_i J_k \tag{27}$$

$$\implies \begin{cases} \partial^j a_j^* = q \iint \partial^j \partial_j J_0 - \partial_0 \partial^j J_j = -q \int J_0 \\ \partial^0 a_j^* - i\epsilon_{jkl}\partial^k a_l^* = q \iint \partial_j \partial^0 J_0 - \partial^0 \partial_0 J_j - \epsilon_{jkl}\partial^k \epsilon_{lcd}\partial_c J_d = -q \int J_j \end{cases} \tag{28}$$

...which allows the following identification with the 4-vector potential of the electromagnetic field, fixed in the Lorenz gauge:

$$\begin{cases} A_0 = \partial^j a_j = q \iint \partial^j \partial_j J_0 - \partial_0 \partial^j J_j \\ A_j = \partial^0 a_j + i\epsilon_{jkl}\partial^k a_l \\ A_\nu = -q \int J_\nu \end{cases} \tag{29}$$

Analogously to (7):

$$\frac{\partial \mathcal{Q}_a}{\partial e_{L\theta}^\dagger} = 0 \implies \int i x^\nu e^{iqa^*\cdot\sigma}\sigma_\nu e_L - y_e \iint \frac{v+h}{\sqrt{2}} e^{iqa^*\cdot\sigma} e_R = 0$$

$$\frac{\partial \mathcal{Q}_a}{\partial e_{R\theta}^\dagger} = 0 \implies \int i x^\nu e^{iqa\cdot\tilde{\sigma}}\tilde{\sigma}_\nu e_R - y_e \iint \frac{v+h^*}{\sqrt{2}} e^{iqa\cdot\tilde{\sigma}} e_L = 0$$

$$\implies \begin{cases} \iint e^{iqa^*\cdot\sigma}[i\sigma_\nu \overleftrightarrow{\partial^\nu} e_L - m_e e_R] = 0 \\ \iint e^{iqa\cdot\tilde{\sigma}}[i\tilde{\sigma}_\nu \overleftrightarrow{\partial^\nu} e_R - m_e e_L] = 0 \end{cases} \tag{30}$$

where $m_e = y_e v/\sqrt{2}$ and we have omitted the term in h for the sake of clarity.

$$\implies \begin{cases} \left[i\sigma_\nu \partial^\nu - q\sigma_j \sigma_\nu (\partial^\nu a_j^*)\right] e_L - m_e e_R = 0 \\ \left[i\tilde{\sigma}_\nu \partial^\nu - q\tilde{\sigma}_j \tilde{\sigma}_\nu (\partial^\nu a_j)\right] e_R - m_e e_L = 0 \end{cases} \tag{31}$$

..where $j = 1,2,3$ since $\partial^\mu a_0 = 0$, the conserved charge in (24) being a_0 $(= a_0^*)$.

$$\implies \begin{cases} \left[i\sigma_\nu \partial^\nu - q(\partial^j a_j^* - \sigma_j[\partial^0 a_j^* + i\epsilon_{jkl}\partial^k a_l^*])\right] e_L - m_e e_R = 0 \\ \left[i\tilde{\sigma}_\nu \partial^\nu - q(\partial^j a_j - \tilde{\sigma}_j[\partial^0 a_j + i\epsilon_{jkl}\partial^k a_l])\right] e_R - m_e e_L = 0 \end{cases} \tag{32}$$

Using (29), this reduces to the Dirac equation for an electron in an electromagnetic field:

$$\implies \begin{cases} \sigma_\nu[i\partial^\nu - qA^\nu]e_L - m_e e_R = 0 \\ \tilde{\sigma}_\nu[i\partial^\nu - qA^\nu]e_R - m_e e_L = 0 \end{cases} \tag{33}$$

.

6 Leptons and electroweak bosons

We will now return to a consideration of (20) without the restriction $z_\nu, w_\nu = 0$.

$$\frac{\partial \mathcal{Q}}{\partial z_j} = 0 \implies$$

$$z_j = \int x^\nu \left[g_z \nu_L^\dagger \sigma_\nu \sigma_j \nu_L - {g'}_z e_L^\dagger \sigma_\nu \sigma_j e_L + g' \sin\theta_W e_R^\dagger \tilde{\sigma}_j \tilde{\sigma}_\nu e_R\right] - g_z^2 |v+h|^2 z_j \tag{34}$$

...which expresses the fact that the Z-boson has a mass of $M_Z = g_z v$ and couples to both left- and right-handed fermionic currents. Similarly,

$$\frac{\partial \mathcal{Q}}{\partial w_j^+} = 0 \implies w_j^+ = \int g x^\nu \nu_L^\dagger \sigma_\nu \sigma_j e_L - g^2 |v+h|^2 w_j^+ \tag{35}$$

$$\frac{\partial \mathcal{Q}}{\partial w_j^-} = 0 \implies w_j^- = \int g x^\nu e_L^\dagger \sigma_\nu \sigma_j \nu_L - g^2 |v+h|^2 w_j^- \tag{36}$$

..which express the fact that the $W^\pm$-bosons have a mass $M_W = gv$ and couple to left-handed fermions only.

$$\frac{\partial \mathcal{Q}}{\partial \psi_{L\theta}^\dagger} = 0 \implies \iint e^{\Theta_L^*} \left[i\sigma_j \partial^\nu \psi_L - \Phi \psi_R \right] = 0$$

$$\sigma_\nu \begin{pmatrix} i\partial^\nu + g_z Z^\nu & gW^{+\nu} \\ gW^{-\nu} & i\partial^\nu - g'_z Z^\nu - qA^\nu \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} - (m_e + y_e h) e^{\Theta_R - \Theta_L^*} \begin{pmatrix} 0 \\ e_R \end{pmatrix} = 0$$

where, in addition to the identities of (24), we have:

$$\begin{cases} Z_0 = \partial^j z_j^* \\ Z_j = \partial^0 z_j^* - i\epsilon_{jkl} \partial^k z_l^* \\ W_0^\pm = \partial^j w_j^{\pm *} \\ W_j^\pm = \partial^0 w_j^{\pm *} - i\epsilon_{jkl} \partial^k w_l^{\pm *} \end{cases} \tag{37}$$

$$Z_\mu = \int g_z \nu_L^\dagger \sigma_\mu \nu_L + (g' \sin\theta_W - g\cos\theta_W) e_L^\dagger \sigma_\mu e_L + g' \sin\theta_W e_R^\dagger \tilde{\sigma}_\mu e_R - M_Z^2 Z_\mu$$

$$W_\mu^+ = \int g \nu_L^\dagger \sigma_\mu e_L - M_W^2 W_\mu^+$$

$$W_\mu^- = \int g e_L^\dagger \sigma_\mu \nu_L - M_W^2 W_\mu^-$$

Similarly

$$\frac{\partial \mathcal{Q}}{\partial \psi_{R\theta}^\dagger} = 0 \implies \iint e^{\Theta_R^*} \left[i\tilde{\sigma}_j \partial^\nu \psi_R - \Phi \psi_L \right] = 0$$

$$\tilde{\sigma}_\nu \begin{pmatrix} i\partial^\nu & 0 \\ 0 & i\partial^\nu + g' \sin\theta_W Z^\nu - qA^\nu \end{pmatrix} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} - (m_e + y_e h) e^{\Theta_L - \Theta_R^*} \begin{pmatrix} 0 \\ e_L \end{pmatrix} = 0$$

The mass terms are clearly modified with respect to the SM. It is not clear whether this will give rise to observable effects.

7 Conclusion

We have demonstrated the existence of a representation of particle physics from which derivative field operators are initially absent and in which all three categories of particle field (fermionic, bosonic and scalar) appear in transformed guise: $\psi \to \psi_\theta$, $A_\mu \to \partial^\mu \Theta_{\mu\nu}$, $\phi \to \Phi_\nu$. The central role of the lagrangian density ($\mathcal{L}$) is here played by a quasi-action $\mathcal{Q}$ whose (dimensionless) terms are recognizable analogues of those appearing in $\mathcal{L}$. The conceptual simplifications accruing from this novel approach include the unification of the Higgs with the gauge bosons, the unification of the boson field intensities with the conserved charges, and the disappearance of unphysical gauge modes of freedom and FP ghosts.

By subsuming the Poincare group into cones upon which the $SO(3,1)$ Lorentz group is realised as an additional internal symmetry, our approach is incompatible with SUSY. It therefore predicts that no superpartners exist.

References

[1] H.Fritsch, P.Minkowski, "Unified interactions of leptons and hadrons", *Ann. Phys.* **93**, 193-266 (1975)