

Who needs wave equations?

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6 December 2018

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Abstract: This paper further explores a dual interpretation of the Uncertainty Principle as applied to the classical Rutherford-Bohr calculations of the geometry of the hydrogen electron orbitals. It shows the Rutherford-Bohr model has some advantages over the quantum-mechanical model (Schrödinger's equation for the hydrogen atom). As such, it basically continues a development started in my previous paper (<http://vixra.org/abs/1812.0028>). What is novel in this paper is the exploration of the mathematical equivalence between both models.

Keywords: Bohr model, photon model, Uncertainty Principle, rest matter oscillation, electron orbitals, wavefunction interpretations.

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Who needs wave equations?

Introduction

In one of his introductory *Lectures* on electrodynamics (*Lectures*, Volume II, Chapter 5), Feynman briefly discusses the Rutherford-Bohr model of an atom. He duly notes the model's key advantage over the preceding static models (the electrons are kept from falling in toward the nucleus by their orbital motion), but then dismisses it based on the usual objection: "With such motion, the electrons would be accelerating (because of the circular motion) and would, therefore, be radiating energy. They would lose the kinetic energy required to stay in orbit and would spiral in toward the nucleus." He then sums up the quantum-mechanical model of an atom as follows:

"The electrostatic forces pull the electron as close to the nucleus as possible, but the electron is compelled to stay spread out in space over a distance given by the Uncertainty Principle. If it were confined in too small a space, it would have a great uncertainty in momentum. But that means it would have a high expected energy—which it would use to escape from the electrical attraction. The net result is an electrical equilibrium not too different from the idea of Thompson—only is it the negative charge that is spread out, because the mass of the electron is so much smaller than the mass of the proton."

This explanation is a bit sloppy, and one has to patiently wait for Feynman to introduce Schrödinger's equation and the related derivation of the electron orbitals to get the following clarification:

"The wave function $\psi(r)$ for an electron in an atom does not describe a smeared-out electron with a smooth charge density. The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge." (Feynman's *Lectures*, III-21-4)

This leaves us somewhat bewildered, because it is not clear at all how this quantum-mechanical picture is supposed to solve the radiation problem! Indeed, if the pointlike charge is sometimes here, and sometimes there, then it must – logically – also go from here to there once in a while, and then it should generate some electromagnetic radiation too. In addition, the quantum-mechanical model comes with its own set of interpretation problems. The solutions for the spherical shells, for example, imply that the electron spends most of its time *in* the nucleus itself – which is a solution we would probably want to avoid. Most importantly, the quantum-mechanical picture feels like a *deus ex machina*: we have a wave equation – and the solutions to it – but we have no idea what those solutions actually *are*. There is no mechanism. In short, there is no *physical* explanation, really.

We, therefore, want to re-examine the Rutherford-Bohr model but add some whistles and bells to address its perceived shortcomings.

The Rutherford-Bohr model

In our previous papers, we developed a physical model of an electron in free space. We want to combine these with an augmented Bohr model of an atom.¹ The illustrations below depict the hypothetical geometry of the two situations – which can be described by the same mathematical object (the elementary wavefunction – Euler’s function, in other words) but are very different.

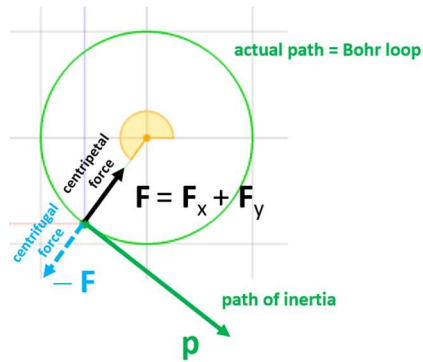


Figure 1: The position, force and momentum vector in a Bohr loop

The Bohr model has a positively charged nucleus at its center and its electron has an effective rest mass: the radial velocity $v = a \cdot \omega$ of the electron is, therefore, some *fraction* of the speed of light ($v = \alpha \cdot c$). It also has some non-zero momentum $p = m \cdot v$ which we can relate to the electrostatic centripetal force using the simple classical formula $F = p \cdot \omega = mv^2/a$. In contrast, the model of an electron in free space is based on the presumed *Zitterbewegung*, which combines the idea of a very high-frequency circulatory motion with the idea of a pointlike *charge* which – importantly – has *no inertia* and can, therefore, move at the speed of light ($v = c$).

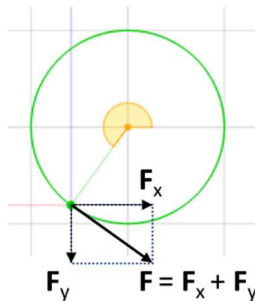


Figure 2: The *Zitterbewegung* model of a free electron

The center of the *Zitterbewegung* is plain nothingness and we must, therefore, assume some two-dimensional oscillation makes the charge go round and round. The angular frequency of the *Zitterbewegung* rotation is given by the Planck-Einstein relation ($\omega = E/\hbar$) and we get the *Zitterbewegung* radius (which is just the Compton radius $a = r_c = \hbar/mc$) by equating the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equation.² The energy and, therefore, the (equivalent) mass is in the oscillation and we,

¹ Jean Louis Van Belle, *Bohr’s Atom, the Photon, and the Uncertainty Principle*, 3 December 2018 (<http://vixra.org/pdf/1812.0028v1.pdf>).

² Jean Louis Van Belle, *The Metaphysics of Physics*, 30 November 2018 (<http://vixra.org/pdf/1811.0399v3.pdf>).

therefore, should associate the momentum $p = E/c$ with the electron as a whole or, if we would really like to associate it with a single mathematical *point* in space, with the center of the oscillation – as opposed to the rotating massless *charge*.

We should note that the distinction between the pointlike *charge* and the electron is subtle, perhaps, but essential. The electron is the *Zitterbewegung* as a whole: the pointlike charge has no rest mass, but the electron as a whole does. In fact, that is the whole point of our *Zitterbewegung* model: we explain the rest mass of an electron by introducing a *rest matter oscillation*. The model cannot be verified because of the extreme frequency ($f_e = \omega_e/2\pi = E/h \approx 0.123 \times 10^{-21}$ Hz) and sub-atomic scale ($a = r_C = \hbar/mc \approx 386 \times 10^{-15}$ m). It is, therefore, a *logical* model only: it gives us the right values for the angular momentum ($L = \hbar/2$), the magnetic moment ($\mu = (q_e/2m) \cdot \hbar$), and the gyromagnetic factor ($g = 2$).

The formulas in the Bohr-Rutherford model are derived from the quantum-mechanical that angular momentum comes in units of $\hbar = h/2\pi$. We rephrased that rule as: physical action comes in unit of h . We also associated Planck's quantum of action with a *cycle*: one rotation will pack some energy over some time (the cycle time) or – what amounts to the same – some momentum over some distance (the circumference of the loop). We wrote:

$$S = h = E \cdot T = L \cdot 2\pi \cdot r_B$$

Using the $v = \alpha \cdot c$ and $r_C = \alpha \cdot r_B$ relations³ one can easily verify this for the momentum formulation:

$$S = p \cdot 2\pi \cdot r_B = mv \cdot (r_C/\alpha) = m\alpha c \cdot \frac{2\pi\hbar}{\alpha mc} = h$$

We can also calculate S by calculating the force and then multiply the force with the distance and the time. The force is just the (centripetal) electrostatic force between the charge and the nucleus

$$F = \frac{q_e^2}{4\pi\epsilon_0 r_B^2} = \alpha \cdot \frac{\hbar c}{r_B^2}$$

We can then recalculate S as:

$$S = F \cdot r_B \cdot T = \alpha \cdot \frac{\hbar c}{r_B^2} \cdot r_B \cdot \frac{2\pi r_B}{v} = \alpha \cdot \frac{\hbar c}{\alpha c} = h$$

All is consistent. However, we should note the implied energy concept is somewhat surprising:

$$S = h = E \cdot T = E \cdot \frac{2\pi r_B}{v} = E \cdot \frac{\hbar}{\alpha mc} \Leftrightarrow E = \alpha^2 mc^2$$

This is *twice* the ionization energy of hydrogen ($Ry = \alpha^2 mc^2/2$), and it is also *twice* the kinetic energy ($\hbar^2/2ma^2 = \alpha^2 mc^2/2$). It is also just a fraction ($\alpha^2 \approx 0.00005325$) of the rest energy of the electron.⁴ This

³ These relations come out of the model. They are, therefore, not some new hypothesis. The α in the formula is the fine-structure constant. It pops up in (almost) all of the equations we get. As such, it does appear as some magical dimensionless number that relates almost all (physical) dimensions of the electron (radii, circumferences, energies, momenta, etcetera).

⁴ The reader can check the conversion of the Rydberg energy in terms of the fine-structure constant and the rest mass (or rest energy) of the electron.

somewhat odd result can be explained if we would actually be thinking of a two-dimensional oscillation here. In that case, we would effectively write the force as $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ (as suggested in the illustration above) in a moment) and, hence, we should therefore *add* the kinetic and potential energy of *two* oscillators. As the reader might simply switch off now (read: stop reading), we would like to redraw his or her attention to an alternative view of Schrödinger's equation. Before we do, however, we would like to sum up the key results in regard to the values we got for the mentioned quantum-mechanical qualities of the electron – as a free particle (or a *spin-only* electron, we might say) in the *Zitterbewegung* model or, alternatively, as an electron in a Bohr orbital. We should mention that the calculated values for the angular momentum and the magnetic moment for a Bohr orbital assume *one electron only*. They are, therefore, the angular momentum and the magnetic moment of a theoretical *no-spin* electron in the 1st, 2nd, 3rd, etc. orbital of the hydrogen atom.⁵

The model of a two-dimensional oscillator is inherent in both the *zbw* model as well as the Bohr model of the electron orbitals. Now, the professional physicist may reluctantly want to envisage such oscillator model but he or she will – or should! – immediately ask: what is the propagation mechanism here? If the real and imaginary part of the wavefunction are to be interpreted as field vectors, how do they propagate? We will address this in the next section.

The equivalence with Schrödinger's equation

The *Zitterbewegung* model and the Bohr model are very different but complementary at the same time – as we will try to show here by exploring the idea of Schrödinger's equation as a diffusion equation. Feynman (III-16-1) summarized that idea as follows:

“We can think of Schrödinger's equation as describing the diffusion of the probability amplitude from one point to the next. [...] But the imaginary coefficient in front of the derivative makes the behavior completely different from the ordinary diffusion such as you would have for a gas spreading out along a thin tube. Ordinary diffusion gives rise to real exponential solutions, whereas the solutions of Schrödinger's equation are complex waves.”

We would like to think the behavior is different – but not all that different. Schrödinger's equation is one equation but, because the wavefunction is complex, one gets two equations for the price of one, and we may, therefore, try to compare them to Maxwell's equations in free space (no charges, no currents). Note that we will use the Schrödinger equation in free space (no potential) to not complicate matters for the time being.⁶

$$\frac{\partial \psi}{\partial t} = i \frac{\hbar}{2m} \nabla^2 \psi$$

⁵ For the detailed calculations, see: Jean Louis Van Belle, *The Metaphysics of Physics*, 30 November 2018 (<http://vixra.org/pdf/1811.0399v3.pdf>).

⁶ The potential only transforms linear motion into circular motion and, hence, we do not need to consider it when focusing on the propagation mechanism only.

We get the following schema⁷:

$$\begin{array}{c}
 \text{Re}(\partial\psi/\partial t) = -(1/2) \cdot (\hbar/m_{\text{eff}}) \cdot \text{Im}(\nabla^2\psi) \\
 \text{Im}(\partial\psi/\partial t) = (1/2) \cdot (\hbar/m_{\text{eff}}) \cdot \text{Re}(\nabla^2\psi) \\
 \partial\mathbf{B}/\partial t = -\nabla \times \mathbf{E} \\
 \partial\mathbf{E}/\partial t = c^2 \nabla \times \mathbf{B}
 \end{array}$$

Figure 3: Propagation mechanisms

The red arrows visualize the propagation mechanism. For an electromagnetic wave, we have Maxwell's equations in free space (no charges, no potential): a changing electric field ($\partial\mathbf{E}/\partial t$) will cause some (infinitesimal) circulation of \mathbf{E} , so that is some curl ($\nabla \times \mathbf{E}$), and that causes a change in the magnetic field ($\partial\mathbf{B}/\partial t$), and so we have some curl of a magnetic field ($\nabla \times \mathbf{B}$), and so that is equivalent to some $\partial\mathbf{E}/\partial t$ again. The mechanism for Schrödinger's equations is different, of course. A curl operator ($\nabla \times$) is not a Laplacian (∇^2). So, let us just get through it. How should we think of a Laplacian operator with the real and imaginary 'operators' $\text{Re}(z)$ and $\text{Im}(z)$? Laplacians pop up in energy diffusion equations, like the heat diffusion equation, which is easy to represent geometrically. However, here we have cyclical functions. Let us do the calculations for the elementary wavefunction $ae^{i\theta} = ae^{i(kx - \omega t)}$:

$$\begin{aligned}
 \text{Re}\left(\frac{\partial\psi}{\partial t}\right) &= \text{Re}(-i\omega e^{i\theta}) = \text{Re}(-i\omega(\cos\theta + i\sin\theta)) = \text{Re}(-i\omega\cos\theta - i^2\omega\sin\theta) = \omega\sin\theta \\
 \text{Im}(\nabla^2\psi) &= \text{Im}\left(\frac{\partial\left(\frac{\partial(ae^{i\theta})}{\partial x}\right)}{\partial x}\right) = \text{Im}\left(\frac{\partial(iake^{i\theta})}{\partial x}\right) = \text{Im}(-ak^2e^{i\theta}) = \text{Im}(-ak^2(\cos\theta + i\sin\theta)) = -ak^2\sin\theta \\
 \text{Im}\left(\frac{\partial\psi}{\partial t}\right) &= \text{Im}(-i\omega e^{i\theta}) = \text{Im}(-i\omega(\cos\theta + i\sin\theta)) = \text{Im}(-i\omega\cos\theta - i^2\omega\sin\theta) = -i\omega\cos\theta \\
 \text{Re}(\nabla^2\psi) &= \text{Re}\left(\frac{\partial\left(\frac{\partial(ae^{i\theta})}{\partial x}\right)}{\partial x}\right) = \text{Re}\left(\frac{\partial(iake^{i\theta})}{\partial x}\right) = \text{Re}(-ak^2e^{i\theta}) = \text{Re}(-ak^2(\cos\theta + i\sin\theta)) = -ak^2\cos\theta
 \end{aligned}$$

What can we do with this? If we substitute the *de Broglie* relations ($E = \hbar \cdot \omega$ and $p = \hbar \cdot k$), we can get the energy concept that is used in Schrödinger's equation – which is just kinetic energy:

$$\text{Re}\left(\frac{\partial\psi}{\partial t}\right) = -\frac{\hbar}{2m} \text{Im}(\nabla^2\psi) \Leftrightarrow \omega\sin\theta = \frac{\hbar}{2m} ak^2\sin\theta \Leftrightarrow \frac{\hbar}{2m} k^2 = \omega \Leftrightarrow \frac{\hbar}{2m} \frac{p^2}{\hbar^2} = \frac{E}{\hbar} \Leftrightarrow E = \frac{p^2}{2m}$$

Substituting once more ($\omega = E/\hbar = (p^2/2m)/\hbar$ and $p = \hbar \cdot k$), we get the *equivalent* two-equations expression of the Schrödinger equation once again:

⁷ Two complex numbers are $a+ib$ and $c+id$ are equal if their real and imaginary parts are equal. One also needs to use the $i(c+id)=ic+i^2d = -d+ic$ equality here.

$$\begin{aligned} \operatorname{Re}\left(\frac{\partial\psi}{\partial t}\right) &= a\omega\sin\theta = a\frac{p^2}{2m}\frac{1}{\hbar}\sin\theta = a\frac{k^2\hbar^2}{2m\hbar}\sin\theta = -\frac{\hbar}{2m}\operatorname{Im}(\nabla^2\psi) \\ \operatorname{Im}\left(\frac{\partial\psi}{\partial t}\right) &= a\omega\cos\theta = a\frac{p^2}{2m}\frac{1}{\hbar}\cos\theta = a\frac{k^2\hbar^2}{2m\hbar}\cos\theta = \frac{\hbar}{2m}\operatorname{Re}(\nabla^2\psi) \end{aligned}$$

However, we should note that we have to be consistent here: the energy E in the $E = \hbar\cdot\omega$ expression *excludes* the rest energy. It must because, if we would use the $E = mc^2$ equations, the equations become contradictory:

$$E = mc^2 \neq \frac{p^2}{2m} = \frac{m^2v^2}{2m} = \frac{mv^2}{2}$$

Note that the $1/2$ factor is there even for $v = c$. Hence, if we would want Schrödinger's equation to also fit the *Zitterbewegung* model, then we have to interpret half of the $E = mc^2$ as kinetic and half of it as potential – and we would, once again, consider the kinetic energy only.⁸ However, the *Zitterbewegung* model is not what we are interested in here. We will want to substitute the energy and momentum formulas we get for the Bohr atom ($E = \alpha^2mc^2/2$ and $p = \hbar\cdot k = mac$ or $k = mac/\hbar$). We get:

$$\begin{aligned} \operatorname{Re}\left(\frac{\partial\psi}{\partial t}\right) &= a\omega\sin\theta = a\frac{\alpha^2mc^2}{2\hbar}\sin\theta = a\frac{k^2\hbar}{2m}\sin\theta = -\frac{\hbar}{2m}\operatorname{Im}(\nabla^2\psi) \\ \operatorname{Im}\left(\frac{\partial\psi}{\partial t}\right) &= a\omega\cos\theta = a\frac{\alpha^2mc^2}{2\hbar}\cos\theta = a\frac{k^2\hbar}{2m}\cos\theta = \frac{\hbar}{2m}\operatorname{Re}(\nabla^2\psi) \end{aligned}$$

That is good. It had better be because the $E = \alpha^2mc^2/2$ expression is, of course also compatible with the (kinetic) energy expression: $E = p^2/2m = m^2\alpha^2c^2/2m = \alpha^2mc^2/2$. So how should we interpret this? We interpret the sine and cosine functions as an oscillation in two dimensions. The sine and cosine are, effectively, the same function but with a 90-degree phase difference. The *kinetic* energy of the first oscillator is equal to $a^2\omega^2\sin^2\theta$ and, hence, the energy *transfer* from this oscillator to the other is given by:

$$2a^2\omega^2\cdot\sin\theta\cdot d(\sin\theta)/d\theta = 2a^2\omega^2\cdot\sin\theta\cdot\cos\theta$$

This is absorbed by the other oscillator, whose motion is given by the $\cos\theta$ function, which is equal to $\sin(\theta+\pi/2)$. Hence, its kinetic energy is equal to $a^2\omega^2\cos^2\theta = \sin^2(\theta+\pi/2)$, and how it *changes* – as a function of θ – will be equal to:

$$2a^2\omega^2\cdot\sin(\theta+\pi/2)\cdot\cos(\theta+\pi/2) = -2a^2\omega^2\cdot\cos\theta\cdot\sin\theta$$

All that remains to be done is to simplify the $2a^2\omega^2$ by substituting a for the Bohr radius radius ($a = r_B$) and ω for the angular frequency of the rotation ($\omega = v/r_B$). We get the following:

$$a^2\omega^2 = r_B^2\frac{v^2}{r_B^2} = \alpha^2c^2$$

⁸ This may sound phony to the reader and, to some extent, it effectively is: the Schrödinger equation is non-relativistic and should, therefore, not be used in the context of the *Zitterbewegung* model. It should also *not* be used to model a photon. In short, Schrödinger's equation assumes a non-relativistic tangential velocity.

We get the following equation: $E = \alpha^2 \cdot m \cdot c^2 = m \cdot a^2 \cdot \omega^2$. All is wonderfully consistent. The total energy that is stored in the system is the sum of the kinetic and potential energies of the two oscillators:

$$ma^2\omega^2 = \frac{ma^2\omega^2}{2} + \frac{ma^2\omega^2}{2} = \alpha^2 mc^2$$

What about the argument of the wavefunction? It must have a consistent interpretation too. It has. The model circular model implies that the position and momentum vector are always at right angles to each other. Hence, the vector dot product $\mathbf{p} \cdot \mathbf{x}$ disappears, and the argument of the wavefunction therefore reduces to $(E/\hbar) \cdot t = \omega \cdot t$.⁹ In fact, even if the $\mathbf{p} \cdot \mathbf{x}$ product would not disappear, we have an invariant four-vector product here that we can re-write as $E' \cdot t'$ (the product of the energy and time in the stationary reference frame) and because velocities are non-relativistic here, t and t' will not be (very) different. Hence, the $ae^{i\theta} = ae^{i(kx-\omega t)}$ wavefunction just gives us where the electron actually *is* at every point in time. There is nothing magical about it. Who needs wave equations?

The meaning of the complex conjugate

We should make one more point here. Schrödinger's equation does *not* work for the complex conjugate of the elementary wavefunction. That is an aberration. Indeed, most introductory courses on quantum mechanics will show that both $\psi = \exp(i\theta) = \exp[i(kx-\omega t)]$ and $\psi^* = \exp(-i\theta) = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)] = -\psi$ are acceptable waveforms to describe a particle that is propagating in the x -direction. Both have the required mathematical properties—as opposed to, say, some real-valued sinusoid.¹⁰ We would then think some proof should follow of why one would be better than the other or, preferably, one would expect as a discussion on what these two mathematical possibilities might represent—but, no. That does not happen. The physicists conclude that “the choice is a matter of convention and, happily, most physicists use the same convention.”¹¹ Instead of making a choice here, we could, perhaps, use the various mathematical possibilities to incorporate spin in the description, as real-life particles – think of electrons and photons here – have two spin states (*up* or *down*), as shown below.

Table 1: Matching mathematical possibilities with physical realities?¹²

Spin and direction of travel	Spin up	Spin down
Positive x-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative x-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

⁹ Even if the $\mathbf{p} \cdot \mathbf{x}$ product would not disappear, we have an invariant four-vector product here that we can re-write as $E' \cdot t'$: this is the product of the energy and time in the stationary reference frame. Because velocities are non-relativistic here, t and t' will not be (very) different.

¹⁰ The argument is based on whether or not the superposition of similar waveforms gives us a sensible composite waveform. Our formulas only give the *elementary* wavefunction. The wave *packet* will be a Fourier sum of such functions.

¹¹ See, for example, the MIT's edX Course 8.04.1x (Quantum Physics), Lecture Notes, Chapter 4, Section 3.

¹² Of course, the formulas only give the *elementary* wavefunction. The wave *packet* will be a Fourier sum of such functions.

This seems to make sense. Theoretical spin-zero particles do not exist and we should therefore, perhaps, use the extra degree of freedom in the mathematical description to describe the spin state of our particle. An important added benefit here is that the weird 720-degree symmetry of spin-1/2 particles collapses into an ordinary 360-degree symmetry and that we would, therefore, have no need, perhaps, to describe them using spinors and other complicated mathematical objects. We have written about this at length elsewhere¹³ and so we won't repeat ourselves here.

By now, the reader might wonder: what is the point? The point is: we do not need the Schrödinger equation to construct a hydrogen atom model. It does not *explain* anything. The Rutherford-Bohr model does. Let us, therefore, now return to it.

How to understand the atom

The Rutherford-Bohr model gave us the following formulas¹⁴:

Table 2: Matching mathematical possibilities with physical realities?

Spin-only electron (free electron)	Orbital electron (Bohr atom)
$\mu = I \cdot \pi r_C^2 = \frac{q_e}{2m} \hbar$	$\mu = I \cdot \pi r_B^2 = n \cdot \frac{q_e}{2m} \hbar$
$L = I \cdot \omega = \frac{\hbar}{2}$	$L = I \cdot \omega = n \cdot \hbar$
$g = \frac{2m \mu}{q_e L} = 2$	$g = \frac{2m \mu}{q_e L} = 1$

The n is the orbital number, of course. Note that the 1/2 factor in the angular momentum (L) is due to the form factor – which is inherent to the very different model (*Zitterbewegung* – or rest matter oscillation – versus Bohr orbitals). Also note that we assume that the Bohr atom has one electron only (the solutions to Schrödinger's equation also model a hydrogen atom only).

Now we need to combine the magnetic moments from the spin and orbital angular momentum. The magnetic moment will be measured in units of the Bohr magneton: $\mu_B = q_e \hbar / 2m$. Students are often confused here, because the electron is said to be a spin-1/2 particle. The table above shows the free electron will, effectively, have an angular momentum that is equal to $\pm \hbar/2$, but its magnetic moment will be equal to the Bohr magneton itself. That is because the gyromagnetic factor is equal to 2. Hence, in an *external* magnetic field (B), it will get an energy that is equal to:

$$\mu = -\mu_B \cdot B$$

¹³ Jean Louis Van Belle, *Euler's wavefunction* (<http://vixra.org/abs/1810.0339>, accessed on 30 October 2018)

¹⁴ The I and the I in the equations stand for the current and the moment of inertia respectively. These are two very different concepts but we did not want to invent new symbols.

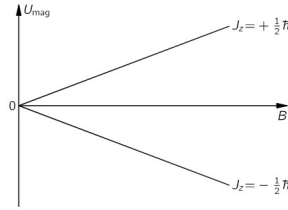


Figure 4: Spin values and magnetic energy of an electron¹⁵

What about a Bohr atom with one electron? Here things are very interesting because we now need to *combine* the spin and orbital angular momentum somehow. We have three possibilities here, as shown below.

Orbital angular momentum, l	z -component, m
0	0
1	+1
	0
	-1

Figure 5: Combining orbital angular momentum with spin angular momentum¹⁶

We took a screenshot from an actual physics book, which refers to the spin angular momentum as the z -component, to show that the Rutherford-Bohr model does incorporate the same quantum numbers as the Schrödinger model. The question, of course, is: how can the spin angular momentum be zero? The answer is: it could, possibly, be orthogonal to the magnetic field.

If this is how it goes, how do we explain a silver atom has only spin 1/2? Some think the magnetic moment of the various orbitals will cancel each other out because of their respective orientation. That is a little bit vague. Let me quote a more precise explanation: “There are 47 electrons surrounding the silver atom nucleus, of which 46 form a closed inner core of total angular momentum zero – there is no orbital angular momentum, and the electrons with opposite spins pair off, so the total angular momentum is zero, and hence there is no magnetic moment due to the core. The one remaining electron also has zero orbital angular momentum, so the sole source of any magnetic moment is that due to the intrinsic spin of the electron.”¹⁷

We can combine angular and spin orbital momentum in various ways. For example, the ground state of a helium atom (which has two electrons) will actually be that of a spin-zero particle. This is, obviously, because both the orbital as well as the spin orbital angular momenta (and the associated magnetic moments) for the *two* electrons cancel each other out.

¹⁵ The illustrations in this section come out of Feynman’s *Lectures* (II-34 and III-19).

¹⁶ The illustrations in this section come out of Feynman’s *Lectures* (II-34 and III-19).

¹⁷ See: J D Cresser, 2009, <http://physics.mq.edu.au/~jcresser/Phys301/Chapters/Chapter6.pdf>.

Conclusions

This was a lot of fun. The only objective of this paper is to show that we actually do not need symmetric and anti-symmetric wavefunctions or complicated wave equations to explain electron orbitals and the associated atomic behavior. The Rutherford-Bohr model is simple, intuitive and – most importantly – it explains *how* things work – rather than just *describe* them.

Jean Louis Van Belle, 6 December 2018

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to

1. Feynman's *Lectures on Physics* (<http://www.feynmanlectures.caltech.edu>). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.
4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, <https://arxiv.org/pdf/0802.2728.pdf>.
5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, <https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf>

The illustrations in this paper are open source and have been *augmented* by the author. References and credits – including credits for open-source Wikipedia authors – have been added in the text.

One reference that has not been mentioned in the text is:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

The latter work is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: “Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research.” (p. 1-10)

Last but not least, we also mentioned the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (<https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html>). However, we have not gone into the nitty-gritty of their work and, therefore, do not want to pretend we have.