Bohr's atom, the photon and the [Un]Certainty Principle

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Abstract: This is a didactic exploration of a possible dual interpretation of the Uncertainty Principle as applied to the classical Rutherford-Bohr calculations of the geometry of the hydrogen electron orbitals. It highlights, in particular, a classical mistake in regard to the interpretation of atoms as atomic oscillators – and the calculation of their Q. It also offers a substantial correction to the model of a photon that was presented in a previous paper.

Keywords: Bohr model, photon model, Uncertainty Principle, rest matter oscillation, electron orbitals, wavefunction interpretations.

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Introduction

In one of his introductory *Lectures* on electrodynamics (*Lectures*, Volume II, Chapter 5), Feynman briefly discusses the Rutherford-Bohr model of an atom. He duly notes the model's key advantage over the preceding static models (the electrons are kept from falling in toward the nucleus by their orbital motion), but then dismisses it based on the usual objection: "With such motion, the electrons would be accelerating (because of the circular motion) and would, therefore, be radiating energy. They would lose the kinetic energy required to stay in orbit and would spiral in toward the nucleus." He then sums up the quantum-mechanical model of an atom as follows:

> "The electrostatic forces pull the electron as close to the nucleus as possible, but the electron is compelled to stay spread out in space over a distance given by the Uncertainty Principle. If it were confined in too small a space, it would have a great uncertainty in momentum. But that means it would have a high expected energy—which it would use to escape from the electrical attraction. The net result is an electrical equilibrium not too different from the idea of Thompson—only is it the negative charge that is spread out, because the mass of the electron is so much smaller than the mass of the proton."

This explanation is a bit sloppy, and one has to patiently wait for Feynman to introduce Schrödinger's equation and the related derivation of the electron orbitals to get the following clarification:

> "The wave function $\psi(\boldsymbol{r})$ for an electron in an atom does not describe a smeared-out electron with a smooth charge density. The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge." (Feynman's *Lectures*, III-21-4)

This leaves us somewhat bewildered, because it is not clear at all how this quantum-mechanical picture is supposed to solve the radiation problem! Indeed, if the pointlike charge is sometimes here, and sometimes there, then it must – logically – also go from here to there once in a while, and then it should generate some electromagnetic radiation too! Let us, therefore, re-examine Feynman's 'in-between model' of the Bohr atom, which is based on the idea that the Uncertainty Principle should compel the electron to stay spread in space. It might have some hidden advantage.

Feynman's atom

Feynman (III, 2-4) derives the Bohr radius of an atom from the momentum-space expression of the Uncertainty Principle which we may loosely state as follows: the product of the uncertainty in the momentum (Δp) and the uncertainty in the position (Δx) has an order of magnitude that is equal to Planck's quantum (h). His equation is the following:

$$\mathrm{p}\cdot a \approx \hbar \Leftrightarrow \mathrm{p} \approx \hbar/a$$

This allows him to write the kinetic energy of the electron as $\mathrm{m}v^2/2 = \mathrm{p}^2/2\mathrm{m} = \hbar^2/2\mathrm{m}a^2$. The potential energy is just the electrostatic energy $-\mathrm{e}^2/a$.[1] The idea is then that the configuration must minimize the total energy $\mathrm{E} = \hbar^2/2\mathrm{m}a^2 - \mathrm{e}^2/a$. The variable is the radius a and, hence, we get a by calculating the $d\mathrm{E}/da$ derivative and equating it to zero. We thus get the correct Bohr radius:

$$r_{\mathrm{Bohr}} = \frac{\hbar^2}{\mathrm{me}^2} = \frac{4\pi\varepsilon_0\hbar^2}{\mathrm{mq_e}^2} \approx 53\times10^{-12}\ \mathrm{m} = \frac{1}{\alpha}\cdot r_{\mathrm{Compton}}$$

We can also calculate the ionization energy of hydrogen (Rydberg) by using the Bohr radius to calculate the energy $\mathrm{E} = \hbar^2/2\mathrm{m}a^2 - \mathrm{e}^2/a$:

$$\mathrm{E_R} = \frac{1}{2}\frac{\hbar^2}{\mathrm{m}}\frac{\mathrm{m}^2\mathrm{e}^4}{\hbar^4} - \mathrm{e}^2\frac{\mathrm{me}^2}{\hbar^2} = -\frac{1}{2}\frac{\mathrm{me}^4}{\hbar^2} \approx -13.6\ \mathrm{eV}$$

Note that the Rydberg constant can be re-written in terms of the fine-structure constant and the electron energy:

$$\mathrm{E_R} = -\frac{1}{2}\frac{\mathrm{me}^4}{\hbar^2} = -\frac{1}{2}\frac{\mathrm{e}^2}{r_{\mathrm{Bohr}}} = -\frac{1}{2}\frac{\alpha\mathrm{e}^2}{r_{\mathrm{Compton}}} = -\frac{1}{2}\frac{\alpha\mathrm{m}c^2}{\hbar c}\frac{\mathrm{q_e}^2}{4\pi\varepsilon_0} = -\frac{1}{2}\alpha^2\mathrm{m}c^2$$

This amount equals the kinetic energy ($\hbar^2/2\mathrm{m}a^2 = \alpha^2\mathrm{m}c^2/2$). The electrostatic energy itself is *twice* that value ($-\mathrm{e}^2/r_{\mathrm{Bohr}} = -\alpha^2\mathrm{m}c^2$).

The argument is impeccable. The only problem is the interpretation: Feynman equates the *uncertainty* in the momentum as the momentum itself (Δp = p) and the uncertainty in the position as a radius. We offer an alternative interpretation. If Planck's constant is, effectively, a physical constant ($h \approx 6.626\times10^{-34}$ N · m · s), then we should interpret it as such. If physical action – some force over some distance over some time – comes in units of h, then the relevant distance here is the loop, so that is $2\pi\cdot r_{\mathrm{Bohr}}$. We would, therefore, like to re-write Feynman's $\mathrm{p}\cdot a \approx \hbar$ assumption as:

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[1] The e^2 in this formula is the squared charge of an electron ($\mathrm{q_e}^2$) divided by the electric constant ($4\pi\varepsilon_0$). The formula assumes the potential is zero when the distance between the positively charged nucleus and the electron is infinite, which explains the minus sign. We also get the minus sign, of course, by noting the two charges (electron and nucleus) have equal magnitude but opposite sign. One should note that the formulas are non-relativistic. This is justified by the fact that the velocities in this model are non-relativistic (the electron velocity in the Bohr orbital is given by $v_e = \alpha\cdot c \approx 0.0073\cdot c$. This is an enormous speed but still less than 1% of the speed of light.

$$h = p \cdot 2\pi \cdot r_{Bohr} = p \cdot \lambda$$

The λ is, of course, the circumference of the loop. The equation resembles the *de Broglie* equation $\lambda = h/p$. How should we interpret this?

Planck's quantum as the (minimum) action in a cycle

Planck's quantum of action or, more generally, the concept of *physical* action, is expressed in N · m · s: force times distance times time. We know some force over some distance is energy, and force times time is momentum. Hence, we can think of action – and of the quantum of action itself – in two ways: (1) some energy over some time, (2) some momentum over some distance. The illustration below gives us the presumed geometry of the situation. The momentum $p = m \cdot v$ of the pointlike electron should make it follow the path of inertia, but the centripetal electrostatic force ensures it follows the Bohr loop instead.

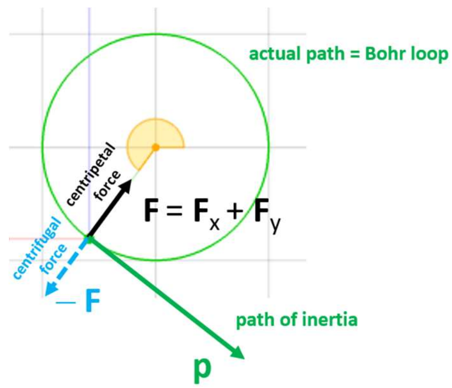

Figure 1: Different paths in spacetime

Let us do some calculations. The Bohr model gives us a classical velocity for the electron in an electron orbital: $v = v_e = \alpha \cdot c$. The ratio α is the fine-structure constant. It is a mysterious number because it also relates (1) the Bohr radius and the Compton radius (aka the reduced Compton wavelength) and (2) the Compton radius and the Lorentz radius (aka the classical electron radius). We write:

$$v_e = \alpha \cdot c \approx 0.0073 \cdot c$$

$$r_C = \alpha \cdot r_B \approx 386 \times 10^{-15}\ \text{m}$$

$$r_L = \alpha \cdot r_C = \alpha^2 \cdot r_B \approx 2.82 \times 10^{-15}\ \text{m}$$

In previous papers, we developed a *Zitterbewegung* model of a free stationary electron which gave us the Compton radius $a = r_C$ from boldly equating the $E = mc^2$, $E = m \cdot a^2 \cdot \omega^2$ and $E = \hbar \cdot \omega$ equations.[2] That model may or may not make sense but, the very

[2] See: Jean Louis Van Belle, *The Metaphysics of Physics*, 30 November 2018 (http://vixra.org/pdf/1811.0399v3.pdf).

least, it is fun to note that it is *consistent* with the classical formula for the velocity of an electron in the Bohr atom:

$$v = \alpha \cdot c = \alpha \cdot r_C \cdot \omega = \alpha \cdot \frac{\hbar}{\mathrm{m}c} \cdot \frac{\mathrm{m}c^2}{\hbar} = \alpha \cdot c$$

Let us now calculate the action in one loop:

$$S = \mathrm{p} \cdot 2\pi \cdot r_{\mathrm{B}} = \mathrm{m}v \cdot 2\pi \cdot r_{\mathrm{B}} = \mathrm{m} \cdot \alpha \cdot c \cdot 2\pi \frac{\hbar^2}{4\pi\varepsilon_0 \mathrm{m} {\mathrm{q_e}}^2} = \mathrm{m} \cdot \frac{{\mathrm{q_e}}^2}{4\pi\varepsilon_0 \hbar c} \cdot c \cdot 2\pi \frac{4\pi\varepsilon_0 \hbar^2}{\mathrm{m}{\mathrm{q_e}}^2} = h$$

This is, of course, exactly what we want it to be. We should now expect to get the Bohr orbitals for $S = 2h$, $3h$, etc. This is, of course, nothing but an *interpretation* of the quantum-mechanical angular momentum rule, which says that angular momentum should always come in units of $\hbar$:

$$S_{\mathrm{n}} = \mathrm{p_n} \cdot 2\pi \cdot a_{\mathrm{n}} = \mathrm{n} \cdot h \Leftrightarrow L_{\mathrm{n}} = \mathrm{m_e} \cdot v_{\mathrm{n}} \cdot a_{\mathrm{n}} = \mathrm{n} \cdot \hbar$$

The $\mathrm{p} \cdot a = \hbar$ identity becomes $\mathrm{p_n} \cdot a_{\mathrm{n}} = \mathrm{n} \cdot \hbar$, and the kinetic energy is, therefore, equal to $\mathrm{m}{v_{\mathrm{n}}}^2/2 = {\mathrm{p_n}}^2/2\mathrm{m} = \mathrm{n}^2\hbar^2/2\mathrm{m}{a_{\mathrm{n}}}^2$. The formula for the potential energy ($-\mathrm{e}^2/a_{\mathrm{n}}$) does not change, of course, and the $d\mathrm{E}/da_{\mathrm{n}} = 0$ condition for minimal energy now becomes:

$$\frac{d\mathrm{E}}{da_{\mathrm{n}}} = 0 \Leftrightarrow -\frac{\mathrm{n}^2\hbar^2}{\mathrm{m}{a_{\mathrm{n}}}^3} + \frac{\mathrm{e}^2}{{a_{\mathrm{n}}}^2} \Leftrightarrow a_{\mathrm{n}} = \frac{\mathrm{n}^2\hbar^2}{\mathrm{me}^2} = \mathrm{n}^2 r_{\mathrm{Bohr}} = \frac{\mathrm{n}^2}{\alpha} r_{\mathrm{Compton}} = \frac{\mathrm{n}^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$$

Let us highlight the formula for the radius of the nth Bohr orbital because we will use it quite often:

$$a_{\mathrm{n}} = \mathrm{n}^2 r_{\mathrm{Bohr}} = \frac{\mathrm{n}^2}{\alpha} r_{\mathrm{Compton}} = \frac{\mathrm{n}^2}{\alpha}\frac{\hbar}{\mathrm{m}c}$$

The calculations above are familiar[3], but their *interpretation* might not be: we substituted an uncertainty principle for an *exact* expression – some kind of *Certainty* Principle, perhaps[4]:

$$S_{\mathrm{n}} = \mathrm{n} \cdot h$$

Physical action always comes in units of $\boldsymbol{h}$.[5]

The reader might think: of course, it does ! This is just another way of stating the quantum-mechanical rule that angular momentum comes in units of $\hbar = h/2\pi$! The difference is subtle, however: the quantum-mechanical rule doesn't tell us *why* this should

[3] Note, however, that our formula gives the Bohr radius as a_1, while physicists would usually note it as a_0. We think the $a_{\mathrm{n}} = a_1$ for n = 1 notation makes more sense because we also have formulas for $\mathrm{p_n}$ and v_{n}. We are also discussing the *first* Bohr orbital here and, hence, we can just think of the concept of the zeroth orbital as zero.

[4] This poor joke explains the title of the paper. It is *very* nice that the editors of the viXra.org pre-publishing site only judge papers on sense and logic – which may or may not be the same concepts in this paper.

[5] The subtle difference may be noted: S (action) comes in units of h (the non-reduced constant), but angular momentum comes in units of $\hbar = h/2\pi$.

be so. In contrast, what we are presenting here is a physical (geometric) *interpretation* of the quantum-mechanical rule.

Currents and magnetic moments

The Bohr model associates increasing angular momentum with increasing orbital radius, but what about the magnetic moment? Let us calculate the currents. We have circular motion with a radius and a velocity. We can, therefore, calculate the frequency ($\omega = v/a$) and, hence, we get the following rather elegant formula for the current:

$$\mathrm{I}_{\mathrm{Bohr}} = \mathrm{q_e}f = \mathrm{q_e}\frac{v}{2\pi a} = \mathrm{q_e}\frac{\alpha c}{2\pi\frac{r_C}{\alpha}} = \mathrm{q_e}\frac{\alpha^2 c}{2\pi\frac{\hbar}{\mathrm{m}c}} = \mathrm{q_e}\alpha^2\frac{\mathrm{m}c^2}{h}$$

The formula is elegant because it reflects the formula for the *Zitterbewegung* (zbw) current[6]:

$$\mathrm{I}_{\mathrm{zbw}} = \mathrm{q_e}f = \mathrm{q_e}\frac{\mathrm{E}}{h} = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \approx (1.6\times10^{-19}\ \mathrm{C})\frac{8.187\times10^{-14}\ \mathrm{J}}{6.626\times10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

The only difference is the α^2 factor, and we need it – because the loop is much larger. To be precise, it is $1/\alpha \approx 137$ times larger. Now, despite the household-current (almost 2 *ampere*), we found a consistent result when calculating the magnetic moment for the *zbw* electron (read: the stationary electron in free space). To be specific, we calculated the magnetic moment as the current times the area of the loop (πa^2), and we got the following elegant result:

$$\mu = \mathrm{I}\cdot\pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h}\cdot\pi a^2 = \mathrm{q_e}c\frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

The result is elegant because it gave us the correct gyromagnetic ratio (g = 2) for the *pure* spin moment of an electron. What do we get for the Bohr orbital? Let us first calculate the magnetic moment.

$$\mu = \mathrm{I}\cdot\pi a^2 = \mathrm{q_e}\alpha^2\frac{\mathrm{m}c^2}{h}\cdot\pi(\frac{r_C}{\alpha})^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{2\pi\hbar}\frac{\pi\hbar^2}{\mathrm{m}^2c^2} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

This is exactly the same. What do we get for the gyromagnetic ratio? To calculate the g-factor, we must use the right formula for the angular momentum. The *zbw* model assumed that the effective mass of the electron was spread over a circular disk and we, therefore, used the 1/2 form factor for the moment of inertia (I).[7] Here we can just use the formula for a rotating point mass: $I = \mathrm{m}r^2$. We get the following result:

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[6] See the reference above (http://vixra.org/pdf/1811.0399v3.pdf).

[7] Symbols may be confusing. For example, I refers to the current, but I refers to the moment of inertia. Likewise, E refers to energy, but E may also refer to the magnitude of the electric force. We could have introduced new symbols but the context should make clear what we are talking about. We also try to use italics consistently. Note that bold letters (***F*** versus *F*, for example) will usually denote a vector, i.e. a quantity with a magnitude (F) and a direction.

$$L = I \cdot \omega = ma^2 \cdot \frac{v}{a} = m \cdot a \cdot v = \hbar$$

We now get the correct g-factor for the *orbital* spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \Leftrightarrow \mathrm{g} = 1$$

Problem solved ! But can we generalize this result for $S_n = n \cdot h$? In other words, do we get the right values for the other orbitals? To calculate the current, we will need a formula for the velocity, which we get as follows:

$$v_\mathrm{n} = \frac{\mathrm{p_n}}{\mathrm{m}} = \frac{\mathrm{n}\hbar}{a_\mathrm{n}\mathrm{m}} = \frac{\mathrm{n}\hbar}{\mathrm{n}^2 a_1 \mathrm{m}} = \frac{1}{\mathrm{n}}\frac{\hbar}{a_1\mathrm{m}} = \frac{1}{\mathrm{n}}v_1$$

Let us now re-calculate the *Bohr current*:

$$\mathrm{I_{Bohr}} = \mathrm{q_e}f = \mathrm{q_e}\frac{v}{2\pi a} = \frac{1}{\mathrm{n}^3}\mathrm{q_e}\frac{v_1}{2\pi a_1} = \frac{1}{\mathrm{n}^3}\cdot \mathrm{q_e}\alpha^2\frac{\mathrm{m}c^2}{h}$$

The magnetic moment is then going to be equal to:

$$\mu = \mathrm{I}\cdot\pi a^2 = \frac{1}{\mathrm{n}^3}\cdot\mathrm{q_e}\alpha^2\frac{\mathrm{m}c^2}{h}\cdot\pi(\mathrm{n}^2\frac{r_C}{\alpha})^2 = \mathrm{n}\cdot\mathrm{q_e}\frac{\mathrm{m}c^2}{2\pi\hbar}\frac{\pi\hbar^2}{\mathrm{m}^2c^2} = \mathrm{n}\cdot\frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

Likewise, we get the following value for the angular momentum:

$$L = I\cdot\omega = ma_\mathrm{n}{}^2\cdot\frac{v_\mathrm{n}}{a_\mathrm{n}} = m\cdot a_\mathrm{n}\cdot v_\mathrm{n} = \frac{\mathrm{n}^2}{\mathrm{n}}m\cdot a_1\cdot v_1 = \mathrm{n}\cdot\hbar$$

Hence, both the magnetic moment and the angular momentum get multiplied by *n* but the gyromagnetic ratio remains the same:

$$\mathrm{g} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mu}{\mathrm{L}} = \frac{2\mathrm{m}}{\mathrm{q_e}}\frac{\mathrm{n}\cdot\frac{\mathrm{q_e}}{2\mathrm{m}}\hbar}{\mathrm{n}\cdot\hbar} = 1$$

Some more calculations

The *Zitterbewegung* interpretation of an electron[8] combines the idea of motion with the idea of a pointlike *charge*, which has no inertia and can, therefore, move at the speed of

[8] Erwin Schrödinger derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this

light. The illustration below described the presumed circular oscillatory motion of the charge (the *Zitterbewegung*). We got wonderful results. The most spectacular result is the explanation for the *rest* mass of an electron: it is the equivalent mass of what we referred to as the *rest matter oscillation*.

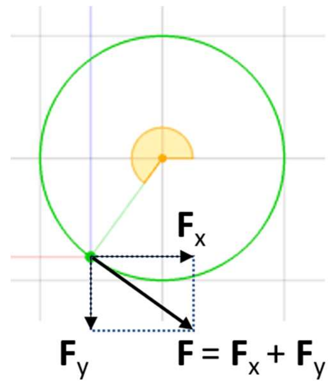

Figure 2: The *Zitterbewegung* model of an electron

We also got all of the properties above – angular momentum, magnetic moment, g-factor, etc. – but we got them for the stationary electron in free space. Indeed, the reader should keep his wits about him[9] here: the *Zitterbewegung* model should not be confused with our Bohr atom. We do not have any centripetal force here. There is no nucleus or other charge at the center of the *Zitterbewegung*. Instead of a tangential momentum vector, we have a tangential *force* vector (**F**), which we thought of as being the resultant force of two perpendicular oscillations.[10] This led us to boldly equate the $E = mc^2$, $E = m \cdot a^2 \cdot \omega^2$ and $E = \hbar \cdot \omega$ equations – which gave us all the results we wanted. The *zbw* model – which, as we have mentioned in the footnote above, is inspired by the solution(s) for Dirac's wave equation for free electrons – tells us the velocity of the pointlike *charge* is equal to *c*. Hence, *if* the *zbw* frequency would be given by Planck's energy-frequency relation ($\omega = E/\hbar$), *then* we can easily combine Einstein's $E = mc^2$ formula with the radial velocity formula ($c = a \cdot \omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$r_{\text{Compton}} = \frac{\hbar}{mc} = \frac{\lambda_e}{2\pi} \approx 0.386 \times 10^{-12}\ \text{m}$$

The oscillator model allowed us to calculate the force. Indeed, because the energy in the oscillator has to be equal to the magnitude of the force times the length of the loop, we could calculate the magnitude of the force as follows:

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consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

[9] The him could be a her, of course.

[10] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

$$E = F\lambda_e \Leftrightarrow F = \frac{E}{\lambda_e} \approx \frac{8.187 \times 10^{-14}\ \text{J}}{2.246\ \times 10^{-12}\ \text{m}} \approx 3.3743 \times 10^{-2}\ \text{N}$$

It is a rather enormous force in light of the sub-atomic scale – the distance scale is actually *less* than sub-atomic – but is consistent with the model. More in particular, we found that the action per cycle is equal to:

$$S_{\text{Compton}} = F \cdot \lambda_C \cdot T = \frac{E}{\lambda_C} \cdot \lambda_C \cdot \frac{h}{E} = h$$

This confirms our hypothesis: **(physical) action always comes in units of *h*.**

We refer to our paper[11] for the other calculations (angular momentum, magnetic moment and g-factor). The question here is: can we do similar calculations for the *force* in the Bohr model? Of course, we can. In fact, the value of the centripetal force (F) is easy to calculate. It is just the electrostatic force:

$$F = \frac{{q_e}^2}{4\pi\varepsilon_0 a^2}$$

Now, in models with a centripetal force, we have the following relation between a centripetal force F, a mass m, a radius r, and an angular frequency ω:

$$F = m \cdot v \cdot \omega = m \cdot r \cdot \omega^2 = p \cdot \omega \Leftrightarrow p = \frac{F}{\omega}$$

Hence, we can write the action also as:

$$S_{\text{Bohr}} = p \cdot 2\pi a = \frac{F}{\omega} \cdot 2\pi a = \frac{{q_e}^2}{4\pi\varepsilon_0 a^2} \cdot \frac{2\pi a}{\omega} = \frac{{q_e}^2}{4\pi\varepsilon_0} \cdot \frac{2\pi}{v} = \frac{{q_e}^2}{4\pi\varepsilon_0} \cdot \frac{2\pi}{\alpha c} = \frac{{q_e}^2}{4\pi\varepsilon_0} \cdot \frac{4\pi\varepsilon_0 \hbar}{{q_e}^2} 2\pi = h$$

All is consistent. No surprises. Let us now write the action as the product of energy and time. When we do so, we only get a sensible result when our energy concept is equal to $\alpha^2 mc^2$, as is evidenced from the calculation below:

$$S = h = E \cdot T = E \cdot \frac{2\pi a}{v} = E \cdot \frac{\frac{h}{\alpha mc}}{v}$$

$$\Leftrightarrow E = \alpha m v c = \alpha^2 m c^2$$

This is *twice* the ionization energy of hydrogen (Rydberg), and it is also *twice* the kinetic energy ($\hbar^2/2ma^2 = \alpha^2 mc^2/2$). This is a somewhat odd result and, hence, it requires some interpretation. We will come back to this. The general idea is that we are actually looking at some two-dimensional oscillation here (we will write the force as $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$ in a moment) and that we should, therefore, add the kinetic and potential energy of *two* oscillators. Note that we can write $E = \alpha^2 mc^2$ as the product of the force over the radius, as shown below[12]:

[11] See the reference above (http://vixra.org/pdf/1811.0399v3.pdf)

[12] The calculation uses the formula for the fine-structure constant, which we get from the $\alpha = r_C/r_B$ ratio.

$$F \cdot a = \frac{q_e^2}{4\pi\varepsilon_0 a^2} \cdot a = \frac{q_e^2}{4\pi\varepsilon_0 \hbar c} \cdot \frac{\hbar c}{a} = \alpha \cdot \frac{\alpha\hbar m c^2}{\hbar} = \alpha^2 m c^2$$

The interesting thing here is that, in order to be consistent, we should apparently calculate the energy as the force over the *radius*. This is different from our *Zitterbewegung* model, where we took the force over the loop. This has to do with the direction of the force, which is centripetal in the Bohr model, as shown below.

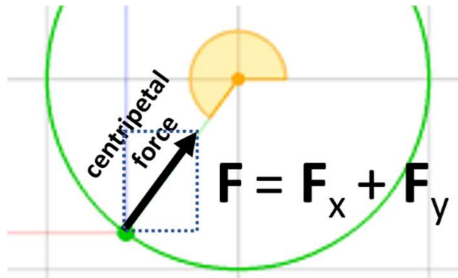


Figure 3: The oscillator model of the Bohr orbital

Having said that, we could think of the Bohr model in terms of a two-dimensional oscillation. We would then look at the centripetal force as a *resultant* force[13]:

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

Needless to say, the **boldface** here indicates *vectors*: objects with a magnitude as well as a direction. We can now use the elementary wavefunction again – what a wonderful mathematical object Euler has given us ! – to develop a dual view of what is going on. On the one hand, Euler's function would describe the physical position (i.e. the *x*- and *y*-coordinates) of the electron. This is the green dot in the illustration, whose motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

On the other hand, we can also use Euler's wavefunction to describe the force(s). The Bohr model implies the circular motion of the pointlike *electron* is driven by (1) its inertia – because the pointlike *charge* acquired some mass as a result of its *Zitterbewegung*[14] – and (2) a centripetal force (because of the presence of a nucleus with the opposite charge). As mentioned above, we can deconstruct this force into an $\mathbf{F}_x$ and $\mathbf{F}_y$ force (with magnitudes equal to F_x and F_y respectively). We write:

$$\boldsymbol{F} = -F_x \cdot \cos(\omega t) - i \cdot F_x \cdot \sin(\omega t) = -F \cdot e^{i\theta}$$

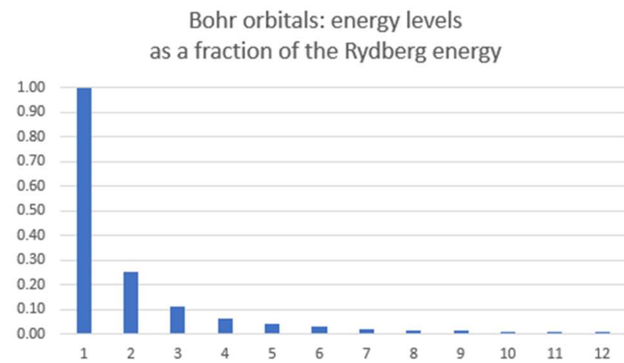
Before we move on, we should generalize the force and energy formula for all of the Bohr orbitals. Let us start with the energy formula:

[13] Again, boldface indicates *vectors*: objects with a magnitude as well as a direction.
[14] This is quite different from the *Zitterbewegung* model. The green dot in the *Zitterbewegung* model is a pointlike *charge* with *no inertia to motion*, which is why its speed can be equal to *c*. In the Bohr model, the electron moves at a non-relativistic speed: $v = \alpha c$ with $\alpha \approx 0.0073$.

$$S_n = n \cdot h = \mathrm{E}_n \cdot \mathrm{T}_n = \mathrm{E}_n \cdot \frac{2\pi a_n}{v_n} = \mathrm{E}_n \cdot \frac{n^2 2\pi a_1}{\frac{v_1}{n}} = \mathrm{E}_n n^3 \cdot \frac{\frac{h}{\alpha \mathrm{m}c}}{\alpha c} = \mathrm{E}_n n^3 \cdot \frac{h}{\alpha^2 \mathrm{m}c^2}$$

$$\Leftrightarrow \mathrm{E}_n = \frac{1}{n^2} \cdot \alpha^2 \mathrm{m}c$$

This, too, is a familiar formula. We visualized the $1/n^2$ fraction below.



Note that we can also write the energy, once again, as the product of the force and the radius:

$$\mathrm{F}_n \cdot a_n = \frac{\mathrm{q_e}^2}{4\pi\varepsilon_0 {a_n}^2} \cdot a_n = \frac{\mathrm{q_e}^2}{4\pi\varepsilon_0 \hbar c} \cdot \frac{\hbar c}{n^2 a_1} = \frac{1}{n^2} \alpha \cdot \frac{\alpha \hbar \mathrm{m}c^2}{\hbar} = \frac{1}{n^2} \cdot \alpha^2 \mathrm{m}c^2$$

Finally, we can also calculate the *difference* between the successive energy levels as:

$$\mathrm{E}_n - \mathrm{E}_{n-1} = \frac{1}{n^2} \cdot \alpha^2 \mathrm{m}c - \frac{1}{(n-1)^2} \cdot \alpha^2 \mathrm{m}c = [\frac{1}{n^2} - \frac{1}{(n-1)^2}] \cdot \alpha^2 \mathrm{m}c$$

Feynman's photon

We now want to do something very difficult: when an electron moves from one orbital to another, it is going to emit or absorb some energy in the form of a *photon*. What *is* a photon? We don't know. All we know is that they pack some electromagnetic oscillation and, therefore, some energy. We also know they are absorbed and emitted by atoms[15], which we model as oscillators. In fact, that's what we have been doing above: we look at an atom as an oscillator.

Oscillators have a Q: the idea is that any oscillation will actually be *transient*. It will die out after a while. The most intuitive definition of the Q of an oscillator is, quite simply, the ratio of (1) the energy stored in the oscillating resonator to (2) the energy that is being dissipated per cycle[16]:

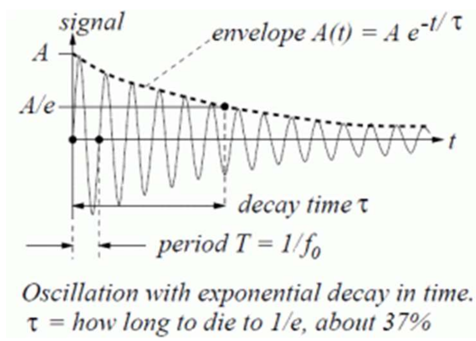
[15] We should also allow for emission and absorption of photons by *molecules*, of course. The idea of *molecular* electron orbitals should be quite similar to the idea of atomic electron orbitals.
[16] The formal definition of the Q includes

$$Q = \frac{E_{total}}{E_{cycle}} = \frac{\text{energy stored}}{\text{energy dissipated per cycle}}$$

It is easy to see that an oscillation like this will have a limited duration. If T is the cycle time (and T is the inverse of the frequency *f*, of course), then we can calculate the decay time τ as:

$$\tau = Q \cdot T = \frac{Q}{f}$$

Of course, we will probably want to think that we do not lose the exact *same* amount of energy in a cycle as time goes by: the idea is probably that, as the oscillator loses energy, we have some kind of exponential *decay* of the oscillation. Such assumption will lead to a slightly different interpretation of the decay time: it is going to be the time τ by which, after Q oscillations, the *amplitude* of the wave will have died by a factor $1/e \approx 1/2.718 = 0.368$.



It is interesting that the assumption of exponential decay will also involve Euler's number (e) but we have no time to dwell on that here. The point is: if we think of a photon as a transient electromagnetic wave, then we might think that its amplitude in time, or in space, could have the following shape.

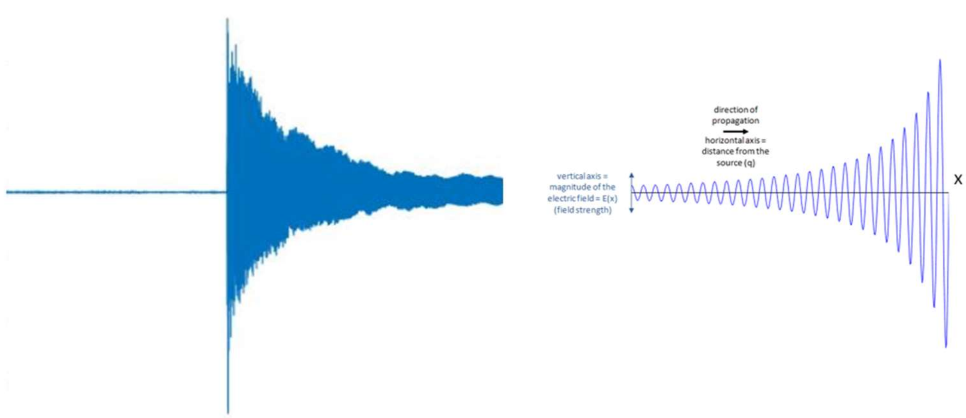


Figure 4: A wave as a transient[17]

[17] The image on the left depicts the amplitude of a musical note from a guitar string. The *density* (in space or in time) depends on the frequency and the scale, of course.

Note that the shape reverses depending on whether we take the horizontal axis to be time (t) or spatial position (x). We can, of course, easily relate this decay time to the Q of the system. The decay time is, obviously, the *length* of the wave in time, and if we have the length of the wave in time – and its speed, of course – then we can easily calculate its length in space. So, the question is: can we calculate a Q – or a decay time – for an atomic oscillator?

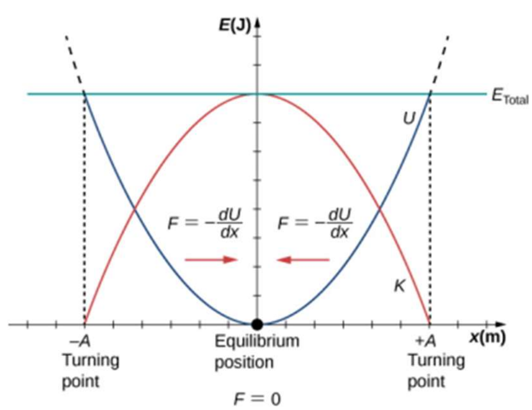
Feynman does that in one of his famous *Lectures* (Volume I, Chapter 32, Section 3). He thinks about a sodium atom – which emits and absorbs sodium light, of course[18] – and, based on various assumptions, he gets a Q of about 5×10^7. Now, the frequency of sodium light is about 500 THz (500×10^{12} oscillations per second). Hence, the *decay time* of the radiation is of the order of 10^{-8} seconds. So that means that, after 5×10^7 oscillations, the amplitude will have died by a factor $1/e \approx 0.37$.

That seems to be very short, but it still makes for 5 million oscillations and, because the wavelength of sodium light is about 600 nm (600×10^{-9} meter), we get a wave train with a considerable length:

$$(5\times10^6) \cdot (600\times10^{-9} \text{ meter}) = 3 \text{ meter}$$

Surely you're joking, Mr. Feynman! A photon with a length of 3 meter – or longer? We may think relativity theory is going to save us here because relativistic length contraction should cause this length to reduce to zero as the wave train zips by at the speed of light. However, something does not feel right about this. Hence, let us examine Feynman's assumptions.

We first need to think about the energy of our harmonic oscillator. Feynman rightly notes there are *two* energy concept(s) here: kinetic and potential. We know the kinetic and potential energy in a harmonic oscillator are constant: over one cycle, the kinetic energy will go from 0 to its maximum and then back to zero, while the potential energy will go from its maximum value to zero, and the back to its maximum value, as shown below.



[18] Feynman wrote his *Lectures* in the 1960s. That is when high-pressure sodium lamps came on the market for street lighting.

Figure 5: The one-dimensional oscillator[19]

Now, we also know that the kinetic energy and potential energy will vary with the square of the sine or cosine function that describes the motion. We write:

$$\mathrm{KE} = (1/2) \cdot \mathrm{m} \cdot \omega^2 \cdot a^2 \cdot \sin^2(\omega \cdot \mathrm{t})$$

$$\mathrm{PE} = (1/2) \cdot \mathrm{m} \cdot \omega^2 \cdot a^2 \cdot \cos^2(\omega \cdot \mathrm{t})$$

Now, the average value of the squared sine (or cosine) is equal to ½, so the *average* KE and PE are equal to $(1/4) \cdot \mathrm{m} \cdot \omega^2 \cdot a^2$. However, the idea is that the oscillator will lose all *all* of its energy – kinetic and potential – so Feynman suggests the $\mathrm{E} = (1/2) \cdot \mathrm{m} \cdot \omega^2 \cdot a^2$ for the energy of the oscillator. The question then becomes: what values do we use for m, a and ω? This is where Feynman goes inexplicably wrong: he uses the electron rest mass for m (which we understand) but then uses the frequency of the emitted light for ω. To make things worse, he then uses the *classical electron radius* (rather than the Bohr radius) for the amplitude a. Unsurprisingly, we get a value for the energy that does *not* correspond to the energy of a sodium light photon, which is about 2.3 eV. So, yes, Feynman must have been joking. The question is: can we do better?

We can.

The Rutherford-Bohr photon

Let us try the following. The orbitals are separated by a amount of *action* that is equal to h. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. Our photon will have to pack that, somehow. It will also have to pack the related energy which – in this particular example – is equal to $0.75 \cdot \alpha^2 \mathrm{m}c^2 \approx$ 20.4 eV. If the total action is equal to h, then the time T must be equal to:

$$\mathrm{E} \cdot \mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135 \times 10^{-15} \mathrm{eV} \cdot \mathrm{s}}{20.4 \ \mathrm{eV}} \approx 0.2 \times 10^{-15} \ \mathrm{s}$$

This is much shorter than 10^{-8} seconds ! It corresponds to a wave train with a length of $(3\times10^8 \ \mathrm{m/s}) \cdot (0.2\times10^{-15} \ \mathrm{s}) = 6\times10^{-8}$ m. This is the nanometer scale, which is the size of large molecules (e.g. C_{60} fullerene) and, therefore, much more reasonable. It is, in fact, equal to the wavelength $\lambda = c/\mathrm{f} = c \cdot \mathrm{T} = \mathrm{h}c/\mathrm{E}$. In fact, let us go all the way and see what picture of the photon we get here. Because of the angular momentum, we may want to think of it as a circularly polarized wave, as shown below.[20] We may represent this by the elementary wavefunction – but in a different interpretation. Indeed, this time we do not need to interpret Euler's number: it represents the rotating electric field vector itself or, remembering the $\mathrm{F} = \mathrm{q_e E}$ equation, the force field. Can we calculate them? Yes, we can.

[19] I should find the source of this illustration but it is so common I forgot.

[20] Note that the wave could be either left- or right-handed.

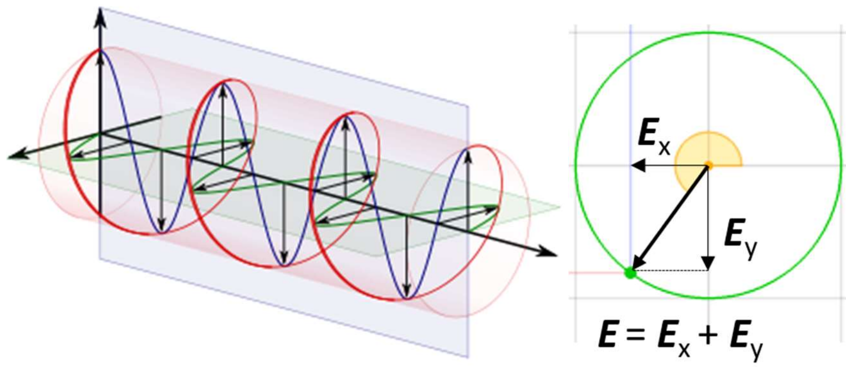


Figure 6: The Rutherford-Bohr photon

This model of a photon is delightfully simple. The photon is just one cycle traveling through space and time. It packs one unit of h (angular momentum), which gives us its frequency through the Planck-Einstein relation: $f = 1/\mathrm{T} = \mathrm{E}/h$. We can, of course, do what we did for the electron, so let us express h in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Note we can rewrite the $\mathrm{E} = \mathrm{m}c^2$ as $\mathrm{p} = \mathrm{m}c = \mathrm{E}/c$, which we use above. Energy is some force over a distance, and the distance is going to be the same wavelength, of course ! We can, therefore, calculate the magnitude of the electric field vector (E) as follows[21]:

$$\mathrm{E} = \mathrm{F}\lambda = \mathrm{q_e}E\lambda \Leftrightarrow E = \frac{\mathrm{E}}{\mathrm{q_e}\lambda} = \frac{\mathrm{E}^2}{\mathrm{q_e}hc} = \frac{\mathrm{m}^2c^3}{\mathrm{q_e}h}$$

Strange formula? The reader can check its physical dimension: we do get something expressed in *newton* per *coulomb* units: force per unit charge. Let us calculate its value for our 20.4 eV photon. The mass is, obviously, the photon mass, and the charge is the electron charge – equally obviously, because there is no other candidate for the unit charge, is there? We get:

$$E = \frac{\mathrm{m}^2c^3}{\mathrm{q_e}h} = \frac{(20.4\ \mathrm{eV}/c^2)^2c^3}{(1.6 \times 10^{-19}\ \mathrm{C}) \cdot (4.135 \times 10^{-15}\ \mathrm{eV} \cdot s)} = 62.9 \times 10^{34}\,\frac{\mathrm{eV/m}}{\mathrm{C}}$$

[21] The symbol is confusing, but we hope the italics (E) – and the context of the formula, of course ! – will be sufficient to distinguish the electric field vector (E) from the energy (E).

This looks pretty monstrous but let us convert the eV unit in *joule* or newton · meter (1 J = N · m) to try to interpret this value:

$$E = (62.9 \times 10^{34}\,\frac{\frac{\text{eV}}{\text{m}}}{\text{C}}) \cdot (1.6 \times 10^{-19}\,\frac{\text{J}}{\text{eV}}) \approx 0.1 \times 10^{18}\,\frac{\text{N}}{\text{C}}$$

This is gargantuan. The energy densities involved should cause spacetime curvature and therefore, would probably warrant an examination of this result from the perspective of general relativity: the Rutherford-Bohr photon might be a tiny black hole ! However, we are not very well versed in general relativity and so we will leave this for future research.

There is a final check on consistency that we could try to make: the energy of any oscillation will always be proportional to (1) its amplitude (a) and (2) its frequency (f). Do we get any meaningful result when we apply that principle here? If we write the proportionality coefficient as k, we could write something like this:

$$\text{E} = k \cdot a^2 \cdot \omega^2$$

It would be wonderful if this would give some meaningful result – and even more so if we could interpret the proportionality coefficient k as the mass m. Why? Because we have used the $\text{E} = \text{m} \cdot a^2 \cdot \omega^2$ equation before: it gave us this wonderful interpretation of the *Zitterbewegung* as what we referred to as the *rest matter oscillation*. We also vaguely started to apply the idea of a two-dimensional oscillation to the Rutherford-Bohr model, and it might work ! So, can we repeat the trick? The idea is the same as for the *Zitterbewegung* model: the Planck-Einstein relation gives us the frequency, and then we need to find the amplitude from that $\text{E} = \text{m} \cdot a^2 \cdot \omega^2$ formula.

The idea is really simple: we have two degrees of freedom (the amplitude a and the frequency ω) and so we always need to find some extra formula to calculate both. Can we do it? We tried to plug in the amplitudes of F and E but it turns out that doesn't get us anywhere – but perhaps the reader can try again ! However, we do get something quite obvious when we use the wavelength λ for the amplitude here. Look at this:

$$\text{E} = ka^2\omega^2 = k(2\pi a)^2\frac{\text{E}^2}{h^2} = k\lambda^2\frac{\text{E}^2}{h^2} = k\frac{h^2c^2}{\text{E}^2}\frac{\text{E}^2}{h^2} = kc^2 \Leftrightarrow k = \text{m and E} = \text{m}c^2$$

Sometimes physics can actually be *just nice*: I think we have a pretty good photon model here.

Conclusions

What is the point that we wanted to make? It is the following. We readily acknowledge the defect of the Rutherford-Bohr model of an atom: how can we explain a rotating charge does not radiate its energy away? However, we do *not* see how the mainstream Copenhagen interpretation solves it[22]. In fact, the solutions to Schrödinger's equation come with their own problems: the spherically symmetric solutions to the Schrödinger equation assume an electron spends most of its time right on top of the nucleus ! That is not very logical either, is it? Hence, we feel the mainstream Copenhagen interpretation might be improved upon too.

We, therefore, wanted to revisit the Rutherford-Bohr interpretation and show how wonderfully consistent it is – especially in regard to explain the physics of photon emission and absorption. In fact, we really need to figure out why the modern (mainstream) quantum-mechanical view would be any more – or any less – consistent. Indeed, the Schrödinger model also does *not* explain why oscillating electric charges do not radiate their energy away. As such, the Rutherford-Bohr model is and remains a pretty sound model – if only because it tries to provide a geometric or physical explanation of what might actually be happening !

Jean Louis Van Belle, 3 December 2018

[22] It is an important point. In fact, The spherically symmetric solutions to the Schrödinger equation assume an electron spends most of its time right on top of the nucleus. That is not logical. We should develop some better model.

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

One should also mention the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Francesco Celani:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.

4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.

5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

The illustrations in this paper are open source and have been *augmented* by the author. References and credits – including credits for open-source Wikipedia authors – have been added in the text.

One reference that has not been mentioned in the text is:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

The latter work is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Last but not least, we also mentioned the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html). However, we have not gone into the nitty-gritty of their work and, therefore, do not want to pretend we have.