Riemann Hypothesis Resolution using Game theory

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Note : This paper is based on my discussion with Prof .Ken Ono, Celebrated Number Theorist Professor University of Virginia)(Former VP at American Mathematical Society) way back in 2019 .This paper was written in 2019 and before when I saw it few days ago and thought to publish it publicly.

Abstract :

Riemann Hypothesis is TRUE if we look at the Functional Equation satisfied by the Riemann Zeta function upon analytical continuation in Game Perspective way as visualized by David Hilbert. The functional equation already shows the existence of trivial zeros . Here, in this paper I try to use the same functional equation to find out the location of non-trivial zeros and hence show that Riemann hypothesis is true for Riemann Zeta function. It uses technical game theoretical concept of Nash Equilibrium. There is need to imagine the Foundational Principles underlying Mathematics . In other words, it's the game of arranging Zeros on the complex plane using the functional equation.

INTRODUCTION:

In this paper, I will be looking functional equation satisfied by Riemann zeta function actually a non-cooperative game between its constituent terms(here different mathematical functional symbols) in which the best strategy adopted by each player to locate zeros on mathematical field leads to discovering the most stable arrangement of physical location non-trivial zeros of Riemann zeta function, which in turn leads to TRUTHFULNESS OF RIEMANN HYPOTHESIS..

As visualized by David Hilbert- Mathematics is actually a game between different mathematical symbols, where different symbols follow certain defined rules.

The mathematical theory of games was invented by John von Neumann and Oskar Morgenstern (1944).Game theory is the study of the ways in which *strategic interactions* among *agents* produce *outcomes* with respect to the *preferences* (or *utilities*) of those agents, where the outcomes in question might have been intended by none of the agents..All situations in which at least one agent can only act to maximize his utility through anticipating (either consciously, or just implicitly in his behavior) the responses to his actions by one or more other agents is called a *game*. Agents involved in games are referred to as *players*. If all agents have optimal actions regardless of what the others do, as in purely parametric situations or conditions of monopoly or perfect competition we can model this without appeal to game theory; otherwise, we need it.

Each player in a game faces a choice among two or more possible *strategies*. A strategy is a predetermined 'programme of play' that tells her what actions to take in response to *every possible strategy other players might use*. I will prominently use the tools of game theory to find out different Nash equilibrium stage in this functional game played between mathematical symbols.

Here, in particular, I visualize the functional equation satisfied by Riemann zeta function as game between different constituent terms which are connected through multiplication sign on both side of equality sign.. I would be finding the Nash Equilibrium which will be the solution and prove the Riemann Hypothesis to be True.

. As this has exactly 1 NE stage corresponding to the location of non-trivial zeros on the critical line in 0<R(s) <1.

So, what I would be doing is- finding the locations of trivial & non-trivial zeros by looking the arithmetic structure of Riemann zeta function and by applying the two basic arithmetic of numeric '0' to find out different set of possibilities of taking zero value by different constituent terms.

In a nutshell, I will NOT go into finding the zeros of this functiuon, rather I will be visualizing the arithmetic structure of FUNCTIONAL EQUATION ,in which different constituent terms are connected through multiplicative sign and using game theory find the NE stage to locate zeros. So, it has hardly anything to do with anything else than game theory and arithmetic of numeric 0.

The Riemann zeta function ((s) is a function of a complex variable s = o + it (here, s, o and t are traditional notations associated to the study of the Ç-function). The following infinite series converges for all complex numbers s with real part greater than 1, and defines (s) in this case:

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \qquad \sigma = \Re(s) > 1.$$

The Riemann zeta function is defined as the analytic continuation of the function defined for o > 1 by the sum of the preceding series.

The Riemann zeta function satisfies the functional equation

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s),$$

where T(s) is the gamma function which is an equality of meromorphic functions valid on the whole complex plane. This equation relates values of the Riemann zeta function at the points s and 1 – s. The gamma function has a simple pole at every non-positive integer, therefore, the functional equation implies that ((s) has a simple zero at each even negative integer s = – 2n Pi these are the trivial zeros of ((s).

$$\eta(s) = \sum_{n=1}^{\infty} (-1)^{n-1} = (1 - 2^{1-s})\zeta(s).$$

n^s

Incidentally, this relation is interesting also because it actually exhibits ((s) as a Dirichlet series (of the y-function) which is convergent (albeit non-absolutely) in the larger half- plane o > 0 (not just o > 1), up to an elementary factor.

<u>Statement of Riemann Hypothesis</u>:

All non-trivial zeros of Riemann zeta function in the critical space 0<R(s)<1 lies on R(s)=1/2 .

Here we look at Game theoretic aspects of how to arrange the Zeros on this plane.

, I visualize numbers and their mathematical functions playing the game of symbols .

In context of functional equation game played by Riemann zeta functions in the game there are two players A & B where A corresponds to sin() NOT =0 and B corresponds to sin() =0.

A solution concept in game theory :
Nash Equilibrium which corresponds to the solution,here the physical location of non-trivial zeros of Riemann zeta function.

<u>PROOF</u>:

Functional equation satisfied by Players Ç (s) & Ç(1-s) in the entire complex domain 'C' is

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\, \zeta(1-s).$$

As one and only one term on each side of "=" sign can and must be zero as 0*0 = 0 &
0 *non-zero number= 0

2^s(Pi)^(s-1) and Gamma function terms can never be equal to 0 ,so we can skip that here as they will not contribute to becoming 0 using the functional equation.
And by coordinate transformation, s & 1-s can be transformed to ½-s and ½+s

Note: I have used "f(s)" in place of the function in the Riemann Zeta functional equation further for simplicity.

Notations

- A = Function f(s) for { C: s :: sin(Pi*s/2)=0,s‡0} as s=0 is the location for pole

i.e. those values of s for which Sin(Pi*s/2) is not equal to 0.

- B = Function f(s) for { C-A,s‡0} i.e. those values of s for which Sin(Pi*S/2) =0

Player A (for which Sin () term is not 0) has also two options .It can also exercise one of the two.

1. Ç(s)=0 for R(s)> 1/2 and simultaneously for R(s)<1/2 (Both sides 0 simultaneously)

2. Ç(s)=0 for s= ½+it But, Ç(s)‡ 0 for R(s)>1 (Or none of the sides will be 0)
   i.e. C(s) ‡0 for R(s)<1/2 and C(s) ‡0 for R(s) >1/2

Similarly,

Player B (for which Sin() term =0) has two options to exercise in the game .It can exercise only one of the two.

1. Ç(s) =0 for R(s)>1/2, Ç(s)= 0 for R(s)<1/2 i.e.(Both sides will be 0)
2. Ç(s)=0 for R(s)<1/2, Ç(s)‡0 for R(s)>1/2 (Left side of R(s)=1/2 will be Zero ,Right side will not be zero)

Now, we look at the different permutations of strategies adopted in this game and find their payoff matrix.

Payoff matrix of this game for the Riemann Zeta function

	Player A exercises 1st option	**Player A exercises 2nd option**
Player B exercises 1st option	**0,0(All the points 0)Impossible as it means f(s)=0 for all s**	**0,0 (0 on both sides for Sin()=0 Impossible because it is already proven that there are no zeros for R(s)>1 for Riemann Zeta function.**
Player B exercises 2nd option	**0,0(All one side points = 0) Impossible as only trivial zeros already known. Impossible**	**1,1 (Possible location for 0) The only possible way to gain the stability and maximizes the payoff. Equilibrium Stage for Riemann Zeta function.**

By looking at the table Payoff is maximum i.e.(1,1) when A exercises 2nd and B also exercises 2nd option to locate Zeros.
The players A & B (the sub players derived from Sine function) both similarly exercise their respective options uniformly.
That's the Nash equilibrium state by looking when both the players exercise the 2nd options. Which means that f(s) =0 in the critical strip 0< R(s)<1/2 will not exist either on the left side of R(s)=1/2 nor right side. So, the only possible location for the Non-Trivial Zeros would be R(s)=1/2 for Riemann Zeta function.

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This asserts the truthfulness of the Riemann hypothesis for Riemann Zeta function that trivial zeros lie on the points s=2k,k<0 and non-trivial zeros will lie on the R(s)=1/2 .Thus,

It implies that

Ç(s) =0 for R(s)=1l2+it for 0<R(s)<1 and
also Ç(s) ‡ 0 for R(s)>1/2

QED

References:

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3. Riemann hypothesis by Enrico Bombieri
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Explanations to the possibilities of Counterexamples as demonstrated (Those functions satisfying the same one variable Riemann zeta functional equation but Riemann Hypo. NOT TRUE for them) in terms of the strategies in the game.

Here I am showing that there may exist possibilities for the counter examples in this above game based under constraints i.e. cooperative/coalition behavior of the variable within the game. In such cases, the strategy will be different in the same game.

The one variable generic functional equation found for Riemann Zeta function implies to the fact Riemann Hypothesis is TRUE. But there may exist some counter examples to this as pointed out by Prof.Ken Ono.

In any case of counter examples, I have shown below the possible type of strategy in the same game that will be followed by those specific functions whether L-functions or any other type of functions in general. There may be many such functions as counterexamples. One has to study those functions deeply separately to find out the sub players classification details in that strategy for the counter examples.

If certain functions e.g L-functions (two-variables) or other functions are forcibly made to satisfy the same one variable functional equation, this leads to the external constraints in the functional equation game and changes the fundamental aspect of the game from non-cooperative to cooperative under external constraints.. As a result of this, the strategies in the same game will differ from the original basic one derived for the (Riemann Zeta) RZ function. This enforces constraints in terms of other variables/parameters/other aspects on the behavior of the function for the variable.

[Kindly note that the two variables generic functional equations involving S and Dirichlet parameters X(n) for L-functions is different without any constraints.]

Coming to the discussion of the strategies for the counterexamples mentioned above. There could be possible many counter examples when there exist some external constraints in terms of variable, parameters or regrouping of some parts etc. . In that case there will be added sub players in the game for the variable s depending upon the external constraint/variable say of L-function or any other counter example functions.

Let me explain the possible case just in context of the counterexample

Player A : When sin() not equals 0, there are two options originally if f(1/2-s) =0, then f(1/2+s) also equals 0

Or if f(1/2-s) Not equals 0, then f(1/2+s) also Not equals 0.

But the external constraints leads to subplayers for the player A & B for variable S namely say A(a),A(b) & B(a),B(b).

When sin() not equals 0,

For the subplayer A(a),

F(1/2-s) =0 and f(1/2+s)=0 and

For the sub player A(b)

(1/2-s) not equals 0 and f(1/2+s) not equals 0.

In this case Riemann Hypothesis may NOT necessarily be TRUE!

Now the sub players A(a),A(b),B(a)& B(b) for the variable S will depend upon the various counter examples-. This needs to be discovered for each counterexample case separately. Hence the different sub players (i.e. different group of values of s) will follow the strategies.

Euler product form based counter examples : Everything comes into the functional equation. I am completely looking at the one variable generic functional equation derived and satisfied for Riemann Zeta function. Euler product contains another function called multiplicative functions, which could lead to a constraint . If some of the terms of the Euler product form are modified externally to satisfy the one variable RZ functional equation to produce some counter examples , it will create new sub players for the player variable S depending upon the coalition characteristics and behavior of the modified terms and hence will enforce external constraint. So again different strategies will arise for different subplayers in the game. The generic functional equation satisfying those modified functions without any constraint would be different. The entire set of sub players and the generic functional equations satisfied requires to be found on case to case basis for various counterexamples.

So, then the solution i.e. the equilibrium point will be decided upon considering the strategies of sub players. This is infact technically cooperative game where due to external constraints, the formation of coalition for sub players is formed and in that case the equilibrium and solution is calculated by taking the various combinations of coalition of sub players. But in that case as the strategies will be different for sub players , it can lead to the violation of Riemann Hypothesis Truthfulness as all the non-trivial 0s will not lie on the critical line R(s)=1/2 because of possibility of one more sub option where both the f(1/2-s) and f(1/2+s) becomes 0 when sin() not equals 0.

To summarize , under external constraints and cooperative behavior of sub-players of the variable due to modifications or whatsoever, the case of counter examples becomes a case of further cooperative subgames and will be dealt accordingly separately. But in that case RH needs not be TRUE.

In the list of strategies mentioned below, I am showing the strategy within the game for all the counter examples in general .

Notations for the Sub players.

Player A will now have two sub players A(a) & A(b) based on the characteristic of their behavior in variable S with each having two options.

Similarly, Player B will now have two sub players B(a) & B(b) with each having two options

Now various combinations of Payoff Matrix of the Game under Constraints would be like this as follows:
I am showing the possible type of strategy against Riemann hypothesis.(i.e. It may NOT be True).

##The strategies of the Game and their Payoffs as follows:

1)A (a) 1st option + A(b) 1st option + B(a) 1st option +B(b) 1st option

(In this case f(s)=0 ,impossible)

2) A(a) 1st option +A(b)1st option + B(a)2nd option +B(b)2nd option

In this case all one side to the left f(s)=0,impossible.)

3)A(a) 2^{nd} option + A(b)2^{nd} option +B(a) 1^{st} option +B(b)1^{st} option

(In this case Both sides 0, RH may be True for some other functions apart from Riemann zeta function also but impossible for Riemann Zeta function as shown in the matrix payoff earlier as it has no trivial 0s for R(s)>1).

4)A(a)2^{nd} option+ A(b) 2^{nd} option + B(a)2^{nd} option+ B(b)2^{nd} option

(In this case RH True for Riemann Zeta function as shown earlier in the matrix payoff in the original paper)

5)A(a)1 option +A(b) 2^{nd} option + B(a) 1^{st} option +B(b) 2^{nd} option.

(In this case RH may not be true as for various counter examples)

Hence, the last one is the possible strategy corresponding to the various counter examples satisfying the one variable Riemann Zeta functional Equation but violating Riemann hypothesis.

So, the functional equation game shows that Riemann hypothesis will be True for Riemann Zeta function and some other functions but may NOT be TRUE for various counterexamples functions.