# An Analysis and Proof on Beal's Conjecture

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## Abstract

In this article, the author first classify A, B and C according to their respective odevity, and thereby get rid of two kinds which belong not to $A^X+B^Y=C^Z$. Then, affirm the existence of $A^X+B^Y=C^Z$ in which case A, B and C have at least a common prime factor by several concrete equalities. After that, prove $A^X+B^Y\neq C^Z$ in which case A, B and C have not any common prime factor by the mathematical induction with the aid of the distinct odd-even relation on the premise whereby even number $2^{W-1}H^Z$ as symmetric center of positive odd numbers concerned after divide the inequality in four. Finally, reach a conclusion that the Beal's conjecture holds water via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

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## 1. Introduction

The Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.



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The conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society, [1]. Yet it is still both unproved and un-negated a conjecture hitherto.

## 2. Analyzing $A^X$, $B^Y$ and $C^Z$, and Illustrating $A^X+B^Y=C^Z$ when A, B and C have a common prime factor

First regard limits of values of A, B, C, X, Y and Z in indefinite equation $A^X+B^Y=C^Z$ of the Beal's conjecture as given requirements for indefinite equations and inequalities concerned after this.

Then classify A, B and C according to their respective odevity, and thereby exclude following two kinds which belong not to $A^X+B^Y=C^Z$.

**(α)**A, B and C, all are positive odd numbers.

**(β)**A, B and C are two positive even numbers and a positive odd number.

After that, merely continue to have following two kinds of $A^X+B^Y=C^Z$ under the given requirements.

**(γ)** A, B and C, all are positive even numbers.

**(δ)** A, B and C are two positive odd numbers and a positive even number.

For the indefinite equation $A^X+B^Y=C^Z$ which satisfies aforesaid either set of qualifications, in fact, it has many sets of the solution with A, B and C as positive integers, and illustrate with examples as follows respectively.

When A, B and C all are positive even numbers, if let A=B=C=2 and X=Y≥3, then $A^X+B^Y=C^Z$ is changed into $2^X+2^X=2^{X+1}$ where Z=X+1.

Obviously the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 2, 2 and 2, and that A, B and C have common prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3 and Z=4, then $A^X+B^Y=C^Z$ is changed into $162^3+162^3=54^4$. So the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3 and Z=5, then $A^X+B^Y=C^Z$ is changed into $3^3+6^3=3^5$. So the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 3, 6 and 3, and that A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7 and Z=3, then $A^X+B^Y=C^Z$ is changed into $7^6+7^7=98^3$. So the indefinite equation $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as positive integers 7, 7 and 98, and that A, B and C have common prime factor 7.

Therefore the indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either set of qualifications is able to hold water, but A, B and C must have at least a common prime factor.

By this token, if we can prove that there is only $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then the conjecture is tenable definitely.

Since A, B and C have the common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, yet another has not, then it will lead up to $A^X+B^Y\neq C^Z$ according to the unique factorization theorem of natural number.

Unquestionably, let following two inequalities add together, then they are able to replace fully $A^X+B^Y\neq C^Z$ under the given requirements plus the set of qualifications that A, B and C are two positive odd numbers and a positive even number without a common prime factor.

**(μ)** $A^X+B^Y\neq 2^Z G^Z$ under the given requirements plus the set of qualifications that A and B are two positive odd numbers, and G is a positive integer, and that they have not a common prime factor.

**(ν)** $A^X+2^Y D^Y\neq C^Z$ under the given requirements plus the set of qualifications that A and C are two positive odd numbers, and D is a positive integer, and that they have not a common prime factor.

For $A^X+B^Y\neq 2^Z G^Z$, it can be divided into two inequalities as follows.

**(1)** $A^X+B^Y\neq 2^W$ where A and B are positive odd numbers without a common prime factor, and that X, Y and W are integers ≥3.

**(2)** $A^X+B^Y \neq 2^W H^Z$ where A, B and H are positive odd numbers without a common prime factor, and H ≥3, and that X, Y, Z and W are integers ≥3.

For $A^X+2^Y D^Y \neq C^Z$, it can be divided into two inequalities as follows.

**(3)** $A^X+2^W \neq C^Z$ where A and C are positive odd numbers without a common prime factor, and that X, W and Z are integers ≥3.

**(4)** $A^X+2^W R^Y \neq C^Z$ where A, R and C are positive odd numbers without a common prime factor, and R ≥3, and that X, Y, Z and W are integers ≥3.

Regard limits of values of A, B, C, H, R, X, Y, Z and W in listed above four inequalities and their co-prime relation in each inequality as known requirements for inequalities or indefinite equations concerned after this.

Thus it can be seen, that the proof of $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove listed above four inequalities under the known requirements. For this purpose, it is necessary to expound beforehand some circumstances relating to proofs for them, ut infra.

First classify all positive odd numbers into two kinds, i.e. Φ and Ω. Namely the form of Φ is 1+4n, and the form of Ω is 3+4n, where n ≥ 0.

As thus, positive odd numbers from small to large form infinitely many cycles of Φ plus Ω, to wit Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; Φ, Ω; …

After that, add even numbers $2^{W-1}H^Z$ among the sequence of positive odd numbers, where H is an odd number ≥1, and W, Z ≥ 3.

Again regard each of $2^{W-1}H^{Z}$ as a symmetric center of positive odd numbers concerned, then positive odd numbers on the left side of a symmetric center and positive odd numbers near the symmetric center and on the right side of the symmetric center are one-to-one bilateral symmetries at the number axis or in the sequence of natural numbers, [2]. Such symmetric relations of positive odd numbers indicate that for any of $2^{W-1}H^{Z}$ as a center of symmetry, it can only symmetrize one of Φ and one of Ω, yet can not symmetrize two of either kind, and that start from any concrete symmetric center, there are both finitely many cycles of Ω plus Φ leftwards until Ω=3 and Φ=1, and infinitely many cycles of Φ plus Ω rightwards. Clearly two distances from a symmetric center to bilateral symmetric Φ and Ω on two sides of the symmetric center are either two equilong segments at the number axis, or two identical differences in the sequence of natural numbers.

Consequently, the sum of two bilateral symmetric odd numbers is equal to the double of even number as the symmetric center. Yet over the left, a sum of two non-symmetric odd numbers is surely unequal to the double of even number as the symmetric center.

Hereinafter, we will use such a distinct odd-even relation between two odd numbers and an even number as the symmetric center to ascertain whether an algebraic expression is the equality.

In addition, for a positive odd number, it is able to be expressed as one of

$O^V$ where V expresses the greatest common divisor of exponents of distinct prime factors of the positive odd number and $V\geq 1$, and O is a positive odd number. When V=1 or 2, write $O^V$ to $O^{1\sim 2}$.

By now, set about proving aforesaid 4 inequalities by the mathematical induction with the aid of the distinct odd-even relation explained in advance, one by one.

## 3. Proving $A^X+B^Y\neq 2^W$ under the Known Requirements

Regard $2^{W-1}$ as a center of symmetry of positive odd numbers concerned to prove $A^X+B^Y\neq 2^W$ under the known requirements by the mathematical induction thereinafter.

**(1)** When W-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric centers $2^{W-1}$ are listed below successively.

$1^6$, 3, $(2^2)$, 5, 7, $(2^3)$, 9, 11, 13, 15, $(2^4)$, 17, 19, 21, 23, 25, $3^3$, 29, 31, $(2^5)$, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, $(2^6)$, 65, 67, 69, 71, 73, 75, 77, 79, $3^4$, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, $5^3$, 127

By this token, there are not two of $O^V$ with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}$ as a symmetric center where W-1=2, 3, 4, 5 and 6. Namely there are $A^X+B^Y\neq 2^3$, $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements.

**(2)** When W-1=K with $K\geq 6$, suppose that there are not two of $O^V$ with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^K$ as a symmetric center. Namely suppose $A^X+B^Y\neq2^{K+1}$ under the known requirements.

**(3)** When W-1=K+1, prove that there are not two of $O^V$ with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center. Namely prove $A^X+B^Y\neq2^{K+2}$ under the known requirements.

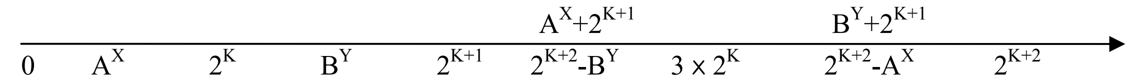
***Proof·*** Since odd numbers whereby $2^{W-1}$ including $2^K$ plus $2^{K+1}$ as a symmetric center are possessed of one-to-one symmetric relations, then positive odd numbers whereby $2^K$ as a symmetric center are positive odd numbers on the left side of symmetric center $2^{K+1}$.

Thus, for positive odd numbers whereby $2^{K+1}$ as a center of symmetry, their a half retains still on original places after move the symmetric center to $2^{K+1}$ from $2^K$, and that the half lies on the left side of $2^{K+1}$. While, another half is formed from $2^{K+1}$ plus each of positive odd numbers whereby $2^K$ as a symmetric center, and that the half lies on the right side of $2^{K+1}$.

Suppose that $A^X$ and $B^Y$ are a pair of bilateral symmetric positive odd numbers whereby $2^K$ as a symmetric center, then there is $A^X+B^Y=2^{K+1}$ according to the distinct odd-even relation explained in advance.

Since there are not two of $O^V$ with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^K$ as a symmetric center according to second step of the mathematical induction, so tentatively let $A^X$ as one of $O^V$ with $V\geq3$, and $B^Y$ as one of $O^{1\sim2}$, i.e. let $X\geq3$ and Y=1 or 2.

By now, let $B^Y$ plus $2^{K+1}$ to make $B^Y+2^{K+1}$. Please, see also a simple illustration at the number axis as follows.




Since there is only $A^X+B^Y \neq 2^{K+1}$ under the known requirements according to second step of the mathematical induction, therefore there is inevitably $A^X+B^Y=2^{K+1}$ under the known requirements except for Y and Y=1 or 2.

As thus, this can deduce $B^Y+2^{K+1}=A^X+2B^Y=2^{K+2}-A^X$ from $A^X+B^Y=2^{K+1}$.

Also $A^X$ and $2^{K+2}-A^X$ are bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center from $A^X+(2^{K+2}-A^X)=2^{K+2}$, according to the distinct odd-even relation explained in advance.

So $A^X$ and $A^X+2B^Y$ are two bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center too, and there are $A^X+(2^{K+2}-A^X)=A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y and Y=1 or 2.

But then, there is only $A^X+B^Y \neq 2^{K+1}$ under the known requirements, thus it has $A^X+ [A^X+2B^Y] =2[A^X+B^Y] \neq 2^{K+2}$ in that case.

In any case $A^X+2B^Y$ can only be a positive odd number. So let $A^X+2B^Y=D^E$ where E expresses the greatest common divisor of exponents of distinct prime factors of the positive odd number, and D is a positive odd number, then it has $A^X+[A^X+2B^Y] =A^X+D^E \neq 2^{K+2}$ under the known requirements.

That is to say, no matter what positive integer which E equals, and no matter what positive odd number which D equals, there is $A^X+D^E \neq 2^{K+2}$ under the known requirements invariably. Namely $A^X$ and $D^E$ under the known requirements are not two bilateral symmetric odd numbers whereby

$2^{K+1}$ as a symmetric center.

Whereas, under the known requirements except for Y and Y=1 or 2, $A^X$ and $A^X+2B^Y$ are indeed a pair of bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center, and there is $A^X+[A^X+2B^Y]=2^{K+2}$ according to the above result got, i.e. there is $A^X+D^E=2^{K+2}$ due to there is $A^X+2B^Y=D^E$.

Such being the case, provided you slightly change a bit of valuation of any letter of $A^X+2B^Y$, then it at once is not original that $A^X+2B^Y$ under the known requirements except for Y and Y=1 or 2. Of course, now it lies not on the place of the symmetry with $A^X$ either.

That is to say, $A^X$ and $A^X+2B^Y$ under the known requirements are not two bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center, because Y=1 or 2 has been changed into Y≥3.

Thus there is $A^X+[A^X+2B^Y]=A^X+D^E\neq 2^{K+2}$ under the known requirements according to the distinct odd-even relation explained in advance.

Moreover, from $A^X+B^Y=2^{K+1}$ under the known requirements except for Y and Y=1 or 2 get $A^X+2B^Y=2^{K+1}+B^Y$, in addition $A^X$ has been supposed as one of $O^V$ with V≥3 on the left side of symmetric center $2^{K+1}$, so $A^X+2B^Y$ i.e. $D^E$ lies on the right side of symmetric center $2^{K+1}$ surely.

For the inequality $A^X+D^E\neq 2^{K+2}$, substitute D by B, since B and D are any positive odd number; also substitute Y for E where E≥3, and Y≥3.

After such substitutions, get $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

In this proof, if $B^Y$ is one of $O^V$ with V≥3, then $A^X$ is surely one of $O^{1\sim 2}$.

And that a conclusion concluded from this is one and the same with $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

If $A^X$ and $B^Y$ are bilateral symmetric two of $O^{1\sim 2}$ whereby $2^K$ as a symmetric center, then whether $A^X$ and $A^X+2B^Y$, or $B^Y$ and $B^Y+2A^X$, they are a pair of bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center. But, no matter what positive odd number which $A^X+2B^Y$ or $B^Y+2A^X$ equals, it can not change the pair of bilateral symmetric odd numbers into two of $O^V$ with $V\geq 3$, because $A^X$ or $B^Y$ in the pair is not one of $O^V$ with $V\geq 3$ originally.

Overall, the author has proven that when W-1=K+1 with $K\geq 6$, there is only $A^X+B^Y\neq 2^{K+2}$ under the known requirements. In other words, there are not two of $O^V$ with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{K+1}$ as a symmetric center.

By the preceding way of doing things, can continue to prove that when W-1=K+2, K+3…up to every integer $\geq 3$, there surely are $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to $A^X+B^Y\neq 2^W$ under the known requirements.

## 4. Proving $A^X+B^Y\neq 2^W H^Z$ under the Known Requirements

Proving $A^X+B^Y\neq 2^W H^Z$ under the known requirements by the mathematical induction successively, and point out $H\geq 3$ emphatically.

**(1)** When H=1, $2^{W-1}H^Z$ is $2^{W-1}$. Of course, we have seen the proof of $A^X+B^Y\neq 2^W$ under the known requirements in the preceding section already. Namely there are not two of $O^V$ with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}$ as a symmetric center.

**(2)** When H=J and J is an odd number ≥1, $2^{W-1}H^Z$ i.e. $2^{W-1}J^Z$, suppose $A^X+B^Y\neq 2^WJ^Z$ under the known requirements. Namely suppose that there are not two of $O^V$ with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}J^Z$ as a symmetric center.

**(3)** When H=K with K=J+2, $2^{W-1}H^Z$ i.e. $2^{W-1}K^Z$, prove $A^X+B^Y\neq 2^WK^Z$ under the known requirements. Namely prove that there are not two of $O^V$ with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center.

***Proof·*** Since after regard $2^{W-1}H^Z$ as a symmetric center, the sum of every pair of bilateral symmetric odd numbers is equal to $2^WH^Z$, while a sum of two odd numbers of no symmetry is unequal to $2^WH^Z$.

In addition, there are not two of $O^V$ with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}J^Z$ as a symmetric center. Namely there is $A^X+B^Y\neq 2^WJ^Z$ under the known requirements according to second step of the mathematical induction.

Such being the case, so suppose that $A^X$ and $B^Y$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}J^Z$ as a symmetric center, also tentatively let Y ≥3 and X=1 or 2, then there is $A^X+B^Y=2^WJ^Z$ undoubtedly.

On the other, after regard $2^{W-1}K^Z$ as a symmetric center, $B^Y$ and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers from $B^Y+(2^WK^Z-B^Y)=2^WK^Z$ according to the distinct odd-even relation explained in advance.

By now, let $A^X$ plus $2^W(K^Z-J^Z)$ to make $A^X+2^W(K^Z-J^Z)$. Since it has $A^X+B^Y=2^WJ^Z$, then there are $A^X+2^W(K^Z-J^Z)=A^X+2^WK^Z-2^WJ^Z=2^WK^Z-(2^WJ^Z-A^X)=2^WK^Z-B^Y$ under the known requirements except for X and X=1 or 2.

Now that there is $A^X+2^W(K^Z-J^Z)=2^WK^Z-B^Y$ under the known requirements except for X and X=1 or 2; in addition $B^Y$ and $2^WK^Z-B^Y$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center, therefore $B^Y$ and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center.

So there is $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X and X=1 or 2.

From $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$ and beforehand supposed $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, can get $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^WK^Z-2^WJ^Z\neq 2^WK^Z$ under the known requirements.

Thus it can be seen, that $B^Y$ and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center since there is $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^WK^Z$ according to the distinct odd-even relation explained in advance.

It is obvious that $A^X+2^W(K^Z-J^Z)$ in aforesaid two cases expresses two disparate odd numbers, that is due to $X\geq 3$ in one, and X=1 or 2 in another.

From $A^X+2^W(K^Z-J^Z)=2^WK^Z-(2^WJ^Z-A^X)$ and $2^WJ^Z-A^X\neq B^Y$ under the known requirements, can get $A^X+2^W(K^Z-J^Z)\neq 2^WK^Z-B^Y$.

In any case, $A^X+2^W(K^Z-J^Z)$ can only be a positive odd number, thus let

$A^X+2^W(K^Z-J^Z)=F^V$ where V expresses the greatest common divisor of exponents of distinct prime factors of the positive odd number, and F is a positive odd number. So there is $F^V\neq 2^WK^Z-B^Y$ on the basis of $A^X+2^W(K^Z-J^Z)=F^V$ and $A^X+2^W(K^Z-J^Z)\neq 2^WK^Z-B^Y$ under the known requirements. That is to say, there is $B^Y+F^V\neq 2^WK^Z$ under the known requirements.

Since $B^Y$ and $A^X+2^W(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center from $B^Y+[A^X+2^W(K^Z-J^Z)]=2^WK^Z$ under the known requirements except for X and X=1 or 2 according to the result got in advance.

Such being the case, provided you slightly change a bit of valuation of any letter of $A^X+2^W(K^Z-J^Z)$, then it at once is not original that $A^X+2^W(K^Z-J^Z)$ under the known requirements except for X and X=1 or 2. Of course, now it lies not on the place of the symmetry with $B^Y$ either.

That is to say, $B^Y$ and $A^X+2^W(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center, because X=1 or 2 has been changed into $X\geq 3$.

As thus, there is $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^WK^Z$ under the known requirements according to the distinct odd-even relation explained in advance.

Namely there is $B^Y+F^V\neq 2^WK^Z$ under the known requirements due to there is $A^X+2^W(K^Z-J^Z)=F^V$.

For inequality $B^Y+F^V\neq 2^WK^Z$, substitute F by A, since A and F express any positive odd number; also substitute X for V where $V\geq 3$, and $X\geq 3$. After pass such substitutions, get $A^X+B^Y\neq 2^W K^Z$ under the known requirements.

In this proof, if $A^X$ is one of $O^V$ with $V\geq 3$, then $B^Y$ is one of $O^{1\sim 2}$ surely.

And that a conclusion concluded from this is one and the same with $A^X+B^Y\neq 2^W K^Z$ under the known requirements.

If $A^X$ and $B^Y$ are bilateral symmetric two of $O^{1\sim 2}$ whereby $2^{W-1}J^Z$ as a symmetric center, then whether $A^X+2^W(K^Z-J^Z)$ and $B^Y$, or $B^Y+2^W(K^Z-J^Z)$ and $A^X$, they are a pair of bilateral symmetric odd numbers whereby $2^{W-1}K^Z$ as a symmetric center. But, no matter what positive odd number which $A^X+2^W(K^Z-J^Z)$ or $B^Y+2^W(K^Z-J^Z)$ equals, it can not change the pair of bilateral symmetric odd numbers into two of $O^V$ with $V\geq 3$, because $B^Y$ or $A^X$ in the pair is not one of $O^V$ with $V\geq 3$ originally.

On balance, the author has proven $A^X+B^Y\neq 2^W K^Z$ with K=J+2 under the known requirements. In other words, when H=J+2, there are not two of $O^V$ with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}(J+2)^Z$ as a symmetric center.

By the above way of doing things, can continue to prove that when H=J+4 , J+6… up to every positive odd number, there surely are $A^X+B^Y\neq 2^W(J+4)^Z$, $A^X+B^Y\neq 2^W(J+6)^Z$…up to $A^X+B^Y\neq 2^W H^Z$ under the known requirements.

## 5. Proving $A^X+2^W\neq C^Z$ under the Known Requirements

On the basis of the certain conclusion got, continue to prove $A^X+2^W\neq C^Z$ under the known requirements by the mathematical induction.

**(1)** When W=3, 4, 5, 6 and 7, bilateral symmetric odd numbers on two sides of symmetric centers $2^3$, $2^4$, $2^5$, $2^6$ and $2^7$ are listed below successively.

$1^7$, 3, 5, 7, $(2^3)$, 9, 11, 13, 15, $(2^4)$, 17, 19, 21, 23, 25, $3^3$, 29, 31, $(2^5)$, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, $(2^6)$, 65, 67, 69, 71, 73, 75, 77, 79, $3^4$, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, $5^3$, 127, $(2^7)$, 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, $3^5$, 245, 247, 249, 251, 253, 255.

Ut supra, there is only higher power's $1^7$ on the left side of symmetric center $2^3$;

There is only higher power's $1^7$ on the left side of symmetric center $2^4$;

There are altogether higher power's $1^7$ and $3^3$ on the left side of symmetric center $2^5$;

There are altogether higher power's $1^7$and $3^3$ on the left side of symmetric center $2^6$;

There are altogether higher power's $1^7$, $3^3$, $3^4$ and $5^3$ on the left side of symmetric center $2^7$.

Clearly, it is observed that it has $1^7+2^3\neq C^Z$; $1^7+2^4\neq C^Z$; $1^7+2^5\neq C^Z$, $3^3+2^5\neq C^Z$; $1^7+2^6\neq C^Z$, $3^3+2^6\neq C^Z$; $1^7+2^7\neq C^Z$, $3^3+2^7\neq C^Z$, $3^4+2^7\neq C^Z$ and $5^3+2^7\neq C^Z$.

Therefore there are $A^X+2^3\neq C^Z$, $A^X+2^4\neq C^Z$, $A^X+2^5\neq C^Z$, $A^X+2^6\neq C^Z$ and $A^X+2^7\neq C^Z$ under the known requirements.

**(2)** When W=N with N≥7, suppose that there is $A^X+2^N\neq C^Z$ under the known requirements, where $A^X < 2^N < C^Z$.

**(3)** When W=N+1, prove that there is $A^X+2^{N+1}\neq C^Z$ under the known requirements, where $A^X < 2^{N+1} < C^Z$.

***Proof·*** Since there is $(2^{N+1}+A^X)+(2^{N+1}-A^X) = 2^{N+2}$, so $2^{N+1}+A^X$ and $2^{N+1}-A^X$ are a pair of bilateral symmetric odd numbers whereby $2^{N+1}$ as a symmetric center according to the distinct odd-even relation explained in advance.

Also there is the inequality $2^{N+1}-A^X \neq O^V$ i.e. $A^X+O^V \neq 2^{N+1}$ where $V \geq 3$ according to proven $A^X+B^Y \neq 2^W$ under the known requirements, so that $2^{N+1}-A^X$ can only be one of $O^{1\sim2}$.

Now that $2^{N+1}-A^X$ is one of $O^{1\sim2}$, then $2^{N+1}-A^{1\sim2}$ contain both some of $O^{1\sim2}$ and all of $O^V$ with $V \geq 3$ under even number $2^{N+1}$.

In addition, $2^{N+1}+A^{1\sim2}$ and $2^{N+1}-A^{1\sim2}$ are two bilateral symmetric odd numbers whereby $2^{N+1}$ as a symmetric center from $(2^{N+1}+A^{1\sim2})+(2^{N+1}-A^{1\sim2}) = 2^{K+2}$ according to the distinct odd-even relation explained in advance.

Therefore $2^{N+1}+A^{1\sim2}$ contains both some of $O^{1\sim2}$ and all of $O^V$ with $V \geq 3$ under even number $2^{N+2}$.

Now that $2^{N+1}-A^X$ within $(2^{N+1}+A^X)+(2^{N+1}-A^X) = 2^{N+2}$ is one of $O^{1\sim2}$, as thus if from this aspect alone to consider, then $2^{N+1}+A^X$ is either one of $O^{1\sim2}$ or one of $O^V$ with $V \geq 3$ under even number $2^{N+2}$, because there are not two of $O^V$ with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{W-1}$ as a symmetric center.

But, since $2^{N+1}+A^{1\sim2}$ contains all of $O^V$ with $V \geq 3$ under even number $2^{N+2}$, therefore $2^{N+1}+A^X$, i.e. $A^X+2^{N+1}$ can only be one of $O^{1\sim2}$.

On the other, $C^Z$ is one of $O^V$ with $V \geq 3$ according to stipulations of values of C and Z within the known requirements.

Consequently there is $A^X+2^{N+1} \neq C^Z$ under the known requirements.

By the preceding way of doing things, can continue to prove that when $W=N+2$, $N+3$…up to every integer$\geq 3$, there surely are $A^X+2^{N+2}\neq C^Z$, $A^X+2^{N+3}\neq C^Z$…up to $A^X+2^W\neq C^Z$ under the known requirements surely.

## 6. Proving $A^X+2^W R^Y\neq C^Z$ under the Known Requirements

On the basis of the certain conclusion got, continue to prove $A^X+2^W R^Y\neq C^Z$ under the known requirements by the mathematical induction.

**(1)** When $R=1$, $2^W R^Y$ is exactly $2^W$, and that there is proven $A^X+2^W\neq C^Z$ under the known requirements in the preceding section.

**(2)** When $R=J$ and $J$ is an odd number $\geq 1$, $2^W R^Y$ i.e. $2^W J^Y$, suppose that there is $A^X+2^W J^Y\neq C^Z$ under the known requirements, where $A^X<2^W J^Y< C^Z$.

**(3)** When $R=K$ with $K=J+2$, $2^W R^Y$ i.e. $2^W K^Y$, prove that there is $A^X+2^W K^Y \neq C^Z$ under the known requirements, where $A^X< 2^W K^Y < C^Z$.

***Proof·*** Since there is $(2^W K^Y+A^X)+(2^W K^Y-A^X)=2^{W+1}K^Y$, then $2^W K^Y+A^X$ and $2^W K^Y-A^X$ are a pair of bilateral symmetric odd numbers whereby $2^W K^Y$ as a symmetric center according to the distinct odd-even relation explained in advance.

Also there is the inequality $2^W K^Y-A^X\neq O^V$ i.e. $A^X+O^V\neq 2^W K^Y$ where $V\geq 3$, according to proven $A^X+B^Y\neq 2^W H^Z$ under the known requirements, so that $2^W K^Y-A^X$ can only be one of $O^{1\sim 2}$.

Now that $2^W K^Y-A^X$ is one of $O^{1\sim 2}$, then $2^W K^Y-A^{1\sim 2}$ contains both some of $O^{1\sim 2}$ and all of $O^V$ with $V\geq 3$ under even number $2^W K^Y$.

In addition, $2^WK^Y+A^{1\sim2}$ and $2^WK^Y-A^{1\sim2}$ are a pair of bilateral symmetric odd numbers whereby $2^WK^Y$ as a symmetric center from $(2^WK^Y+A^{1\sim2})+(2^WK^Y-A^{1\sim2})=2^{W+1}K^Y$ according to the distinct odd-even relation explained in advance.

Therefore $2^WK^Y+A^{1\sim2}$ contains both some of $O^{1\sim2}$ and all of $O^V$ with $V\geq3$ under even number $2^{W+1}K^Y$.

Now that $2^WK^Y-A^X$ within $(2^WK^Y+A^X)+(2^WK^Y-A^X)=2^{W+1}K^Y$ is one of $O^{1\sim2}$, as thus, if from this aspect alone to consider, then $2^WK^Y+A^X$ is either one of $O^{1\sim2}$ or one of $O^V$ with $V\geq3$ under even number $2^{W+1}K^Y$ because there are not two of $O^V$ with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^WH^Z$ as a symmetric center.

But, since $2^WK^Y+A^{1\sim2}$ contains all of $O^V$ with $V\geq3$ under even number $2^{W+1}K^Y$, therefore $2^WK^Y+A^X$ i.e. $A^X+2^WK^Y$ can only be one of $O^{1\sim2}$.

On the other, $C^Z$ is one of $O^V$ with $V\geq3$ according to stipulations of values of C and Z within the known requirements.

Consequently there is $A^X+2^WK^Y\neq C^Z$, i.e. $A^X+2^W(J+2)^Y\neq C^Z$ under the known requirements.

By the above way of doing things, can continue to prove that when R=J+4, J+6…up to every positive odd number, there surely are $A^X+2^W(J+4)^Y\neq C^Z$, $A^X+2^W(J+6)^Y\neq C^Z$… up to $A^X+2^WR^Y\neq C^Z$ under the known requirements.

To sum up, the author has proven every kind of $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor. In addition to this, he has proven $A^X+B^Y=C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor in the front of this article.

Such being the case, so long as make a comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements, at once reach inevitably such a conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the given requirements is that A, B and C must have at least a common prime factor. The proof was thus brought to a close. As a consequence, the Beal's conjecture holds water.

## PS. Proving Fermat's last theorem From Beal's Conjecture

Fermat's last theorem is a special case of the Beal's conjecture, [3]. If Beal's conjecture is proved to hold water, then let X=Y=Z, so indefinite equation $A^X+B^Y=C^Z$ is changed into indefinite equation $A^X+B^X=C^X$.

In addition, divide three terms of $A^X+B^X=C^X$ by greatest common divisor of the three terms, then get a set of the solution of positive integers without a common prime factor. Obviously this conclusion is in contradiction with proven Beal's conjecture, as thus, we have proved Fermat's last theorem by reduction to absurdity as easy as the pie.

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