Classify Positive Integers to Prove Collatz Conjecture by the Mathematical Induction

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Abstract

In this article, the author uses the mathematical induction, classifies positive integers gradually, and passes necessary operations by the operational rule to achieve finally the purpose proving Collatz conjecture.

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1. Introduction

The Collatz conjecture also called the *3x+1* mapping, *3n+1* problem, Hasse's algorithm, Kakutani's problem, Syracuse algorithm, Syracuse problem, Thwaites conjecture and Ulam's problem, etc. Yet it is still both unproved and un-negated a conjecture ever since named after Lothar Collatz in 1937, [1].

2. A Few Bits of Basic Concepts

The Collatz conjecture states that take any positive integer n, if n is even, divide it by 2 to get $n/2$; if n is odd, multiply it by 3 and add 1 to get $3n+1$. Repeat the process indefinitely, then, no matter what positive integer you start with, you will always eventually reach a result of *1*, [2] and [3].

Let us regard aforesaid operational stipulations as the operational rule.

Begin with any positive integer or integer's expression to operate by the operational rule continuously, so form successive integers or integer's expressions. We regard such consecutive integers or integer's expressions plus synclastic arrowheads among them as an operational route.

If a positive integer's expression P_{ie} or integer P exists at an operational route, then may term the operational route "an operational route via P_{ie} or P". Generally speaking, integer's expressions at an operational route have a common variable or some variables which can transform into a variable. Two operational routes via P_{ie} branch from P_{ie} or an integer's expression after pass operations of P_{ie}.

3. Judging Criteria and the Classified Proof

Prove the Collatz conjecture by the mathematical induction [4], as follows.

(1) From 1→4→2→1; 2→1; 3→10→5→16→8→4→2→1; 4→2→1; 5→16→8→4→2→1; 6→3→10→5→16→8→4→2→1; 7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 8→4→2→1; 9→28→14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 10→5→16→8→4→2→1; 11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 12→6→3→10→5→16→8→4→2→1; 13→40→20→10→5→16→8→4→2→1; 14→7→22→11→34→17→52→26→13→40→20→10→5→16→8→4→2→1; 15→46→23→70→35→106→53→160→80→40→20→10→5→16→8→4→2→1; 16→8→

$4 \rightarrow 2 \rightarrow 1$; $17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$; $18 \rightarrow 9 \rightarrow 28 \rightarrow 14 \rightarrow 7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$ and $19 \rightarrow 58 \rightarrow 29 \rightarrow 88 \rightarrow 44 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$, get that every positive integer≤ 19 suits the conjecture.

(2) Suppose that n suits the conjecture, where n is an integer≥ 19.

(3) Prove that positive integer $n+1$ suits the conjecture too.

Before make the proof, it is necessary to prepare judging criteria concerned.

Theorem 1· If an integer's expression at an operational route via integer's expression P_{ie} is smaller than P_{ie}, and that P_{ie} contains $n+1$, then $n+1$suits the conjecture.

For example, if $P_{ie}=31+3^2\eta$, and P_{ie} contains $n+1$, where $\eta\geq 0$, then from $27+2^3\eta \rightarrow 82+3\times 2^3\eta \rightarrow 41+3\times 2^2\eta \rightarrow 124+3^2\times 2^2\eta \rightarrow 62+3^2\times 2\eta \rightarrow 31+3^2\eta > 27+2^3\eta$, conclude that $n+1$ suits the conjecture. For another example, if $P_{ie}=5+2^2\mu$, and P_{ie} contains $n+1$, where $\mu\geq 0$, then from $5+2^2\mu \rightarrow 16+3\times 2^2\mu \rightarrow 8+3\times 2\mu \rightarrow 4+3\mu < 5+2^2\mu$, conclude that $n+1$ suits the conjecture.

Proof· Suppose that there is an integer's expression C_{ie} at an operational route via P_{ie}, and $C_{ie} < P_{ie}$.

Since P_{ie} and C_{ie} exist at an operational route, and $C_{ie} < P_{ie}$, then, when their common variable equals some fixed value such that $P_{ie}=n+1$, let C_{ie}=integer m, so it has $m<n+1$. Undoubtedly operations of $n+1$ can pass operations of m at the operational route via $n+1$ to reach 1, because each and every positive integer $< n+1$ was supposed to suit the conjecture.

Lemma 1˙ If an integer's expression or an integer at an operational route suits the conjecture, then each and every integer's expression or each and every integer at the operational route suits the conjecture too.

Theorem 2˙ If an operational route via integer's expression Q_{ie} and an operational route via integer's expression P_{ie} intersect, and that an integer's expression at the operational route via Q_{ie} is smaller than P_{ie}, in addition P_{ie} contains $n+1$, then $n+1$ suits the conjecture, where $P_{ie}\neq Q_{ie}$.

For example, $P_{ie}=95+3^2\times2^7\varphi$, and P_{ie} contains $n+1$, where $\varphi\geq0$, then from

$95+3^2\times2^7\varphi\to286+3^3\times2^7\varphi\to143+3^3\times2^6\varphi\to430+3^4\times2^6\varphi\to215+3^4\times2^5\varphi\to646+3^5\times2^5\varphi$ $\to323+3^5\times2^4\varphi\to970+3^6\times2^4\varphi\to485+3^6\times2^3\varphi\to1456+3^7\times2^3\varphi\to728+3^7\times2^2\varphi\to$ $364+3^7\times2\varphi\to182+3^7\varphi\to\ldots$

$\uparrow121+3^6\times2\varphi\leftarrow242+3^6\times2^2\varphi\leftarrow484+3^6\times2^3\varphi\leftarrow161+3^5\times2^3\varphi\leftarrow322+3^5\times2^4\varphi\leftarrow107+3^4\times2^4\varphi\leftarrow$ $214+3^4\times2^5\varphi\leftarrow71+3^3\times2^5\varphi<95+3^2\times2^7\varphi$, conclude that $n+1$ suits the conjecture.

***Proof*˙** Suppose that there is an integer's expression D_{ie} at an operational route via Q_{ie} and $D_{ie}<P_{ie}$, in addition the operational route via Q_{ie} and an operational route via P_{ie} intersect at A_{ie}.

When their common variable equals some fixed value such that $P_{ie}=n+1$, let D_{ie}=integer μ, and A_{ie}=integer ξ, so it has $\mu<n+1$. Then, operations of ξ can pass operations of μ at the operational route via ξ to reach 1 according to Lemma 1. Like that, operations of $n+1$ can pass operations of ξ at the operational route via $n+1$ to reach 1.

Lemma 2˙ If an operational route via integer's expression Q_{ie} and an operational route via integer's expression P_{ie} are at indirect connection, and that an integer's expression at the operational route via Q_{ie} is smaller than P_{ie}, in addition P_{ie} contains $n+1$, then $n+1$ suits the conjecture.

What is called the indirect connection? Such as an operational route via Q_{ie} intersects an operational route via R_{ie}, and the operational route via R_{ie} intersects an operational route via P_{ie}, yet the operational route via Q_{ie} intersects not the operational route via P_{ie}, then the operational route via Q_{ie} and the operational route via P_{ie} are at the indirect connection.

Lemma 3· If an integer's expression at an operational route suits the conjecture, then each and every integer's expression at every operational route that intersects directly the operational route and connects indirectly with the operational route suits the conjecture.

For example, on the supposition that integer's expression C_{ie} at an operational route via integer's expression A_{ie} suits the conjecture, if the operational route via A_{ie} and an operational route via integer's expression B_{ie} intersect on integer's expression X_{ie}, then X_{ie} suits the conjecture according to Lemma 1; like the reason, each and every integer's expression at the operational route via B_{ie} suits the conjecture.

If the operational route via A_{ie} and an operational route via integer's expression P_{ie} are at indirect connection, then each and every integer's expression at the operational route via P_{ie} suits the conjecture, as long as apply continuously aforementioned way of doing the thing, in line for.

By now, let us set to prove the Collatz conjecture progressively, ut infra.

Proof· According to fore-prepared theorems and lemmas, on balance, must classify positive integers, then find out a relation between each class which possibly contains $n+1$ and another class which is smaller than the former, to prove that $n+1$ suits the conjecture.

It is well known that positive integers are divided into positive even numbers and positive odd numbers.

For even numbers $2k$ with $k\geq 1$, from $2k\rightarrow k<2k$, conclude that if $n+1\in 2k$, then $n+1$ suits the conjecture according to Theorem 1.

Secondly, for all unproved positive odd numbers, divide them into two genera, i.e. $5+4k$ and $7+4k$, where $k\geq 4$.

For $5+4k$, from $5+4k\rightarrow 16+12k\rightarrow 8+6k\rightarrow 4+3k<5+4k$, conclude that if $n+1\in 5+4k$, then $n+1$ suits the conjecture according to Theorem 1.

Also divide $7+4k$ into 3 sorts, i.e. $15+12c$, $19+12c$ and $23+12c$, where $c\geq 0$.

For $23+12c$, from $15+8c\rightarrow 46+24c\rightarrow 23+12c<15+8c$, conclude that if $n+1\in 23+12c$, then $n+1$ suits the conjecture according to Theorem 1.

For $15+12c$ and $19+12c$ when $c=0$, they were proved to suit the conjecture fore. So only need us to prove $15+12c$ and $19+12c$ where $c\geq 1$.

For $15+12c$ and $19+12c$ where $c\geq 1$, we will operate them right along, so that expound the relation that they act in accordance with fore-prepared several judging criteria.

Firstly, operate $15+12c$ by the operational rule successively, as follows.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)   e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦   e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)   **…**
♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ **…**
f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→**…**
g=2h: 322+4374h→**… …**

g=2h: 86+243h (**5**)
♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→**…**
f=2g: 304+1458g→152+729g ↓→**…**
…
…
♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ **…**   **…**
e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ **…**
f=2g: 20+243g↓→g=2h: 10+243h (**6**)   **…**
g=2h+1:790+1458h→395+729h↑→ **…**

Annotation:

(1) Each of letters c, d, e, f, g, h …etc at listed above operational routes expresses each of natural numbers plus 0.

(2) Also, there are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦.

(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

In the course of operation for *15+12c/19+12c* by the operational rule, if an operational result is smaller than a kind of *15+12c/19+12c*, and that it first appears at an operational route of *15+12c/19+12c*, then let us term the operational result “№1 satisfactory operational result”.

Hereupon conclude six kinds of *15+12c* derived monogamously from №1 satisfactory operational results at the bunch of operational routes of *15+12c*. If *n+1*∈a kind in them, then *n+1* suits the conjecture, as listed below.

From c=2d+1 and d=2e+1, get c=2d+1=2(2e+1)+1=4e+3, then 15+12c=51+48e $=51+3\times2^4e\to154+3^2\times2^4e\to77+3^2\times2^3e\to232+3^3\times2^3e\to116+3^3\times2^2e\to58+3^3\times2e\to$ 29+27e where mark (**1**), and 29+27e < 51+48e, so if *n+1*∈*51+48e*, then *n+1* suits the conjecture according to Theorem *1*.

From $c=2d+1$, $d=2e$ and $e=2f+1$, get $c=2d+1=4e+1=4(2f+1)+1=8f+5$, then $15+12c=75+96f=75+3\times2^5f\to226+3^2\times2^5f\to113+3^2\times2^4f\to340+3^3\times2^4f\to170+3^3\times2^3f\to85+3^3\times2^2f\to256+3^4\times2^2f\to128+3^4\times2^1f\to64+81f$ where mark (**2**), and $64+81f<75+96f$, so if $n+1\in75+96f$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d$, $d=2e+1$ and $e=2f+1$, get $c=2d=4e+2=4(2f+1)+2=8f+6$, then $15+12c=87+96f=87+3\times2^5f\to262+3^2\times2^5f\to131+3^2\times2^4f\to394+3^3\times2^4f\to197+3^3\times2^3f\to592+3^4\times2^3f\to296+3^4\times2^2f\to148+3^4\times2^1f\to74+81f$ where mark (**3**) , and $74+81f<87+96f$, so if $n+1\in87+96f$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d+1$, $d=2e$, $e=2f$, $f=2g+1$ and $g=2h+1$, get $c=2d+1=4e+1=8f+1=8(2g+1)+1=16g+9=16(2h+1)+9=32h+25$, then $15+12c=315+384h=315+3\times2^7h\to946+3^2\times2^7h\to473+3^2\times2^6h\to1420+3^3\times2^6h\to710+3^3\times2^5h\to355+3^3\times2^4h\to1066+3^4\times2^4h\to533+3^4\times2^3h\to1600+3^5\times2^3h\to800+3^5\times2^2h\to400+3^5\times2^1h\to200+243h$ where mark (**4**), and $200+243h<315+384h$, so if $n+1\in315+384h$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d$, $d=2e+1$, $e=2f$, $f=2g+1$ and $g=2h$, get $c=2d=2(2e+1)=4e+2=8f+2=8(2g+1)+2=16g+10=32h+10$, then $15+12c=135+384h=135+3\times2^7h\to406+3^2\times2^7h\to203+3^2\times2^6h\to610+3^3\times2^6h\to305+3^3\times2^5h\to916+3^4\times2^5h\to458+3^4\times2^4h\to229+3^4\times2^3h\to688+3^5\times2^3h\to344+3^5\times2^2h\to86+243h$ where mark (**5**), and $86+243h<135+384h$, so if $n+1\in135+384h$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d$, $d=2e$, $e=2f$, $f=2g$ and $g=2h$, get $c=2d=32h$, then $15+12c=15+384h=15+3\times2^7h\to46+3^2\times2^7h\to23+3^2\times2^6h\to70+3^3\times2^6h\to35+3^3\times2^5h\to106+3^4\times2^5h\to53+3^4\times2^4h\to60+3^5\times2^4h\to80+3^5\times2^3h\to40+3^5\times2^2h\to10+243h$ where mark (**6**), and

10+243h<15+384h, so if *n+1*∈*15+384h*, then *n+1* suits the conjecture according to Theorem *1*.

Secondly, operate *19+12c* by the operational rule successively, as follows.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**)　　　　e=2f:37+81f (**β**)
♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥
c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠
d=2e+1:94+81e↓→e=2f:47+81f (**γ**)
e=2f+1:526+486f ♦

g=2h: 119+243h (**δ**)　　　　**…**
f=2g+1:238+243g↑→g=2h+1:1444+1458h→722+729h↑→**…**
♥466+486f→233+243f↑→f=2g:700+1458g→350+729g↓→g=2h+1:3238+4374h↓
g=2h: 175+729h↓→**…**　**…**
…

g=2h+1:172+243h (**ε**)
f=2g: 101+243g↑→g=2h: 304+1458h→**…**
e=2f+1:202+243f↑→f=2g+1:1336+1458g→**…**
♠322+486e→161+243e↑→e=2f:484+1458f→**…**

♦526+486f→263+243f↓→f=2g: 790+1458g→**…**
f=2g+1: 253+243g↓→g=2h+1: 248+243h (ζ)
g=2h: 760+1458h→**…**

Like that, conclude six kinds of *19+12c* derived monogamously from №1 satisfactory operational results at the bunch of operational routes of *19+12c*. If *n+1*∈a kind in them, then *n+1* suits the conjecture, as listed below.

From c=2d and d=2e, get c=2d=4e, then $19+12c=19+48e=19+3\times2^4e\to58+3^2\times2^4e\to29+3^2\times2^3e\to88+3^3\times2^3e\to44+3^3\times2^2e\to22+3^3\times2e\to11+27e$ where mark (**α**), and 11+27e<19+48e, so if *n+1*∈*19+48e*, then *n+1* suits the conjecture according to Theorem *1*.

From c=2d, d=2e+1 and e=2f, get c=2d=2(2e+1)=4e+2=8f+2, then $19+12c=43+96f=43+3\times2^5f\to130+3^2\times2^5f\to65+3^2\times2^4f\to196+3^3\times2^4f\to98+3^3\times2^3f\to49+3^3\times2^2f\to148+3^4\times2^2f\to74+3^4\times2^1f\to37+81f$ where mark (**β**), and 37+81f < 43+96f, so if

$n+1 \in 43+96f$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d+1$, $d=2e+1$ and $e=2f$, get $c=2d+1=4e+3=8f+3$, then $19+12c=55+96f$ $=55+3\times2^5f\rightarrow166+3^2\times2^5f\rightarrow83+3^2\times2^4f\rightarrow250+3^3\times2^4f\rightarrow125+3^3\times2^3f\rightarrow376+3^4\times2^3f\rightarrow$ $188+3^4\times2^2f\rightarrow 94+3^4\times2^1f\rightarrow47+81f$ where mark (γ), and $47+81f < 55+96f$, so if $n+1 \in 55+96f$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h$, get $c=2d=2(2e+1)=4e+2=4(2f+1)+2$ $=8f+6=8(2g+1)+6=16g+14=32h+14$, then $19+12c=187+384h=187+3\times2^7h\rightarrow562+$ $3^2\times2^7h\rightarrow281+3^2\times2^6h\rightarrow844+3^3\times2^6h\rightarrow422+3^3\times2^5h\rightarrow211+3^3\times2^4h\rightarrow634+3^4\times2^4h\rightarrow317+$ $3^4\times2^3h\rightarrow952+3^5\times2^3h\rightarrow476+3^5\times2^2h\rightarrow238+3^5\times2^1h\rightarrow119+243h$ where mark ($\boldsymbol{\delta}$), and $119+243h<187+384h$, so if $n+1 \in 187+384h$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d+1$, $d=2e$, $e=2f+1$, $f=2g$ and $g=2h+1$, get $c=2d+1=4e+1=4(2f+1)+1=8f+5$ $=16g+5=16(2h+1)+5=32h+21$, then $19+12c=271+384h=271+3\times2^7h\rightarrow814+3^2\times2^7h\rightarrow$ $407+3^2\times2^6h\rightarrow1222+3^3\times2^6h\rightarrow611+3^3\times2^5h\rightarrow1834+3^4\times2^5h\rightarrow917+3^4\times2^4h\rightarrow2752+3^5\times2^4h$ $\rightarrow1376+3^5\times2^3h\rightarrow688+3^5\times2^2h\rightarrow344+3^5\times2^1h\rightarrow172+243h$ where mark ($\boldsymbol{\varepsilon}$), and $172+243h<271+384h$, so if $n+1 \in 271+384h$, then $n+1$ suits the conjecture according to Theorem *1*.

From $c=2d+1$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h+1$, get $c=2d+1= 2(2e+1)+1=$ $4e+3=4(2f+1)+3=8f+7=8(2g+1)+7=16(2h+1)+15=32h+31$, then $19+12c=391+384h$ $=391+3\times2^7h\rightarrow1174+3^2\times2^7h\rightarrow587+3^2\times2^6h\rightarrow1762+3^3\times2^6h\rightarrow881+3^3\times2^5h\rightarrow2644+$ $3^4\times2^5h\rightarrow1322+3^4\times2^4h\rightarrow661+3^4\times2^3h\rightarrow1984+3^5\times2^3h\rightarrow992+3^5\times2^2h\rightarrow496+3^5\times2^1h\rightarrow$ $248+243h$ where mark (ζ), and $248+243h < 391+384h$, so if $n+1 \in 391+384h$, then $n+1$ suits the conjecture according to Theorem *1*.

It is obvious that if $n+1$ belongs within a kind of $15+12c/19+12c$ derived from a №1 satisfactory operational result, then $n+1$ suits the conjecture.

It is observed that variables d, e, f, g, h ... etc. of integer's expressions appear at two bunches of operational routes of $15+12c$ and $19+12c$, in fact, the purpose which substitutes c by them is that in order to avoid the confusion and for convenience. On the contrary, let χ represent variables d, e, f, g, h, etc. intensively, but χ can not represent the variable c directly.

After substitute variables d, e, f, g, h and otherwise by variable χ, the odevity of part integer's expressions that contain the variable χ at operational routes of $15+12c/19+12c$ is still indeterminate. Or rather, for every such integer's expression, both regard it as an odd number to operate, and regard it as an even number to operate. We thus label such integer's expressions "odd-even expressions".

For any odd-even expression at the bunch of operational routes of $15+12c/19+12c$, two kinds of operations synchronize at itself.

After regard an odd-even expression as an odd number to operate, get a greater operational result > itself. Yet after regard it as an even number to operate, get a smaller operational result < itself.

Moreover, pass operations for each odd-even expression, the bunch of operational routes of $15+12c/19+12c$ adds an operational route inevitably.

Begin with an integer's expression to operate by the operational rule continuously, every operational route via consecutive greater operational results will be getting longer and longer, and that the sum of constant term plus coefficient of χ of integer's expression appeared thereon is getting greater and greater on the whole, along continuation of operations.

On the other, for a smaller operational result in synchronism with a greater operational result, if it can be divided by 2^{μ} with $\mu \geq 2$ to get an even smaller integer's expression, and that once the even smaller integer's expression is first smaller than a kind of *15+12c/19+12c*, then the even smaller integer's expression is the very a №1 satisfactory operational result, accordingly can derive the kind of *15+12c/19+12c* from it, so operations at the operational route may stop at here.

If the even smaller integer's expression is still greater than any kind of *15+12c/19+12c*, or the smaller operational result is still an odd-even expression, then this needs us continue to operate by the operational rule.

By this token, on the one hand, operational routes at the bunch of operational routes of *15+12c/19+12c* increase continually always; on the other hand, operational routes at the bunch of operational routes of *15+12c/19+12c* reduce continually always.

Begin with any kind of *15+12c/19+12c* to operate successively by the operational rule, certainly can educe or discover a №1 satisfactory operational result about itself, so long as the operational route proceed along consecutive smaller operational results.

Can derive at least a kind of *15+12c/19+12c* from a №1 satisfactory operational result, for examples, $15+12(1+2^{57}y)$ derives itself from $23+3^{38}y$, yet $15+12(4+2^{55}\times 3^{2}y)$ and $15+12(8+2^{32}\times 3^{17}y)$ derive themselves from $61+2^{3}\times 3^{37}y$, please, see also their operational routes, as listed below.

(1) From $15+12(1+2^{57}y)=27+2^{59}\times 3y \to 82+2^{59}\times 3^{2}y \to 41+2^{58}\times 3^{2}y \to 124+2^{58}\times 3^{3}y \to 62+2^{57}\times 3^{3}y \to 31+2^{56}\times 3^{3}y \to 94+2^{56}\times 3^{4}y \to 47+2^{55}\times 3^{4}y \to 142+2^{55}\times 3^{5}y \to 71+2^{54}\times 3^{5}y \to 214+2^{54}\times 3^{6}y \to 107+2^{53}\times 3^{6}y \to 322+2^{53}\times 3^{7}y \to 161+2^{52}\times 3^{7}y \to 484+2^{52}\times 3^{8}y \to 242+2^{51}\times 3^{8}y \to 121+2^{50}\times 3^{8}y \to 364+2^{50}\times 3^{9}y{*} \to 182+2^{49}\times 3^{9}y \to 91+2^{48}\times 3^{9}y \to 274+2^{48}\times 3^{10}y \to 137+2^{47}\times 3^{10}y \to 412+2^{47}\times 3^{11}y \to 206+2^{46}\times 3^{11}y \to 103+2^{45}\times 3^{11}y \to 310+2^{45}\times 3^{12}y \to 155+2^{44}\times 3^{12}y \to 466+2^{44}\times 3^{13}y \to 233+2^{43}\times 3^{13}y \to 700+2^{43}\times 3^{14}y \to 350+2^{42}\times 3^{14}y \to 175+2^{41}\times 3^{14}y \to 526+2^{41}\times 3^{15}y \to 263+2^{40}\times 3^{15}y \to 790+2^{40}\times 3^{16}y \to 395+2^{39}\times 3^{16}y \to 1186+2^{39}\times 3^{17}y \to 593+2^{38}\times 3^{17}y \to 1780+2^{38}\times 3^{18}y \to 890+2^{37}\times 3^{18}y \to 445+2^{36}\times 3^{18}y \to 1336+2^{36}\times 3^{19}y \to 668+2^{35}\times 3^{19}y \to 334+2^{34}\times 3^{19}y{**} \to 167+2^{33}\times 3^{19}y \to 502+2^{33}\times 3^{20}y \to 251+2^{32}\times 3^{20}y \to 754+2^{32}\times 3^{21}y \to 377+2^{31}\times 3^{21}y \to 1132+2^{31}\times 3^{22}y \to 566+2^{30}\times 3^{22}y \to 283+2^{29}\times 3^{22}y \to 850+2^{29}\times 3^{23}y \to 425+2^{28}\times 3^{23}y \to 1276+2^{28}\times 3^{24}y \to 638+2^{27}\times 3^{24}y \to 319+2^{26}\times 3^{24}y \to 958+2^{26}\times 3^{25}y \to 479+2^{25}\times 3^{25}y \to 1438+2^{25}\times 3^{26}y \to 719+2^{24}\times 3^{26}y \to 2158+2^{24}\times 3^{27}y \to 1079+2^{23}\times 3^{27}y \to 3238+2^{23}\times 3^{28}y \to 1619+2^{22}\times 3^{28}y \to 4858+2^{22}\times 3^{29}y \to 2429+2^{21}\times 3^{29}y \to 7288+2^{21}\times 3^{30}y \to 3644+2^{20}\times 3^{30}y \to 1822+2^{19}\times 3^{30}y \to 911+2^{18}\times 3^{30}y \to 2734+2^{18}\times 3^{31}y \to 1367+2^{17}\times 3^{31}y \to 4102+2^{17}\times 3^{32}y \to 2051+2^{16}\times 3^{32}y \to 6154+2^{16}\times 3^{33}y \to 3077+2^{15}\times 3^{33}y \to 9232+2^{15}\times 3^{34}y \to 4616+2^{14}\times 3^{34}y \to 2308+2^{13}\times 3^{34}y \to 1154+2^{12}\times 3^{34}y \to 577+2^{11}\times 3^{34}y \to 1732+2^{11}\times 3^{35}y \to 866+2^{10}\times 3^{35}y \to 433+2^{9}\times 3^{35}y \to 1300+2^{9}\times 3^{36}y \to 650+2^{8}\times 3^{36}y \to 325+2^{7}\times 3^{36}y \to 976+2^{7}\times 3^{37}y \to 488+2^{6}\times 3^{37}y \to$

$244+2^5\times3^{37}y\to122+2^4\times3^{37}y\to61+2^3\times3^{37}y\to184+2^3\times3^{38}y\to92+2^2\times3^{38}y\to46+2^1\times3^{38}y$ $\to23+3^{38}y$, get №1 satisfactory operational result $23+3^{38}y$ about the kind of $27+2^{59}\times3y$.

(2) From $15+12(4+2^{55}\times3^2y)=63+2^{57}\times3^3y\to190+2^{57}\times3^4y\to95+2^{56}\times3^4y\to286+2^{56}\times3^5y$ $\to143+2^{55}\times3^5y\to430+2^{55}\times3^6y\to215+2^{54}\times3^6y\to646+2^{54}\times3^7y\to323+2^{53}\times3^7y\to$ $970+2^{53}\times3^8y\to485+2^{52}\times3^8y\to1456+2^{52}\times3^9y\to728+2^{51}\times3^9y\to364+2^{50}\times3^9y$* at operational route $27+2^{59}\times3y\ldots\to61+2^3\times3^{37}y$, get №1 satisfactory operational result $61+2^3\times3^{37}y$ about the kind of $63+2^{57}\times3^3y$.

(3) From $15+12(8+2^{32}\times3^{17}y)=111+2^{34}\times3^{18}y\to334+2^{34}\times3^{19}y$** at operational route $27+2^{59}\times3y\ldots\to61+2^3\times3^{37}y$, get the same №1 satisfactory operational result $61+2^3\times3^{37}y$ about the kind of $111+2^{34}\times3^{18}y$ too.

In some cases, an operational route of $15+12c$ and an operational route of $19+12c$ can intersect, such as when operate $15+12(1+2^{57}y)$ to fifth step, the integer's expression got is exactly $19+12(1+2^{54}\times3^2y)$ in example (1).

Due to $c\geq1$, there are infinitely many odd numbers of $15+12c/19+12c$, whether they belong to infinite many kinds or finite many kinds.

Since there is an operational route between each kind of $15+12c/19+12c$ and №1 satisfactory operational result about the kind itself, so $15+12c/19+12c$ has how many kinds of $15+12c/19+12c$, then there are how many operational routes of $15+12c/19+12c$, yet for each line segment coincided from one another, either it is regarded as component part of some operational routes, or it is irrespective for the others.

Therefore, all operational routes at the bunch of operational routes of *15+12c/19+12c* are formally similar to networking status. That is to say, between each operational route and each of operational routes except for the former, either both intersect directly, or both connect indirectly.

Since setting up the variable *c*, such that all kinds of *15+12c/19+12c* collect at the bunch of operational routes of *15+12c/19+12c*; but then, due to the odevity of χ, such that the bunch of operational routes of *15+12c/19+12c* has infinitely many branches.

Now that №1 satisfactory operational results determine all kinds of *15+12c/19+12c*, so begin with any kind of *15+12c/19+12c* to operate by the operational rule continuously, then a №1 satisfactory operational result about the kind can only first appear under one in following *3* cases.

(1) №1 satisfactory operational result about a kind of *15+12c/19+12c* first appears at an operational route via the kind of *15+12c/19+12c* itself;

(2) An operational route via a kind of *15+12c/19+12c* intersects another operational route that has №1 satisfactory operational result about the kind of *15+12c/19+12c*;

(3) An operational route via a kind of *15+12c/19+12c* indirectly connects with another operational route that has №1 satisfactory operational result about the kind of *15+12c /19+12c*.

In any case, all kind of *15+12c/19+12c* and every №1 satisfactory operational result must coexist at the bunch of operational routes of *15+12c*

/19+12c, otherwise, no matter what integer's expression, it belongs not to the type of *15+12c/19+12c* or №1 satisfactory operational result either.

Operations by the operational rule from any concrete integer within a kind of *15+12c/19+12c* to №1 satisfactory operational result about the concrete integer can only pass finitely more steps.

Because even if an operational route via the kind of *15+12c/19+12c* pass consecutive greater operational results to elongate infinitely, while there is a smaller operational result in synchronism with each such greater operational result, and that operational routes via the smaller operational result will continue to operate along consecutive smaller operational results, up to educe or discover the №1 satisfactory operational result about the kind of *15+12c/19+12c* after pass operations of finite steps.

Accordingly, when their common variable equals some fixed value such that the kind of *15+12c/19+12c* equals the concrete integer, let the №1 satisfactory operational result equals another integer, then from the concrete integer to appearing another integer can only pass finite more steps too.

Now that operational routes of every kind of *15+12c* can only exist at the bunch of operational routes of *15+12c* and operational routes of every kind of *19+12c* can only exist at the bunch of operational routes of *19+12c*, then either any operational route of any kind of *15+12c/19+12c* intersects directly an operational route of any of above-listed six kinds of *15+12c/*

19+12c derived monogamously from №1 satisfactory operational results according to the bunch of operational routes of *15+12c/19+12c*, or any operational route of any kind of *15+12c/19+12c* connects indirectly with an operational route of any of the above-listed six kinds of *15+12c/19+12c*. Yet the six №1 satisfactory operational results and the six kinds of *15+12c/19+12c* have been proven to suit the conjecture. By this token, for each operational route via a smaller operational result, whether it has the termination, this is absolutely unnecessary worry.

Therefore any kind of *15+12c/19+12c* is proven to suit the conjecture according to Lemma *1*, or Lemma *3*.

Consequently, if *n+1* belongs within any kind of *15+12c/19+12c*, then *n+1* suits the conjecture according to Theorem *1*, Theorem *2*, or Lemma *2*.

To sum up, *n+1* has been proved to suit the conjecture, whether *n+1* belongs within which genus, which sort or which kind of odd numbers, or it is exactly an even number.

Likewise, we can too prove positive integers *n+2*, *n+3*, *n+4* etc. up to every positive integer to suit the conjecture in the light of the old way of preceding doing things.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.

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