Dirac and Majorana Field Operators with Self/Anti-Self Charge Conjugate States

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The dates of receipt and acceptance should be inserted later

Key words Field Operators, QFT, Dirac, Majorana, Neutral Particles

We discuss relations between Dirac and Majorana-like field operators with self/anti-self charge conjugate states. The connections with recent models of several authors have been found.

In Refs. (Dvoeglazov 2003,2006,2009,2011,2013,2015, 2016) we considered the procedure of construction of the field operators *ab initio* (including for neutral particles). The Bogoliubov-Shirkov method has been used, Ref. (Bogoliubov & Shirkov 1984).

In the present article we investigate the spin-1/2 case for self/anti-self charge conjugate states. We look for interrelations between the Dirac field operator and the Majorana field operator. It seems that the calculations give mathematically and physically reasonable results in the helicity basis only.

We write the charge conjugation operator into the form:

$$C = e^{i\theta_c}\begin{pmatrix} 0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & i & 0 & 0\\ -i & 0 & 0 & 0\end{pmatrix}\mathcal{K} = -e^{i\theta_c}\gamma^2\mathcal{K}\,. \tag{1}$$

It is the anti-linear operator of charge conjugation. $\mathcal{K}$ is the complex conjugation operator. We define the *self/anti-self* charge-conjugate 4-spinors in the momentum space (Ahluwalia 1996):

$$C\lambda^{S,A}(\mathbf{p}) \;=\; \pm\lambda^{S,A}(\mathbf{p})\,, \tag{2}$$

$$C\rho^{S,A}(\mathbf{p}) \;=\; \pm\rho^{S,A}(\mathbf{p})\,. \tag{3}$$

Thus,

$$\lambda^{S,A}(p^\mu) = \begin{pmatrix} \pm i\Theta\phi_L^*(\mathbf{p})\\ \phi_L(\mathbf{p})\end{pmatrix}\,, \tag{4}$$

and

$$\rho^{S,A}(\mathbf{p}) = \begin{pmatrix} \phi_R(\mathbf{p})\\ \mp i\Theta\phi_R^*(\mathbf{p})\end{pmatrix}\,. \tag{5}$$

ϕ_L, ϕ_R can be boosted with the Lorentz transformation $\Lambda_{L,R}$ matrices.[1]

The rest $\lambda-$ and $\rho-$ spinors are:[2]

$$\lambda^S_\uparrow(\mathbf{0}) \;=\; \sqrt{\frac{m}{2}}\begin{pmatrix}0\\ i\\ 1\\ 0\end{pmatrix}\,,\; \lambda^S_\downarrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}-i\\ 0\\ 0\\ 1\end{pmatrix}\,, \tag{7}$$

$$\lambda^A_\uparrow(\mathbf{0}) \;=\; \sqrt{\frac{m}{2}}\begin{pmatrix}0\\ -i\\ 1\\ 0\end{pmatrix}\,,\; \lambda^A_\downarrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}i\\ 0\\ 0\\ 1\end{pmatrix}\,, \tag{8}$$

$$\rho^S_\uparrow(\mathbf{0}) \;=\; \sqrt{\frac{m}{2}}\begin{pmatrix}1\\ 0\\ 0\\ -i\end{pmatrix}\,,\; \rho^S_\downarrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}0\\ 1\\ i\\ 0\end{pmatrix}\,, \tag{9}$$

$$\rho^A_\uparrow(\mathbf{0}) \;=\; \sqrt{\frac{m}{2}}\begin{pmatrix}1\\ 0\\ 0\\ i\end{pmatrix}\,,\; \rho^A_\downarrow(\mathbf{0}) = \sqrt{\frac{m}{2}}\begin{pmatrix}0\\ 1\\ -i\\ 0\end{pmatrix}\,. \tag{10}$$

Thus, in this basis the explicit forms of the 4-spinors of the second kind $\lambda^{S,A}_{\uparrow\downarrow}(\mathbf{p})$ and $\rho^{S,A}_{\uparrow\downarrow}(\mathbf{p})$ are:

$$\lambda^S_\uparrow(\mathbf{p}) \;=\; \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} ip_l\\ i(p^-+m)\\ p^-+m\\ -p_r\end{pmatrix}\,, \tag{11}$$

$$\lambda^S_\downarrow(\mathbf{p}) \;=\; \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} -i(p^++m)\\ -ip_r\\ -p_l\\ (p^++m)\end{pmatrix}\,,$$

$$\lambda^A_\uparrow(\mathbf{p}) \;=\; \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} -ip_l\\ -i(p^-+m)\\ (p^-+m)\\ -p_r\end{pmatrix}\,,$$

$$\lambda^A_\downarrow(\mathbf{p}) \;=\; \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} i(p^++m)\\ ip_r\\ -p_l\\ (p^++m)\end{pmatrix}\,,$$

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[1] Such definitions of 4-spinors differ, of course, from the original Majorana definition in x-representation:

$$\nu(x) = \frac{1}{\sqrt{2}}(\Psi_D(x) + \Psi^c_D(x))\,, \tag{6}$$

$C\nu(x) = \nu(x)$ that represents the positive real $C-$ parity field operator. However, the momentum-space Majorana-like spinors open various possibilities for description of neutral particles (with experimental consequences, see (Kirchbach & Compean & Noriega 2004).

[2] The choice of the helicity parametrization for $\mathbf{p}\to\mathbf{0}$ is doubtful in Ref. (Ahluwalia & Grumiller 2005), and it leads to unremovable contradictions, in my opinion.

$$\rho^S_\uparrow(\mathbf{p}) = \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} p^+ + m\\ p_r\\ ip_l\\ -i(p^+ + m)\end{pmatrix}, \qquad (12)$$

$$\rho^S_\downarrow(\mathbf{p}) = \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} p_l\\ (p^- + m)\\ i(p^- + m)\\ -ip_r\end{pmatrix},$$

$$\rho^A_\uparrow(\mathbf{p}) = \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} p^+ + m\\ p_r\\ -ip_l\\ i(p^+ + m)\end{pmatrix},$$

$$\rho^A_\downarrow(\mathbf{p}) = \frac{1}{2\sqrt{E_p+m}}\begin{pmatrix} p_l\\ (p^- + m)\\ -i(p^- + m)\\ ip_r\end{pmatrix}.$$

As we showed $\lambda-$ and $\rho-$ 4-spinors are not the eigenspinors of the helicity. Moreover, $\lambda-$ and $\rho-$ are not (if we use the parity matrix $P = \begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix} R$) the eigenspinors of the parity, as opposed to the Dirac case. The indices $\uparrow\downarrow$ should be referred to the chiral helicity quantum number introduced in the 60s, $\eta = -\gamma^5 h$, for λ spinors. While

$$Pu_\sigma(\mathbf{p}) = +u_\sigma(\mathbf{p})\,, Pv_\sigma(\mathbf{p}) = -v_\sigma(\mathbf{p})\,, \qquad (13)$$

we have

$$P\lambda^{S,A}(\mathbf{p}) = \rho^{A,S}(\mathbf{p})\,, P\rho^{S,A}(\mathbf{p}) = \lambda^{A,S}(\mathbf{p}) \qquad (14)$$

for the Majorana-like momentum-space 4-spinors on the first quantization level. In this basis one has

$$\rho^S_\uparrow(\mathbf{p}) = -i\lambda^A_\downarrow(\mathbf{p})\,, \rho^S_\downarrow(\mathbf{p}) = +i\lambda^A_\uparrow(\mathbf{p})\,, \qquad (15)$$

$$\rho^A_\uparrow(\mathbf{p}) = +i\lambda^S_\downarrow(\mathbf{p})\,, \rho^A_\downarrow(\mathbf{p}) = -i\lambda^S_\uparrow(\mathbf{p})\,. \qquad (16)$$

The analogs of the spinor normalizations (for $\lambda^{S,A}_{\uparrow\downarrow}(\mathbf{p})$ and $\rho^{S,A}_{\uparrow\downarrow}(\mathbf{p})$) are the following ones:

$$\overline{\lambda}^S_\uparrow(\mathbf{p})\lambda^S_\downarrow(\mathbf{p}) = -im\,, \overline{\lambda}^S_\downarrow(\mathbf{p})\lambda^S_\uparrow(\mathbf{p}) = +im\,, \qquad (17)$$

$$\overline{\lambda}^A_\uparrow(\mathbf{p})\lambda^A_\downarrow(\mathbf{p}) = +im\,, \overline{\lambda}^A_\downarrow(\mathbf{p})\lambda^A_\uparrow(\mathbf{p}) = -im\,, \qquad (18)$$

$$\overline{\rho}^S_\uparrow(\mathbf{p})\rho^S_\downarrow(\mathbf{p}) = +im\,, \overline{\rho}^S_\downarrow(\mathbf{p})\rho^S_\uparrow(\mathbf{p}) = -im\,, \qquad (19)$$

$$\overline{\rho}^A_\uparrow(\mathbf{p})\rho^A_\downarrow(\mathbf{p}) = -im\,, \overline{\rho}^A_\downarrow(\mathbf{p})\rho^A_\uparrow(\mathbf{p}) = +im\,. \qquad (20)$$

All other conditions are equal to zero.

The $\lambda-$ and $\rho-$ spinors are connected with the $u-$ and $v-$ spinors by the following formula:

$$\begin{pmatrix}\lambda^S_\uparrow(\mathbf{p})\\ \lambda^S_\downarrow(\mathbf{p})\\ \lambda^A_\uparrow(\mathbf{p})\\ \lambda^A_\downarrow(\mathbf{p})\end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & i & -1 & i\\ -i & 1 & -i & -1\\ 1 & -i & -1 & -i\\ i & 1 & i & -1\end{pmatrix}\begin{pmatrix} u_{+1/2}(\mathbf{p})\\ u_{-1/2}(\mathbf{p})\\ v_{+1/2}(\mathbf{p})\\ v_{-1/2}(\mathbf{p})\end{pmatrix} \qquad (21)$$

provided that the 4-spinors have the same physical dimension.[3]

We construct the field operators on using the Bogoliubov-Shirkov procedure with $\lambda^S_\eta(p)$:

$$\begin{aligned}\Psi(x) &= \frac{1}{(2\pi)^3}\int d^4p\,\delta(p^2-m^2)e^{-ip\cdot x}\Psi(p) =\\ &= \frac{1}{(2\pi)^3}\sum_{\eta=\uparrow\downarrow}\int d^4p\,\delta(p_0^2-E_p^2)e^{-ip\cdot x}\sqrt{m}\\ &\quad [\lambda^S_\eta(p_0,\mathbf{p})c_\eta(p_0,\mathbf{p})] = \qquad (22)\\ &= \frac{\sqrt{m}}{(2\pi)^3}\int\frac{d^4p}{2E_p}[\delta(p_0-E_p)+\delta(p_0+E_p)]\\ &\quad [\theta(p_0)+\theta(-p_0)]e^{-ip\cdot x}\sum_{\eta=\uparrow\downarrow}\lambda^S_\eta(p)c_\eta(p)\\ &= \frac{\sqrt{m}}{(2\pi)^3}\sum_{\eta=\uparrow\downarrow}\int\frac{d^4p}{2E_p}[\delta(p_0-E_p)+\delta(p_0+E_p)]\\ &\quad \big[\theta(p_0)(p)\lambda^S_\eta(p)c_\eta(p)e^{-ip\cdot x}+\\ &+\ \theta(p_0)\lambda^S_\eta(-p)c_\eta(-p)e^{+ip\cdot x}\big]\\ &= \frac{\sqrt{m}}{(2\pi)^3}\sum_{\eta=\uparrow\downarrow}\int\frac{d^3\mathbf{p}}{2E_p}\theta(p_0)\\ &\quad \Big[\lambda^S_\eta(p)c_\eta(p)|_{p_0=E_p}e^{-i(E_pt-\mathbf{p}\cdot\mathbf{x})}+\\ &+\ \lambda^S_\eta(-p)c_\eta(-p)|_{p_0=E_p}e^{+i(E_pt-\mathbf{p}\cdot\mathbf{x})}\Big]\end{aligned}$$

Thus, comparing with the Dirac field operator we have 1) instead of $u_h(\pm p)$ we have $\lambda^S_\eta(\pm p)$; 2) possible change of the annihilation operators, $a_h \to c_\eta$. Apart, one can make corresponding changes due to normalization factors. Thus, we should have

$$\sum_{\eta=\uparrow\downarrow}\lambda^A_\eta(p)d^\dagger_\eta(p) = \sum_{\eta=\uparrow\downarrow}\lambda^S_\eta(-p)c_\eta(-p)\,. \qquad (23)$$

Multiplying by $\overline{\lambda}^A_{-\kappa}(p)$ or $\overline{\lambda}^S_{-\kappa}(-p)$, respectively, we find surprisingly:

$$d^\dagger_\kappa(p) = -\frac{ip_y}{p}\sigma^y_{\kappa\tau}c_\tau(-p)\,, \qquad (24)$$

$$c_\kappa(-p) = -\frac{ip_y}{p}\sigma^y_{\kappa\tau}d^\dagger_\tau(p)\,. \qquad (25)$$

The above-mentioned contradiction may be related to the possibility of the conjugation which is different from that of Dirac. Both in the Dirac-like case and the Majorana-like case ($c_\eta(p) = e^{-i\varphi}d_\eta(p)$) we have difficulties in the construction of field operators (Dvoeglazov 2018b).

The bi-orthogonal anticommutation relations are given in Ref. (Ahluwalia 1996). See other details in Ref. (Dvoeglazov 1995a, 1997). Concerning with the P,C and T properties of the corresponding states see Ref. (Dvoeglazov 2011) in this model.

Similar formulations have been presented in Refs. (Markov 1937), and (Barut & Ziino 1993). Namely, the reflection properties are different for some solutions of relativistic equations therein. Two opposite signs at the mass terms have been taken into account. The group-theoretical basis for such doubling has been given in the papers by

[3] The change of the mass dimension of the field operator has no sufficient foundations because the Lagrangian can be constructed on using the coupled Dirac equations, see Ref. (Dvoeglazov 1995). After that one can play with $\sqrt{m}$ to reproduce all possible mathematical results, which may (or may not) answer to the physical reality.

Gelfand, Tsetlin (1957) and Sokolik (1957), who first presented the theory of 5-dimensional spinors (or, the one in the 2-dimensional projective representation of the inversion group) in 1956 (later called as “the Bargmann-Wightman-Wigner-type quantum field theory” in 1993). The corresponding connection with the time reversion has been clarified therein. It was one of the first attempts to explain the K-meson decays. M. Markov proposed *two* Dirac equations with opposite signs at the mass term (Markov 1937) to be taken into account:

$$[i\gamma^\mu \partial_\mu - m]\,\Psi_1(x) \;=\; 0\,, \tag{26}$$
$$[i\gamma^\mu \partial_\mu + m]\,\Psi_2(x) \;=\; 0\,. \tag{27}$$

In fact, he studied all properties of this relativistic quantum model (while the quantum field theory has not yet been completed in 1937). Next, he added and subtracted these equations. What did he obtain?

$$i\gamma^\mu \partial_\mu \varphi(x) - m\chi(x) \;=\; 0\,, \tag{28}$$
$$i\gamma^\mu \partial_\mu \chi(x) - m\varphi(x) \;=\; 0\,. \tag{29}$$

Thus, the corresponding φ and χ solutions can be presented as some superpositions of the Dirac 4-spinors $u-$ and $v-$. These equations, of course, can be identified with the equations for the Majorana-like $\lambda-$ and $\rho-$, which we presented in Ref. (Dvoeglazov 1995b).[4]

$$i\gamma^\mu \partial_\mu \lambda^S(x) - m\rho^A(x) \;=\; 0\,, \tag{30}$$
$$i\gamma^\mu \partial_\mu \rho^A(x) - m\lambda^S(x) \;=\; 0\,, \tag{31}$$
$$i\gamma^\mu \partial_\mu \lambda^A(x) + m\rho^S(x) \;=\; 0\,, \tag{32}$$
$$i\gamma^\mu \partial_\mu \rho^S(x) + m\lambda^A(x) \;=\; 0\,. \tag{33}$$

Neither of them can be regarded as the Dirac equation. However, they can be written in the 8-component form as follows:

$$[i\Gamma^\mu \partial_\mu - m]\,\Psi_{(+)}(x) \;=\; 0\,, \tag{34}$$
$$[i\Gamma^\mu \partial_\mu + m]\,\Psi_{(-)}(x) \;=\; 0\,, \tag{35}$$

with

$$\Psi_{(+)}(x) = \begin{pmatrix}\rho^A(x)\\ \lambda^S(x)\end{pmatrix}, \Psi_{(-)}(x) = \begin{pmatrix}\rho^S(x)\\ \lambda^A(x)\end{pmatrix}, \tag{36}$$
$$\Gamma^\mu = \begin{pmatrix}0 & \gamma^\mu\\ \gamma^\mu & 0\end{pmatrix}. \tag{37}$$

It is possible to find the corresponding Lagrangian, projection operators, and the Feynman-Dyson-Stueckelberg propagator. For example,

$$\begin{aligned}\mathcal{L} \;=\; & \frac{i}{2}\,[\overline{\Psi}_{(+)}\Gamma^\mu \partial_\mu \Psi_{(+)} - (\partial_\mu \overline{\Psi}_{(+)})\Gamma^\mu \Psi_{(+)} + \\ + \;& \overline{\Psi}_{(-)}\Gamma^\mu \partial_\mu \Psi_{(-)} - (\partial_\mu \overline{\Psi}_{(-)})\Gamma^\mu \Psi_{(-)}] - \\ - \;& m[\overline{\Psi}_{(+)}\Psi_{(+)} - \overline{\Psi}_{(-)}\Psi_{(-)}]\,.\end{aligned} \tag{38}$$

The projection operator P_+ can be easily found, as usual,

$$P_+ = \frac{\Gamma_\mu p^\mu + m}{2m}\,. \tag{39}$$

[4] Of course, the signs at the mass terms depend on, how do we associate the positive- or negative- frequency solutions with λ and ρ.

However, due to the fact that P_- satisfies the Dirac equation with the opposite sign, we cannot have $P_+ + P_- = 1$. This is not surprising because the corresponding states $\Psi_\pm$ do not form the complete system of the 8-dimensional space. One should consider the states $\Gamma_5\Psi_\pm(\mathbf{p})$ too. See also (Dvoeglazov 2018a) for the methods of obtaining the propagators in the non-trivial cases.

In the previous papers I explained: the connection with the Dirac spinors has been found (Dvoeglazov 1995b; Kirchbach & Compean & Noriega 2004) through the unitary matrix, provided that the 4-spinors have the same physical dimension.[5] Thus, this represents itself the rotation of the spin-parity basis. However, it is usually assumed that the $\lambda-$ and $\rho-$ spinors describe the neutral particles, meanwhile, the $u-$ and $v-$ spinors describe the charged particles. Kirchbach, Compean and Noriega (2004) found the amplitudes for neutrinoless double beta decay ($00\nu\beta$) in this scheme. It is obvious from (21) that there are some additional terms comparing with the standard calculations of those amplitudes. One can also re-write the above equations into the two-component forms. Thus, one obtains the Feynman and Gell-Mann (1958) equations.

Barut and Ziino (1993) proposed yet another model. They considered γ^5 operator as the operator of the charge conjugation. In their case the self/anti-self charge conjugate states are, at the same time, the eigenstates of the chirality. Thus, the charge-conjugated Dirac equation has a different sign compared with the ordinary formulation:

$$[i\gamma^\mu \partial_\mu + m]\Psi^c_{BZ} = 0\,, \tag{40}$$

and the so-defined charge conjugation applies to the whole system, fermion + electromagnetic field, $e \to -e$ in the covariant derivative. The superpositions of the Ψ_{BZ} and Ψ^c_{BZ} also give us the “doubled Dirac equation”, as the equations for $\lambda-$ and $\rho-$ spinors. The concept of the doubling of the Fock space has been developed in the Ziino works, cf. (Gelfand & Tsetlin 1957; Sokolik 1957; Dvoeglazov 1998) in the framework of the quantum field theory (Ziino 1996). Next, it is interesting to note that we have for the Majorana-like field operators ($a_\eta(\mathbf{p}) = b_\eta(\mathbf{p})$):

$$\begin{aligned}\left[\nu^{^{ML}}(x^\mu) + \mathcal{C}\nu^{^{ML}\,\dagger}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p} \\ \sum_\eta \left[\begin{pmatrix} i\Theta\phi^{*\,\eta}_{_L}(p^\mu)\\ 0\end{pmatrix} a_\eta(p^\mu)e^{-ip\cdot x} + \right. \\ \left. + \begin{pmatrix}0\\ \phi^\eta_L(p^\mu)\end{pmatrix} a^\dagger_\eta(p^\mu)e^{ip\cdot x}\right],\end{aligned} \tag{41}$$

$$\begin{aligned}\left[\nu^{^{ML}}(x^\mu) - \mathcal{C}\nu^{^{ML}\,\dagger}(x^\mu)\right]/2 = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p} \\ \sum_\eta \left[\begin{pmatrix}0\\ \phi^\eta_{_L}(p^\mu)\end{pmatrix} a_\eta(p^\mu)e^{-ip\cdot x} + \right. \\ \left. + \begin{pmatrix} -i\Theta\phi^{*\,\eta}_{_L}(p^\mu)\\ 0\end{pmatrix} a^\dagger_\eta(p^\mu)e^{ip\cdot x}\right].\end{aligned} \tag{42}$$

[5] The reasons of the change of the fermion mass dimension are unclear in the recent works on *elko*.

This naturally leads to the Ziino-Barut scheme of massive chiral fields. See, however, the recent paper (Dvoeglazov 2018b) which deals with the problems of the Majorana field operator.

Acknowledgements. I acknowledge discussions with colleagues at recent conferences. I am grateful to the Zacatecas University for professorship.

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