Elements 2 : The Integral Formula

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24-11-2018 9:21:12

abstract

This note presents a elementary integral formula.

Formula

$$\int_0^{1/4} \sqrt[4]{\frac{1-2x-2x^2+\sqrt{1-4x}}{x^2}}\,dx-\int_{\left(\sqrt{2}-1\right)/2}^{1/4} \sqrt[4]{\frac{1-2x-2x^2-\sqrt{1-4x}}{x^2}}\,dx=$$

$$=\frac{\sqrt[4]{2}\left(\Gamma\left(1/4\right)\right)^2}{8\sqrt{\pi}}-\frac{\pi}{4\sqrt[4]{2}}+\frac{1}{4\sqrt[4]{2}}\ln\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right)+\frac{\sqrt[4]{2}\left(\sqrt{2}-1\right)}{2}$$

Remarks:

$$\pi=4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}=3.141592...$$

$$\Gamma\left(1/4\right)=4\prod_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{1/4}\left(1+\frac{1}{4n}\right)^{-1}=3.625609...$$

References

1. Gradshteyn, I.S. and Ryzhik, I.M.: Table of Integrals, Series and Products. Seventh Edition, edited by Alan Jeffrey and Daniel Zwillinger. Academic Press, 2007.
2. Ramanujan, S.: Notebooks (2 volumes), Tata Institute of Fundamental Research, Bombay, 1957.