

Elements 3 : Elementary Infinite Product

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Abstract. This note presents a elementary infinite product.

Infinite Product

$$\frac{2(9+2\pi\sqrt{3})}{27} = \prod_{n=1}^{\infty} \left(1 + \frac{1}{\binom{2n+2}{n+1} s_n} \right) = \prod_{n=1}^{\infty} \left(1 + \frac{1}{c_n} \right)$$

$$s_{n+1} = \frac{5n+3}{4n+2} s_n - \frac{n+1}{4n+2} s_{n-1}, \quad s_0 = 0, s_1 = 1/2$$

$$c_{n+1} = \frac{(2n+3)(5n+3)}{(2n+1)(n+2)} c_n - \frac{2(2n+3)}{n+2} c_{n-1}, \quad c_0 = 0, c_1 = 3$$

Remark: $\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.141592\dots$

References

1. Ramanujan, S.: Modular equations and approximations to π , Quart. J. Math. (Oxford) 45, 1914, 350-372.
2. Ramanujan, S.: Proof of certain identities in combinatory analysis, Proc. Cambridge Philos. Soc. 19, 1919, 214-216.
3. Ramanujan, S.: Notebooks (2 volumes), Tata Institute of Fundamental Research, Bombay, 1957.