The metaphysics of physics

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Abstract: This is a didactic exploration of the basic assumptions and concepts of the *Zitterbewegung* interpretation of quantum mechanics. Its novelty is in applying the concepts to photons and relating it to other uses of the wavefunction. As such, we could have chosen another title for this paper: the physics of quantum physics. However, we only present interpretations, hypotheses and assumptions. As such, we thought we should stick to the title above: the metaphysics of physics. It sounds *somewhat* less arrogant.

Keywords: *Zitterbewegung*, Uncertainty, mass-energy equivalence, wavefunction interpretations.

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The metaphysics of physics

1. The wavefunction of an electron

The *Zitterbewegung* model

The wavefunction is a wonderful mathematical object which, we argue, has several physical interpretations. In the *Zitterbewegung* interpretation of an electron[1], it will describe the circular oscillatory motion of an electron (the *Zitterbewegung*) or – possibly – of any charged particle, as depicted below.

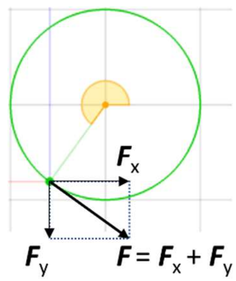

Figure 1: The force and position vector

The illustration above makes it clear that, *within* this rather particular interpretation of the wavefunction, we need to develop a dual view of the reality of the real and imaginary part of the wavefunction. On the one hand, they will describe the physical position (i.e. the *x*- and *y*-coordinates) of the pointlike charge – the green dot in the illustration, whose motion is described by:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

As such, the (elementary) wavefunction is viewed as an implicit function: it is equivalent to the $x^2 + y^2 = a^2$ equation, which describes the same circle.

On the other hand, the *zbw* model implies the circular motion of the pointlike charge is driven by a tangential force, which we write as:

$$\boldsymbol{F} = F_x \cdot \cos(\omega t+\pi/2) + i \cdot F_x \cdot \sin(\omega t+\pi/2) = F \cdot e^{i(\theta+\pi/2)}$$

[1] Erwin Schrödinger derived the *Zitterbewegung* as he was exploring solutions to Dirac's wave equation for free electrons. In 1933, he shared the Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

The line of action of the force is the orbit because a force needs something to grab onto, and the only thing it can grab onto in this model is the oscillating (or rotating) charge.

We think of $\boldsymbol{F}$ as a composite force: the resultant force of two perpendicular oscillations.[2] This leads us to boldly equate the $\mathrm{E} = \mathrm{m}c^2$ and $\mathrm{E} = \mathrm{m} \cdot a^2 \cdot \omega^2$ formulas. We can think of this as follows. The *zbw* model – which is derived from Dirac's wave equation for free electrons – tells us the velocity of the pointlike charge is equal to c. If the *zbw* frequency is given by Planck's energy-frequency relation ($\omega = \mathrm{E}/\hbar$), then we can combine Einstein's $\mathrm{E} = \mathrm{m}c^2$ formula with the radial velocity formula ($c = a \cdot \omega$) and find the *zbw* radius, which is nothing but the (reduced) Compton wavelength:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_e}{2\pi} \approx 0.386 \times 10^{-12}\ \mathrm{m}$$

Because the energy in the oscillator must be equal to the magnitude of the force times the length of the loop, we can calculate the magnitude of the force, which is rather enormous in light of the sub-atomic scale:

$$\mathrm{E} = \mathrm{F}\lambda_e \Leftrightarrow F = \frac{\mathrm{E}}{\lambda_e} \approx \frac{8.187 \times 10^{-14}\ \mathrm{J}}{2.246\ \times 10^{-12}\ \mathrm{m}} \approx 3.3743 \times 10^{-2}\ \mathrm{N}$$

The associated current is equally humongous:

$$\mathrm{I} = \mathrm{q_e} f = \mathrm{q_e}\frac{\mathrm{E}}{h} \approx (1.6 \times 10^{-19}\ \mathrm{C})\frac{8.187 \times 10^{-14}\ \mathrm{J}}{6.626 \times 10^{-34}\ \mathrm{Js}} \approx 1.98\ \mathrm{A}\ (ampere)$$

A household-level current at the sub-atomic scale? The result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

It is also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle. As the oscillator model implies the effective mass of the electron will be spread over the circular disk, we should use the 1/2 form factor for the moment of inertia (I).[3] We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

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[2] A metaphor for such oscillation is the idea of two springs in a 90-degree angle working in tandem to drive a crankshaft. The 90-degree ensures the independence of both motions. See: Jean Louis Van Belle, *Einstein's mass-energy equivalence relation: an explanation in terms of the Zitterbewegung*, 24 November 2018 (http://vixra.org/pdf/1811.0364v1.pdf).

[3] Alternatively, we may think of the mass of the electron being spread over a hoop. We will come back to this. Note that symbols may be confusing. For example, I refers to the current, but I refers to the moment of inertia. Likewise, E refers to energy, but E may also refer to the magnitude of the electric force. We could have introduced new symbols but the context should make clear what we are talking about. We also try to use italics consistently. Note that bold letters ($\boldsymbol{F}$ versus F, for example) will usually denote a vector, i.e. a quantity with a magnitude (F) and a direction.

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right)\mathbf{L} \Leftrightarrow \frac{q_e}{2m}\hbar = g\frac{q_e}{2m}\frac{\hbar}{2} \Leftrightarrow g = 2$$

The vector notation for **μ** and **L** (boldface) in the equation above should make us think about the plane of oscillation. This question is related to the question of how we should analyze all of this is a moving reference frame. This is a complicated question. The Stern-Gerlach experiment suggests we may want to think of an oscillation plane that might be perpendicular to the direction of motion, as illustrated below.

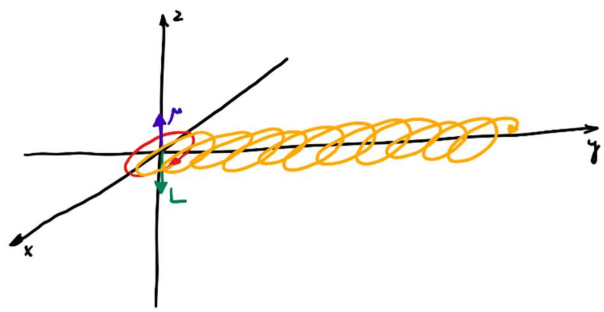

Figure 2: The *zbw* electron traveling through a Stern-Gerlach apparatus?

Of course, the Stern-Gerlach experiment assumes the application of a (non-homogenous) magnetic field. In the absence of such field, we may want to think of the plane of oscillation as something that is rotating in space itself. The idea, then, is that it sort of snaps into place when an external magnetic field is applied. We will discuss this idea when introducing Uncertainty in Section 4 of this paper.

As for the question of how we should look at the motion in a moving reference frame – and, in particular, when the electron would move at a relativistic speed, this will be discussed more in detail in Section 3. We first want to think about how we can use the wavefunction concept to interpret the nature of a photon, which we do in Section 2.

Before we move on, however, we should add a few remarks here.

The black-hole model

First, we should, perhaps, think some more about the nature of the force. The assumption is that the force grabs onto a pointlike charge. Hence, the force must be electric. We write:

$$\boldsymbol{F} = q_e\boldsymbol{E}.$$

Because the force is humongous (a force of 0.0375 N is equivalent to a force that gives a mass of 37.5 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second), and the charge is tiny), we get an absurd field strength:

$$E = \frac{F}{q_e} \approx \frac{3.3743 \times 10^{-2}\ \text{N}}{1.6022\ \times 10^{-19}\ \text{C}} \approx 0.21 \times 10^{18}\ \text{N/C}$$

This is an implausible number because the most powerful accelerators may only reach field strengths of the order of 10^9 N/C (1 GV/m).[4] The associated energy density can be calculated as:

$$u = \epsilon_0 E^2 \approx 8.854 \times 10^{-12} \cdot (0.21 \times 10^{18})^2 \frac{\mathrm{J}}{\mathrm{m}^3} = 0.36 \times 10^{24} \ \frac{\mathrm{J}}{\mathrm{m}^3} = 0.63 \times 10^{24} \ \frac{\mathrm{J}}{\mathrm{m}^3}$$

This amounts to about 7 kg per mm^3 (cubic *milli*meter). Do this make any sense? Maybe. Maybe not. The rest mass of the electron is tiny, but then the *zbw* radius of an electron is also exceedingly small. We will leave it to the reader to verify the calculation and – perhaps – make some more sense of it. It would be very interesting, for example, to think about what happens to the curvature of spacetime with such mass densities: perhaps our pointlike charge goes round and round on a geodesic in its own (curved) space. We are not well-versed on this and we can, therefore, only offer some general remarks here[5]:

1. If we would pack all of the mass of an electron into a black hole, then the Schwarzschild formula gives us a radius that is equal to:

$$r_s = \frac{2\mathrm{Gm}}{c^2} \approx 1.35 \times 10^{-57} \mathrm{m}\ (meter)$$

This exceedingly small number has no relation whatsoever with the Compton radius. In fact, its scale has no relation with whatsever distance one encounters in physics: it is *much* beyond the Planck scale, which is of the order of 10^{-35} meter and which, for reasons deep down in relativistic quantum mechanics[6], physicists consider to be the smallest possibly sensible distance scale.

2. We are intrigued, however, by the suggestion that the Schwarzschild formula should not be used as it because an electron has angular momentum, a magnetic moment and other properties, perhaps, that do not apply when calculating, say, the Schwarzschild radius of the mass of a baseball. To be precise, we are particularly intrigued by models that suggest that, when incorporating the above-mentioned properties of an electron, the Compton radius might actually be the radius of an electron-sized black hole.

One of these models[7] also describes the electron as a disk with a thickness of the order of the *classical* electron radius[8] and with a radius that is equal to $\hbar/2\mathrm{m}c$, which is *half* the

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[4] Such field strength assumes a very-high frequency oscillation of the field. To be precise, the oscillations should be in the 30-50 GHz range to reach such field strength. This is high, but the *zbw* frequency is *much* higher: $f_e = \omega_e/2\pi = E/\hbar \approx 0.123\times10^{-21}$ Hz.

[5] These remarks are based solely on the Wikipedia article on the black-hole electron (https://en.wikipedia.org/wiki/Black_hole_electron) and should, therefore, be read with circumspection.

[6] The phrase is Feynman's (Lectures, III-4-2), and he uses it in the context of an extremely convoluted discussion on the dichotomy between fermions (read: matter-particles) and bosons (read: photons). This particular lecture of Feynman is the only one which comes across as plain weird: how would particles (bosons or fermions) *know* when to add or when to subtract their amplitudes? In our humble view, the fundamental difference between matter-particles and photons is explained sufficiently by the interpretation we are offering here. There is no need for an outlandish explanation here.

[7] See: Alexander Burinskii, *The Dirac–Kerr–Newman electron*, 19 March 2008 (https://arxiv.org/abs/hep-th/0507109).

[8] The classical electron radius is also known as the *Lorentz* radius of an electron, or the Thomson scattering length. It is equal to $r_e = e^2/\mathrm{m}c^2 \approx 2.82\times10^{-15}$ m. The ratio between the Thomson and Compton radius is equal to the fine-structure constant: $r_e/a = (2.82\times10^{-15}\,\mathrm{m})/(386\times10^{-15}\,\mathrm{m}) = \alpha \approx 1/137 \approx 0.0073$. We may note that we can use the α ratio once again to get the size of the electron orbital (the Bohr radius of the hydrogen atom): $a_0 = r_{\mathrm{Bohr}} = (386/\alpha)\times10^{-15}$ m $\approx 53\times10^{-12}$ m. This ratio (the fine-structure constant α)

Compton radius that we calculated above. The 1/2 factor might be explained by the above-mentioned interpretation issue: should we think of the mass of an electron as being spread out over a disc (which we think we should do) or, alternatively, as being spread over a hoop.

These so-called Dirac-Kerr-Newman models involves extreme mathematical gymnastics but the results are intuitive and interesting because of two reasons. First, combining the *zbw* model with such black-hole model might explain why the pointlike charge does not radiate its energy away: the radiation cannot escape.[9]

The second reason why these models are interesting is that they do give us some *volume* to use to calculate the total energy from the obtained energy density. Let us try to do that. The issue is that the form factor is not very intuitive. Should we integrate over a hoop, over a disk, or over a more complicated shape – such as an oblate ellipsoid, perhaps?[10] Let us check if an integration over the energy density over some volume makes sense at all. The total energy should be given by the volume times the density and it should add up to $\mathrm{E} = \mathrm{m}c^2$. Let us examine the various factors that would go into the formula.

First, we have the density $u = \varepsilon_0 E^2 = \varepsilon_0 F^2/\mathrm{q_e}^2 = \varepsilon_0(\mathrm{E}^2/\lambda_\mathrm{e})^2/\mathrm{q_e}^2 = (2\pi)^{-2}\varepsilon_0\mathrm{E}^2/(a^2\mathrm{q_e}^2)$. Let us now multiply this with the volume that we get when using the simple formula for a disk of thickness $r_\mathrm{e} = \mathrm{e}^2/\mathrm{m}c^2$ and radius $a = \hbar/\mathrm{m}c$. This formula is $\mathrm{V} = \pi R^2 r$, so we get the following result:

$$\mathrm{E} = u \cdot \mathrm{V} = \frac{\varepsilon_0}{4\pi^2}\frac{\mathrm{m}^2 c^4}{a^2 \mathrm{q_e}^2} \cdot \pi a^2 \cdot \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q_e}^2}{\mathrm{m}c^2} = \frac{\mathrm{m}c^2}{16\pi^2}$$

The $1/16\pi^2$ amounts to about 0.006333 which, obviously, suggests we are not using the right volume here. Let us try the formula for a hoop (or a *torus*), which is $\mathrm{V} = 2\pi^2 r^2 R$. The r and R in this formula are the inner and outer radius respectively. Note that, in this formula, we square the *small* radius. To facilitate the calculations, we will use the remarkable formula that relates the Lorentz with the Compton radius: $r_\mathrm{e} = \alpha \cdot a$. We now get the following:

$$\mathrm{E} = u \cdot \mathrm{V} = \frac{\varepsilon_0}{4\pi^2}\frac{\mathrm{m}^2 c^4}{a^2 \mathrm{q_e}^2} \cdot 2\pi^2\alpha^2 a^3 = \frac{\varepsilon_0}{2}\frac{\mathrm{m}^2 c^4}{\mathrm{q_e}^2} \cdot \alpha^2 a = \frac{\varepsilon_0}{2}\frac{\mathrm{m}^2 c^4}{\mathrm{q_e}^2} \cdot \alpha \cdot \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{q_e}^2}{\mathrm{m}c^2} = \frac{\alpha}{8\pi}\mathrm{m}c^2$$

The result is – as one might expect – even worse: the $\alpha/8\pi$ factor amounts to about 0.00029. Of course, these results should not discourage us because we have not been consistent in our approach: we derived the force and, therefore, the energy density from our $\mathrm{E} = \mathrm{m}c^2 = \mathrm{F}\lambda_\mathrm{e}$ formula, which assumes the volume is not a disc but a one-dimensional loop.

The point is: there is no wire to confine the motion of our pointlike charge. Hence, the *Zitterbewegung* model has to be complemented by more advanced approaches – such as

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between these three radii is arguably the most intriguing geometric relation in all of physics. It should be noted that the e^2 in the $r_\mathrm{e} = \mathrm{e}^2/\mathrm{m}c^2$ formula is equal to $\mathrm{e}^2 = \mathrm{q_e}^2/4\pi\varepsilon_0$.

[9] Of course, for the mass of the black hole to be constant, it should not only absorb but also emit radiation. Hence, this model would require some equilibrium between the so-called Hawking radiation rate (which causes blackhole evaporation if the blackhole has no mass or energy to absorb which, we argue, is not the case here) and the absorption rate.

[10] In the Dirac-Kerr-Newman electron model of Burinskii (2008, see the above-mentioned reference), the volume is, effectively, that of highly oblate ellipsoid.

the Dirac-Kerr-Newman model, for example – to *explain why the pointlike charge does not spin away.*

We cannot say all that much about it here: we can only hope our paper will motivate more advanced people to integrate it all. In the meanwhile, we should move on within our own mental limits. Before we do so, we should make one more note on the results we have obtained above. We get the following result when calculating the product of the calculated force, the *zbw* circumference (Compton wavelength) and the *zbw* frequency:

$$F \cdot \lambda_e \cdot T = \frac{E}{\lambda_e} \cdot \lambda_e \cdot \frac{1}{f_e} = E \cdot \frac{h}{E} = h$$

Hence, the amount of (physical) action here is equal to Planck's constant ($h \approx 6.626 \times 10^{-34}$ N · m · s). It is *not* equal to the angular momentum, which we calculated as ħ/2.

How should we interpret this? We are not sure. All that we should note here is that, despite the huge force, the total amount of action is quite modest: h is a small unit. If anything, the obtained result shows that h is a small unit only because the SI distance and time units (*meter* and *second*) are rather astronomical when discussing what might be happening at the (sub-)atomic scale.

The nature of the oscillation

Let us go back to the idea of a two-dimensional oscillation. The $E = ma^2\omega^2 = mc^2$ is intuitive: the energy of any oscillation will be proportional to the square of (*i*) the (maximum) amplitude of the oscillation and (*ii*) the frequency of the oscillation, with the mass as the proportionality coefficient. At the same time, we should wonder: what could it possibly *mean*?

This question is difficult to answer. Is there any other idea – we mean: other than the idea of a two-dimensional oscillation – to explain the *Zitterbewegung*? We do not see any. We explored the basic ideas elsewhere[11] and, hence, we will not dwell on them here. We will only make one or two remarks below which may or may not help the reader to develop his or her own interpretation of what might be going on *in reality*.

The first remark is this: when everything is said and done, we should admit that the bold $c^2 = a^2 \cdot \omega^2$ assumption interprets spacetime as a relativistic aether. It is a term that is, unfortunately, taboo but, fortunately, some respected academics, such as Nobel Prize Laureate Robert Laughlin[12], are still defending it. This interpretation is inspired by the most obvious implication of Einstein's $E = mc^2$ equation, and that is that the ratio between the energy and the mass of *any* particle is always equal to c^2:

$$\frac{E_{electron}}{m_{electron}} = \frac{E_{proton}}{m_{proton}} = \frac{E_{photon}}{m_{photo}} = \frac{E_{any\ particle}}{m_{any\ particle}} = c^2$$

This reminds us of the $\omega^2 = C^{-1}/L$ or $\omega^2 = k/m$ of harmonic oscillators – with one key difference, however: the $\omega^2 = C^{-1}/L$ and $\omega^2 = k/m$ formulas introduce *two* (or more) degrees of freedom.[13] In contrast, $c^2 = E/m$ for *any* particle, *always*. This is the point: we

[11] See the reference above (Jean Louis Van Belle, 2018).

[12] Robert Laughlin (2005), as quoted in the Wikipedia article on aether theories (https://en.wikipedia.org/wiki/Aether_theories).

[13] The $\omega^2 = 1/LC$ formula gives us the natural or resonant frequency for an electric circuit consisting of a resistor (R), an inductor (L), and a capacitor (C). Writing the formula as $\omega^2 = C^{-1}/L$ introduces the concept of elastance, which is the equivalent of the mechanical stiffness (k) of a spring. We will usually also

can modulate the resistance, inductance and capacitance of electric circuits, and the stiffness of springs and the masses we put on them, but we live in *one* physical space only: *our* spacetime. Hence, the speed of light c emerges here as *the* defining property of spacetime. It is, in fact, tempting to think of it as some kind of *resonant frequency* but the $c^2 = a^2 \cdot \omega^2$ hypothesis tells us it defines both the frequency as well as the amplitude of what we will now refer to as *the rest energy oscillation*.

We are now ready to look at how we can use the very same wavefunction to describe the conceptual opposite of matter: the photon.

2. The wavefunction of a photon

Photons may or may not have a wavefunction but, if they do, we would probably want to visualize it as a circularly polarized wave, as illustrated below: a rotating electric field vector ($\boldsymbol{E}$) which can be analyzed as the sum of two orthogonal components: $\boldsymbol{E} = \boldsymbol{E}_x + \boldsymbol{E}_y$.

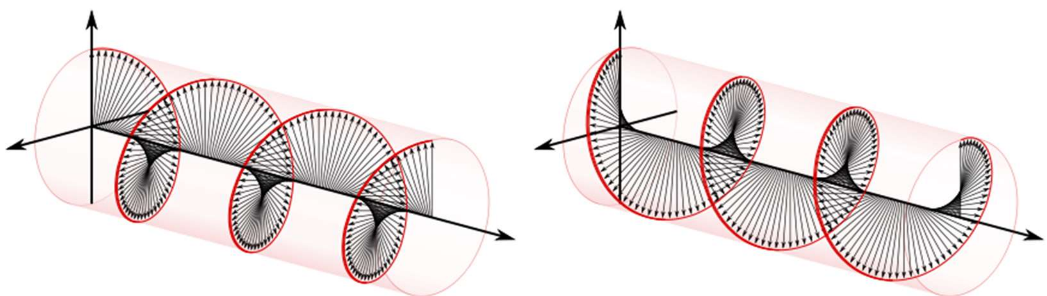

Figure 3: LHC- and RHC-polarized light

This is a very different view of the (elementary) wavefunction. It is *not* an implicit function anymore. It is a proper function now. To be precise, we think of $a \cdot e^{i\theta}$ as a function from some *domain* ($\Delta\boldsymbol{x}$, Δt) to an associated *range* of values $a \cdot e^{i\theta}$. We may write this as:

$$(\boldsymbol{x},\, t) \rightarrow a \cdot e^{i\theta} = a \cdot e^{i(\omega \cdot \mathrm{t} - \mathbf{k} \cdot \mathbf{x})} = a \cdot \cos(\omega \cdot \mathrm{t} - \mathbf{k} \cdot \mathbf{x}) + i \cdot a \cdot \sin(\omega \cdot \mathrm{t} - \mathbf{k} \cdot \mathbf{x})$$

Hence, while the domain of this wavefunction has to be *limited* in space and in time, the wave itself will, effectively, occupy some space at any point in time and, conversely, will only have non-zero values over a limited time *interval* at any point in space. Of course, the amplitude is not necessarily uniform. If you have ever recorded someone playing the guitar (yourself, perhaps), then you are probably aware of how an actual wave packet looks like: it is a transient oscillation, as shown below. Note that its shape reverses depending on whether we take the horizontal axis to be time (t) or spatial position (x).

include a resistance in an electric circuit to introduce a damping factor or, when analyzing a mechanical spring, a drag coefficient. Both are usually defined as a fraction of the *inertia*, which is the mass for a spring and the inductance for an electric circuit. Hence, we would write the resistance for a spring as γm and as R $= \gamma L$ respectively. This is a third degree of freedom in classical oscillators.

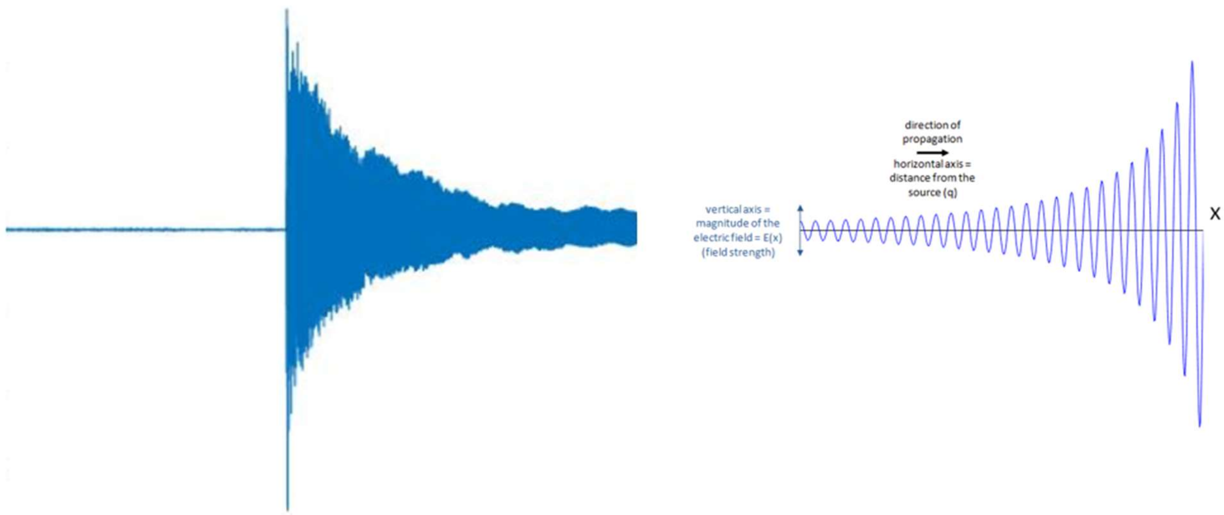

Figure 4: An actual wave is usually a transient[14]

This may look outlandish but it makes sense if we think photons are emitted – and absorbed – by an atomic transition from one energy state to another. We think of these atoms at atomic oscillators, and we can calculate their Q: it's of the order of 10^8 (see, for example, Feynman's *Lectures*, I-33-3), which means that, after about as many oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. Let us give an example, because it gives rise to interesting questions. For sodium light – which has a frequency of 500 THz (500×10^{12} oscillations per second) and a wavelength of 600 nm (600×10^{-9} meter) – the *decay time* of the radiation will be some 3.2×10^{-8} seconds. That makes for about 16 million oscillations. Now, the wavelength is small but the speed of light is huge. The length of the wave train is, therefore, still quite considerable: about 9.6 meter.

This sounds lunatic: a photon with a length of 9.6 meter? Yes. Fortunately, we are saved by relativity theory: as this wave train zips by at the speed of light, relativistic length contraction reduces its *length* to zero. What about the field strength? Because the electric field is perpendicular to the direction of propagation, we like to think the amplitude remains what it is. However, that requires, perhaps, a more careful consideration.

At this point in the argument, we have no choice but to think about relativistic transformations of the wavefunction or, to be precise, relativistic transformations of its argument. Before we do so, we need to make one more note. It should, intuitively, be obvious that the energy of a photon – the energy of the wave train, really – is packed over many oscillations. Zillions, literally. Each of these oscillations will, therefore, pack an exceedingly small (but real) amount of energy. As any oscillation, each oscillation takes some time (the cycle time) and, in the case of the photon, some space (the wavelength). In contrast, the electron picture was different: *one* oscillation – one cycle, really – packs all of the energy $E = F\lambda_e = m_e c^2$. Hence, the magnitude of the associated electric field is humongous as compared to the amplitude of the oscillations of our photon.

Let relativity enter the picture now.

[14] The image on the left depicts the amplitude of a musical note from a guitar string.

3. The relativistic invariance of the wavefunction

Let us consider the idea of a particle traveling in the positive x-direction at constant speed v. This idea implies a pointlike concept of position and time: we think the particle will be somewhere at some point in time. The *somewhere* in this expression does not necessarily mean that we think the particle itself will be dimensionless or pointlike. It just implies that we can associate some *center* with it. Think of the *zbw* model here, for example: we have an oscillation around some center, but the oscillation has a *physical* radius, which we refer to as the Compton radius of the electron.[15] Of course, two extreme situations may be envisaged: $v = 0$ or $v = c$. However, let us not consider these right now (we will do so later, of course).

The point is: in our reference frame, we have a position – a mathematical *point* in space, that is – which is a function of time: $x(t) = v \cdot t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t'. Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t', and the position and time in the two reference frames will be related as follows:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows:

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \frac{E_0}{\hbar}t'$$

We have just shown that the argument of the wavefunction is relativistically invariant.[16] It makes us think that of the argument of the wavefunction and – therefore – the wavefunction itself – might be more real – in a *physical* sense, that is – than the various wave equations (Schrödinger, Dirac, Klein-Gordon) for which it is some solution. Let us, therefore, further explore this. We have been interpreting the wavefunction as an implicit function again: for each x, we have a t, and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by $x(t) = v \cdot t$. We will get some linear motion. If we look at the $\psi = a \cdot \cos(p \cdot x/\hbar - E \cdot t/\hbar) + i \cdot a \cdot \sin(p \cdot x/\hbar - E \cdot t/\hbar)$ once more, we can write $p \cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what x this phase factor $\Delta = p \cdot x/\hbar$ will be equal to 2π. Hence, we write:

$$\Delta = p \cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi \cdot \hbar/p = h/p = \lambda$$

---

[15] The language is quite subtle: the Compton *radius* is the reduced Compton wavelength: $a = r_C = \lambda_e/2\pi$.

[16] E_0 is, obviously, the rest energy and, because $p' = 0$ in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself.

We now get a meaningful interpretation of the *de Broglie* wavelength. It is the distance between the crests (or the troughs) of the wave, so to speak, as illustrated below.

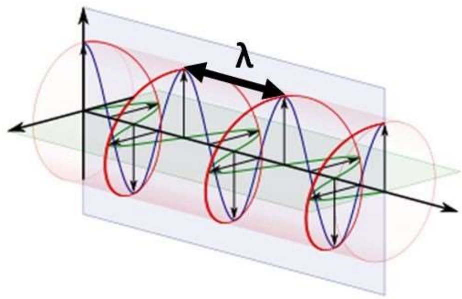

Figure 5: An interpretation of the *de Broglie* wavelength

Of course, we should probably think of the plane of oscillation as being *perpendicular* to the plane of motion – or as oscillating in space itself – but that doesn't matter. Let us explore some more. We can, obviously, re-write the argument of the wavefunction as a function of *time* only:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}vt\right) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)t = \sqrt{1-\frac{v^2}{c^2}}\cdot\frac{\mathrm{E}_0}{\hbar}t$$

We recognize the *inverse* Lorentz factor here, which goes from 1 to 0 as v goes from 0 to c, as shown below.

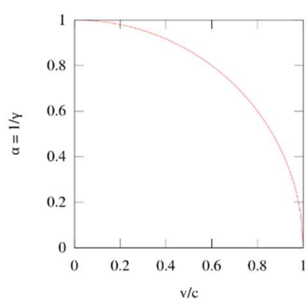

Figure 6: The inverse Lorentz factor as a function of (relative) velocity (v/c)

Note the shape of the function: it is a simple circular arc. This result should not surprise us, of course, as we also get it from the Lorentz formula:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t - \frac{v^2}{c^2}t}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{1-\frac{v^2}{c^2}}\cdot t$$

What does it all mean? We can go through a simple numerical example to think this through. Let us assume that, for example, that we are able to speed up an electron to, say, about one tenth of the speed of light. Hence, the Lorentz factor will then be equal to $\gamma = 1.005$. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0: $\mathrm{E}_v = \gamma\mathrm{E}_0$

≈ 1.005 · 0.511 MeV ≈ 0.5135 MeV. The relativistic momentum will then be equal to $m_v v$ = (0.5135 eV/c^2) · (0.1 · c) = 5.135 eV/c. We get:

$$\theta = \frac{E_0}{\hbar}t' = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = 0.955\frac{E_0}{\hbar}t$$

This is interesting, and then it is not. A more interesting question is what happens to the *radius* of the oscillation. Does it change? It must, but how should we interpret this? In the moving reference frame, we measure higher mass and, therefore, higher energy – as it includes the kinetic energy. The $c^2 = a^2 \cdot \omega^2$ identity must now be written as $c^2 = a'^2 \cdot \omega'^2$. Instead of the rest mass m_0 and rest energy E_0, we must now use $m_v = \gamma m_0$ and $E_v = \gamma E_0$ in the formulas for the Compton radius and the Einstein-Planck frequency, which we just write as m and E in the formula below:

$$m a'^2 \omega'^2 = m\frac{\hbar^2}{m^2c^2}\frac{m^2c^4}{\hbar^2} = mc^2$$

This is easy to understand intuitively: we have the mass factor in the denominator of the formula for the Compton radius, so it must increase as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, increase with velocity. It is interesting to note that we have a simple (inverse) proportionality relation here. The idea is visualized in the illustration below (for which credit goes to the modern *zbw* theorists Celani et al.), which depicts an *accelerating* electron: the *radius* of the circulatory motion must effectively diminish as the electron gains speed. Once again, however, we should warn the reader that he or she should also imagine the plane of oscillation to be possibly parallel to the direction of propagation, in which case the circular motion becomes elliptical.

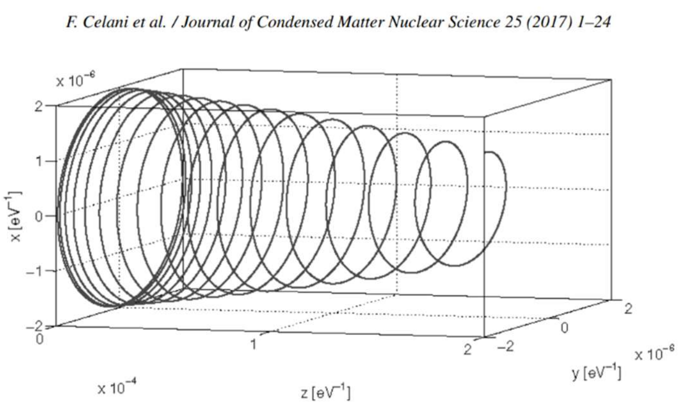

Figure 7: The Compton radius must decrease with increasing velocity

Can the velocity go to c? This is where the analysis for an electron – or any other matter-particle – and for a photon part ways. In the *zbw* model, we have a rest energy which is explained by the *Zitterbewegung* of a pointlike electric charge. Hence, relativity tells us we can never accelerate it to the speed of light, because its mass – a measure for inertia to movement – becomes infinite.

In contrast, we have no such constraint for a photon. In fact, it does not have any rest energy (or rest mass). All of its energy is in its motion. What happens to the argument of the wavefunction? If we still think of the photon just like we would think of a particle –

i.e. in terms of it being at some specific point in space at some specific point in time – then the argument vanishes:

$$\theta = \frac{1}{\hbar}(\mathrm{E} \cdot t - \mathrm{p} \cdot x) = \frac{1}{\hbar}(\mathrm{p}c \cdot t - \mathrm{p} \cdot ct) = 0$$

What does this tell us? In our humble view, this tells us that we should *not* think of a photon as we think of a particle. If we want to associate a wavefunction with a photon, then we should not think of it as an implicit function, but as a proper function, i.e. a function from some *domain* ($\Delta \boldsymbol{x}$, Δt) to an associated *range* of values $a \cdot e^{i\theta}$. We can then use the superposition principle to shape it anyway we would want to shape it, and we should probably think of some transient here, rather than a nice symmetrical wave packet.

4. Planck's constant and the concept of Uncertainty

It may look like we do not have any uncertainty in the wavefunction concept as we have used it so far. The particle is always somewhere at some point in time. As for the photon, we assume that – in its own frame of reference – we have some precise value for the electromagnetic field at any point in space and in time. This is true and not true at the same time. Our model is deterministic but, at the same time, we do not have a precise position for the pointlike *charge*: it goes round and round but we don't have a zero point for its motion. Hence, it is somewhere but also everywhere.[17]

As for the photon, it appears both as an incredibly long string (in its own frame of reference) as well as a point (in our frame of reference). Hence, we may ask the same question: what is the reality of the position here?

Is this, then, the concept of Uncertainty? We could summarize this concept as: the particle is *some*where, but we don't know where – because it is some oscillation and we do not have the $t = 0$ point for this oscillation.

Perhaps it is. Perhaps it is not. We will only offer a few remarks here that may or may not help the reader to develop his or her own views on it. Let us look at the argument of the elementary wavefunction once again:

$$\theta = \frac{1}{\hbar}(\mathrm{E} \cdot t - \mathrm{p} \cdot x)$$

Planck's constant (ħ) appears a scaling constant here: it looks like Nature is telling us to measure $\mathrm{E} \cdot t - \mathrm{p} \cdot x$ in units of ħ. Planck's constant is the quantum of action. The concept of (physical) action is not often used and may, therefore, not be intuitive. One of its uses is in the Principle of Least Action, which is used in classical mechanics as well as in quantum physics. We will come back to that. As for now, we should note that action is measured in N · m · s, so that is a force times some distance times some time. We know force times distance is energy, and force times time is momentum. Hence, we can think of action – and of the quantum of action itself – in two ways:

1. Some energy that is available for some time.
2. Some momentum that is available over some distance.

[17] Cf. the intuitive interpretation of probabilities of having to be proportional to mass and/or energy densities, *somehow*.

It is not obvious to intuitively understand what this means. Think of the Sun as a huge reservoir of energy. One day, its energy will be depleted, and we can calculate its life span because we have some idea of the *power* it delivers, which is the energy it delivers *per second*. We understand energy and we understand power – because power times time gives us energy again – but what is energy times time?

The same goes for momentum. We can think of a 5,000 kg lorry traveling at 70 km per hour and associate the related $p = m \cdot v$ momentum with that idea, but what is momentum times distance?

In fact, it may be easier to just take the physical dimension of action and think of it for what it is: some force over some distance over some time. Let us take an example from our *Zitterbewegung* model. The action – which we will denote by S as per the usual convention – that is associated with one loop along the *zbw* circumference over its cycle time is equal to:

$$S = F \cdot \lambda_e \cdot T = \frac{E}{\lambda_e} \cdot \lambda_e \cdot \frac{1}{f_e} = E \cdot \frac{h}{E} = h$$

We already commented on this: h is a small unit but small and large are relative. We have a rather enormous force here, but a tiny time and distance scale.

The example involves circular motion. The reader will be more familiar with examples involving *linear* motion. Let us assume that we move some object over some distance x. To make it simple, we will assume that we move in free space, so there is no potential. We apply some force F which will give the object an acceleration a. The acceleration is just the ratio between the force and the mass ($a = F/m$). The $x = a \cdot t^2/2$ then gives us the following equation for the time that is needed:

$$t = \sqrt{\frac{2 \cdot m \cdot x}{F}}$$

This shows that if we want to *halve* the time t, we need to *quadruple* the force F. The distance remains the same, so the total amount of physical *action*, which we will write as S, doubles. We get the following formula for the action associated with some distance x and some force F:

$$S = F \cdot x \cdot t = F \cdot x \cdot \sqrt{\frac{2 \cdot m \cdot x}{F}} = \sqrt{F} \cdot x^{3/2} \cdot \sqrt{2m}$$

Conversely, we can also write the action in terms of F and t:

$$S = F \cdot x \cdot t = F \cdot \frac{F \cdot t^2}{m \cdot 2} \cdot t = \frac{F^2 \cdot t^3}{2m}$$

The reader can fill in some numbers. For example, a force equal to 2 N that is acting on a 2 kg mass over a distance of 2 m amounts gives us $S = 8\ N \cdot m \cdot s$. Does this make us any wiser? Maybe. Maybe not. Whatever the perception of the reader here, he or she will have to admit that the formulas do help us to intuitively understand what the concept of physical action implies: it embodies some real event – but incorporating all aspects of it: not only distance, but also time, and vice versa.

We can think of this in three different ways: (1) some energy over some time (F · λ_e times T), (2) some momentum over some distance (F · T times λ_e) and (3) some force over some distance over some time. As such, it is associated with some path in space *and* in time – i.e. a path in spacetime. In the path integral formulation of quantum mechanics, we will associate different paths in spacetime with different amounts of action. Let us illustrate this.

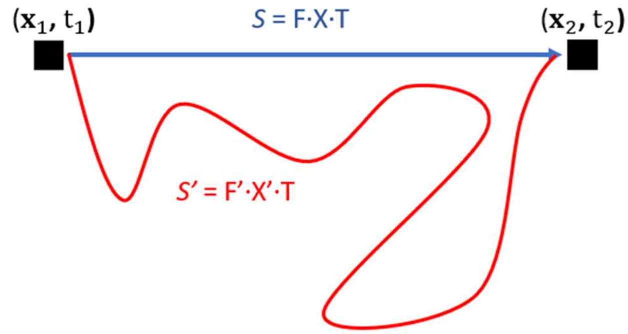

Figure 8: Different paths in spacetime

We can take some mass m – ourselves, perhaps – from point ($\mathbf{x}_1$, t_1) to point ($\mathbf{x}_2$, t_2) in two ways: we can take the straight-line path, or we can take the weird curved path. If we want to get from $\mathbf{x}_1$ to $\mathbf{x}_2$ in the same time $T = t_2 - t_1$, then we will need a lot more action along the curved path: we will need to go a lot faster and, therefore, we will need a lot more force. Hence, the curved path will be associated with more action. We write:

$$\Delta S = S' - S = F' \cdot X' \cdot T - F \cdot X \cdot T = (F' \cdot X' - F \cdot X) \cdot T = (E' - E) \cdot T$$

Can we take *any* path ? In classical mechanics, we can. However, in quantum mechanics, we may want to think action comes in *discrete* amounts: h, 2h, 3h,… n · h,... This, then, explains Feynman's explanation for diffraction (see below): not enough arrows. We should read this as: we only have a discrete set of possible paths in a limited space, and the relevant scale factor is given by Planck's constant: the amount of action that is associated with these paths differs by h, 2h, 3h,… n · h, etcetera.

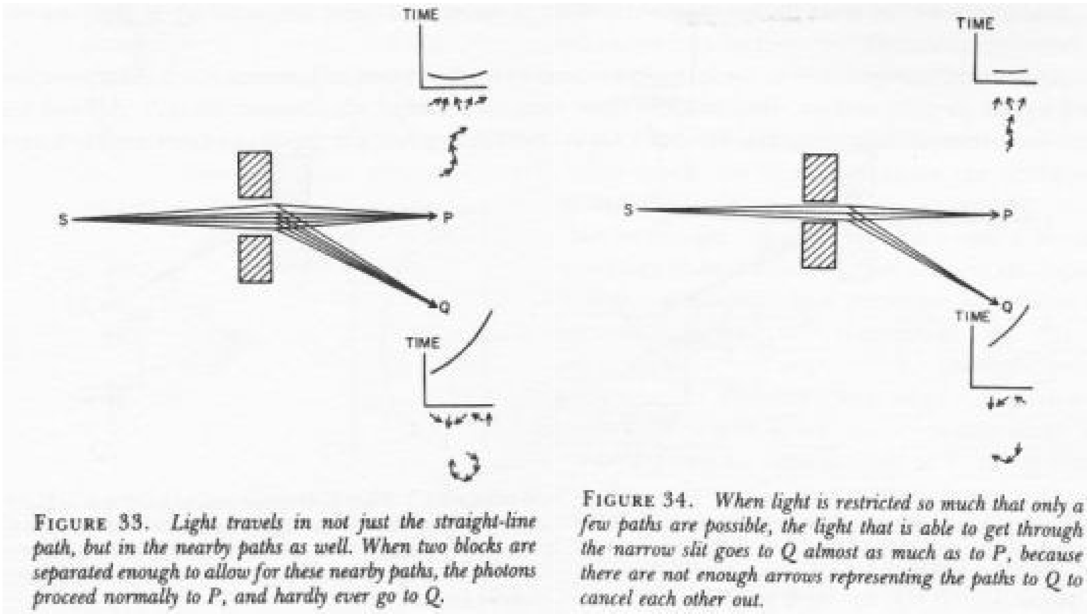

FIGURE 33. *Light travels in not just the straight-line path, but in the nearby paths as well. When two blocks are separated enough to allow for these nearby paths, the photons proceed normally to P, and hardly ever go to Q.*

FIGURE 34. *When light is restricted so much that only a few paths are possible, the light that is able to get through the narrow slit goes to Q almost as much as to P, because there are not enough arrows representing the paths to Q to cancel each other out.*

Figure 9: Explaining diffraction[18]

Here, we have yet another use of the wavefunction: we will associate each path in spacetime with the related action, and this amount of action is then used to calculate the probability amplitude that is associated with the path. We then add the amplitudes – Feynman's arrows – to get the combined amplitude. The combined amplitude is then given by the propagator function. If our physical interpretations of the wavefunction makes any sense, then we will, somehow, need to relate them to the propagator function.

We cannot say much about this right now, except that the path integral formulation of quantum mechanics has little intuitive appeal. It may explain diffraction and interference *mathematically*, but it does not feel like a *physical* model: the idea that a pointlike particle can, somehow, travel along *different* paths simultaneously does not make much sense.

We think recent theory and experiments focusing on how slits or holes affect wave *shapes* as electrons – or photons – go through them are far more promising in terms of offering some kind of *physical* explanation for interference and/or diffraction.[19] The diagram below illustrates the point that we are trying to make here.

[18] Source: Richard Feynman, *The Strange Theory Of Light and Matter*, 1985

[19] The definition is somewhat random but we think of diffraction if there is only one slit or hole. In contrast, the idea of interference assumes two or more wave sources. The research we refer to is the work of the Italian researchers Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html). The illustration was taken from the same source, but the author of this paper added the explanatory tags.

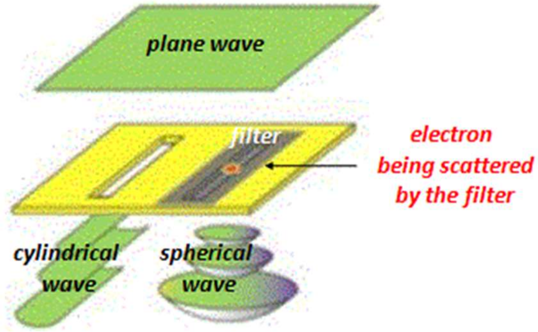

Figure 10: Explaining diffraction

As noted in a previous paper, we do think physicists should seriously explore how (composite) circularly polarized waves are split or built up from/into linearly polarized waves.[20] All of the work in regard to weak measurement seems to indicate the photon (or the electron) does travel through both slits – or along two paths – at the same time. What is actually traveling is probably only one of the *components* of the joint wave. Having said that, these components are as real as the particle as a whole: we only need to explain why the whole oscillation seems to stick together.[21]

Let us look at the argument of the elementary wavefunction once again:

$$\theta = \frac{1}{\hbar}(\mathrm{E} \cdot t - \mathrm{p} \cdot x)$$

As mentioned, Planck's constant (ħ) appears as a mathematical scaling constant: Nature tells us to *measure* $\mathrm{E} \cdot t - \mathrm{p} \cdot x$ in units of $\hbar$. However, we do like to believe Planck's constant is also a *physical* constant: the quantum of action. As such, physical action might, effectively, comes in actual *physical* units equal to $\hbar$. Of course, we know the argument of the wavefunction is expressed in *radians*. Hence, we should probably multiply $\hbar$ with 2π to get a value we can associate with a full cycle:

$$h = 2\pi\hbar$$

What cycle? We do not really know, but the *physical* model of an electron – and a photon – that we have offered here surely give us some clues of what we should be thinking of. Let us return to the idea of a force on a particle combined with some distance or – alternatively – with some time. Let us think about the kinetic energy and the momentum of the particle. In this particular example, we had no potential energy, because we think our particle is moving in free space. Hence, the kinetic energy is the only energy and is given by[22]:

[20] See, for example: Jean Louis Van Belle, 5 November 2018, *Linear and Circular Polarization States in the Mach-Zehnder Interference Experiment*, http://vixra.org/abs/1811.0056.

[21] When an apparatus detects the electron – or the photon – it gets *all* of its energy. Hence, the whole oscillation sort of collapses into one point when the particle is being detected (or measured). Here again, gravitation might explain why this is so, but the detailing of the nitty-gritty of this seems to be a complex task.

[22] We use the non-relativistic formula for the kinetic energy here, but it should be possible to generalize this to the more general relativistic case.

$$E = \frac{mv^2}{2} = \frac{ma^2t^2}{2} = \frac{m\frac{F^2}{m^2}}{2} \cdot \frac{2 \cdot m \cdot x}{F} = F \cdot x$$

Equally unsurprisingly, the momentum is given by:

$$p = mv = mat = m \cdot \frac{F}{m} \cdot t = F \cdot t$$

The force appears as a simple proportionality factor in these formulas. Now, if Nature is probabilistic, and the *order of magnitude* of the uncertainty[23] is given by h or $\hbar$, then we should wonder: which factor captures the uncertainty? The force, the distance, the time? We humbly offer the following reflections here.

The distance x and the time t are mathematical quantities. We may not *know* the actual distance, or the actual time, but there is no uncertainty in the concept as such. In contrast, mass and force – and, therefore, energy and momentum – do describe reality. If there is uncertainty in them – let us denote it by Δ – then the uncertainty has to be in the force. We write:

$$\Delta E = \Delta F \cdot x$$

$$\Delta p = \Delta F \cdot t$$

What about uncertainty in the mass, or in acceleration? The $E = mc^2$ principle tells us the idea of a (im)precise energy implies an (im)precise mass. We write:

$$\Delta E = \Delta\left(\frac{m}{c^2}\right) = \frac{1}{c^2}\Delta m$$

In contrast, an acceleration is – once again – a mathematical concept. There should be no imprecision there.

In short, the uncertainty is in the force. The idea of a force combines a direction as well as a magnitude. Hence, we may give the following some thought:

1. The $\Delta E = \Delta F \cdot x$ cannot be written as a vector equation, because energy has no direction.
2. In contrast, the $\Delta p = \Delta F \cdot t = m \cdot \Delta v \cdot t$ may also be written as $\Delta \mathbf{p} = \Delta \mathbf{F} \cdot t = m \cdot \Delta \boldsymbol{v} \cdot t$.[24] Hence, when we are looking at uncertainty in the position and/or momentum space, then the uncertainty may not be in the magnitude, but in the *direction* of the force.

More in general, we like the idea that any uncertainty in the force will, most likely, involve a combination of uncertainty in its magnitude as well as in its direction. We think these are very simple but also very fundamental distinctions which may explain a variety of phenomena in a manner that might be more straightforward than… Well… The usual explanations.

We would, therefore, propose to re-define Planck's quantum of action as a rotating *vector*:

$$\boldsymbol{h} = h \cdot e^{i\varphi}$$

[23] The idea of physical uncertainty having some *exact* value is a *contradictio in terminis*: we should think of h (or $\hbar$) as a statistical concept. Think of a standard deviation, for example.

[24] The difference between normal and **boldface** may not be obvious but is very substantial.

What is the $\boldsymbol{\varphi}$ phase? We are not sure. We may think – intuitively – that it should be equal to:

$$\varphi = \frac{\mathrm{E}_0}{\hbar} t'$$

But is it? The *t'* factor is the proper time of the particle. We are not quite sure what that is but we could possibly *measure* it in terms of the cycle time T =1/*f* = E_0/h units. Hence, *t'* would be written as some (pure) number T measured in terms of E_0/h units. The formula for Planck's *vector* then reduces to:

$$\boldsymbol{h} = h \cdot e^{i2\pi \cdot \mathrm{T}}$$

T then appears to be some (random) number going from zero to infinity: we don't know its starting point (T = 0), but we know it is ticking away like the seconds on a clock. So that is why Nature appears to be probabilistic, then: we do not know the T = 0 position of a particle in spacetime. Apart from that, Einstein was right: Nature is *not* probabilistic because we actually *can* explain the probabilities.

How? This question is answered by the next question: *can* we know the T = 0 position of a particle in spacetime and, if so, *how*? The answer is very straightforward: yes, we can. We know it when we do the measurement. A measurement amounts to synchronizing the stopwatches of the subject – me or, more generally, the measurement apparatus – and the object: the electron, or the photon, or whatever particle we are trying to push into some corner in spacetime.

Another question is: it T a discrete or a continuous variable? We will let the reader of this paper think about this. We welcome any ideas (our email is on the title page).

Jean Louis Van Belle, 30 November 2018

References

This paper discusses general principles in physics only. Hence, references were mostly limited to references to general physics textbooks. For ease of reference – and because most readers will be familiar with it – we often opted to refer to

1. Feynman's *Lectures* on Physics (http://www.feynmanlectures.caltech.edu). References for this source are per volume, per chapter and per section. For example, Feynman III-19-3 refers to Volume III, Chapter 19, Section 3.

We also mentioned the rather delightful set of Alix Mautner Lectures, although we are not so impressed with their transcription by Ralph Leighton:

2. Richard Feynman, *The Strange Theory of Light and Matter*, Princeton University Press, 1985

Specific references – in particular those to the mainstream literature in regard to Schrödinger's *Zitterbewegung* – were mentioned in the footnotes. We should single out the various publications of David Hestenes and Celani et al., because they single-handedly provide most of the relevant material to work on here:

3. David Hestenes, Found. Physics., Vol. 20, No. 10, (1990) 1213–1232, *The Zitterbewegung Interpretation of Quantum Mechanics*, http://geocalc.clas.asu.edu/pdf/ZBW_I_QM.pdf.

4. David Hestenes, 19 February 2008, *Zitterbewegung in Quantum Mechanics – a research program*, https://arxiv.org/pdf/0802.2728.pdf.

5. Francesco Celani et al., *The Electron and Occam's Razor*, November 2017, https://www.researchgate.net/publication/320274514_The_Electron_and_Occam's_Razor.

In addition, it is always useful to read an original:

6. Paul A.M. Dirac, 12 December 1933, Nobel Lecture, *Theory of Electrons and Positrons*, https://www.nobelprize.org/uploads/2018/06/dirac-lecture.pdf

The illustrations in this paper are open source or – in one or two case – have been created by the author. References and credits – including credits for open-source Wikipedia authors – have been added in the text.

One reference that has not been mentioned in the text is:

7. *How to understand quantum mechanics* (2017) from John P. Ralston, Professor of Physics and Astronomy at the University of Texas.

The latter work is one of a very rare number of exceptional books that address the honest questions of amateur physicists and philosophers upfront. We love the self-criticism: "Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research." (p. 1-10)

Last but not least, we also mentioned the work of Stefano Frabboni, Reggio Emilia, Gian Carlo Gazzadi, and Giulio Pozzi, as reported on the phys.org site (https://phys.org/news/2011-01-which-way-detector-mystery-double-slit.html). However, we have not gone into the nitty-gritty of their work and, therefore, do not want to pretend we have.

The same goes for the references to the work of Alexander Burinskii and colleagues, who are further developing the Dirac-Kerr-Newman electron model (Alexander Burinskii, *The*

Dirac–Kerr–Newman electron, 19 March 2008, https://arxiv.org/abs/hep-th/0507109). We have glanced through the paper – but we have to admit we only understand a fraction of the math.