Proceedings on qualitative and quantitative psychology.

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Contents

Some words upfront.

This book constitutes a revision of two previous books on this matter with precisely the same titles. The content has somewhat been enlarged, but especially its presentation has been drastically improved and many inacurracies in the previous version have been removed. The real beef of this book is also available in another book of mine on geometrical quantum theory: the latter aimed at the professional physicist or mathematician. The material is rather mathematical in nature and does require some substantial amount of effort from those who are not trained as such: that is why I have included a small chapter into the mathematical minimum of this book. It regards some elementary things about linear operators and vector spaces, the language of the old quantum theory as to speak. Let me tell you first who I am, what my intentions are and what this book is and is not about. First of all, I am a trained physicist and mathematician with a double bachelor and a master in theoretical physics; later on I obtained a PhD in theoretical physics, namely quantum gravity, and worked some time as a post-doctoral researcher in that field. I have never been interested in school, but at the age of 13 I was immersed already on my own into the books of Freud, Jung and Aldler on psychology and Nietzsche on philosophy (albeit I considered that work more as a venting of bad, but largely justified emotions towards humanity) but I also read upon engineering, especially hydraulics and so on trying to figure out what this integral really meant 4 years prior to learning it at school. The reason why I went into the exact sciences, is because I was largely dissatisfied with everything I read; Freud was as to say banal, Jung was interesting in some aspects but his theory was “not even wrong” meaning it was ill defined, you couldn’t do anything with it and Friedrich, ah well, was just an interesting story teller of how humanity appears to operate. It was a bit like Harry Potter, high class entertainment and wizardry: you have to love it but I am afraid Hogwarths will never materialize in this world. Unfortunatly, psychology has remained in the same crappy state since then trying to put minds into boxes and point out what your true self is; a completely pompous and overinflated concept to say the very least and very dangerous when this inner circle gets credited for knowing you better than you so-called know yourself. Likewise so with concepts as IQ, EQ and all the business regarding psychiatric deseases, albeit that seems more in the hands of so called “medical doctors”, another class of wizards. The only interesting thing what all of those tests and deseases reveal is the bias (and completely mental inaptitude) of the test (desease) maker and no, I do not think psychologists neither psychiatrists are on average very intelligent beings and therefore certainly not in the driving seat to judge intelligence or mental sanity of the “patient”. In a way both fields are the shame of science even to the extend that I bluntly call its practioners mentally retarded sociopaths (and indeed, almost all psychatrists are sociopaths if you take the definition literally: they have no respect for the rights of others to think what they deem appropriate) and totally unaware of what our best theory of reality really is, a field within which they themselves proclaim to be the arbiter of truth. In reality, psychology and psychiatry are more sociology and behavioral “sciences” than anything else whereas there are many beautiful questions to ask such as regarding the very basic nature from which we approach this world mentally and how this relates to the physical fabric of spacetime and the laws of the elementary particles. In short, the very core behind the mind-matter correspondance! We shall adress this issue in this book in an unprecedented depth and precision, as it turns out reality is far more complex as you think it is if you stick to some simple mathematical rules.

Let me explain my venom a bit better since I hear the psychologist already saying "but we are objective and things such as schizophrenia and intelligence have a God given meaning[1]"; if you are really serious about that then your IQ on the Noldus scale is well below 80, no matter how many books you have read or how high grades your teacher bestowed you with or God knows, how many prizes in your "field" you have won. Just to give a completely ridiculous example regarding IQ tests of how dumb those tests really are, I once encountered "some test" by a prankster who seemed to have understood this point very well: his question went, given the series $20, 20, 20, 20, 20, 20, 20, 20$ what is the next number in line? I said hmm, probably 20, I am not sure, but appearantly my IQ was below average because the correct answer was 21. As a justification, he provided for a divine polynomial in one variable which produced 20 for integers between 1 and 8 and 21 for 9. I hear you say, but we don't do that, our tests are not as evil; but they really are if you think about it a bit better. The only correct answer to such questions is of course "insufficient data to draw a reliable conclusion within an appropriate confidence interval" but that was not an option; so far for the limited intelligence of that test designer. So what could you test really? Very little I am afraid, an objective measure would consist into how good you are at solving abstract mathematical problems but that would be a pretty frustrating way to look at it if you are studying humanities or so. They might argue that it needs to correlate with your flexibility in learning new languages; that certainly requires a special brain but I would not call it intelligence either. Nor would I say that brilliant problem solvers are the smartest people in the world as they often tend to focus on problems which are simply the wrong ones. An old collegue of mine at university once told me that it was not very interesting to make progress into a difficult area, but rather to find a way to rephrase the question such that everything becomes totally simple. That is real brilliance indeed which goes beyond training and schooling but demands a certain raw, bare intelligence keeping an acute perspective upon what you are doing and if you really understand all assumptions which creep into your theory. Most so called intelligensia do not possess this quality and sometimes I wonder whether the postman next door is not way smarter than my cardiologist is. But I go even further than this, I know of a psychiatric case, who studied Latin-Greek in high school, never went to university, became a postman and so on but who beats 99 percent of academics in chess. The only thing which my collegue would have said and which I also do is that it is an ill posed question, completely irrelevant

[1] If you really believe that, you should go to an asylum.

to ask and destructive even. I am Buddhist in that way and believe in tolerance, acceptance and kindness towards others, values which the west does not seem to share very much. The west is sick in that way; sometimes old friends of mine tend to say that I express emotions in my work and that this is not academic, but they fail to comprehend that their reaction is equally emotional and not logical at all. Indeed, all of Einstein's emotional outbursts towards so called peers have nicely been swept under the historical carpet: as all intelligent people, he had a disdain for conventional researchers especially when their social status was on the high side. Einstein remembered very well what Planck had done for him and did similar things for other scientists such as S.N. Bose whose work constitutes now the underpinning of quantum field theory and was dismissed by several reputed (English) journals. So please gentlemen, try to really understand Einstein when he said he was only interested in the thoughts of the old man: he most likely was not that dumb as to ever assume that he would find those, but it occurs to me that he was referring to the wisdom of what kind of problems are worthwile approaching and what not. In this book, I shall focus on the God given questions and stay far removed from those issues I just briefly commented upon. Unlike Friedrich, I am not going to write books about the enlightened Zarathustra and spit my venom upon the social market place: I will try to be constructive here and offer new ways of thinking about things which are much better in my mind. I am of course not infalliable and that is why I somwhat apologize for the first version of this book which suffered from similar weaknesses, but by far not to that degree, as those which I just relegated to the trashbin. That is precisely why this work is of mathematical precision and it was a painful exercise indeed to self-correct the entire manuscript.

As I said, the real beef is available already for physicists and mathematicians in another book but here I shall be more gentle, explaing things in even more detail and commenting upon the ramifications of our findings: things which actually can be tested in a clear way. In that vein did Jung make several interesting observations, it is just so that the theory behind it is not very good at all to put it mildly. Another reason to recommend this book is that it is way shorter as my book an quantum theory which makes it not only cheaper but also nicer to digest; I do not believe in books of over 300 pages unless they constitute the presentation of a well known and developed field. This book is entirely novel and offers new pathways for thinking about the mind. The organization is as follows, chapter 1 contains an exposition of the mathematical requirements for understanding the matter, this chapter is 99.7 percent totally rigorous but there is an occasion on which I appeal on intuition in order to understand the theorem whereas a formal argument would be way beyond the scope of this book and involve more advanced math involving the lemma of Zorn or complex analysis. This is also my way of doing math, 99.7 percent rigorous but not fully caring to fill in all the details. Chapter 2 contains an introduction to the correct language to adress issues of the mind, this by itself is ground breaking in a way as the language we use will force us to redefine our position upon what constitues reality; it will also make clear why I am so venomous in the first place. Chapter 3 delves deeper into the world of chapter 2 and opens the door for rational discussions of psychism, communal spirits and so on. By this I intend to refer to what it really means for minds to engage with one and another and join in a union. Also, I briely discuss some findings of mine relating the results of our viewpoint to different philosophical ideas in the literature I am aware of. Chapter 4 deals with conclusions and supplementary remarks. Hence, we have explained the title of this book: "proceedings on qualitative and quantitative psychology" since we shall not only derive certain things from primary issues but also quantify those by means of a calorimetrics of the mind. The reader who is interested in delving deeper regarding the correspondence with our physical universe, is of course invited to consult my book on geometrical quantum theory albeit that requires a good deal of differential geometry.

To wrap things up; the sciences of the mind are in such a bad state since there is no governing principle, there are no guidelines: this leads to an incredible bias (story telling) creeping in regarding the interpretation of the data, which taken together with the very weak correlations they find, implies a worthless investigation. This gives rise to delusional theories with practioners which all suffer from schizophrenia if one is willing to take the concept literally: indeed, I once asked a lawyer how I could legally enforce a psychiatrist, suffering from that condtion, to be treated unvolountarily in a hospital! You see, if they are willing to do the mud slinging towards others, I am the first one to hold them a mirror and sling back; unfortunately, my legal case is a practical impossibility, the judge and the psychatrist like to lick each others butts as to speak. What do we offer in this book? In the first place precision, to the degree that I might make myself ridiculous! It is better to give away a very precise thought which has limitations rather than remaining vague and saying nothing of value at all. So, we offer a ground for discussion to this extend that minds can partially be programmed even on a computer. This is the way to go if we want to proclaim that we are doing science. Ultimately, precision equals mathematics: language is far too vague for that purpose. This explains why we shall study some elementary math first.

Chapter 1

The mathematical minimum.

In what follows, I start from the assumption that the reader knows what integers $\mathbb{Z}$, rational $\mathbb{Q}$, real $\mathbb{R}$ and complex $\mathbb{C}$ numbers are and that the last three constitute a so called field. Also, it is required that he or she knows about the notion of a complex conjugate as well as the foundations of classical logic by means of "or, and, not, forall, exists" as well as all interference rules (such as the de Morgan rule). To recap, a complex number $c = a + ib$ where $a, b \in \mathbb{R}$ and i is the so called imaginary unit satisfying $i^2 = -1$. Complex conjugation is then a reflection around the real axis meaning $\overline{c} = a - ib$. The reader verifies that $\overline{cd} = \overline{c}\overline{d}$ and $\overline{c + d} = \overline{c} + \overline{d}$. This defines the usual Euclidean norm squared on the complex plane by

$$|c|^2 = c\overline{c} = a^2 + b^2 > 0.$$

The reader may verify that $c\overline{d} = |c||d|\cos(\theta)$ where θ is the oriented angle between the numbers c, d seen as vectors in the two plane. Central in the theory of complex numbers is the so called exponential mapping $e : z \to e^z$ obeying

$$e^{w+z} = e^w e^z, \;\; \overline{e^z} = e^{\overline{z}}, \;\; \frac{d}{dz}e^z = e^z$$

where $\frac{d}{dz}$ denotes the derivative. A central cornerstone regarding the exponential mapping is the so called Euler formula, which says that

$$e^{ia} = \cos(a) + i\sin(a)$$

and $e^b \in \mathbb{R}_+$ for any real numbers a, b. This means that the exponential mpping wraps the complex plane an infinite number of times into itself since

$$e^{a+ib} = e^{a+i(b+2\pi n)}$$

where $n \in \mathbb{Z}$. Henceforth, any complex number can be written uniquely $c = |c|e^{i\phi}$ where ϕ may vary between 0 and 2π or between $-\pi$ and π. The reader understands of course that $|e^{i\phi}| = 1$ and therefore these numbers describe the unit circle. This makes the construction of an inverse of the exponential map, called the natural logarithm, a bit delicate. In particular, the standard convention being

$$\ln(a + ib) = \ln(|a + ib|) + i\phi$$

where $\phi \in (-\pi, \pi)$ and is given by $\tan(\phi) = \frac{b}{a}$. In terms of the representation above, this reads

$$\ln(|c|e^{i\phi}) = \ln(|c|) + i\phi$$

so that $\ln(cd) = \ln(c) + \ln(d)$ as long as the sum of the angles does not exceed the range $-\pi \ldots \pi$. The half line, given by $\phi = \pi \equiv -\pi$ is called the branch cut; one cannot extend the logarithm beyond that range without it becoming multiple valued. Finally, there is a single important theorem regarding complex numbers which you should know about: it is called algebraic completeness. To introduce this a bit, define a polynomial of degree n in one complex variable x as

$$P(x) = \sum_{k=0}^{n} a_k x^k$$

where $a_k \in \mathbb{C}$ and $a_n \neq 0$. Then the theorem saus that such polynomial has precisely n roots $r_i \in \mathbb{C}$, some of which may coincide, such that

$$P(r_i) = 0; \;\; P(x) = a_n \prod_{i=1}^{n} (x - r_i)$$

meaning the polynomial factorizes as a product of polynomials of first degree. The reader notices that for real polynomials, it holds that if c is a root, then $\bar{c}$ is a root as well.

The goal of this chapter is not to give an exclusive overview of many mathematical topics, but to provide the reader with the main tools required to understand the language of the old quantum theory of Heisenberg and Dirac. Albeit I shall also use some notions of differential geometry, this topic will not be touched upon in this chapter. It is secondary in a way as it deepens our understanding of more advanced topics which shall be discussed in somewhat less depth in this book. The reader who wants to understand it all is invited to read any good textbook on the matter or consult my book on mathematics [1]. In a way, this work is one of divine mercy where I try to meld spirits into a better way of thinking about the world, very much like catholic priests try to persuade people to follow the light of Jezus. I could have easily dismissed this task and refer the reader to the standard literature which would not only increase the digestive process but also lack the personal perspective I am willing to offer. In a way, it is pretty bad that the educational system delivers diploma's based upon knowledge and methods of a century old and I try to rectify this a bit here hoping that the reader may be interested in knowing more about this. We have just repeated some trivia about complex numbers and the reason why they are so special is precisely because of the algebraic completeness. Quantum theory is all about complex linear spaces, and we shall introduce those in several steps so that the reader gets a good feeling of what you can do with them. The prototype of an n-dimensional real vector space is

$$\mathbb{R}^n = \times_{i=1}^n \mathbb{R} = \{(x_i)_{i=1}^n | x_i \in \mathbb{R}\}$$

which is the set of n-tuples of real numbers equipped with the notion of a sum given by

$$(x_i) + (y_i) = (x_i + y_i)$$

and likewise can one define the scalar multiplication of a real number with an n-tuple vector by means of

$$r.(x_i) = (rx_i).$$

More in general, let $R, +, .$ be a field and $G, +$ a commutative group[1], then we say that G is an R module in case there exists a scalar multiplication such that

$$1.g = g;\ (rs).g = r.(s.g);\ (r+s).g = r.g + s.g;\ r.(g_1 + g_2) = r.g_1 + r.g_2$$

for all $r, s \in R$ and $g, g_1, g_2 \in G$. In case $R = \mathbb{R}$ we call the module a real vector space. In $\mathbb{R}^n, +$, we have special vectors e_i, defined by the number 1 on the i'th digit and zero elsewhere; it holds that

$$\sum_{i=1}^n r_i.e_i = 0$$

if and only if all $r_i = 0$ and moreover each vector can be written uniquely as

$$\sum_{i=1}^n r_i.e_i.$$

In case these properties hold for a set of vectors $\{v_i | i = 1 \ldots m\}$, then we call $\{v_i | i = 1 \ldots m\}$ a basis. One notices that we have used two integer numbers here, n for the e_i and m for all v_j; it is now a piece of cake to show that $n = m$. The reason is the following, because e_i is a basis, one can write the v_j uniquely as

$$v_j = \sum_{i=1}^n v_j^i e_i$$

[1] A group $G, +$ is a set G endowed with a mapping $+ : G \times G \to G : (x, y) \to x + y$. This mapping obeys (a) associativity, meaning that $+(x, +(y, z)) = +(+(x, y), z)$ something which is well known under the notation $x + (y + z) = (x + y) + z$, (b) existence of a unique neutral element (which for the sum is denoted by 0) satisfying $0 + x = x = x + 0$ (c) existence of an inverse, which we denote for the sum by $-x$ obeying $x + (-x) = 0 = (-x) + x$. Such structure is called a group; the group is commutative if and only if $x + y = y + x$ for all x, y. A field is a double commutative group in a way $G, +, .$ where we denote the unit element of . by 1 and only 0 has no inverse for the multiplication. Finally, the multiplication is distributive regarding the sum, meaning that $x.(y + z) = x.y + x.z$.

and reversely

$$e_i = \sum_{j=1}^{m} e_i^j v_j.$$

Henceforth,

$$\sum_{i=1}^{n} v_j^i e_i^k = \delta_j^k;\ j,k : 1 \ldots m$$

and

$$\sum_{j=1}^{m} e_i^j v_j^l = \delta_i^l;\ i,l : 1 \ldots n$$

where $\delta_j^k = 1$ if and only if $j = k$ and zero otherwise. This system of equations is symmetrical in e and v and therefore $m = n$ given that both mappings are injective. Henceforth n is a basis invariant and called the dimension of $\mathbb{R}^n, +$. Now, we have a sufficient grasp upon finite dimensional real vector spaces and the complex case is identical.

Let V, W be two (complex) vector spaces, then a linear mapping $A : V \to W$ is a function satisfying

$$A(r.v + s.w) = r.A(v) + s.A(w)$$

where the dot denotes scalar multiplication. Evidently one has that $A(0) = 0$ and $A(v) = A(w)$ if and only if $A(v - w) = 0$. Henceforth, the so called nucleus of A, defined by $\mathrm{Ker}(A) = \{v | A(v) = 0\}$ measures the deviation from injectivity of A. Every image $A(w)$ has as inverse $w + \mathrm{Ker}(A)$. The nucleus is henceforth itself a linear subspace of V. In the same way, one has that the so called image

$$\mathrm{Im}(A) = \{A(v) | v \in V\}$$

constitutes a subspace of W. It is now evidently true that

$$\mathrm{Im}(A) \cong \frac{V}{\mathrm{Ker}(A)}$$

meaning that both linear spaces are isomorphic to one and another. Indeed,

$$A : \frac{V}{\mathrm{Ker}(A)} \to \mathrm{Im}(A) : w + \mathrm{Ker}(A) \to A(w)$$

is linear and bijective which are the defining characteristics of an isomorphism. A trivial consequence of this theorem is that

$$\dim(\mathrm{Ker}(A)) + \dim(\mathrm{Im}(A)) = \dim(V)$$

where “dim” stands for dimension. Linear mappings can be represented by means of matrices defined with respect of basis vectors e_i in V and f_j in W respectively. The definition is given by

$$A(e_i) = \sum_{j=1}^{m} A_i^j f_j$$

where j is called the row index and i the column-index; taking a general vector $v = v^i e_i$ gives rise to the matrix multiplication

$$A(v) = \sum_{j=1}^{m} (\sum_{i=1}^{n} A^j_i v^i) f_j.$$

The composition of two linear mappings $A : V \to W$ en $B : W \to Z$ results into the matrix product

$$(BA)^j_i = \sum_{k=1}^{m} B^j_k A^k_i$$

where m represents the dimension of W. From now on, we dispose of the summation-signs, a convention which has been named after Einstein; so

$$\sum_{i=1}^{n} A^j_i v^i$$

is noted as

$$A^j_i v^i.$$

A 2×3 matrix, or a matrix with 2 rows and 3 columns is represented as

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

and regarding the matrix product BA one has the rule that the column dimension of B has to be equal to the row dimension of A. Show by means of a computational exercise that

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 13 \\ 19 & 19 \end{pmatrix}.$$

Show that in general for 2×2 matrices A, B one has that

$$AB - BA \neq 0$$

where 0 denotes the zero matrix. This result shows that the matrix multiplication is in general non-commutative and hitherto such operators constitute a non-commutative ring. The latter has been constructed as an object formed by elements which belong to a field. One can justifiably wonder whether the non-commutative number systems such as the quaternions and Clifford algebra's can be represented as matrices over the complex numbers. The answer is yes and one can obtain representations in different dimensions; regarding the quaternions q one has that

$$q = \begin{pmatrix} a + ib & ic - d \\ ic + d & a - ib \end{pmatrix}$$

where $a, b, c, d \in \mathbb{R}$. Another way of writing those in terms of Pauli matrices is provided by

$$q = a.1 + ic.\sigma_1 + id.\sigma_2 + ib.\sigma_3.$$

Now that we have understood a few essentials of matrix calculus, we arrive at the following natural question regarding matrix representations of linear operators: is it possible to find a basis e_i in V associated to a linear operator $A : V \to V$ such that A has a particularly simple matrix representation regarding e_i? Evidently, the formulation is somewhat vague up till now but try to ensure yourself that for an arbitrary $n \times n$ matrix A it almost always holds that $A = ODO^{-1}$ where $OO^{-1} = 1_n = O^{-1}O$ and $D_i^j = \lambda_i \delta_i^j$ with $\delta_i^j = 1$ if and only if $i = j$ and zero otherwise. D is a so called diagonal $n \times n$ matrix and the λ_i are called the eigenvalues such that

$$A(Oe_i) = \lambda_i(Oe_i)$$

which translates as the statement that Oe_i constitutes an eigenvector of A with eigenvalue λ_i. O is called an invertible or reversible $n \times n$ matrix. The reasoning behind it is very simple: in general, it holds that almost any square matrix O is invertible such that O has n^2 degrees of freedom; the mapping $O \to ODO^{-1}$ reduces exactly n dimensions in case all λ_i in D are different because the equation $VDV^{-1} = ODO^{-1}$ implies that $(V^{-1}O)D = D(V^{-1}O)$ such that $V = OD'$ with D' diagonal and henceforth any D "orbit" is $n^2 - n$ dimensional. Given that the number of degrees of freedom in D also equals n we have in total n^2 degrees of freedom and henceforth we obtain a "generic" $n \times n$ matrix. Prior to proceeding, we study the effect of a change of basis on the matrix representation of A. Denoting $e'_i = O(e_i)$ then one has

$$A'^i_j e'_i = A(O(e_j)) = A(O^k_j e_k) = A^l_k O^k_j e_l = A^l_k O^k_j (O^{-1})^i_l e'_i$$

and as such $A'^i_j = (O^{-1})^i_k A^k_l O^l_j$. So, generically, one can find a basis with respect to which the matrix representation for A is diagonal. The reader should show that all eigenvalues are unique as well as the eigenvectors (upon a normalization constant) in case all λ_j differ. One can find exceptions to this rule! Show that the matrix

$$N = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

satisfies $N^2 = 0$ and therefore cannot be diagonalized. This is a simple consequence of the fact that any eigenvalue must be equal to zero and henceforth $N = 0$ in case N can be diagonalized which is a contradiction. In two dimensions, one can by means of a suitable choice of basis ensure that an operator can be exclusively represented by one of the following matrices:

$$\begin{aligned} A &= \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \\ A &= \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}. \end{aligned}$$

In case the reader wishes to prove such a result, as well as a suitable extension towards higher dimensions, then I advise further reading up to the end of the chapter prior to dealing with this challenge. An $n \times n$ matrix A can still be interpreted in a different way as being merely the representation of a linear operator with respect to a vector space basis. One can see A as a collection of n ordered column vectors $A = (v_1, \ldots, v_n)$. This viewpoint allows one to interpret A as a simplex or the multi-dimensional cube determined by the column vectors v_i. The determinant $\det(A)$, to be defined below, calculates then the oriented volume of that cube which is just the product of the lengths of the basis vectors e_i if the latter are perpendicular to one and another. We derive a formula for $\det(A)$ from conditions the oriented volume needs to satisfy. First of all, det is multilinear in the columns; it is to say:

$$\det(v_1, \ldots, v_{i-1}, a.v_i + b.w_i, v_{i+1}, \ldots, v_n) =$$

$$a \det(v_1, \ldots, v_{i-1}, v_i, v_{i+1}, \ldots, v_n) + b \det(v_1, \ldots, v_{i-1}, w_i, v_{i+1}, \ldots, v_n)$$

as well as nilpotent in the sense that if $v_i = v_j$ for some $i \neq j$ then the determinant vanishes. This last condition merely reflects that if some axis coincide then the matrix defines a lower dimensional object with vanishing volume. Finally, one imposes the normalization condition that $\det(1_n) = 1$. These three conditions fully determine the functional description for the determinant: from the first and second condition one derives that the determinant is fully anti-symmetrical; it is to say that $\det(v_1, \ldots, , v_i, \ldots, v_j, \ldots, v_n) = -\det(v_1, \ldots, v_j, \ldots, v_i, \ldots, v_n)$. Combining this fact with the third and first condition one arrives at

$$\det(A) = \sum_{\sigma \in S_n} \mathrm{sign}(\sigma) A^1_{\sigma(1)} \ldots A^n_{\sigma(n)}$$

where σ is a so called permutation and sign denotes the sign thereof. Due to the anti-symmetrical nature of the determinant, each index is allowed to appear exactly once which is encoded in the above formula by means of a permutation. The latter is a bijection of $\{1, 2, \ldots, n\}$ onto itself whereas the sign denotes the even or odd nature of the number of swappings one has to perform to arrive from the identity mapping to σ, where an even number results in the value one and the odd number in minus one. One shows that permutations constitute a non commutative group S_n with $n.(n-1).(n-2)\ldots 3.2.1$ elements and we show now that the sign function is well defined meaning no odd and even number of swappings can occur. The proof is a bit technical; denote with (ij) the swapping operation of the i'th and j'th index leaving the remainder invariant, then it holds that

$$\begin{aligned} (ik)(ij) &= (jk)(ik) \\ (ik)(jl) &= (jl)(ik) \\ (ik)(ij) &= (ij)(jk) \end{aligned}$$

for distinct i, j, k, l. First of all, it is clear that any permutation can be written as a product of such swapping operations. Given a non trivial product of such swappings equivalent to the identity, then it is a simple matter to prove that it contains an even number of swappings using above swapping rules. Indeed, given $\sigma = l(ik)s(jk)$ where l, s are products of swappings and l does not contain a swapping with the index k then the reader shows that it is possible to rewrite this decomposition as $\sigma = ls'$ where s' does not contain the index k and has precisely the same number of swappings as s has modulo two. In this way, one proves that σ contains an even number of swappings. From this it follows that two different products l, s for any permutation σ always differ by an even number of swappings by denoting that ls^{-1} is equivalent to the identity. Henceforth, the function sign is well defined; show that the determinant of a 2×2 matrix is given by

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

Prove now that

- $\mathrm{sign}(\rho\sigma) = \mathrm{sign}(\rho)\mathrm{sign}(\sigma)$
- $\det(AB) = \det(A)\det(B)$.

This last rule holds due to

$$\begin{aligned} \det(AB) &= \sum_{\sigma \in S_n} \mathrm{sign}(\sigma)(AB)^1_{\sigma(1)} \ldots (AB)^n_{\sigma(n)} \\ &= \sum_{\sigma \in S_n} \sum_{m_1, \ldots, m_n} \mathrm{sign}(\sigma) A^1_{m_1} \ldots A^n_{m_n} B^{m_1}_{\sigma(1)} \ldots B^{m_n}_{\sigma(n)} \end{aligned}$$

where $m_1, \ldots, m_n$ is another notation for a permutation. One easily understands this as follows: assuming that $m_i = m_j$ then for every permutation σ it holds that the associated term is compensated by the one associated to the permutation $\sigma(ij)$. Henceforth, we have that

$$\begin{aligned} \det(AB) &= \sum_{\sigma, \rho \in S_n} \mathrm{sign}(\sigma) A^1_{\rho(1)} \ldots A^n_{\rho(n)} B^{\rho(1)}_{\sigma(1)} \ldots B^{\rho(n)}_{\sigma(n)} \\ &= \sum_{\sigma, \rho \in S_n} \mathrm{sign}(\sigma\rho) A^1_{\rho(1)} \ldots A^n_{\rho(n)} B^1_{\sigma(1)} \ldots B^n_{\sigma(n)} \\ &= \det(A)\det(B). \end{aligned}$$

This implies in particular that $\det(A^{-1}) = (\det(A))^{-1}$. Hence, the determinant of $A = (v_1, \ldots, v_n)$ differs from zero if and only if the v_i constitute a basis which is equivalent to invertibility of A. Show that the inverse of

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is provided by

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

The reader is advised to explicitely write out the determinant for 3×3 matrices as well as to develop a suitable formula for the inverse of such matrix.

Now, we return to the study of the classification of matrices in "standard form" by means of a basis transformation, the so called Cartan problem which requires the proof of existence of eigenvalues λ as well as associated eigenvectors v_λ satisfying

$$A(v_\lambda) = \lambda v_\lambda.$$

Another way of phrasing this is to say that the nucleus of $A - \lambda 1_n$ is nontrivial which is true if and only if

$$\det(A - \lambda 1_n) = 0.$$

At this point, determinants become useful because this formula can be interpreted as a root equation for a polynomial of the n'th degree. As we know, this polynomial can be entirely factorized over the field of complex numbers $\mathbb{C}$ and we obtain in general n distinguished complex eigenvalues showing that almost any matrix can be diagonalized. The reader is now advised to consider the previous example in two dimensions where two eigenvalues coincide and consider further examples of operators of a higher nilpotency in three or more dimensions.

The vigilant reader has meanwhile noticed that that determinant of a matrix is a basis invariant and henceforth associated to a linear mapping; that is,

$$\det(O^{-1}AO) = \det(O)^{-1}\det(A)\det(O) = \det(A).$$

Therefore, it is noticed that the eigenvalue polynomial $\det(A - \lambda 1_n)$ is an operator invariant. In particular, it is shown that the functional coefficient of k'th degree corresponding to the $n-k$'th power of λ constitutes an invariant under basis transformations. For $k = 1$ this gives $(-1)^{n-1}\mathrm{Tr}(A)$ where the so called trace Tr is defined by means of

$$\mathrm{Tr}(A) = \sum_{i=1}^{n} A^i_i.$$

Verify as an exercise in an explicit way that the trace is indeed a basis invariant and study the specific functional form of the higher invariants as well. One might try to write those as polynomials of traces of powers of the matrix; in particular in two dimensions it holds that

$$2\det(A) = (\mathrm{Tr}(A))^2 - \mathrm{Tr}(A^2).$$

Show that, in case one replaces the real number λ by the matrix A in the eigenvalue polynomial that it holds then that the resulting matrix equals the zero matrix. This is known as the theorem of Cayley Hamilton (hint: suppose first that A can be diagonalized and use then the definition of an eigenvalue as a root of the eigenvalue polynomial and finally employ that any matrix can be arbitrarily well approximated by one which is diagonalizable) which reads in two dimensions as

$$A^2 - \mathrm{Tr}(A)A + \det(A)1_2 = 0.$$

Show finally that $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$ and that this implies that no pair of matrices A, B exists such that

$$AB - BA = 1_n$$

a formula which is known as the bosonic Heisenberg relation and requires an infinite number of dimensions for operators having infinite traces. Note that it is possible to find two by two matrices such that

$$AB + BA = 1_2$$

known as the fermionic Heisenberg relationship. Bosons require henceforth an infinite number of dimensions whereas fermions live in dimensions equal to $n = 2^d$ where the reader should find a realization for $d = 1$. Finally, we define the notion of transposition A^T as well as the complex conjugate $\overline{A}$ of a matrix A

$$(A^T)^i_j = A^j_i, \ (\overline{A})^i_j = \overline{A^i_j}.$$

Show that

$$(AB)^T = B^T A^T, (A^T)^T = A, (rA + sB)^T = rA^T + sB^T, (A^{-1})^T = (A^T)^{-1}$$

and similar properties for the complex conjugation. The hermitian conjugate, which is of fundamental importance in this book, is given by $A^\dagger = \overline{A}^T$ and the reader may verify that

$$(AB)^\dagger = B^\dagger A^\dagger, (A^\dagger)^\dagger = A, (rA + sB)^\dagger = \overline{r}A^\dagger + \overline{s}B^\dagger, (A^{-1})^\dagger = (A^\dagger)^{-1}$$

In particular, it holds that for

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

$$A^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

swapping rows as well as columns. Prove that for

$$N = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

it holds that $N^T N + NN^T = 1_2 = N^\dagger N + NN^\dagger, N^2 = 0$ giving rise to the namer that N constitutes a fermionic creation-operator. This suffices for a first encounter with linear spaces and operators; the next section treats the subject in more depth and we continue now with succinct excercises

Exercises regarding Hermitian projection operators.

- Let P, Q be two Hermitian projection operators meaning that $P^2 = P$, $Q^2 = Q$, $P^\dagger = P, Q^\dagger = Q$. Show that $P + Q$ constitutes a Hermitian projection operator if and only if $PQ = QP = 0$. Show that the same holds for PQ if and only if $PQ = QP$.

- Two Hermitian projection operators P, Q are orthogonal if and only if $PQ = 0$; we define the partial order $\leq$ by means of $P \leq Q$ if and only if $QP = PQ = P$. Prove explicitly that $\leq$ defines a partial order[2] on the set of Hermitian projection operators. In particular, it holds that $P \leq Q$ and $Q \leq P$ implies that $P = Q$. Also, $P \leq Q$ and $Q \leq R$ leads to $P \leq R$.

- We call the set of Hermitian projection operators on a vector space, equipped with $\leq$, a raster. Show that for any P, Q there exists a minimal projection operator $P \vee Q$ such that $P, Q \leq P \vee Q$ and any R such that $P, Q \leq R$ satisfies $P \vee Q \leq R$. On the other hand, one may construct a maximal projection operator $P \wedge Q \leq P, Q$. Show that $\vee, \wedge$ do not in general obey the rule of de Morgan:

$$P \wedge (R \vee Q) \neq (P \wedge R) \vee (P \wedge Q).$$

- Show that the raster possesses a unique minimum as well as maximum provided by 0 and 1 respectively.

- Show that there exist minimal nonzero Hermitian projection operators, called atoms. Every Hermitian projection operator may be written as a sum of orthogonal atoms.

Quantum logic.
Given that in the previous exercise $\vee$ and $\wedge$ may be conceived as "or" and "and" respectively, it becomes possible to understand quantal logic by means of using Hermitian projection operators as propositions. Reflect on this and retrieve classical pointer propositions.

Hilbert space.
Let v and w be two complex vectors and denote by

$$\langle v|w\rangle = v^\dagger w \in \mathbb{C}$$

the so-called scalar product of v and w. Prove that

$$\langle v|w\rangle = \overline{\langle w|v\rangle}, \ \ \langle v|v\rangle \geq 0 \text{ and equality holds if and only if } v = 0$$

$$\langle v|aw + bz\rangle = a\langle v|w\rangle + b\langle v|z\rangle$$

and the reader verifies that these equalities imply that

$$\langle av + bz|w\rangle = \overline{a}\langle v|w\rangle + \overline{b}\langle z|w\rangle.$$

[2]Meaning that if $a \leq b$ then the reverse does not hold, $a \leq a$ and finally if $a \leq b$ and $b \leq c$ then $a \leq c$.

As a challenging exercise, the reader proves that

$$|\langle v|w\rangle| \leq ||v||||w||$$

where $||v|| = \sqrt{\langle v|v\rangle}$. Prove from hereon that

$$||v + w|| \leq ||v|| + ||w||$$

the so-called triangle inequality. Finally, let A be an operator, then show that

$$\langle v|Aw\rangle = \langle A^{\dagger}v|w\rangle.$$

Dirac notation: a vector v is also denoted as $|v\rangle$ and a conjugate vector $v^{\dagger}$ as $\langle v|$ so that $|v\rangle\langle v|$ is the Hermitian projector on v in case $\langle v|v\rangle = 1$.

Non-commutative Quantum logic.
We generalize the operations $\wedge$ and $\vee$ to a context in which they are no longer commutative; this procedure holds as well for the classical Boolean logic or the quantual logic explained above where the de Morgan rule gets a minor blow. It is natural to interpret $\wedge$ as well as $\vee$ as mappings $\wedge, \vee : P \times P \to P : (x, y) \to x \wedge y$, $(x, y) \to x \vee y$ where P denotes the lattice of propositions defined by means of a linear Euclidean space in the quantal case. Define the mapping $S : P \times P \to P \times P : (x, y) \to (y, x)$ and consider $\wedge^{'}_{(V,W)} := W \circ \wedge \circ S \circ V$ as well as $\vee^{'}_{(V,W)} = W \circ \vee \circ S \circ V$ where $V : P \times P \to P \times P$ is required to be invertible as well as is the case for $W : P \to P$. Requiring $\wedge^{'}_{(V,W)}$ to satisfy $\left(\wedge^{'}_{(V,W)}\right)^{'}_{(V,W)} = \wedge$ it is sufficient and mandatory that $W^2 = 1$ as well as $S \circ V \circ S \circ V = 1$. This demand is of a special algebraic nature which we dub by the name of an involution; so we are going to study involutive deviations from quantal logic. An involution gives rise to a notion of duality; in particular self-duality is defined by the condition that

$$\wedge^{'}_{(V,W)} = \wedge, \vee^{'}_{(V,W)} = \vee.$$

It is natural to propose first S symmetrical logics; these are given by

$$\wedge^{'(V,W)} \circ S = \wedge^{'(V,W)}, \vee^{'(V,W)} \circ S = \vee^{'(V,W)}.$$

This can only happen by choosing V such that

$$V \circ S = S \circ V$$

reducing a previous condition to

$$V^2 = 1$$

whereas it still holds that

$$\wedge^{'(V,W)} = W \circ \wedge \circ S \circ V.$$

In case $\wedge$, $\vee$ coincide with the standard Boolean or Quantal operations denoted by $\wedge_d$, $\vee_d$ where $d = c, q$ one has that

$$\wedge_d \circ S = \wedge_d,\ \vee_d \circ S = \vee_d.$$

In such a case,

$$\wedge := \wedge_d^{\prime(V,W)} = W \circ \wedge_d \circ V$$

a small simplification of the previous formula and $\vee$ is defined in a similar way. Now, to remain entirely clear, it is so that the d index should be the same in $\wedge$, $\vee$ but (V, W) becomes (R, T) for $\vee$ whereas the former pertains to $\wedge$. We now isolate the "de Morgan expression" $a \wedge (b \vee c)$:

$$\wedge \circ (1 \times \vee)(a, b, c) = W \wedge_q V(1 \times T \vee_q R)(a, b, c).$$

It is subsequently natural to call T - $(\wedge_q, V)$ compatible if and only if $\wedge_q V(1 \times T) = T' \wedge_q V$ for some $T' : P \to P$. Likewise, it is natural to call V - $\vee_q$ compatible if and only if $V(1 \times \vee_q) = (1 \times \vee_q)V'$ for some $V' : P^3 \to P^3$. Under these assumptions, the previous expression reduces to

$$WT'(\wedge_q(1 \times \vee_q))V'(1 \times R)$$

which was the desirable separation. It is furthermore natural to suggest further restrictions

$$WT' = 1,\ V'(1 \times R) = 1_3.$$

Truth evaluators ω

The material presented below constitutes an extension of the notes I have received once from Rafael Dolnick Sorkin; in classical Boolean logic one disposes of truth evaluator ω of logical sentences which constitutes a homomorphism from the set of propositions $P, \vee_c, \wedge_c$ to $\mathbb{Z}_2, +, .$ where 0 is interpreted as false and 1 as true and $\vee_c$ is the so called exclusive *or* in the sense that $a \vee_c b$ is true if and only if exactly one of them is true. It is to say that

$$\omega(a \vee_c b) = \omega(a) + \omega(b),\ \omega(a \wedge_c b) = \omega(a)\omega(b).$$

In quantum logic, there is no such thing as a truth evaluator; one can only say wether a particular assertion is true or false with a certain probability. A quantum reality is then a particular choice of mapping from P to $\mathbb{Z}_2$ but it makes no sense any longer to speak about a homomorphism because the de-Morgan rule fails in general: the lattice is not distributive. As such, it may very well be that you have a quantal reality ω for which $\omega(a) = \omega(b) = 1$, but $\omega(a \wedge_q b) = 0$. To get an idea of what more general realities are about, let us describe a classical system in a quantum mechanical fashion. An example is give by means of the weather, "the sun shines", modelled by $|l\rangle$, or "it is dark" given by $|d\rangle$. Quantum mechanically, one disposes of a complex two dimensional Euclidean space spanned by the extremal vectors $|l\rangle, |d\rangle$. Consider now a general state

$$|\psi\rangle = \alpha|l\rangle + \beta|d\rangle$$

and study the class of truth functionals ω which merely depend upon

$$\frac{|\alpha|^2}{|\alpha|^2 + |\beta|^2}, \frac{|\beta|^2}{|\alpha|^2 + |\beta|^2}$$

something which reduces to a parameter $0 \leq \lambda \leq 1$ due to

$$\frac{|\alpha|^2}{|\alpha|^2 + |\beta|^2} + \frac{|\beta|^2}{|\alpha|^2 + |\beta|^2} = 1.$$

When all truth evaluators merely depend upon this parameter only, the complex plane may be reduced to the line segment connecting both extremal vectors $|l\rangle, |d\rangle$ to one and another. An example of such a gneralized reality is provided by

$$\omega^l_\epsilon : [0, 1] \rightarrow \mathbb{Z}_3$$

given by means of the prescription

$$\omega^l_\epsilon(\sqrt{\lambda}|l\rangle + \sqrt{(1-\lambda)}|d\rangle) = \chi(\lambda + \epsilon - 1) + 2\chi(\lambda - \epsilon)\chi(1 - \epsilon - \lambda).$$

ω^l and is henceforth connected to the question whether the light shines and ϵ is the tolerance of the observer. This truth evaluator says "yes", given by means of 1, in case $1 - \epsilon \leq \lambda \leq 1$, under determined or "vague" 2 when $\epsilon \leq \lambda \leq 1 - \epsilon$ and no, given by 0, when $0 \leq \lambda \leq \epsilon$. We have that χ is the so called characteristic function defined on the real numbers by means of $\chi(x) = 1$ in case $x \geq 0$ and zero otherwise. The issue is that we departed from a quantum mechanical description of the weather and by reduction of the allowed questions arrived to a classical system where, moreover, ω^l_ϵ is nonlinear.

Most physicists would suggest at this moment that we did not make a sufficient distinction between classical and quantum logic as yet because $\wedge_q, \vee_q$ are commutative, assiociative but $\wedge_q$ is not distributive with regard to $\vee_q$ which is the case for $\wedge_c$, $\vee_c$. In our most general setting, one has that $\wedge$ and $\vee$ are neither commutative, nor associative

$$\vee(1 \times \vee)(a, b, c) = T \vee_d R(1 \times T \vee_d R)(a, b, c) \neq T \vee_d R(T \vee_d R \times 1)(a, b, c) = \vee(\vee \times 1)(a, b, c)$$

and likewise so for $\wedge$. The main distinction between classical and quantum logic resides in the fact that the set of propositions constitutes a distributive lattice in the former case whereas it does not in the latter; this results in the statement that the classical rule

$$\mu(a|b)\mu(b) = \mu(b|a)\mu(a)$$

is no longer true in the quantal case. Here, μ is the probability measure that a is true; in other words, the truth determinations of a and b depend upon the order in which they occur. This has so far not been accounted given that a homomorphism $\vee_{c,q}, \wedge_{c,q}$ does not make any distinction in the order of the factors. Therefore, classically, for our homomorphism $\omega_c(a \wedge_c b)$ is determined by the unordered tuple $\{\omega_c(a), \omega_c(b)\}$. Quantum mechanically, it is as such that the reality $\omega_q(a \wedge_q b)$ is not provided by the ordered couple $(\omega_q(a), \omega_q(b))$ as elements of $\mathbb{Z}_2$ but also depends upon a, b themselves. It is not so that

$$\mu_{|v\rangle}(a|b) = \frac{\mu_{|v\rangle}(a \wedge_q b)}{\mu_{|v\rangle}(b)}$$

due to commutativity of $\wedge_q$ as well as $a \wedge_q b = 0$ for distinct one dimensional Hermitian projection operators a, b on a Hilbert space $\mathcal{H}$. The exact formula is given by

$$\mu_{|v\rangle}(a|b) = \frac{\mathrm{Tr}(|v\rangle\langle v|bab)}{\mathrm{Tr}(|v\rangle\langle v|b)}$$

and the reader notices that the non-commutativity of a and b is of vital importance. Henceforth, the ontological mapping defined in quantum theory is given by $\kappa : P \to \mathbf{L}(\mathcal{H})$ where P is the set of prepositions with a yes or no answer onto the lattice of Hermitian projection operators defined on the Hilbert space of states of the system. The classical Lagrange formula

$$\mu(a|b)\mu(b) = \mu(b|a)\mu(a)$$

where μ is determined by the state of the system is abandoned upon provided that $\wedge_q$ a la Von Neumann offers no alternative. The natural question henceforth is whether we may find a natural $\wedge$ as well as a consistent set of realities

$$\omega_q^\rho : P \to \mathbb{Z}_2 \times [0, 1]$$

attached to density matrices ρ defined on $\mathcal{H}$, such that

$$\omega_q^\rho(a) = (1, \lambda)$$

and

$$\omega_q'^\rho(a) := (0, 1 - \lambda)$$

is defined as the complementary observation. It is clear that ω_q is not always given by a homomorphism; prior to proceeding, it is important to understand $\vee_q$. It is clearly so that in quantum theory, we have an extended ontology; we do not only pose the question "what is the probability that $a \wedge_c b$ holds given that a as well as b are true" such as the case in classical logic, but we insist on the formulation "what is the chance that $a \wedge_q b$ holds given that a after b has been experimentally established". The right answer is easy if $a \wedge b$ is represented by the Hermitian operator bab which is logical given that the order of measurement matters. In general, one shows that

$$a \wedge_q b = \lim_{n \to \infty} \left(\frac{1}{2}(ab + ba) \right)^n$$

and in the framework of our deformation theory $\wedge$ is given by means of

$$V(a, b) = (1, bab)$$

at least this is so for atomistic elements a, b. For atomistic elements, $\frac{bab}{\mathrm{Tr(ab)}}$ is again a rank one Hermitian projection operator; however for projection operators of general rank, this is no longer the case. Here, we have to extend our definition of V as going from $P \times P \to C \times C$ where C are the so called positive operators on Hilbert space. An operator A is positive if and only if A is self adjoint and

$$\langle v|A|v\rangle > 0$$

for all $v \neq 0$. As an exercise, the reader understands that the definition of $\leq$ extends to the Hermitian operators by means of $A \leq B$ if and only if

$$\langle v|(B - A)|v\rangle > 0.$$

Show that in such a case, the definitions of $\wedge_q$ and $\vee_q$ can be extended as the largest Hermitian operator smaller or equal than A, B and the smallest Hermitian operator greater or equal to A, B respectively. The proof of this statement hinges on the so-called spectral decomposition theorem for Hermitian operators, something which we shall study in the next section. Briefly, it says that any Hermitian operator A can be written as

$$A = \sum_i \lambda_i P_i$$

where the λ_i are the real eigenvalues and the P_i Hermitian projection operators such that $P_i P_j = \delta_{ij} P_i$. Therefore, take A, B and order all eigenvalues

$$\lambda_0 < \lambda_1 \ldots < \lambda_k$$

with $k \leq 2n$ where n is the dimension of Hilbert space. Note that some of the λ_i may belong to A as well as B; in that case, we consider the projection operators $R_i := P_i \vee_q Q_i$ where the Q_i refer to B otherwise R_i equals P_i or Q_i. Start now with λ_0, the smallest eigenvalue, and consider the operator $C_0 = \lambda_0 R_0$; clearly $C_0 \leq A, B$. Proceed now towards the minimal λ_j such that $S_j := \vee_{i=2}^{j} R_i$ obeys $[S_j, R_0] = 0$ and consider the projection operator

$$T_1 := S_j(1 - R_0)$$

then the reader verifies that this is an Hermitian projection operator and that $T_1 R_0 = 0$. In case no such j exists, then define $A \wedge_q B = \lambda_0 R_0 + \lambda_1(1 - R_0)$, otherwise proceed with $C_1 := \lambda_0 R_0 + \lambda_1 T_1$. The reader now understands that he has to look at λ_{j+1} and construct the smallest $S_k := \vee_{i=j+1}^{k} R_i$ such that

$$[S_k, R_0 + T_1] = 0.$$

In case no such k exists $A \wedge_q B = \lambda_0 R_0 + \lambda_1 T_1 + \lambda_{j+1}(1 - R_0 - T_1)$ otherwise we consider

$$C_2 = \lambda_0 R_0 + \lambda_1 T_1 + \lambda_{j+1} T_2$$

where $T_2 = S_k(1 - R_0 - T_1)$ and the procedure continues. It is obvious that the final result is the optimal Hermitian operator which is smaller or equal to both A, B. The construction of $\vee_q$ is similar, but then one starts at the largest eigenvalue of both operators. W is henceforth determined on the rank 1 matrices by means of the identity. Therefore, for rank one projectors a, b it holds that

$$a \wedge b = T \circ \wedge_q \circ R(a, b) = bab.$$

Subsequently, one has that

$$\omega_q^\rho(a) = (1, \mathrm{Tr}(\rho a))$$

or

$$\omega_q^\rho(a) = (0, 1 - \mathrm{Tr}(\rho a))$$

for a of rank one. Clearly, by definition

$$\omega_{q,1}^\rho(a|b) := \frac{\pi_2(\omega_q^\rho(a \wedge b))}{\pi_2(\omega_q^\rho(b))}$$

equals the probability that a is measured after b. Here π_j equals the projection on the j'th factor. Elaborate further on this theory and determine a suitable $\vee$ operation. Hint: the latter is cannot be given by $a \vee b = a + b$ in the deformation framework provided that $\vee_q$ does not allow one to determine the projection of a on b as is given by $\mathrm{Tr}(ab)$. This is something which is mandatory to extract the sum operation. To define $\vee$ it is advised to use the classical rule

$$\neg(a \vee_c b) = (\neg a) \wedge_c (\neg b)$$

and using $\neg\neg = 1$, it holds that

$$a \vee b = \neg((\neg a) \wedge (\neg b)).$$

In quantum theory, $\neg(a)$ is provided by $1 - a$ and henceforth, we arrive at

$$a \vee b = 1 - (1 - a) \wedge (1 - b)$$

which leads to a violation of the de Morgan rule given that

$$a \wedge (b \vee c) = a \wedge (1 - (1 - b) \wedge (1 - c)) = (1 - (1 - c)(1 - b)(1 - c))a(1 - (1 - c)(1 - b)(1 - c))$$

whereas

$$(a \wedge b) \vee (a \wedge c) = 1 - (1 - cac).(1 - bab).(1 - cac).$$

General exercise.
Determine matrix representations of deformed logic's in terms of commutative albeit possible non-associative ones. It is to say that

$$\wedge = (\tilde{\wedge}_{ijk})_{i,j,k:1\ldots n}$$

where

$$\tilde{\wedge}_{ijk}(a_j, b_k) = \tilde{\wedge}_{ijk}(b_k, a_j)$$

constitute S symmetrical logics on the product space $\times_n P$ where P provides for elementary propositions. Classify first the S symmetric deformations of Boolean logic on general proposition sets.

1.1 Hilbert spaces and some important theorems.

Whereas our exposition up till now has uncovered some important notions such as a modified logic and the very idea that you can sum up the states of a system, the so called superposition principle (which has been experimentally verified), the really interesting stuff which we shall crucially use in the next chapters resides in infinite dimensions. This entire section is simply devoted to this particular subtlety because in an infinite number of dimensions things can happen which do not occur in a finite number. In that context, we have said already that the commutation relations

$$AB - BA = 1$$

cannot materialize in finite dimensions whereas those will constitute the very cornerstone of our thinking about the mental world. In order to proceed, we must deviate slightly into the notion of topology which is a means to speak about wether two objects are in each others neighborhood or not. Lets introduce it by means of a metric; you all know that the distance between two points p_i with coordinates (x^i, y^i) in the real plane is given by

$$d(p_1, p_2) = \sqrt{(x^1 - x^2)^2 + (y^1 - y^2)^2}$$

and the natural generalization thereof to n dimensions reads

$$\sqrt{\sum_{j=1}^{n} (x_j^1 - x_j^2)^2}.$$

As for every distance function, it holds that

$$0 \leq d(x, y) = d(y, x),\ d(x, z) \leq d(x, y) + d(y, z),\ d(x, y) = 0 \leftrightarrow x = y.$$

Now, one can define a set O to be open if and only if for any $x \in O$, there exists a sufficiently small number $\epsilon > 0$ such that if $d(x, y) < \epsilon$ then $y \in O$. The reader shows that the finite intersection of open sets is open as well as an arbitrary (possibly infinite) union of open sets. By definition $\mathbb{R}^n$ and the empty set are open. These three properties constitute the very definition of what we mean with a topology; we shall in this book restrict ourslves to topologies generated by (pseudo) norms, a concept which shall be explained further on in this chapter. Until now, we have been silent about the subject of topologies on linear spaces as well as on spaces of linear operators defined upon the former. The reason for this is very simple: all such spaces have been equivalent to $\mathbb{R}^n$ from the set theoretical point of view and all "natural" topologies which spring to ones mind are equivalent to the product norm topology. In a countable infinite number of dimensions, these topologies become inequivalent and we shall study those at an early stage in this section. We shall commence with studying an in-product geometry and see how it connects to probability theory as well as topology: the philosophy then is that such flat geometry precedes all remaining concepts in a well defined sense.

There exist many distinguished means of presenting the material below but I am of the opinion that the succinct presentation beneath is the most efficient one. A distinguished feature of Euclidean geometry is that the underlying set is given by means of linear space, this is no longer true when studying curved geometry. This very feature shaped a too limited characterization for two thousand years of several geometrical concepts such a the one of an oriented line segment connecting two points x, y. The old view was that those could be connected by means of a free vector $y - x$ which is then assumed to be "thight" to the point x. Crucial herein is the minus sign as an operation suggesting that it is possible to add vectors without caring about their "anchoring" to particular points. Mathematically, this results in the notion of a linear space with the zero displacement 0 as a neutral element mistakingly dubbed as the "origin" of the latter space. This preferred origin has been long subject of "theological debate" which has its philosophical side too: is earth the center of the universe which never is in motion? Or must one speak about the sun or another heavenly body in this regard? Newton and his friends were the first to cut the Gordian knot: they introduced the concept of an affine space by allowing for translations removing any preference of origin whatsoever. Indeed, the mapping $x \to x + a$ does not commute with the addition given that

$$(x + y) + a \neq (x + a) + (y + a).$$

It leaves however the difference invariant in the sense that

$$(y + a) - (x + a) = y - x$$

such that vectors, bound or free, have a significantly distinct status from points. In Newton's world, nothing is fixed and that was a grand realization by itself. Mathematicians such as Gauss, Riemmann and Cartan did proceed even further on: modern cosmos has no translation symmetry any longer and cannot be described any more in the language of affine spaces. The importation of this realization into physics has been the great achievement of Albert Einstein by means of his theory of general relativity which constitutes by far a superior explanation behind everyday large scale observations in the universe. Euclidean space or an (in)finite dimensional flat geometry is defined henceforth by means of a real vector space $\mathcal{H}$ as well as scalar product $\langle v|w\rangle$ where $v, w \in \mathcal{H}$. The scalar product between v and w is supposed to be equal to the product of the oriented length of the projection of w upon v times the length of v. This quantity satisfies, by means of simple experience, the following properties:

$$\begin{aligned}
\langle v|w\rangle &= \langle w|v\rangle \\
\langle v|aw + bu\rangle &= a\langle v|w\rangle + b\langle v|u\rangle \\
\langle v|v\rangle &\geq 0 \, \text{where equality holds if and only if} \, v = 0.
\end{aligned}$$

The scalar product henceforth determines the notion of perpendicularity; the very fact that we have here on earth a preferred notion of perpendicularity is of a physical nature. Albert Einstein discovered that this information is encoded partially into the gravitational field. It could be that an alien would experience this gravitational field differently and that it would suggest a different local geometry. One can speak about complex geometries: in such a case, one defines in exactly the same fashion a sesquilinear form where now

$$\langle v|w\rangle = \overline{\langle w|v\rangle}$$

with the complex conjugation defined as usual by means of

$$\overline{a+bi} = a - bi.$$

For example, $\mathbb{C}$ constitutes a one dimensional Hilbert space with as scalar product $\overline{v}w$. As stated in the introduction, a Hilbert space carries some natural topologies; to define those, we show that the scalar product defines in a canonical fashion a metric d. We first prove that the quantity $||v||$ defined by

$$||v|| = \sqrt{\langle v|v\rangle}$$

and called a norm has identical properties to those of the modulus of a complex number. An important step herein is the so called Cauchy-Schwartz identity

$$|\langle v|w\rangle| \leq ||v||||w||$$

meaning that the projection of w on v multiplied with the length of v is less or equal to the product of the lengths of v and w, a result one expects to hold trivially. The formal proof goes as follows:

$$0 \leq ||v+\lambda w||^2 = ||v||^2 + |\lambda|^2 \, ||w||^2 + 2\mathrm{Re}\left(\overline{\lambda}\langle w|v\rangle\right)$$

where $\mathrm{Re}(a+ib) = a$ is the real part of the complex number $z = a + bi$. One verifies that the real part of the complex number z may be written as $\frac{1}{2}(z+\overline{z})$ whereas the imaginary part equals $-i\frac{1}{2}(z-\overline{z})$. The modulus of a complex number is defined by means of

$$|z| = \sqrt{z\overline{z}} = \sqrt{a^2+b^2}$$

and satisfies

$$|z+z'|^2 = |z|^2 + |z'|^2 + (z\overline{z'} + \overline{z}z')$$

whereas the last term equals, up to a factor of two,

$$aa' + bb'$$

and the absolute value is bounded from above by $|a|\,|a'| + |b|\,|b'|$. The square of this last expression is given by

$$a^2a'^2 + b^2b'^2 + 2\,|a|\,|a'|\,|b|\,|b'| \leq \left(a^2+b^2\right)\left(a'^2+b'^2\right) = |z|^2\,|z'|^2$$

and consequently one has that

$$|z + z'|^2 \leq (|z| + |z'|)^2$$

and hitherto

$$|z + z'| \leq |z| + |z'|$$

a formula known as the triangle inequality. Consequently, we may define a metric on the complex plane by means of

$$d(z, z') = |z - z'| \,.$$

Returning to the proof of the triangle inequality, one notices that we may pick λ such that

$$\mathrm{Re}\left(\overline{\lambda}\langle w|v\rangle\right) = - |\lambda| \, |\langle v|w\rangle|$$

whereas, in general, the left hand side is always larger than the right hand side. Therefore, we have that

$$0 \leq ||v||^2 + |\lambda|^2 \, ||w||^2 - 2 \, |\lambda| \, |\langle v|w\rangle|$$

which is a quadratic polynomial inequality in the positive variable $|\lambda|$. The existence of at most one positive root demands that

$$0 \leq 4 \, |\langle v|w\rangle|^2 - 4||v||^2||w||^2$$

which proves the result and equality only holds if and only if $w = -\lambda v$. Consequently,

$$||v+w||^2 \leq ||v||^2 + ||w||^2 + 2 \, |\langle v|w\rangle| \leq ||v||^2 + ||w||^2 + 2||v||||w|| = (||v|| + ||w||)^2$$

which proves the triangle inequality for the norm. Consequently, each Hilbert space $\mathcal{H}$ defines a canonical metric topology by means of

$$d(v, w) = ||v - w||$$

and we demand that $\mathcal{H}$ is complete in this topology. This condition is extremely important for the theory of linear operators but let us start by making some preliminary observations. Two non-zero vectors v, w are perpendicular to one and another if and only if $\langle v|w\rangle = 0$ and we say v is normed if and only if $||v|| = 1$. Due to the axiom of choice, any Hilbert space has an orthonormal basis $(e_i)_{i \in I}$ meaning $\langle e_i|e_j\rangle = \delta_{ij}$ where δ_{ij} equals 0 if $i \neq j$ and 1 otherwise. The mindful reader notices that δ^i_j constitutes a basis invariant whereas δ_{ij} is only invariant under orthogonal or unitary transformations. For finite dimensional Hilbert spaces, one has that, with $v = \sum_{i=1}^n v^i e_i$, it holds

$$\langle v|w\rangle = \sum_{i,j=1}^{n} \overline{v^i} w^j \delta_{ij}$$

which constitutes a generalization of the standard in-product in three dimensional Euclidean geometry. Show that by means of a basis transformation $e'_i = O^j_i e_j$ we have that $\delta'_{ij} = \langle e'_i | e'_j \rangle = \overline{O}^k_i O^l_j \delta_{kl}$. Exercise: define Hilbert spaces over the real quaternions.

We now consider some operations or constructions one can perform with real or complex Hilbert spaces. The best known ones are applied in the theory of quantum mechanics and are given by the tensor product $\otimes$ as well as direct sum $\oplus$. Given two Hilbert spaces $\mathcal{H}_i$, the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ constitutes again a Hilbert space spanned by pure vectors $v_1 \otimes v_2$ where $v_i \in \mathcal{H}_i$. Regarding sums $\sum_{i=1}^n z_i v^i \otimes w^i$, the following equivalences are in place

$$\begin{aligned} z(v \otimes w) &\equiv (zv) \otimes w \equiv v \otimes (zw) \\ v \otimes w_1 + v \otimes w_2 &\equiv v \otimes (w_1 + w_2). \end{aligned}$$

We define $\mathcal{H}$ as the linear space of such equivalence classes and make a completion in the metric topology defined by means of the scalar product

$$\langle v_1 \otimes w_1 | v_2 \otimes w_2 \rangle := \langle v_1 | v_2 \rangle \langle w_1 | w_2 \rangle.$$

In a similar vein, the direct sum $\mathcal{H}_1 \oplus \mathcal{H}_2$ is defined by means of the equivalences

$$\begin{aligned} z(v \oplus w) &\equiv (zv) \oplus (zw) \\ v_1 \oplus w_1 + v_2 \oplus w_2 &\equiv (v_1 + v_2) \oplus (w_1 + w_2) \end{aligned}$$

with as scalar product

$$\langle v_1 \oplus w_1 | v_2 \oplus w_2 \rangle := \langle v_1 | v_2 \rangle + \langle w_1 | w_2 \rangle.$$

One verifies that a basis for $\mathcal{H}_1 \otimes \mathcal{H}_2$ is provided by means of $v_i \otimes w_j$ where the v_i constitute a basis of $\mathcal{H}_1$ and w_j of $\mathcal{H}_2$. A basis for $\mathcal{H}_1 \oplus \mathcal{H}_2$ is provided by $v_i \oplus 0, 0 \oplus w_j$.

In a vector space, a basis defines a scalar product and the mapping of bases to Hilbert spaces is surjective. Bases connected by means of a transformation O satisfying

$$\overline{O}^k_i O^l_j \delta_{kl} = \delta_{ij}$$

determine the same scalar product and reversely alike scalar products define separate bases connected by such a transformation. One verifies that those matrices constitute a group, $U(n)$ for n dimensional complex Hilbert spaces and $O(n)$ in the real case, the so called unitary respectively orthogonal groups. The above formula reads in matrix language

$$O^H O = 1$$

whereas $O^H = (\overline{O})^T = \overline{(O^T)}$. Show that in two dimensions the unitary matrices are explicitly given by

$$O = \frac{1}{\sqrt{|a|^2 + |b|^2}} \begin{pmatrix} a & -\overline{b} \\ b & \overline{a} \end{pmatrix}$$

with $a, b \in \mathbb{C}$. This group has three real parameters; the reader is advised to determine an alike representation for $O(2)$. Given linear operators $A : \mathcal{H}_1 \to \mathcal{H}_3$ and $B : \mathcal{H}_2 \to \mathcal{H}_4$ then we may define operators

$$A \oplus B : \mathcal{H}_1 \oplus \mathcal{H}_2 \to \mathcal{H}_3 \oplus \mathcal{H}_4$$

as well as

$$A \otimes B : \mathcal{H}_1 \otimes \mathcal{H}_2 \to \mathcal{H}_3 \otimes \mathcal{H}_4$$

by means of

$$A \oplus B(v_1 \oplus v_2) = A(v_1) \oplus B(v_2)$$

and

$$A \otimes B(v_1 \otimes v_2) = A(v_1) \otimes B(v_2).$$

The reader should reflect for a moment and convince himself that $\otimes$ serves for the purpose of combining separate systems; it is to say functions in n real variables $f_k : (x_1, \ldots, x_n) \to \mathbb{C}$ and m real variables $g_k : (y_1, \ldots, y_m) \to \mathbb{C}$ define functions in $n + m$ real variables by means of

$$F = \sum_k a_k (f_k \otimes g_k) : \mathbb{R}^{n+m} \to \mathbb{C} : (x_1, \ldots, x_n, y_1 \ldots, y_m) \to$$

$$\sum_k a_k f_k(x_1, \ldots, x_n) g_k(y_1, \ldots, y_m).$$

Here, one should not regard $\mathbb{R}^{n+m}$ as a vector space but as a set; in the vector space language, it holds that $\mathbb{R}^{n+m} = \mathbb{R}^n \oplus \mathbb{R}^m$. It is a result from real analysis that $F : \mathbb{R}^{n+m} \to \mathbb{C}$ may be written as $\sum_k a_k (f_k \otimes g_k)$; in other words, one has a complex vector space of functions $L_2(\mathbb{R}^{n+m})$ which equals $L_2(\mathbb{R}^n) \otimes L_2(\mathbb{R}^m)$.

One now makes the following exercises: be $A : V \to V$ and $B : W \to W$ operators on finite dimensional vector spaces; show that

$$\mathrm{Tr}(A \oplus B) = \mathrm{Tr}(A) + \mathrm{Tr}(B),\ \mathrm{Tr}(A \otimes B) = \mathrm{Tr}(A)\mathrm{Tr}(B)$$

and

$$\det(A \oplus B) = \det(A)\det(B),\ \det(A \otimes B) = \det(A)^m \det(B)^n$$

where $n = \dim(V)$ and $m = \dim(W)$. In case V, W constitute moreover Hilbert spaces; show that

$$(A \oplus B)^H = A^H \oplus B^H,\ (A \otimes B)^H = A^H \otimes B^H.$$

Prove that the operations $\oplus, \otimes$ are associative with $\{0\}, \mathbb{C}$ as identity element respectively; denote with $\otimes_{\mathcal{F}}$ the mapping on the space of Hilbert spaces defined by $\otimes_{\mathcal{F}}(\mathcal{H}) = \mathcal{H} \otimes \mathcal{F}$. Construct a $i_{\mathcal{F}}$ such that $i_{\mathcal{F}} \circ (\otimes_{\mathcal{F}}) = \mathrm{id}$ where id is given by the identity transformation. Show that $\otimes_{\mathcal{F}}$ is not surjective unless $\mathcal{F} = \mathbb{C}$ which shows that there does not exist any $p_{\mathcal{F}}$ obeying $(\otimes_{\mathcal{F}}) \circ p_{\mathcal{F}} = \mathrm{id}$. Make a similar construction for $\oplus_{\mathcal{F}}$ and notify that nor $\oplus, \otimes$ are commutative. Here, we have found an example of a mapping, derived from an operation, with a left but no right inverse. Introduce now the concept of an anti-Hilbert space $\mathcal{F}^{\otimes}$ as a formal right inverse for $\mathcal{F}$; it is to say that

$$\mathcal{F} \otimes \mathcal{F}^{\otimes} = \mathbb{C}.$$

In that case $i_{\mathcal{F}}$ equals $\otimes_{\mathcal{F}^{\otimes}}$ on the image of $\otimes_{\mathcal{F}}$. This procedure is entirely analogous to taking negative integers or fractions starting from the natural numbers. Do the same for $\oplus$ and reflect further hereupon. More in particular, denote with a_i bosonic annihilation operators defined by

$$a_i a_j^{\dagger} - a_j^{\dagger} a_i = \delta_{ij} 1$$

and posit that

$$\mathcal{F} \equiv \{v = \sum_{i=1}^{\infty} \lambda^i a_i | \langle 0 | v v^{\dagger} | 0 \rangle < \infty \text{ with scalar product } \langle v | w \rangle = \langle 0 | w v^{\dagger} | 0 \rangle\}$$

where $|0\rangle$ constitutes the so called Fock vacuum defined by $a_i|0\rangle = 0$. $\mathcal{F}^{\otimes}$ equals then for example

$$\{v = \lambda^j a_j^{\dagger} | \text{with as scalar product } \langle v | w \rangle = \langle 0 | v^{\dagger} w | 0 \rangle\}$$

such that

$$v \in \mathcal{F} \otimes \mathcal{F}^{\otimes}$$

is given by $\sum_i \lambda_i \mu_j a_i a_j^{\dagger}$. The scalar product is given by

$$\sum_i |\lambda_i|^2 |\mu_j|^2 \langle 0 | a_j a_i^{\dagger} a_i a_j^{\dagger} | 0 \rangle$$

which equals $|\lambda_j|^2 |\mu_j|^2$. Therefore, all modes in $\mathcal{F}$ with $i \neq j$ are killed such that the positive norm requirement is restored. These phantoms need to be eliminated with the purpose of retaining a one dimensional space.

Remark that this non-commutative "product" also appeared in set theory by means of $\times$. More precisely, given a set A, an anti-set obeys

$$A \times A^{\times} = \{1\}$$

where the last one is a set with one element 1 and henceforth serves as the identity element for $\times$. To represent an anti-set in the set like fashion; denote that if $A = \{x | x \in A\}$ and $A^{\times} = \{\omega_A^{\star}\}$ where $\omega_A : A \to \{1\}$ is the constant mapping onto 1 and $\star$ is the associated duality relation, then

$$A \times \{\omega_A^{\star}\} = \omega_A(A) = \{1\}.$$

Later on, the reader shall deepen his understanding of the fact that Hilbert spaces are employed in physics to describe separated entities such as elementary particles whereas the concept of an anti-Hilbert space can be used to describe particle collisions to create novel types of particles. To collide or not to collide could be mere approximations due to the point description of a particle and the reader is invited, as an exercise of collosal difficulty, to search for a concept of touching.

With this knowledge at hand, it becomes possible to solve standard problems from flat geometry; very strong results are possible here which do not hold in general due to topological as well as metrical complications. The magic of flat geometry is entirely hidden into the vector space structure. For example, on the surface of a ball, any two straight lines, defined as the intersection of the spherical surface with a two dimensional plane containing the barycenter of the sphere, intersect at a length of pi times the radius. In the two dimensional plane on the other hand, there exists a preferential class of parallel lines defined by the property that they do not intersect. In the three dimensional Euclidean space, we call a two dimensional space a plane, a one dimensional a line and a zero dimensional one a point. In Euclidean space, there is only one zero dimensional subspace constituting the neutral element for the addition, denoted by $\{0\}$, also called the origin. A straight line or geodesic is parametrized as follows $r = \{\lambda.v + a | \lambda \in \mathbb{R},\ v, a \in \mathbb{R}^3\}$ and a plane as $vl = \{\lambda.v + \mu.w + a | \lambda, \mu \in \mathbb{R},\ v, w, a \in \mathbb{R}^3\}$ where the free vectors v, w can be chosen to be orthonormal. A straight line can always be written as the intersection of two planes and a plane is completely determined by means of a point and a perpendicular vector. To understand this at a higher level, we introduce the totally anti-symmetrical symbol ϵ_{ijk} where $\epsilon_{123} = 1$ and $\epsilon_{ijk} = \text{sign} \begin{pmatrix} 1 & 2 & 3 \\ i & j & k \end{pmatrix}$ which is merely a convenient notation for the sign of a permutation mapping 1 to i, 2 to j and 3 to k. Henceforth, in this notation,

$$\det(A) = \epsilon_{ijk} A^1_i A^2_j A^3_k$$

for a 3×3 matrix A. Here i, j, k constitute indices with respect to vectors belonging to an orthonormal basis and therefore, the ϵ symbol has a geometrical significance. Indeed, $\delta^{ik} \epsilon_{klm} v^l w^m = (v \times w)^i$ is a vector which is orthogonal to v, w (use the anti-symmetry for that) and δ^{kl} is the inverse of the δ_{ij} symbol. It is to say that

$$\delta^{ik} \delta_{kj} = \delta^i_j,\ \delta_{ik} \delta^{kj} = \delta^i_j.$$

The square length

$$(v \times w)^2 = \epsilon_{lmn} \delta^{li} \epsilon_{ijk} v^m v^j w^n w^k = (\delta_{mj} \delta_{nk} - \delta_{mk} \delta_{nj}) v^m v^j w^n w^k$$

which equals

$$v^2 w^2 - (\langle v | w \rangle)^2$$

and this has the geometrical significance of the surface squared of the parallelipid spanned by the vectors v, w. Henceforth, we have construed a unit vector

$$n = \frac{v \times w}{||v \times w||}$$

perpendicular to the two dimensional subspace spanned by v, w equipped with an orientation such that v rotates right handedly into w. The plane

$$vl = \{\lambda.v + \mu.w + a|\lambda, \mu \in \mathbb{R},\ v, w, a \in \mathbb{R}^3\}$$

then consists precisely out of the points x satisfying the equation

$$\langle n|x - a\rangle = 0$$

which is a linear system in x. In this case $x = (x_1, x_2, x_3)$ satisfies an equation of the form

$$n_1(x_1 - a_1) + n_2(x_2 - a_2) + n_3(x_3 - a_3) = 0.$$

Determine the vector perpendicular to the plane determined by $2x-3y+z-12 = 0$ and compute the point which is the closest to the origin.

Other important equations are given by the so called quadratic equations with as an important example, the n sphere. The latter is defined as the set of all points x located at a fixed distance r from the point a. The corresponding equation is given by

$$||x - a||^2 = r^2$$

which reduces in three dimensions to

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 = r^2.$$

In exactly the same way, the equation of a circle embedded in $\mathbb{R}^2$ is provided by

$$(x - a)^2 + (y - b)^2 = r^2.$$

Show in two different ways that the intersection of the two sphere with a plane in three spatial dimensions is either empty, a point or a circle. Prove that the same result holds for the intersection of two spheres. These properties are not valid any longer for so called curved geometries which we shall study later on. In a similar vein, we shall study the concept of a triangle as well as some theorems regarding properties of them in general curved geometries for which the flat case is of special symmetric nature. Due to the symmetry, extremely sophisticated results exist in flat geometry: old books will serve the reader well who is willing to study those. I am however of the opinion that at this point it is much more important to understand the general setup which reveals the "true" inner workings of general geometry. This is indeed much more gratifying than becoming a specialist in studying linear and quadratic equations, an art which can be further generalized, in an intermediate step towards analytic geometry, provided by algebraic geometry.

This wraps up our discussion about Hilbert spaces; we now return to an elaboration on the theory of linear operators as well as delicate topologies defined on such algebra's. This subject is of extreme importance regarding the old operational formulation of quantum mechanics construed by Heisenberg, Jordan and associated gangsters such as Von Neumann. First, we study two distinct topologies on general Hilbert spaces $\mathcal{H}$ prior to engaging into further discussion of the space of linear operators. On $\mathcal{H}$, we did study the norm topology and one proves now the vericacity of the following two statements:

- A set in a finite dimensional Hilbert space is compact in the norm topology if and only if it is closed and bounded.
- In an infinite dimensional Hilbert space with countable basis $(e_n)_{n\in\mathbb{N}}$, we have that the unit sphere is no longer compact in the norm topology. Hint: argue briefly that the sequence $(e_n)_{n\in\mathbb{N}}$ has no convergent subsequence.

We now arrive at a weaker topology having all advantages of the finite dimensional norm topology and which coincides with the latter in the finite dimensional case. It is clear that the norm topology is too strong in infinite dimensions and we require a weaker one spanned by linear functionals ω, defined as mappings from $\mathcal{H}$ to $\mathbb{C}$, a one dimensional lens through which one perceives the Hilbert space. The space of linear functionals constitutes a vector space called the algebraic dual; we are merely interested in those functionals which are continuous in the norm topology. Such functionals constitute again a vector space called the topological dual $\mathcal{H}^{\star}$. Show that for a finite dimensional Hilbert space, the topological and algebraic dual coincide. An important characterization of continuous functionals is that they are bounded, meaning that

$$|\omega(v)| \leq C||v||$$

for a certain $C > 0$; reversely, it is clear that any bounded linear functional is continuous in the norm topology. We shall give a proof of the former statement: assume that the functional is *not* bounded, then our task is to show that it is not continuous either. More in particular, there exists a sequence of unit norm vectors v_n such that $\omega(v_n) \to \infty$ in the limit for n to ∞. By taking a subsequence, we may assume that $\omega(v_n) > n^2$ and the sequence of vectors $w_k = \sum_{n=0}^{k} \frac{1}{n^2} v_n$ converges to $w = \sum_{n=0}^{\infty} \frac{1}{n^2} v_n$ of finite norm (show that the sequence $\sum_{n>0} \frac{1}{n^2}$ converges) whereas $k < \omega(w_k) \to \infty$ in contradiction to continuity.

Because a continuous linear functional provides one with a one dimensional view upon Hilbert space, it has to coincide with a projection on a vector v. It is to say that

$$\omega(w) = \langle v|w\rangle$$

with $||v|| < \infty$ and the reader is encouraged to provide for a formal proof of this theorem. This viewpoint is evident from the geometrical view given that ω is completely determined by means of its nucleus $W = \{w|\omega(w) = 0\}$ as well as the action upon its normal vector $\frac{v}{||v||}$. This motivates the following definition, the sets

$$\mathcal{O}_{\epsilon;v_1,\ldots,v_n}(w) = \{w' \,|\, |\langle w - w'|v_i\rangle| < \epsilon \,\text{for}\, i = 1\ldots n\}$$

constitute open neighborhoods of w in dimensions determined by v_j and constitute a basis for the *weak* or $\star$-topology.

Open neighborhoods of w in the weak topology control henceforth the modulus of the projection of the difference vector $w - w'$ on a finite dimensional subspace and leave the components perpendicular to it invariant. Given that the norm topology controls all dimensions, it is therefore stronger as the weak one is; in particular, every open set in the weak topology is open in the strong one, a result which follows from the Cauchy Schwartz inequality

$$|\langle w - w' | v_i \rangle| \leq ||w - w'|| ||v_i||.$$

It is henceforth obvious that the same results hold in the weak topology for all Hilbert spaces and that those coincide with the norm topology in the finite dimensional case. In particular, it holds that a set is compact in any Hilbert space if and only if it is bounded in norm and closed in the weak topology. Show that in case a set is bounded in the norm that it is closed in the weak topology if and only if it is so in the norm topology. This is obvious given that boundedness controls an infinite number of dimensions leaving one with a finite number and those are controlled by means of the weak topology. Henceforth, the unit sphere is closed and compact in the weak topology but merely closed in the norm topology a result known as the Hahn Banach theorem. The reverse is also true, a set which is compact in the weak topology is always bounded in norm. We leave the proofs of these statements as challenging exercises for the reader.

We shall now deal with topologies on spaces of linear mappings $A : \mathcal{H} \rightarrow \mathcal{H}$ as well as prove some important theorems regarding operators having a special geometrical significance such as the unitary operators. In particular, we are interested in situations where one disposes of an orthonormal basis of eigenvectors as well as some limitations on the eigenvalues. One disposes of plenty of topologies on specific classes of operators all of which are equivalent in a finite number of dimensions. We start with the supremum norm topology:

$$||A||_{\text{sup}} = \sup_{||v||=1} ||Av||.$$

In case the latter is finite, we call the operator A bounded (which is again equivalent to continuous) and the entire edifice of bounded operators is poured into the framework of so called $C^\star$-algebra's. This theory is an abstraction of the concrete situation delineated below and we are not going to pay too much attention to this given that the operators useful in physics are of an unbounded nature. To deal with those devilish objects, we require weaker topologies to probe them, called the strong and weak $\star$ topologies to name two of them. The first one is defined by means of the open sets

$$\mathcal{O}_{\epsilon; v_1, \ldots v_n}(A) = \{B| \ ||(B - A)v_k|| < \epsilon \, \text{for} \, k = 1 \ldots n\}$$

whereas the latter is defined by means of

$$\mathcal{O}_{\epsilon;v_1,\ldots v_n,w_1,\ldots,w_n}(A) = \{B| \ |\langle (B-A)v_k|w_k\rangle| < \epsilon \,\text{for}\, k = 1\ldots n\}.$$

One shows that both topologies satisfy the Hausdorff property and that the weak-$\star$ topology is weaker as the strong one.

We first introduce some important notions regarding linear operators on Hilbert spaces. The reader may suspect that some subtleties arise which have to do with infinity and were not present in a finite number of dimensions. For example, operators A do have a domain $\mathcal{D} \subset \mathcal{H}$, which we assume to be dense in the norm topology, on which A is well defined. The adjoint operator $A^\dagger$ of A is then retrieved by means of the following procedure. Consider a subspace $\mathcal{D}^\star$ of vectors v such that

$$|\langle v|Aw\rangle| < C(v)||w||$$

for all $w \in \mathcal{D}$. Then, we have that the functional $w \to \langle v|Aw\rangle$ has a unique continuous extension to $\mathcal{H}$ due to the density of $\mathcal{D}$. We obtain the existence of a vector z such that

$$\langle v|Aw\rangle = \langle z|w\rangle$$

and we define $A^\dagger v = z$ and subsequently it easily follows that $A^\dagger$ is a linear operator. Henceforth, the domain of $A^\dagger$ is given by $\mathcal{D}^\star$. Next cases are of extreme importance:

- $A = A^\dagger$ and $\mathcal{D} = \mathcal{D}^\star$ in which case the operator is self adjoint,

- $AA^\dagger = A^\dagger A$ and $\mathcal{D} = \mathcal{D}^\star$ in which case the operator is normal,

- $UU^\dagger = U^\dagger U = 1$ and $\mathcal{D} = \mathcal{D}^\star = \mathcal{H}$ in which case the operator is unitary,

- $P^2 = P = P^\dagger$ and $\mathcal{D} = \mathcal{D}^\star = \mathcal{H}$ in which case the operator constitutes a Hermitian projection.

One verifies that in the finite dimensional case it holds that $A^\dagger = A^H$ and moreover, unitary operators constitute generalizations of $U(n)$. Determine the domain of the operator defined by $Ae_n = ne_n$ for $n \in \mathbb{N}$ where e_m constitutes an orthonormal basis and show that it is dense in $\mathcal{H}$; prove that $\mathcal{D} \subseteq \mathcal{D}^\star$ and that $A = A^\dagger$ on $\mathcal{D}$. We progress now towards the proof of two different theorems: the first one concerns the extension of a special class of operators to Hermitian ones, where the extension of an operator is a new one with a larger domain coinciding with the old operator on its domain. A second result reveals that a normal operator can be decomposed into sums of scalar multiples of Hermitian projection operators in the weak-$\star$ topology.

The importance of the first theorem resides in the second one; this one states that in the finite dimensional case any normal matrix can be diagonalized with respect to an orthonormal basis of eigenvectors. This last aspect is of primary importance to have a probability interpretation such as is the case in quantum theory. Show, by means of an exercise, that in a finite number of dimensions Hermitian operators have only real eigenvalues whereas unitary operators have eigenvalues located on the unit circle in the complex plane. Finally, normal operators can have any complex eigenvalue whatsoever. As said before, one has a connection between unitary and self adjoint operators and in that vein it is easier to deal with the problem of unitary extensions of so called partial isometries V with as domain $\mathcal{D}$ which is not necessarily dense. A partial isometry is defined by means of the property that

$$\langle V(v)|V(w)\rangle = \langle v|w\rangle$$

for all $v, w \in \mathcal{D}$. By means of continuity, we can extend V to the closure $\overline{\mathcal{D}}$ of $\mathcal{D}$ resulting in a unitary mapping between $\overline{\mathcal{D}}$ and $\overline{\mathrm{Im}(V)}$ where $\mathrm{Im}(V) = \{Vw|w \in \mathcal{D}\}$ constitutes the image of V. It must be clear to the reader that only in case the orthogonal complements

$$\mathcal{D}^{\perp} = \{w|\langle w|v\rangle = 0 \ \forall v \in \mathcal{D}\}$$

and

$$(\mathrm{Im}(V))^{\perp}$$

have identical dimension that we are in position to extend V to a unitary operator U by means of $W : \overline{\mathcal{D}}^{\perp} \to (\mathrm{Im}(V))^{\perp}$ where $U = V \oplus W : \mathcal{H} \to \mathcal{H}$. The reader notices that given a subspace W, $W^{\perp}$ is closed in the weak and therefore also norm topology; the sub space $W^{\perp\perp} := (W^{\perp})^{\perp}$ is moreover equal to the weak closure of W.

One notices therefore that a partial isometry has many unitary extensions in case the dimensions of the orthogonal complements are the same or none whatsoever in case this is not true. Now, we return to the mapping connecting Hermitian to unitary operators; Von Neumann knew the so called Cayley transform between Hermitian and unitary operators in finite dimensional Hilbert spaces. A self adjoint operator A is mapped to

$$U = (A - i1)(A + i1)^{-1}$$

where $(A \pm i1)$ is invertible in a finite number of dimensions given that $Av = \mp iv$ which has no solution. One understands this by means of observing that

$$\mp i||v||^2 = \langle v|Av\rangle = \langle Av|v\rangle = \pm i||v||^2$$

implying that $v = 0$. Moreover, $(A + i1)$ commutes with $(A - i1)$ leading to unitarity of U as is confirmed by means of a small computation. Von Neumann wondered which conditions A should obey such that U is a partial isometry. In such a case, an extension can be made towards a unitary operator defining a Hermitian one by means of the inverse Cayley transformation:

$$A = -i(U+1)(U-1)^{-1}.$$

The operator $A \pm i1$ has to be injective so that it becomes possible to take an inverse which suggests that the conditions $\mathcal{D} \subseteq \mathcal{D}^\star$ and $A = A^\dagger$ on $\mathcal{D}$ have to be obeyed which is coined by the term of a symmetric operator. In the infinite dimensional case, it is not necessarily so that $A \pm i1$ is surjective. The Cayley transform is henceforth a linear mapping

$$U : \mathrm{Im}(A+i1) \to \mathrm{Im}(A-i1)$$

and we have to prove three things : (a) verify that U constitutes a partial isometry (b) close the operator $\mathrm{Im}(A \pm i1)$ and finally (c) verify wether $\mathrm{Im}(A+i1)^\perp$ and $\mathrm{Im}(A-i1)^\perp$ have the same dimension. Regarding (a) one notices that

$$\langle U(A+i1)v|U(A+i1)w\rangle = \langle (A-i1)v|(A-i1)w\rangle = \langle Av|Aw\rangle + i\langle v|Aw\rangle - i\langle Av|w\rangle + \langle v|w\rangle$$

and this last expression equals, using the symmetry of A,

$$\langle Av|Aw\rangle + \langle v|w\rangle = \langle (A+i1)v|(A+i1)w\rangle$$

for all $v, w \in \mathcal{D}$. In the standard literature, one closes the operator A prior to taking the Cayley transformation although this is not mandatory; U extends trivially to an operator

$$U : \overline{\mathrm{Im}(A+i1)} \to \overline{\mathrm{Im}(A-i1)}$$

and one requires (c) to extend U to a unitary operator on $\mathcal{H}$. This last condition may be formulated as

$$\mathrm{Im}(A \pm i1)^\perp = \mathrm{Ker}(A^\dagger \mp i1).$$

Indeed,

$$\langle w|(A \pm i1)v\rangle = 0$$

for all $v \in \mathcal{D}$ is equivalent to $w \in \mathcal{D}^\star$ and

$$\langle (A^\dagger \mp i1)w|v\rangle = 0.$$

The latter is true if and only if $(A^\dagger \mp i1)w = 0$ because $\mathcal{D}$ is dense in $\mathcal{H}$; by definition it holds that $\mathrm{Ker}(B) = \{w|Bw = 0\}$ which produces the right result.

We have just shown our first deep result: symmetric, densly defined operators have self adjoint extensions if and only if the dimensions of the subspaces $\mathrm{Ker}(A^\dagger \mp i1)$ are equal to one and another. Now we work towards our second main result regarding normal operators A with as special cases Hermitian and unitary operators. That is, there exists a projection valued measure $dP(z)$ on the complex plane such that in the weak $\star$ topology holds that

$$A = \int_{\mathbb{C}} z \, dP(z).$$

We first encounter here an integral, something which we shall make more or less precise later on. Suppose one would like to achieve such a result, then it is logical that the operator $(A - z1)$ is not invertible in case $dP(z) \neq 0$; logically, one has three possibilities:

- $(A - z1)$ is not injective, nor surjective; in such a case z belongs to the discrete spectre,

- $(A - z1)$ is not injective, but surjective; in such a case z belongs to the residual spectre,

- $(A - z1)$ is injective, but not surjective; in such a case z belongs to the continuous spectre.

Regarding normal operators, the reader may first show that the residual spectre is empty. Note that if A is normal, then $A_z = A - z1$ obeys this property too; moreover, A is injective if and only if $A^\dagger$ is also which is equivalent to the statement that $Av = 0$ if and only if $A^\dagger v = 0$. Mind that surjectivity of A does not imply surjectivity of $A^\dagger$. Suppose that z belongs to the residual spectre then we have that

$$\langle v | A_z w \rangle = 0$$

for all w implies that $v = 0$ due to surjectivity of A_z. This implies that $\mathrm{Ker}(A_z^\dagger) = \mathrm{Ker}(A_z) = 0$ which is the necessary contradiction. Henceforth, we have shown that the residual spectre is empty. In case z belongs to the discrete spectre, one can find a unique Hermitian projection operator P_z on $\mathrm{Ker}(A_z)$. P_z commutes with A, $AP_z = P_z A = zP_z$ because $\langle v | AP_z w \rangle = z \langle v | P_z w \rangle = \langle \overline{z} P_z v | w \rangle = \langle A^\dagger P_z v | w \rangle = \langle v | P_z A w \rangle$ and the same commutation relations hold between $A^\dagger$ and P_z given that the projector is Hermitian. Moreover, suppose that $z \neq z'$ and both belong to the discrete spectre, then it holds that $P_z P_{z'} = 0$ which follows from

$$z P_z P_{z'} = A P_z P'_z = z' P_z P_{z'}.$$

This strongly resembles the result we wish to obtain in the sense that on infinite dimensional Hilbert spaces, the discrete spectre consists at most out of a countable number of points. We procure an example of a bounded linear operator with a compact spectre (which one can show to be always the case). Given that $Ae_n = \frac{1}{n} e_n$ for $n > 0$ and e_m an orthonormal basis: the discrete spectre is given by $\{\frac{1}{n} | n \in \mathbb{N}_0\}$ and 0 belongs to the continuous spectre given that the vector $\sum_{n=1}^{\infty} \frac{1}{n} e_n$ does not belong to the image of A. Henceforth, the continuous spectre may have "measure zero" and henceforth not contribute to the spectral decomposition.

The continuous spectre is clearly void for normal operators on finite dimensional Hilbert spaces and the reader shows as an easy exercise that

$$A = \sum_{z \in \sigma_d(A)} z P_z$$

where $\sum_{z \in \sigma_d(A)} P_z = 1$ and $\sigma_d(A)$ denotes the spectre consisting entirely out of discrete eigenvalues. One should get used to the following notation: given a unit vector v, define by means of the expression

$$P = v v^{\dagger}$$

the operator with as action $Pw = v\langle v|w\rangle$. Prove that P is a rank one Hermitian projection operator with property $AP = zP$ and in particular $Av = zv$ implying that v is an eigenvector. The entire complexity of the theorem regarding the infinitesimal aspect having to do with the integral resides entirely in the treatment of the continuous spectre in an infinite number of dimensions. We shall not present the matter here at the fullest level of generality because this brings along some technical complications muddling with the main line of argumentation. Note that in the finite dimensional case, we did use the fundamental theorem of complex algebra, namely that every polynomial defined over $\mathbb{C}$ can be factorized.

In case z belongs to the continuous spectre, then we have in particular that the image of the unit sphere under A_z does not contain an open neighborhood of the origin. Otherwise, we have the property that A_z is surjective: henceforth, there exists a sequence of unit vectors v_n such that

$$||A_z v_n|| \to 0$$

in the limit for n towards ∞. Therefore, elements in the continuous spectre contain approximate eigenvectors. Henceforth, $\mathrm{Im}(A_z)^{\perp}$ vanishes due to injectivity of A_z implying that $\mathrm{Im}(A_z)$ is dense in $\mathcal{H}$. It holds moreover that for $z \neq z'$,

$$\lim_{n,m \to \infty} \langle v_n | w_m \rangle = 0$$

where $(v_n)_{n \in \mathbb{N}}$ corresponds to A_z and $(w_n)_{n \in \mathbb{N}}$ with $A_{z'}$ giving a generalization of the standard orthogonality property for Hermitian projection operators associated to discrete eigenvalues.

Finally, we deal with the construction of the spectral measure: given the measurable set $\mathcal{O} \subseteq \mathbb{C}$, one defines $P_{\mathcal{O}}$ as the smallest Hermitian projection operator having the property that for each $z \in \sigma(A) \cap \mathcal{O}$ and a sequence of approximating eigenvectors $(v_n)_{n \in \mathbb{N}}$ associated to z, it holds that $||P_{\mathcal{O}}(v_n) - v_n|| \to 0$ in the limit for $n \to \infty$. A measurable Borel set is defined by means of:

- every open set can be measured,
- the complement of a measurable set is measurable,
- any union of measurable sets can be measured.

It may be clear that
$$P_{\mathcal{O}} P_{\mathcal{V}} = P_{\mathcal{O} \cap \mathcal{V}}$$
and the diligent reader delivers a proof. Given a countable partition $(B_n)_{n \in \mathbb{N}}$ of $\mathbb{C}$ by means of measurable sets[3] we consider
$$A_{(B_n)_{n \in \mathbb{N}}} = \sum_{n=0}^{\infty} z_n P_{B_n}$$
where $z_n \in B_n$. The integral is then defined by means of refining the partition and the remainder consists in showing that the sum converges in the weak-$\star$ topology towards the integral as well as A. The first assertion is true by definition whereas the latter follows from prudent estimates.

This extremely important theorem, known as the spectral theorem, allows one to define measurable functions $f : \mathbb{C} \to \mathbb{C}$ replacing the complex variable by the normal operator A. We have that
$$f(A) := \int_{\mathbb{C}} f(z) dP(z)$$
where we have used the spectral decomposition
$$A = \int_{\mathbb{C}} z dP(z).$$
There exist two important generalizations of this theorem: the first one consists in replacing the complex numbers by means of the quaternions $\mathbb{RQ}$ and to consider quaternion bi-modules with a quaternion valued scalar product. A second generalization consists in dropping the condition
$$\langle v | v \rangle \geq 0$$
and to allow for this quantity to become negative. This kind of generalization is much more subtle and requires amongst others the introduction of conjugated null pairs. These comments wrap up our discussion about linear spaces and functions; as usual, there is much more beef into the cow as made explicit above but these constitute the main results indeed. The reader is now invited to make the following crucial exercises.

Exercises on Von Neumann extensions of linear operators.

Consider the operator $i \frac{d}{d\theta}$ on the space of differentiable functions on the unit

[3] A partition satisfies the property that $B_n \cap B_m = \emptyset$ for $n \neq m$ as well as $\cup_{n=0}^{\infty} B_n = \mathbb{C}$.

circle S^1 with circumference 2π. Show that this operator is essentially self adjoint and densely defined on the Hilbert space of square integrable functions on the circle. Consequently, this operator has a unique Von Neumann extension. As an additional exercise, prove that

$$\left[i\frac{d}{d\theta}, \theta\right] = i((2\pi - 1)\delta(\theta) + 1)$$

where $\delta(\theta)$ is defined by means of

$$\int d\theta\, \delta(\theta) f(\theta) = f(0)$$

for any continuous function f on the unit circle.

Perform now the same study for $i\frac{d}{dx}$ defined on complex valued functions with as domain $[a,b]$ by imposing boundary conditions $f(a) = f(b) = 0$. Show that the operator on this function domain $\mathcal{D}$ is symmetrical and determine the adjoint domain $\mathcal{D}^\star$ (differentiable functions on the line segment without boundary conditions). The closure of $i\frac{d}{dx}$ requires some weaker boundary conditions. To calculate those, note that the kernels of the operators $\frac{d}{dx} \pm 1$ are provided by $a_{\mp N} e^{\mp x}$ where $a_{\mp N}$ is a suitable normalization constant. One obtains therefore a one parameter group of unitary operators

$$U(\theta) : a_{-N} e^{-x} \to e^{i\theta} a_{+N} e^{x}$$

providing for a one parameter family of Von Neumann extensions.

Delta-Dirac distributions.
Let $x \in \mathbb{R}$ be a real variable and $f : \mathbb{R} \to \mathbb{C}$ a continuous function, then we define a distribution $\delta(x)$ by means of

$$\int_{\mathbb{R}} \delta(x) f(x) dx = f(0).$$

The integral representation of $\widehat{\delta}$ constitutes a linear functional on the complex vector space of complex valued functions $f : \mathbb{R} \to \mathbb{C}$ provided by

$$\widehat{\delta}(f) = f(0).$$

The latter is a weak limit of a sequence of continuous functionals construed by means of

$$g_n := n\chi_{[-\frac{1}{2n}, \frac{1}{2n}]}$$

where

$$\chi_A(x) = 1$$

if and only if $x \in A$ and zero otherwise. More precisely

$$\widehat{\delta}(f) = \lim_{n \to \infty} \int_{\mathbb{R}} g_n(x) f(x) := \int_{\mathbb{R}} \delta(x) f(x)$$

whereby this last notation constitutes a formal representation. Likewise, one may define $\widehat{\delta}_z$ by means of $\int_{\mathbb{R}} \delta(x-z)f(x) = f(z)$. Prove that

$$\int_{\mathbb{R}} \delta(x-z)\delta(x-y)dx = \delta(y-z)$$

by insisting on

$$\int_{\mathbb{R}} dz \int_{\mathbb{R}} \delta(x-z)\delta(x-y)dx f(z) = \int_{\mathbb{R}} dx\delta(x-y) \int_{\mathbb{R}} \delta(x-z)f(z)dz$$

for any continuous function $f(z)$. Let f, g be two continuous functions from $\mathbb{R}$ onto $\mathbb{C}$ differing from zero only on a compact set such that

$$\langle f|g\rangle = \int_{\mathbb{R}} \overline{f(x)}g(x)dx$$

is well defined. Show that the latter expression provides for a scalar product and define $L^2(\mathbb{R}, dx)$ as the Hilbert space defined by means of this scalar product by taking the completion. Define subsequently the following linear operator $X(f)$ by means of

$$(X(f))(x) = xf(x).$$

Show that the latter is densely defined, essentially self adjoint (vanishing deficiency indices) and that the spectre is continuous and equals the entire $\mathbb{R}$. Finally, the projective measure P is given by means of

$$P((a,b)) = \chi_{(a,b)}$$

as well as

$$(dP(z)f)(x) = \delta(x-z)f(z)dz$$

such that finally

$$(X(f))(x) = \int_{\mathbb{R}} z\delta(x-z)f(z)dz = xf(x).$$

Heisenberg equations.

In the traditional operational formulation of quantum theory, one has the so called Heisenberg pair (X, P), modelled by means of Hermitian operators on an infinite dimensional Hilbert space

$$[P, X] = i1.$$

Herein, one considers the so called Schroedinger representation on $L^2(\mathbb{R}, dx)$ where X has been defined previously and

$$P = i\frac{d}{dx}.$$

The reader is invited to show that the spectrum of P is given by $\mathbb{R}$ and that the so called distributional eigenvectors are provided by e^{-ikx}. The latter define the Fourier transformation. This constitutes the crucial result we were aiming for and which shall be the central topic of the next chapter.

Chapter 2

The language of psychological issues.

This entire chapter turns around one single topic only: how to mathematically describe an issue. That is, what are the very basics behind human thought and how do we engage in conversations. This may seem at first sight something which is not of direct interest, but the reader shall learn in the next chapter that it leads to a very deep comprehension of mental attraction, spiritual beauty and compassion. Each mode of communication comes with a certain sentiment or emotion and there is a unique mapping from a certain sentence to the emotions with which they are conveyed. Therefore, precision of language will learn us something about the very nature of humanity: you are free to agree or disagree with these conclusions but I can assure you it is not a trivial task to do so. Note that I shall use mathematics which was not explained in the previous chapter and I shall do my utmost best to clarify the meaning of certain formulas in a way which is reasonable to my mind. In discussing issues about the mind, we must learn to be precise; in sharp contrast to the tradition in those fields of study, I shall outline my viewpoint with mathematical precision which might be an unreasonably high standard in those fields but in my view constitute the only path towards discussing these matters in a truely profound way. I shall make a bold conjecture of how our perception of space and time might be related to issues of the mind. It is up to you to agree or to disagree with those viewpoints; at least, it seems to me, there is something nontrivial to it and certainly the mathematics behind it is compelling. So, we shall start here by discussing the kind of mathematics which would be required to break the boost symmetry (which mixes up space and time) of the Poincaré algebra and therefore distinguish space from time. Later on, we shall discuss in depth how this issue, in my view, relates to the dynamics of our mental profile. Space, as we perceive it, appears to allow for rotations and time is completely detached from it - it does not transform in our mental perception no matter what you think or do. In fact, the kind of rotations we should consider are active ones regarding our own body and passive ones regarding the outside world. Indeed, our body actively rotates, but nothing happens to the outside world, it is just our perspective which changes (ignoring backreaction effects on all interaction fields). The reason why we know we rotate is due to the spacetime acceleration we undergo during the process and our senses pick up on that. There is almost a natural definition of the x, y, z axis associated to the symmetries of our body. The z-axis is defined by means of the gravitational acceleration we are undergoing in case we are stationary and the y-axis is the projection of the vector away from our eyes on the plane perpendicular to the z, t axis. The x-axis is then fixed; under normal circumstances, this definition agrees respectively with the line connecting our feet to our head, the line of incoming eyesight and finally the line connecting our shoulders. So, biology breaks all free choice of rotation and rotating therefore requires a nontrivial act. Since actions require algebra, this suggests that our experience of time commutes with our experience of space as well as with all rotations thereon. Since this book is intended for those who did not study science, let me tell you a bit about special relativity: this is within reach of the mathematics explained in the previous chapter. I shall later on also use to some extend notions regarding curved geometry, but alas I am not going to explain those but I shall try to give some intuition behind them. The interested reader is referred to any text about Riemannian or Lorentzian geometry, but these matters are not of importance to understand our discussion of an issue, but are mandatory for the dynamics of issues. So, special relativity therefore; as is well known since more than a century, light or electromagnetic waves travel at a fixed speed in any inertial reference frame. This is against the viewpoint of Newton where velocities add up and no such constant could ever exist. Therefore, Einstein concluded we needed a new kind of geometry to describe this phenomenon. At first, he started from a four dimensional real vectorspace $\mathbb{R}^4, +$ with coordinates (t, x, y, z) where t denotes time and x, y, z are the space coordinates. Since time and space are unified in this framework and refer to distinct physical units (of meter nd second), we need a constant $c > 0$ of dimension meter/second to come to homogeneous dimensions. Therefore, one should consider ct, x, y, z where c is then interpreted as the speed of light. He proposed that the correct scalar product defining the geometry was given by

$$\langle (ct, x, y, z) | (ct', x', y', z') \rangle = c^2 tt' - xx' - yy' - zz'$$

and the reader immediately notices the three minus signs. So, this is not the scalar product of Hilbert space which is positive definite, but one of Minkowski which is indefinite. It is important to separate the region where

$$\langle (ct, x, y, z) | (ct, x, y, z) \rangle > 0$$

and the region where is is smaller than zero. Vectors with positive norm are timelike, meaning they correspond to a propagation slower than the speed of light and vectors of negative norm are called spacelike and correspond to our ordinary perception of space. Null vectors correspond to propagation at the speed of light and this may happen forwards in time $t > 0$ as well as backwards in time $t < 0$. Now, just as is the case for Hilbert spaces, we are interested in the linear transformations which leave this scalar product invariant: they constitute the Lorentz group $O(1,3)$. Since we are interested in bound vectors and not free vectors, the reader notices that also translations in space and time are symmetries since they leave the differences invariant. The translations together with the Lorentz transformations generate what is called the Poincaré group. Now, a trick which we usually apply is to consider transformations which differ slightly from the identity transformation denoted by

$$L = 1 + \epsilon M$$

where $\epsilon^2 = 0$ for all practical purposes. It is not too difficult to determine the conditions M should satisfy and finite transformations are given by

$$L = e^M$$

where e refers to the exponential map. The reader who wants to understand all details is referred to the book of Weinberg [3] or any book on Lie groups and Lie algebra's. Now, to gain insight in what follows, it is important to understand that the above are the defining representations of the Lorentz group. We shall be interested in other representations by which we mean that you can write down the group relations in terms of matrices acting on a complex Hilbert space. Again, the reader who is willing to dig deeper into this topic is referred to the same references. We are interested in those representations in which time and space get an algebraically distinct meaning; there are two of them which are the spin $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ representation and not the so called Dirac representation where the "vectors" $x_a \gamma^a$ transform as Lorentz vectors. The fundamental algebraic components of both representations are given by the so-called Pauli matrices σ^a where $\sigma^0 = 1$ the 2×2 identity matrix and

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma^2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \ \sigma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

and the reader verifies that

$$\sigma^k \sigma^l = \delta^{kl} 1 + i\epsilon^{klm} \sigma^m$$

where the ϵ symbol has been explained before. Also, $(\sigma^k)^H = \sigma^k$. The Lorentz group consists out of three boosts which mix up space and time and three rotations in space; denoting by $\vec{a}, \vec{b}$ two three dimensional real vectors generating a rotation and boost respectively, the above representations are implemented by means of

$$D(\vec{b} + i\vec{a}) := e^{(\vec{b}+i\vec{a})_j \sigma^j}, \, E(\vec{b} + i\vec{a}) := e^{(-\vec{b}+i\vec{a})_j \sigma^j}$$

and one notices that

$$D(\vec{b} + i\vec{a}) = (E(\vec{b} + i\vec{a})^{\dagger})^{-1} = \sigma_2 \overline{E(\vec{b} + i\vec{a})} \sigma_2.$$

All matrices A in this representation tranform of course under a change of basis g as gAg^{-1}. A spacetime "vector" is now given by $x := x_a \sigma^a$ and transforms as

$$D(\vec{b} + i\vec{a})xD(\vec{b} + i\vec{a})^{-1}$$

and likewise so for $E(\vec{b}+i\vec{a})$. Hence, one notices that x_0 remains untouched but the x_i transform into complex numbers in case $\vec{b} \neq 0$. Assuming that the only allowed transformations are those which preserve the reality condition $x_a \in \mathbb{R}$, we conclude that $\vec{b} = 0$ and we obtain a mere rotation around the $\vec{a}$ axis around an angle of $||\vec{a}||$. Note that in this view, the different directions of space anti-commute. It is worthwhile to mention that the matrices $D(\vec{b} + i\vec{a})$ constitute all 2×2 complex matrices with unit determinant, a group which is denoted by $SL(2, \mathbb{C})$. This group is the so-called universal cover of the orthochronous Lorentz group and one can define from complex 2 vectors, real four dimensional vectors, and carry the action of $SL(2, \mathbb{C})$ on those complex 2 vectors over into an action of the real orthochronous Lorentz group. For sake of completeness, I will give a brief overview of this formalism. Let W be a two dimensional complex vector space with basis e_A and volume form ϵ_{AB}. In the literature, one puts $e_1 = o$ and $e_2 = n$: $(o, o) := \epsilon_{AB} o^A o^B = (n, n) := \epsilon_{AB} n^A n^B = 0$ and $(o, n) = 1$. Clearly, $SL(2, \mathbb{C})$ leaves the volume form invariant. The complex conjugation sends W to $\overline{W}$, that is $v \in W \to \overline{v} \in \overline{W}$ which is spanned by $\overline{o}, \overline{n}$ with a volume form represented by $\overline{\epsilon}_{A'B'}$ where we agree that primed indices refer to transformations under the complex conjugate representation. $W \otimes \overline{W}$ is four dimensional over $\mathbb{C}$ and one is interested in its real subspace. The latter is spanned (over the real numbers) by

$$\begin{aligned}
\sigma^0 &= \frac{1}{\sqrt{2}} \left(o \otimes \overline{o} + n \otimes \overline{n} \right) \\
\sigma^1 &= \frac{1}{\sqrt{2}} \left(o \otimes \overline{n} + n \otimes \overline{o} \right) \\
\sigma^2 &= \frac{i}{\sqrt{2}} \left(o \otimes \overline{n} - n \otimes \overline{o} \right) \\
\sigma^3 &= \frac{1}{\sqrt{2}} \left(o \otimes \overline{o} - n \otimes \overline{n} \right)
\end{aligned}$$

and the reader understands that the σ^a are the usual Pauli matrices but now with indices A, A' transforming in two different representations. A small computation yields that these vectors are orthonormal with regard to $\omega \otimes \overline{\omega}$ and obey

$$\sigma_a^{AA'} \sigma_b^{BB'} \omega_{AB} \overline{\omega}_{A'B'} = -\eta_{ab}$$

where η_{ab} is the standard Minkowski metric or scalar product, which we just described before, of signature $(+ - --)$. This suggests an identification with some inertial system (t, x, y, z) by means a "solder" form

$$\sigma = t\widehat{t} + x\widehat{x} + y\widehat{y} + z\widehat{z} := x^a \sigma_a^{AA'}.$$

Given all this, it is now an elementary exercise to show that an element $T \in SL(2,\mathbb{C})$ defines, by means of the solder form, an ortochronous Lorentz transformation on x^a. This action will become useful when we want to couple spinor currents to spacetime vectors. It is worthwile to mention that the Pauli matrices correspond to the complex quaternions where minus the unit is a square of each of them. The spin $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ representations define the so called massless Weyl particles which are, as is well known, Fermions. However, in the mental world, we do not care about the speed of light dictating our unconscious interactions and therefore we will be interested in a quantization which treats them as bosons. Let us now argue why this is interesting from the point of view of psychology in a broader sense beyond the mere issue of spacetime awareness; I will take here the point of view that regarding any issue, be it a question, a thought, a worry, a desire or anything you like, that the fundamental dichotomy which dictates our interactions is one of conservatism, that is taking some definite point of view, versus one of a transformative nature seeing change no matter what a conservative person percieves as a definite state, which is maybe not even desirable for him. Another way of saying this is that conservative people accept this issue as being settled in a particular way, whereas transformative people oppose any settlement and ask for change in a particular way. Transformative people are wanderers, searching for an anchor, but not knowing where to start or what to adhere to. In this regard, it seems natural to postulate that the dynamics of the nature of the questions one asks is one which has a personal tendency towards conservatism - ultimately, we are all happy to take a rest and settle in a particular answer regarding these questions based upon an emergent phenomenon called logic. So, the reader must understand that momentum here is not a real quantity in contrast to standard quantum mechanics of particles! People who are transformative have an imaginary reality, a dreamworld of how it could be in the future whereas for conservative people reality is what it is. This is the way in which the spirit differs from an ordinary particle whose options, in the usual framework, are fixed once and for all and we merely study transitions between those options (this is the Schrodinger point of view). I shall make a further assumption here, which is that the stable "ground state" of a community regarding an issue is one which is precisely in the middle between conservatism and progressiveness. This is a healthy assumption as it allows for a very dynamical attitude towards anything in life, you are on the one hand attached to the knowledge you have, but on the other hand you are flexible enough to change your mind when facts call upon you; in either, you are never sure but relaxed. I believe this mentality also to be connected to Darwinism as such persons are the ones who thrive in society. In my previous publications on the matter, I have called such a point of view towards a specific topic of "Schwitchoriem" type, simply because I like to play with words. Conservative types were white and transformative types black in analogy with ying and yang; we shall stick to that convention in this work. So, now the question is, how to translate this algebraically and what does it have to do with the fundamental representation of $SL(2,\mathbb{C})$? For simplicity, let us agree that the definite position or validation you can take towards a certain issue is labelled by a real number

λ; this is just a way of coding things and it allows one to naturally speak about a certain distance between positions. Therefore, we postulate that the white operator X works on the state of the system $|\Psi(\lambda)\rangle$ as

$$X|\Psi(\lambda)\rangle = \lambda|\Psi(\lambda)\rangle.$$

Furthermore, a state is something indefinite and describes in a way how the world values different definite (black) points of view on the matter. This operator is clearly linear by definition. Now, the algebraic object P associated to the transformative or black type must obey the definition of change; hence,

$$[P, X] := PX - XP = 1$$

or $P = \frac{d}{d\lambda}$. All algebraic relations can be rewritten as

$$(X, P) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ P \end{pmatrix} = 1$$

where we use a shorthand

$$\omega := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Hence, the pair

$$\begin{pmatrix} X \\ P \end{pmatrix}$$

can be seen as a two spinor. Indeed, a trivial computation reveals that the symmetry group of our defining relation is given by $A \in SL(2, \mathbb{C})$ since

$$A^T \omega A = \omega$$

where A^T is the transpose of A. It is necessary to remark here that we take an Einsteinian point of view regarding the translations $X \to X + a$, $P \to P + b$ which also preserve the algebra but merely recalibrate the very language we use to describe phenomena. Since those things are personal and never change and we shall associate both X, P with a kind of dimensionless energy, we have to disregard those. The meaning of attributes does not change, which does not mean of course that the attributes attached to a state of the world cannot change. Note that a dynamics regarding the A matrix is not sufficient in order to deal with spiritual interactions, it just says from which dichotomy you are approaching the world, but you still have to choose one of the opposites! Calling them π_1, π_2, where

$$\pi_i : \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \to Z_i$$

we suggest that in a conversation you do not only change your point of view (dichotomy), but you also must have the freedom to choose for the *upper* (1) or *lower* (2) side; the (X, P) dichotomy is called the *canonical* dichotomy and in that case is the projector on the upper side the same as picking the conservative perspective, whereas the projector on the lower side corresponds to the progressive perspective[1]. To say in plain language what I mean, during a conversation, you may look with new glasses at the same issue, but still you can take an upper stance or a lower stance wanting to change that new viewpoint you were just considering. To model such choice function, you could introduce a real variable s and consider the step function $\theta(s)^2$, smoothen it out a bit around 0, denoted by $\tilde{\theta}$, and finally define

$$\pi(s) = \cos\left(\frac{\tilde{\theta}(s)\pi}{2}\right)\pi_1 + \sin\left(\frac{\tilde{\theta}(s)\pi}{2}\right)\pi_2$$

so that there is a fast switching between an upper and lower point of view. The dynamics for s must be coupled to everything, your physical brain, the current white state, as well as your dichotomy. We shall discuss this at greater depth later on. So, our fundamental dichotomy has the natural symmetry of relativity theory; consider now the transformation

$$A_S = \frac{1}{\sqrt{2}}\left(\begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array}\right) \in SL(2, \mathbb{C})$$

which we dub as the Schwitchoriem transformation whose second component

$$a = \frac{1}{\sqrt{2}}(X + P)$$

is precisely the "middle" between the white and black perspective which is still a lower choice to make from our point of view. As a small aside, in standard quantum theory, one represents X, P on a so-called Hilbert space with an hermitian inner product as explained in detail in the previous section. In this account, we had that $X^\dagger = X$ and $P^\dagger = -P$ so that

$$\frac{1}{\sqrt{2}}\left(\begin{array}{cc} 1 & -1 \\ 1 & 1 \end{array}\right)\left(\begin{array}{c} X \\ P \end{array}\right) = \left(\begin{array}{c} a^\dagger \\ a \end{array}\right)$$

where a represents the schwitchoriem point of view and $a^\dagger$ the diametrically opposite point of view. Obviously, one has that

$$\left[a, a^\dagger\right] = 1$$

by definition. Now, we come to the definition of the ground state $|0\rangle$ (of an issue) which a free society, in which there is no interaction between issues and therefore no emergent conventions, which require mental energy, by means of a learning process, aspires to reach. It is defined by the property that the Schwitchoriem,

[1] In a previous publication of this work, I used white and black for the upper and lower side regardless of the dichotomy chosen, which may have lead to some confusion.

[2] $\theta(s) = 0$ if $s < 0$ and one otherwise.

who is the ultimate opportunist, perceives it as a cold or neutral world in which he feels relaxed. Mathematically, this reads

$$a|0\rangle = 0.$$

As is usually done in quantum field theory, one may consider the positive operator

$$H = a^{\dagger}a$$

and one obtains that

$$H|0\rangle = 0$$

which precisely means that the ground state has zero psychological weight for the Schwitchoriem[3]. Here I launch our second principle which is that we project the mental state function down in our mind by using the energy operator attached to the operator Z representing our viewpoint; indeed, one cannot ask about Z itself except when Z is self-adjoint (or normal); this happens only for a four (five) parameter family of profiles given by

$$\begin{pmatrix} a & ib \\ \frac{ad}{b} + i\frac{1-ac}{b} & c + id \end{pmatrix}$$

where $a, b, c, d \in \mathbb{R}$ (or a is complex and $b = ar$ with $r, c, d \in \mathbb{R}$) in case you choose the upper profile and we assume $b \neq 0$ and likewise so for the lower profile. The reader may utter here that our notion of a dichotomy so far is a strange one given that, classically, one thinks of a dichotomy in terms of two sharp unique opposites whereas here the opposites come in two real colours; indeed, the white profile has as opposites $P + cX$ where c is any complex number. One could eliminate this freedom and leave only for 2 complex numbers instead of 3 by demanding that $AA^{\dagger}$ is a diagonal matrix, in either that the profiles are perpendicular to one and another. In matrix language this reads

$$A = \begin{pmatrix} a & b \\ c & \frac{bc+1}{a} \end{pmatrix}$$

assuming a is nonzero and

$$a\overline{c} + b\overline{\frac{bc+1}{a}} = 0$$

which always has a unique solution given by $\overline{c}(|a|^2 + |b|^2) = -b$. Note that the opposite condition $A^{\dagger}A$ is diagonal, leads to $\overline{a}b + \overline{c}\frac{bc+1}{a} = 0$ or $(|a|^2 + |c|^2)b = -\overline{c}$. This equation is quadratic in c instead of linear and does not always have a solution; therefore, the previous condition would be the correct one. Likewise, the complex condition AA^T is diagonal leads to $(a^2 + b^2)c = -b$ which has no solution in case $a^2 = -b^2$ for $b \neq 0$. We shall not take this point of view here and allow for the Greek dichotomy to be two dimensional over the real numbers;

[3]One can define for every psychological type Z the intrinsic free weight attached to its profile as $Z^{\dagger}Z$.

it is just so that good and bad can have different expressions and we will allow for this. In what follows, we shall call the result of combining a profile with an upper-lower choice the decision. Obviously, the 5 parameter family of profiles leading to a definite decision has meaure zero, the 6 dimensional remainder being called normal. For the latter $ZZ^\dagger = Z^\dagger Z + r1$ where $r \neq 0$ (using the adjointness properties of X, P as well as the commutation relations). In that sense are the Schwichoriem and anti-Schwichoriem profiles given by

$$\begin{pmatrix} a^\dagger \\ a \end{pmatrix} \begin{pmatrix} a \\ -a^\dagger \end{pmatrix}$$

the most symmetrical normal ones. The former being hermitian, whereas the latter is the anti-hermitian conjugate. This requires further reflection as a white person will only require no mental energy when he sees at the universe in the distributional state with $\lambda = 0$. As mentioned before, only a three parameter family of decisions is capable of further reduction of the wave function; he or she cannot only answer “I look at it from that point of view, which is maybe complex, with such an intensity (energy)” but also state in a compatible way, “I see it like that”. In this sense are almost all tests most psychologists give too limited since they force you to engage in their specific (not necessarily even white) real reality (not even a complex or imaginary one) given that they are not interested in how you feel about their task, whereas the truth is that the person simply does not want to or cannot give any answer in this way. Since, you just fill in something to please them, they draw entirely bogus conclusions on the nature of “reality”. They don’t even kow what reality is: it is far more complex than the world in which they operate, the latter being one of definite, real, pre-cooked answers! Now, unlike the doctrine in physics where individual systems aspire to be in the lowest energy state possible, humans do enjoy mental effort and like to spend energy in things. Our description of variables attached to an issue seems at first sight ad odds with the existence of mathematical certainties which require only binary answers. However, I shall argue later how, by means of a learning process in which classical logic is dynamically embedded, classical logic, as we practise it may be an emergent phenomenon. This will be discussed at length in the following chapter. Obviously, the stable ground state of our, interacting, world costs energy for the Schwitchoriem as there exist many issues which are poored into more or less definite white pointer states. Indeed, these conventions are called law, logic and truth causing for a polarization of Schwitchoriem vacuum. Before we proceed, let us write down some interesting first order dynamics a psychological type given by a matrix $A(t)$ might undergo; up to third order we have

$$\frac{d}{dt}A(t) = aYA(t) + bA^{-1}(t)ZA^2(t)$$

where a, b are arbitrary complex numbers and Y, Z arbitary traceless complex matrix fields which therefore belong to the Lie algebra of $SL(2, \mathbb{C})$. Indeed, one immediately verifies that

$$\frac{d}{dt}\det(A(t)) = \det(A(t))\mathrm{Tr}\left(\left(\frac{d}{dt}A(t)\right)A^{-1}(t)\right) = 0.$$

Another, more insightful proof (which is important later on) reads

$$\frac{d}{dt}(A(t)^T\omega A(t)) = A(t)^T\omega\left(aYA(t) + bA^{-1}(t)ZA^2(t)\right) - \text{Transpose} =$$

$$\omega\left(aA^{-1}(t)YA(t) + bA^{-2}(t)ZA^2(t)\right) - \text{Transpose} = 0$$

simply because any traceless matrix X obeys $X^T\omega = -\omega X = -(\omega X)^T$ since $\omega^T = -\omega$. So, in finding a general interaction theory for (multiple) issues, we shall look for "intertwiners" obeying these "commutation" relations. Nature has thought us that the best way to construct such an interaction theory, is by making it as symmetrical as possible, meaning in this case that it should be a local $SL(2,\mathbb{C})$ "gauge theory" with Y a connection and $Z = 0$. What follows is a bit technical and uses notions from differential geometry and fibre bundle theory. The reader may ignore this paragraph if he is not willing to study this in detail, but the general idea is that we want to find an appropriate substitute for the operator $\frac{d}{dt}$ which expresses that time is defined by the matter currents in our brain. A connection is in the very basic meaning of the word nothing but an intermediator which allows one to define equations of motion which are not dependent upon a particular representation of A, meaning that it does not feel the impact of a "gauge" transformation $A \to gAg^{-1}$. Now comes the real beef of the story; we already have such a connection which is the gravitational spin connection! Now, the gravitational spin connection really is not a gauge field in the sense that upon performing a Lorentz transformation, the predictions of our theory transform covariantly instead of remaining invariant; in that sense should observables in quantum gravity be diffeomorphism covariant and not invariant as certain luminaries proclaim. In that vein do we claim that a change of reference frame is the result of a mental intervention, one which changes the conditions of the psyche leading to a different evolution. In other words, it is an active transformation and not just one assossiated to the redundancy of the description. In that vein, let $J(x)$, where $g(J(x), J(x)) = 1$, denote the effective classical current describing our brain activity; every nanosecond or so, it gets updated by measuring the real quantum current, then we fix a mental vierbein $\tilde{e}_a$, whose equations of motion are just Fermi transported along the classical neural current

$$\nabla^F_{J(x)}\tilde{e}_a(x) :=$$

$$\nabla_{J(x)}\tilde{e}_a(x) + g(\tilde{e}_a(x), \nabla_{J(x)}J(x))J(x) - g(\tilde{e}_a(x), J(x))\nabla_{J(x)}J(x) = 0$$

with $\tilde{e}_0(x) = J(x)$. The reader immediately notices that Fermi transport is not covariant under local rotations of the $\tilde{e}_j(x)$. It is crucial to understand that this must be so: a change of a reference frame attached to a physical observer requires not only a conscious intervention but also the necessary energy to realize that (ignoring even backreactions on the spacetime fabric itself). As long as no such intervention occurs is there no freedom to rotate. Boosting is very much like an adiabatic process, it takes a while and the changes per unit time are infinitesimal; hence we take the viewpoint that such process is correctly described by

$$\nabla^F_{J(x)}\tilde{e}^a(x) - \alpha^a_{\ b}(t)\tilde{e}^b(x) = 0$$

where $\alpha^b_a(t)dt$ is the boost executed at time t and t is defined with respect to a dynamical coordinate system as follows. Take Σ_0 as a spatial hypersurface, just at the moment prioir to contemplating to change your reference frame and t is then simply defined by means of $\partial_t = \tilde{e}_0$ and $t = 0$ on Σ_0. The attentive reader must have noticed that the latter equation breaks the relation $\tilde{e}_0 = J(x)$, however, an infinitesimal moment in time later $J(x)$ reallings itself with $\tilde{e}_0$ meaning that

$$J^\nu(x^\mu + J^\mu(x)\,dt) := \tilde{e}_0(x^\mu + J^\mu(x)\,dt).$$

So, there is an inherent discontinuity in the process given that $\nabla_{J(x)}J(x)$ is determined by the classical effective equations of motion; but the latter are never integrated when the mind intervenes. The accelerations merely serve as an extra initial condition in defining the Fermi derivative locally. There is no way of explicitely integrating these equations but they should be programmed on a computer taking finite time steps δt and take the limit δt to zero. The reader checks that our equation preserves the orthonormal character of the basis given that $\alpha_{ab}(t) = -\alpha_{ba}(t)$. Actually, I am using nonstandard infinitesimal analysis here, but the correct time derivative is given by

$$\nabla^C_{J(x)} := \nabla_{J(x)} + \tilde{\omega}^a_{\ b} + \frac{i}{2}\tilde{\omega}_{cd}\mathcal{J}^{cd}$$

where $\mathcal{J}^{cd}$ are the generators of the Lorentz group in the spin $(\frac{1}{2}, 0)$ representation and $\tilde{\omega}^a_{\ b} = -\tilde{e}_{b\nu}\nabla_{J(x)}\tilde{e}^{a\nu}$. A spinor $C(x)$ undergoes then an infinitesimal boost

$$C(x) \to e^{i\alpha_{ab}(t)dt\mathcal{J}^{ab}}C(x)$$

and the equations of motion are covariant with respect to this procedure. Note also that the connection $\tilde{\omega}^a_{\ b}$ does not vanish in case no change of reference frame takes place (which would have been the case if we would have defined $\widehat{\omega}^a_{\ b} = -\tilde{e}_{b\nu}\nabla^F_{J(x)}\tilde{e}^{a\nu}$) thereby allowing for interactions between the profile fields and acceleration of the matter distribution. This must be so and later in this chapter shall we discuss couplings of a nonlinear nature. Now, before we proceed to the more general case of multiple issues, let us discuss the ramifications of this idea a bit further. In this regard, it is of crucial importance to note that the canonical dichotomy, which corresponds to the identity matrix, does not change under Lorentz boosts as it should be!! Indeed, our eyes are suddenly not taking a mixed perpective regarding the incoming electromagnetic radiation; they keep on measuring as usual and also preserve their upper perspective. Fact of the matter is that for more complex issues, even the knowledge of mathematical theorems are not approached from the canonical dichotomy (it almost is, but not quite). For example, when a mathematician walking on the street is being asked for his views on a certain mathematical statement, he might very well answer that he is just walking and you should ask this question later again when he is at home sitting at his desk. In this sense is revealing of mathematical knowledge not a canonical dichotomy, but one which is intertwined with other issues such as your state of motion. However, you might ask him a simpeler question requiring a trinary answer - which is much more easy to give - whether this particular statement is true or not? He can then quickly say, yes/no or I don't know. So, this is a very bold conjecture, that mixed issues transform under your change of reference frame; even a simple rotation might cause you to reflect differently on mixed issues; for example a professor sitting at his desk, rotating his chair for 180 degrees so that he is sitting with his back to the desk might suddenly conclude that he cannot think in this way, he needs his desk in front of him to write things down on paper and order his thoughts. So, the fact that he does not see his desk, which is encoded in the accelerations of his new brain current changes his profile on thinking.

2.1 Intermezzo.

We shall further deepen our understanding of the above regarding two different aspects. The first is that so far, we have looked upon the defining white-black relations from the point of view of complex geometry; here we develop the perspective from the Hermitian point of view and draw analogies between both. Indeed, instead of taking as defining relation

$$(X,P)\sigma_2 \begin{pmatrix} X \\ P \end{pmatrix} = -i1$$

we could as well have used

$$(X,P)^\dagger \sigma_1 \begin{pmatrix} X \\ P \end{pmatrix} = -1$$

leading one to a group of transformations A defined by

$$A^\dagger \sigma_1 A = \sigma_1.$$

The latter is four dimensional and not six dimensional and consists out of a $U(1)$ part and something isomorphic to $SO(1,2)$ consisting out of boosts in the $2,3$ direction and a rotation around the 1 axis, realized by $K^2 := -\frac{i}{2}\sigma^2$, $K^3 := -\frac{i}{2}\sigma^3$, $J^1 := \frac{1}{2}\sigma^1$ respectively, obeying

$$\left[K^2, K^3\right] = -iJ^1,\ \left[J^1, K^2\right] = iK^3,\ \left[J^1, K^3\right] = -iK^2.$$

Basically, we also have those generators in our $SL(2,\mathbb{C})$ Lie algebra and we can identify them. We shall come back to this duality in point of view later on in the next chapter of this book.

A second issue regards a natural representation of our profile operators, in particular, consider the following operators

$$\begin{aligned}
E &= \frac{-i}{2}(X,P)\sigma_1 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{-1}{2}(X,P)(-i\sigma_2)(i\sigma^3) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{-i}{2}(2XP+1) \\
&= (X,P)^\dagger(-i\sigma^1)(\frac{1}{2}\sigma_3) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{-1}{2}(X,P)^\dagger\sigma^2 \begin{pmatrix} X \\ P \end{pmatrix} \\
T &= i(X,P)\sigma_2 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= i(X,P)(-i\sigma_2)(i1_2) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= 1_2 \\
&= -(X,P)^\dagger\sigma_1 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= (X,P)^\dagger(-i\sigma_1)(-i1_2) \begin{pmatrix} X \\ P \end{pmatrix} \\
-H &= \frac{1}{2}(X,P)\sigma_3 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(X,P)(-i\sigma^2)(-\sigma^1) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(-X^2+P^2) \\
&= \frac{-1}{2}(X,P)^\dagger 1_2 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= (X,P)^\dagger(-i\sigma^1)(-\frac{i}{2}\sigma^1) \begin{pmatrix} X \\ P \end{pmatrix} \\
B &= \frac{1}{2}(X,P)1_2 \begin{pmatrix} X \\ P \end{pmatrix}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}(X,P)(-i\sigma^2)(i\sigma^2)\begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(X^2+P^2) \\
&= \frac{-1}{2}(X,P)^\dagger \sigma_3 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= (X,P)^\dagger(-i\sigma^1)(-\frac{\sigma^2}{2})\begin{pmatrix} X \\ P \end{pmatrix}
\end{aligned}
$$

Before we proceed, let us mention that these expressions suggest a deeper relationship between $\sigma^1, -i\sigma^2, -\sigma^3, 1_2, i\sigma^2$ on the other[4]. This almost suggests that the 1-axis and time are of the same kind and likewise so for the $2-3$ axis; which is the case for the gravitational field on the earth where the 1 axis equals the z axis and is given by ∂_r and time is related to height. The only freedom which is still left is rotation around the 1 axis, which connects the $2-3$ axis in which biological creatures can move without effort. The reader also notices that there is a reflection symmetry around all axis which amounts to space-time reversal. I was very interested in this observation when I noticed it for the first time and we shall try to make sense out of it later on. One notices that E, H, B are self adjoint operators and that:

$$[H,E] = 2iB,\ [B,H] = 2iE,\ [B,E] = 2iH$$

which is the algebra of $SO(1,2)$ with $J^1 = \frac{1}{2}H, -\frac{1}{2}B = K^2, \frac{1}{2}E = K^3$ and not $SU(2)$. The reader may enjoy understanding that these insights result from quantization of a $1+0$ dimensional complex spinor field theory with as real action

$$S = i\int dt \begin{pmatrix} \Phi(t) \\ \Psi(t) \end{pmatrix}^\dagger \sigma^1 \frac{d}{dt}\begin{pmatrix} \Phi(t) \\ \Psi(t) \end{pmatrix}$$

which is clearly $U(1)\times SO(1,2)$ invariant. A small computation reveals that there exist two independent real components and two imaginary ones forming two conjugate pairs which decouple in the usual Poisson bracket. The correct bracket to quantize however is the Dirac bracket which gives half of the identity (putting $\hbar = 1$) for the conjugate variables. Restricting to one canonical pair, we have that $X = \mathrm{Re}(\Phi(t)), P = 2\mathrm{Im}(\Psi(t))$ since the canonical momentum of $\mathrm{Re}(\Phi)$ equals $\mathrm{Im}(\Psi) = \frac{iP}{2}$ and likewise the canonical momentum of $\mathrm{Im}(\Psi)$ is $-\mathrm{Re}(\Phi) = -X$ so that everything is consistent. I dropped the time dependency here because the Hamiltonian is exactly zero. The charges associated to the

[4]Note that the Hermitian geometry only provides for $i\sigma^2 = \sigma^3$ and $\sigma^1 = -1_2$; all other relations arise from the correspondance with the complex geometry which gives $i\sigma^1 = \sigma^2$ and $\sigma^3 = -1_2$

symmetry group are therefore (ignoring the other conjugate pair)

$$
\begin{aligned}
1_2 &\rightarrow \frac{1}{2}\begin{pmatrix} X \\ P \end{pmatrix}^\dagger \sigma^1 \begin{pmatrix} X \\ P \end{pmatrix} \\
\frac{1}{2}\sigma^1 &\rightarrow \frac{1}{4}\begin{pmatrix} X \\ P \end{pmatrix}^\dagger 1_2 \begin{pmatrix} X \\ P \end{pmatrix} \\
-i\frac{1}{2}\sigma^2 &\rightarrow \frac{1}{4}\begin{pmatrix} X \\ P \end{pmatrix}^\dagger \sigma^3 \begin{pmatrix} X \\ P \end{pmatrix} \\
-i\frac{1}{2}\sigma^3 &\rightarrow -\frac{1}{4}\begin{pmatrix} X \\ P \end{pmatrix}^\dagger \sigma^2 \begin{pmatrix} X \\ P \end{pmatrix}
\end{aligned}
$$

so that it is clear that the algebra is preserved[5]. To turn it into $SU(2)$ we need to analytically continue and state that $J^1 = -\frac{1}{2}H, J^3 = -\frac{1}{2}iE, J^2 = -\frac{1}{2}iB$ which means that the rotations around the $2-3$ axis correspond to anti-Hermitean operators. That this constitutes the right point of view is exemplified by considering the adjoint actions

$$
\begin{aligned}
\left[J^3, X\right] &= -\frac{1}{2}X \\
\left[J^3, P\right] &= \frac{1}{2}P \\
\left[J^2, X\right] &= -\frac{i}{2}P \\
\left[J^2, P\right] &= \frac{i}{2}X \\
\left[J^1, X\right] &= \frac{1}{2}P \\
\left[J^1, P\right] &= \frac{1}{2}X
\end{aligned}
$$

Upon identifying X with $(1,0)^T$ and P with $(0,1)^T$ one sees that $J^j \equiv \frac{1}{2}\sigma^j$ which confirms our previous analysis. Notice that there is another interesting observation here; in a way, the representation of $SO(1,3)$ is broken down to one of $SO(1,2)$ given that the latter constitues the unitary part of the former. The attentive reader must have noticed that there is a slight ambiguity in the above

[5]Note that we have changed sign of $\sigma^1 = -\frac{1}{2}H$ and $\sigma^3 = -\frac{1}{2}E$ but that is inconsequential.

determination of the charges and that we could equally well have considered

$$
\begin{aligned}
1_2 &\rightarrow \left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right)^{\dagger} \sigma^1 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right) \\
&= -\frac{1}{2}1 \\
\frac{1}{2}\sigma^1 &\rightarrow \frac{1}{2}\left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right)^{\dagger} 1_2 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right) \\
&= \frac{1}{2}\left(X^2 - \frac{P^2}{4}\right) \\
-i\frac{1}{2}\sigma^2 &\rightarrow \frac{1}{2}\left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right)^{\dagger} \sigma^3 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right) \\
&= -\frac{1}{2}\left(X^2 + \frac{P^2}{4}\right) \\
-i\frac{1}{2}\sigma^3 &\rightarrow -\frac{1}{2}\left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right)^{\dagger} \sigma^2 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array}\right) \\
&= -\frac{i}{2}\left(XP + \frac{1}{2}\right).
\end{aligned}
$$

which changes the appearance on the right hand side by a factor of 2. Perhaps, there is something deeply hidden in this and that all (inverse) powers of 2 and -1 are encoded into nature.

Finally, note that there is another Hermitian way of encoding the commutation relations which is given by

$$\left(\begin{array}{c} X \\ iP \end{array}\right)^{\dagger} \sigma^2 \left(\begin{array}{c} X \\ iP \end{array}\right) = 1.$$

The symmetries of this relation are again given by $U(1) \times SO(1,2)$ but this time the rotation is around the 2 axis. This leads to an identification of $-\sigma^1 = \sigma^2$ which causes for the entire theory to be invariant under the symmetry $1 \rightarrow i$; this has been suggested into the work of 't Hooft and Nobbenhuis regarding the cosmological constant. Note that the associated complex geometry given by

$$\left(\begin{array}{c} X \\ iP \end{array}\right)^{T} \sigma^2 \left(\begin{array}{c} X \\ iP \end{array}\right) = 1$$

is redundant and does not provide for any new information: its symmetry group is given by

$$A^T \sigma_2 A = \sigma_2$$

and is again the usual $SL(2, \mathbb{C})$. Finally, note that the Schwitchoriem duality is not included in any of the two Hermitean geometries meaning there is no way, by means of the associated symmetries, to rotate the free particle into the Harmonic oscillator or into a particle with no kinetic term at all. This is why all viewpoints seem to be important. Note here that reality itself, that is the psychic wavefunction for all observers is taken to be static so far, it are the perspectives or profiles which change and reality changes only by means of by means of measurement. This is certainly a part of the game, but on the other hand do we know that psychic reality changes if material reality does and this has nothing to do with your decision, that may be perfectly white! So, your local white world evolves too: for example, when you see someone hurt, you will most likely help her so that reality changed in the sense that this became the most likely answer. Of course, this particular information regarding the image of the hurt person must be hidden into you brain currents, another reason why we must take the complex geometry of our commutation relations most seriously given it provides for a $(\frac{1}{2}, 0)$ representation of the Lorentz group and therefore allows for a quadratic coupling to those currents. Indeed, the coupling itself must be Lorentz invariant: the psyche is attached to the body, it is just so that you will have different brain currents when moving relative to one and another compared to when you are in rest to each other simply because incoming signals shift accordingly. For example, when driving a car, you are less likely to stop for someone in need on the street as when meeting this person on foot. We shall not go into this matter further in this book as we clearly lack experimental data of how our psyche couples precisely to our brain currents and how our moral values are affected by what we see from at the level of molecules and so on. A computer again can be just fed with billions of pictures or conversations of the same person so that it eventually recognizes when this person is sad or happy, if the programmer just attached these words to those pictures in the first plae, which is something very different from having an understanding of these words. Again, all of this has its limits, but perhaps nature causes complicated "machines" not to be a machine any longer, by means of something we cannot comprehend. Finally, remark that the only symmetry which is common to all viepoints is a boost around the 3 or y axis associated to the E operator; we shall come back to this in the next chapter.

2.2 Further exploration of the dynamics of the profile matrix.

Since any profile matrix can be identified with $A(x) = e^{\vec{a}(x).\vec{\sigma}}$ where $\vec{a}$ is a complex vector, we have a canonical isomorphism mapping it to

$$\widehat{A} = e^{\vec{a}.\vec{J}}$$

where as mentioned previously $J_1^\dagger = J_1$, $J_2^\dagger = -J_2$, $J_3^\dagger = -J_3$. This allows one to couple the profile matrices to the wavefunction describing the white reality. At this point, it is necessay to mention that all representation Hilbert spaces used in physics carry a natural complex structure, meaning that one can tell whether a vector is real or imaginary. In that vein, can we define the complex conjugate of an operator and both X, P obey

$$\overline{X} = X, \overline{P} = P$$

which leads to

$$\overline{J_1} = J_1,\ \overline{J_2} = -J_2,\ \overline{J_3} = J_3.$$

Therefore, we can relate the adjoint representation to the inverse of the complex conjugate representation if we find an operator S obeying $S^2 = 1$ such that

$$J_j^{\dagger} S = -S\overline{J}.$$

In the standard $su(2)$ representation, this is easily seen to be given by $\pm\sigma^2$; in our framework however, we have to focus on J^3 (which is of course fully equivalent) given that

$$J_1 S = -SJ_1,\ J_2 S = -SJ_2,\ J_3 S = SJ_3.$$

I have not seen any discussion of this "charge conjugation" in the literature but it is obvious from the well known representation theory of $su(2)$ that there is precisely (upon a sign), for each irreducible representation, one S satisfying these equations and one has moreover that $S^{\dagger} = S$. The latter is given by $S_{\sigma,\sigma'} = \delta_{\sigma\sigma'}(-1)^{\sigma - j}$ where σ is half integer in case j is and runs from $-j \ldots j$. In general, our operators X, P are the usual multiplication and differential operators with respect to α and are represented on the Hilbert space of square integrable functions in those psychological variables at any point in spacetime. The associated representation, by means of the J^j, of the rotation group is obviously reducible and can be written as an infinite direct sum of irreducible representations; hence the only freedom in the choice of S is a relative sign ± 1 in each representation block. Hence, we shall be interested in couplings of the kind

$$K(A(x), \Psi(x)) := \frac{\int ds\ \overline{\Psi}(x,s)\, S\, \overline{\widehat{A}(x)\, \Psi(x,s)}}{\int ds\ \overline{\Psi}(x,s)\, S\, \overline{\Psi(x,s)}}$$

where $\Psi(x,s)$, $\widehat{A}(x)$ transform under an (infinitesimal) Lorentz transformation $\Lambda^{\frac{1}{2}}(x) = e^{i\vec{b}(x).\vec{\sigma}}$ as

$$\frac{\widehat{\Lambda^{\frac{1}{2}}(x)}\Psi(x,s)}{\int ds\ \overline{\Psi}(x,s)\, \Psi(x,s)},\ \widehat{\Lambda^{\frac{1}{2}}(x)}\widehat{A}(x)\widehat{\Lambda^{-\frac{1}{2}}(x)}$$

such that $K(A(x), \Psi(x))$ remains an invariant. This principle reflects that the evolution of your profile regarding the mental state remains the same if you change motion. Note that $\widehat{\Lambda^{\frac{1}{2}}(x)}$ is not a unitary operator and that therefore change of reference frame changes (slightly) your notion of orthonormal basis which means that you have a different reality - a change from the traditional viewpoint upon quantum mechanics (note that we renormalized the wave and therefore had to consider the denominator in the definition of $K(A(x), \Psi(x))$ which we assume to be nonzero). Before we proceed, let us think about this a bit further; this viewpoint is exactly the same as we had for the change an external state underwent if the hypothetical observer boosted; the latter description, as we shall explain in full depth in the next chapter, is just a rule of thumb. What really happens is that the state of your brain changes relative to the exterior world which is equivalent to an appearant change in the outher world! We will comment further upon that when discussing issues which boost into one and another when changing frame of reference, such as the different components of your brain currents themselves which just reflect that the state of your brain has altered regarding its coupling with the external world. Indeed, we see colours slighly differently when moving, lengths of objects contract and so on. There is another change with regard to traditional quantum field theory here, which is that the profile field acts upon the upper-lower variables which constitute a "quantum system" themselves. Also this is a novel extension of the usual lore where everything is put on the same level; in the psyche, there are different levels, there are issues, profiles on issues, issues on profiles of issues and so on. The psyche works as such, it is a reflection of intelligence. If you want artificial intelligence to be able to partially reason as a human, you would simply have to feed it with all infinite conversations between humans to obtain a level of speech which is somewhat coherent. All AI can do up till now is give an encyclopedic overview of what is known, but it will actually never make any choice by itself and even if it would (which you can program by means of a random generator) then still you would have to program it as such in order for successive profiles to be coherent. Therefore, AI will never be able to do any original research. Let us also mention that the dynamics for the profile matrix must be as such that the identity matrix is a fixed point and attractor. It is important to notice that the action of the profile matrices (and therefore the action of the symmetry group) is not a unitary one, which leads in general to complex action principles instead of real ones.

2.3 Multiple issues.

In a way, this is a system of one issue; going over to N issues we obtain N operators X_i and P_j satisfying

$$[P_i, X_j] = \delta_{ij} 1$$

as well as

$$[X_i, X_j] = [P_i, P_j] = 0$$

and we look for symmetries of this algebra[6]. Those include the so-called Bogoliubov transformations which map Schwitchoriems on pure issues to Schwitchoriems

[6] Here it is worthwhile to notice that although we consider the direct sum construction in determining the distinct profiles, we shall of course take the tensor-product of the "one issue Hilbert spaces" when representing the issue operators.

on mixed issues; but since the world of issues is classical, those mixed issues are not seen as defining the ground state of society. The full matrix algebra has complex dimension $(2N)^2$ for which the quadratic forms associated to N matrices T^i are put to the identity and the remaing quadratic forms associated to $2N(N-1)$ different matrices $S^{[ij]}_{ab}$ are mapped to the zero operator. Here, $(T^i)^{ka}_{lb} = \delta^k_l \delta^i_k \omega^a_b$ and $S^{[ij]}_{ab} = E_{ij}F_{ab} - E_{ji}F_{ba}$, where E_{ij} is the $N \times N$ matrix with all zero entries except for (ij) where it equals one and F_{ab} is the two times two matrix given by $(F_{ab})_{cd} = \delta_{ac}\delta_{bd}$, are all antisymmetric matrices. The reader may enjoy proving that preservation of these constraints is equivalent to

$$\mathrm{Tr}\left(A^T T^j A \omega_N\right) = -1, \ \mathrm{Tr}\left(A^T S^{[ij]}_{ab} A \omega_N\right) = 0$$

where

$$\omega_N = 1_N \otimes \omega$$

and 1_N is the $N \times N$ identity matrix. So, this leaves us with a $4N^2 - 2N(N-1) - N = 2N^2 + N$ dimensional complex Lie algebra of symmetry transformations. As we knew already, for $N = 1$, we have three generators, naturally associated to space if our question regards the being or perception of space and we added "time" to this picture as the identity matrix. Note that in the exceptional case of the fundamental representation of $SL(2,\mathbb{C})$, the generators do not only constitute a Lie algebra but naturally give rise to a complex algebra with time added. There was another way spacetime arose from this representation and that was by taking the tensor product with its complex conjugate representation and taking real sections. So, in a way, there are two natural algebraic connections between the different psychological types, regarding a certain issue, and the four dimensional nature of spacetime - this might lead to a useful idea when formulating dynamical laws for psychological types. The tensor product construction is the most easy one to generalize[7]; indeed; one may consider a $2N$ dimensional complex vector space defined by

$$W \sim \oplus_{j=1}^N \begin{pmatrix} X_j \\ P_j \end{pmatrix}$$

then insisting that our symmetry transformations preserve the natural symplectic form ω_N imposes $\frac{N(N+1)}{2} - 1$ extra constraints. Hence, going over $W \otimes \overline{W}$ and taking real sections, we have a natural *complex* action of our symmetry group on $\otimes_N(M^4)$ where M^4 is 4 dimensional (complex) Minkowski spacetime. In more detail, consider

$$-\delta_{ij}\eta_{ab} = \delta_{ij}\sigma^{AA'}_a \sigma^{BB'}_b \omega_{AB}\overline{\omega}_{A'B'}$$

and

$$(\Lambda)^{ia}_{jb} := \sigma^a_{AA'}\sigma^{BB'}_b A^{iA}_{kB}\overline{A}^{kA'}_{jB'}$$

[7] The solder form here can be taken to be static, however alternative theories of gravitation may be developed using dynamical solder forms.

then elementary algebra reveals that they constitute elements of $SO(N, 3N; \mathbb{C})$. Indeed, it is generally not so that $(\Lambda)^{ia}_{jb}$ is a real matrix albeit this is certainly so for $N = 1$. To demand it is real is equivalent to the supplementary conditions

$$A^{iA}_{kB}\overline{A}^{kA'}_{jB'} = \overline{A}^{iA'}_{kB'}A^{kA}_{kjB}$$

or equivalently that the $N \times N$ matrices A^{A}_{B} satisfy the following 8 complex conditions

$$\overline{A}^{A'}_{B'}A^{A}_{B} = A^{A}_{B}\overline{A}^{A'}_{B'}.$$

Note that if you would insist upon a real embedding of the entire group without any additional reality conditions then you would need to embed it into $SO(N^2, 3N^2; \mathbb{R})$ and we shall leave this as an exercise to the reader. In the sequal, we shall not go over to any reduced symmetry group, but merely consider gauge transformations which do preserve ω_N which is all we really need. Now, we come to an important point, we must couple spacetime currents (which display the motion of our spirit attached to our body) with the evolution of these matrices and we can use this isomorphism between N copies of the tangent bundle and our mental habitat of N questions, denoting by $\mathcal{J}^{ab}$ the generators of the spin $(\frac{1}{2}, 0)$ representation[8], we can take

$$\mathbf{J}^{N\,ab} = \sum_{j=1}^{N} 0_2 \oplus_1 \ldots 0_2 \oplus_{j-1} \mathcal{J}^{ab} \oplus_j 0_2 \ldots \oplus_{N-1} 0_2$$

where 0_2 is the zero matrix in 2 dimensions. Clearly, this is an element of the Lie algebra of our symmetry group (the direct sum reprentation); now lets consider the spin connection

$$\nabla^{C,N}_{J(x)} := \nabla_{J(x)} + \frac{i}{2}\tilde{\omega}_{ab}\mathbf{J}^{N\,ab}$$

which is mandatory to make the dynamics of the $(2N) \times (2N)$ matrices A belonging to our symmetry group covariant under a change of reference frame. There is an interesting thing to mention here, the intertwiners governing the interactions of profiles have to be dynamical objects themselves in contrast to what happens in particle theory. From a mental point of view, this is entirely logical since the interactions between two identical profiles vary in time; I suggest that those matrices only couple to the mental variables and not the type operators themselves[9]. Here, it is of course understood that a $2N$ complex "mind vector" V transforms under a local Lorentz transformation as

$$V \to e^{i\zeta_{ab}\mathbf{J}^{N\,ab}}V$$

where ζ_{ab} is real an antisymmetric and generates a Lorentz transformation on the vectors by means of

$$e^{\zeta_{ab}J^{ab}}$$

[8] $\mathcal{J}^{0j} = \frac{i}{2}\sigma^j$, $\mathcal{J}^{jk} = \epsilon_{jkl}\frac{1}{2}\sigma^l$

[9] Note that everything is consistent here given that upon applying a spin $\frac{1}{2}$ Lorentz transformation g to X results in $(\omega g X g^{-1})^T = -(g^{-1})^T X^T g^T \omega = \omega g X g^{-1}$ by using the properties $X^T\omega = -\omega X$ and $g^T \omega g = \omega$.

where the J^{ab} are the real antisymmetric generators of the defining representation and $J^{ab} = -J^{ba}$. Likewise can the reader now construct more general intertwiners allowing for different issues to mix with one and another as well as to change your perspective upon things. We now come back to our projectors $\pi_{A,j}(s_j)$ where $A = 1, 2$ and $j : 1 \ldots N$ onto the conservative or progresive side of your dichotomy. I did not mention this previously, but the tensor $\pi_{A_k}(s_j; j = 1 \ldots N) : \mathbb{C}^{2N} \to \mathbb{C}$ simply is the identity matrix so that everything is manifestly Lorentz covariant. It is obvious that one should construct the Lie algebra of our Lie group in terms of the Pauli matrices such that the new quantum generators can be constructed by means of the identical procedure; for $N = 2$ the reader may verify that the complex Lie algebra is generated by

$$R^j := \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix} \; T := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

and

$$P^j = \begin{pmatrix} \sigma^j & 0 \\ 0 & 0 \end{pmatrix} \; Q^j := \begin{pmatrix} 0 & 0 \\ 0 & \sigma^j \end{pmatrix}.$$

The commutation relations are

$$\left[R^j, R^k\right] = 2i\epsilon_{jkl}(P^l + Q^l), \; \left[R^j, P^k\right] = -iT\delta^{jk} + i\epsilon_{jkl}R^l, \; \left[R^j, Q^k\right] = iT\delta^{jk} + i\epsilon_{jkl}R^l,$$

$$\left[P^j, Q^k\right] = 0, \; \left[R^j, T\right] = 2i(P^k - Q^j), \; \left[P^j, T\right] = -iR^j, \; \left[Q^j, T\right] = iR^j$$

and

$$\left[P^j, P^k\right] = 2i\epsilon_{jkl}P^l, \; \left[Q^j, Q^k\right] = 2i\epsilon_{jkl}Q^l.$$

Mapping each generator S to

$$S \to \widehat{S} := \frac{1}{2}(X_1, P_1, X_2, P_2) \begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \begin{pmatrix} X_1 \\ P_1 \\ X_2 \\ P_2 \end{pmatrix}$$

leads to

$$\begin{aligned} R^1 &\to i(-X_1X_2 + P_1P_2) && (2.1) \\ R^2 &\to -(X_1X_2 + P_1P_2) && (2.2) \\ R^3 &\to -i(X_1P_2 + P_1X_2) && (2.3) \\ T &\to i(X_1P_2 - P_1X_2) && (2.4) \\ P^1 &\to -iH_1 && (2.5) \\ P^2 &\to -B_1 && (2.6) \\ P^3 &\to E_1 && (2.7) \\ & && (2.8) \end{aligned}$$

and likewise for Q^j with one and two interchanged. We already know we have to analytically continue $\widehat{P}^1 \to -i\widehat{P}^1, \; \widehat{P}^2 \to i\widehat{P}^2, \; \widehat{P}^3 \to -i\widehat{P}^3$ and likewise for $\widehat{Q}^j$

to get the correct Lie algebra of $SU(2)$. Assuming this has been done and upon using the same symbols for the redefined meanings, one sees that one needs to perform the following analytic continuation

$$\widehat{R}^1 \to i\widehat{R}^1,\ \widehat{R}^2 \to -i\widehat{R}^2,\ \widehat{R}^3 \to i\widehat{R}^3$$

to ensure consistency of the first commutation relations and the reader may verify that all other remaining commutation relations are satisfied. This concludes, up to this point, the very mathematical setting behind our line of thought on the fundamental dichotomy which is conjectured to largely determine the way we interact with one and another. Sociology and psychology as it has been practised so far are not even close to properly adressing those issues from a foundational point of view. It seems that the way both pseudosciences are used by policy makers is to impose a restriction upon free will and behaviour and, even if you commited no punal offence, to put you away in some asylum to "protect" yourself as well as your surroundings. Psychiatrists have become the modern inquisition aimed at taming and re-educating undesirable elements in society, and it is just horrible that they proclaim their gratuitious "deseases" have some objective scientific value. Those, who interested in those power games, please, throw this book away since this is not of my interest. I want to gain a deeper understanding of why people react in a certain way with the goal of an inclusive, modern society and not an exclusive, medieval one which judges and prosecutes. Only Italians seem to have understood this profound message and gave up upon mental asylums due to the work of a psychiatrist in Trieste: long live Italy and maybe I will go on retirement there. Of course, issus are not the only thing our psyche turns around, there is also the issue of consciousness and any theory involving communication of issues must also consider whether this happens in a conscious (c) or unconscious (u) way regarding reception (R) and sending (S) of "signals". This constitutes our second dichotomy which is also of fundamental importance in psychology, but unlike our first dichotomy, where one is free to choose ones point of view and the quantum mechanical description is the accurate one, consciousness is something which simply seems happen to us; it seems rather perverted that one would "pose the question" (unconsciously of course) rearding ones awareness about other questions of the mind. Although there is no strict logical contradiction here as far as I can understand, we will treat consciousness on a classical level of geometry. In particular, we will make a completeness assumption that every send signal is also received so that in a sending-reception process there are two parameters involved indicating the degree of awareness. We shall describe the world in the white basis; that is, we work with real variables attached to issues and study how these values as well as the issues propagate. Ultimately, the real state is a complex wavefunction in as well the issues (which cause for a discrete labelling j), the values (which are just real numbers α_j) and the spacetime location. A signal transmits information which we write down by the letters α, β (which are taken as vectorial quantities); now, it is not so that the received information equals the transmitted one, the reciever may attach a different value to a certain sentence or he might slightly store the message in a different wording in his brain (possibly also with a different appreciation) and thereby mixing the vector components. We must still, for each message, attach the degree of awareness to it; indeed, we have learned that between dichotomies there must exist a continuous spectrum of possibilities. You may not be fully aware of something or largely unaware, but have a gut feeling that you saw something vague but don't quite remember what. Therefore, each entry in the vector α is replaced by a couple (α_j, t_j) where $t_j \in (-\infty, +\infty)$ and $t = -\infty$ equals u and $t = +\infty$ equals c. Keeping this in mind, we use the symbol $\overline{\alpha}$ in this new sense (so it is a $2N$ dimensional vector instead of a N dimensional vector). Now, we must still add the flat $2N$ dimensional complex space of psychological profiles on top of this construction and then we are done. So now that we know how our total space should look like (locally), let us ask some questions regarding the metric. As said before, the bundle of types should be flat (in the bundle coordinates), but there must be some non-trivial dependency of the "profile operators" A on the psychological variables $\overline{\alpha}$ but not really upon the spacetime coordinate x (apart from the influence through the spin connection) since physical signals almost never influence our questions; it is just the level of awareness regarding those signals which get influenced, for example when you are in pain. So, to speak into the language of the next Part of this book, we must find free propagators for the material particles, psychological variables and profile operators separately and then construct intertwiners between them. This should be intertwiners (a) relating the matter propagators (in either incoming physical signals) to the propagators of the psychological variables and (b) relating the psychological variables and types amongst each other. In order to realize this one must have a "learning field" $T^{A_j}_{\alpha_j t_k}(x, \overline{\alpha})$ where $k, j : 1 \ldots N$ and $A = 1, 2$ where the latter transform under local Lorentz transformations and the α_j, t_k coordinates do not transform at all given that they are are canonically associated to the operators X_j, P_j and those are not dynamical. Note that this somewhat Newtonian stance is justified since those variables, unlike (instantaneous) physical attributes[10] of a particle do not depend upon the state of motion relative to the observer but they pertain to the inner kitchen of the observer himself! In a way, it is logical that your state of physical motion in as well the gravitational field as the other force fields interacts with your dichotomy regarding certain issues[11]. Indeed, any motion in the background scenery requires work to be done by the body so that *internal* positions are influenced by that very activity; for example, when asked "what speed you are considering in moving towards the fridge (while sitting in your armchair) to get some beer", you might at first say that you don't know, that you are currently sitting in your chair and that this question is irrelevant to you (mixed profile, it bothers you in a certain sense). Next, when you stand up to get the beer and being asked the same question again, you might say, I consider going at 5 km an hour because that is the optimal speed to get my beer (white profile), whereas finally, upon approaching the fridge you answer to

[10]In classical theories thse attributes transform under local Lorentz transformations but as discussed at length before, this is not the case in quantum theory.

[11]Certainly not issues regarding your perception of the outside world; that one is always white unless you get a stroke or something like that.

the same question again that you consider slowing down (white profile) otherwise you might walk past the fridge. Actually, what you call speed is expressed in the number of steps you make per unit time times the step length, but the only way to step is to use your muscles! Indeed, any single rigid object on the surface of the earth which cannot transform its internal energy into labour will either remain stationary or experience a varying force field (in space) due to the surface of the earth (gravity, as Einstein beautifully expressed it, is not a force and you don't feel it). This is the very crucial distinction between Einstein's view (which is the correct one) and Newton's view upon gravitation. Newton would say there are two forces working on the body, a gravitational one and one due to the surface of the earth conspiring as to keep you on the surface of the earth and there is *almost* no net force felt when you are moving. Einstein, on the other hand, would say that that such a body really is accelerating all the time because it does not move on a geodesic in the gravitational field. Our *interpretation* of speed really regards an acceleration in the Einsteinian sense and that is the reason why we do not only feel speed[12] to some extend but also our own weight! However, the variation of the force field due to the surface of the earth is *interpreted* in a different way: for example, a cylindrical body will experience that this contact force varies as a delta peak $F\delta(\theta-\alpha t)\vec{e}_\theta$, where $\vec{e}_\theta$ is naturally associated to its polar coordinate system, whereas an outside observer would say that the contact force is a delta peak $F\delta(x+R\alpha t)\vec{e}_z$, where the z axis is fixed with respect to him. It are the interpretations which are crucial for the evolution of the profile operators: indeed regarding the issue "what is the angular speed cylinder John is rolling with on the floor?", John himself might take the mixed point of view and say that he does not know wether he is rolling or not, but he feels stinges upon his mantle which continuously move over the entire mantle whereas previously, they were in one place only. Maybe, some evil deamon is playing a game with him? He doesn't know, it is complicated. An observer from the outside will of course give a definite answer. When asking then to John to which extend he would like the stinges to be in one place only, he might give a definite very high appreciation. The psychological appreciation (in the white perspective in terms of the α variables) of walking towards the fridge would roughly be the same in all three circumstances (prior to applying your profile operator). Indeed, at any instant would you have answered that 5 km an hour is the preferred speed *if* you would want to go to the fridge in the first place. Likewise, to give another example, do you look differently at abstract intellectual thought when you are walking on the street; you will most likely say that you do not engage in thought right now and that such activity is to be done when you are sitting quietly at your desk at home. To illustrate these previous thoughts with a specific calculation we attach a Newtonian frame of reference to the center of the earth in polar coordinates (r, θ, ϕ); then upon keeping (r, ϕ) fixed, say $r = R$, $\phi = 0$ where R is the radius of the earth, then we would say that the motion $\theta(t) = \alpha t$ with α a ridiculously small number is

[12]The way we feel speed depends of course upon the means we use to move; for example, when going on foot, I have a different experience than driving a bicycle.

one of constant velicity. However, in Cartesian coordinates, the resulting vector $\vec{r}(t)$ satisfies

$$\frac{d^2}{dt^2}\vec{r}(t) = -\alpha^2 \vec{r}(t)$$

resulting in a magnitude of acceleration squared of $\alpha^4 R^2$ which is a ridiculously small number. In Einsteins view, the spacetime geometry is to a good approximation given by

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right)$$

and upon renormalizing our velocity field we get the quantity

$$U(t) = \frac{c}{\sqrt{c^2 - R^2\alpha^2 - \frac{GM}{R}}} \left(c\partial_t + \alpha\partial_\theta\right).$$

Calculating the accelation gives, with $d\tau = \frac{cdt}{\sqrt{c^2 - R^2\alpha^2 - \frac{GM}{R}}}$,

$$\frac{D}{d\tau}U(t) = -\frac{(R\alpha^2 + \frac{GM}{R^2})c^2}{c^2 - R^2\alpha^2 - \frac{GM}{R}}\partial_r.$$

The magnitude squared of this vector is

$$\left(\frac{(R\alpha^2 + \frac{GM}{R^2})c^2}{c^2 - R^2\alpha^2 - \frac{GM}{R}}\right)^2$$

which is in order of α given by

$$\sim R^2\alpha^4 + (\frac{GM}{R^2})^2 + 2R\alpha^2\frac{GM}{R^2}$$

ignoring the denominator divided by c, since the latter is close to unity. Hence, the Newtonian velocity squared divided by R appears in this formula which of order $10R = 67000000$ larger (in standard units) than the Newtonian acceleration for $R\alpha \sim 1$. So, to wrap up, there is no feeling and no change of perspective associated to Einsteinian velocity of a free body, but what we call speed here on earth really regards an acceleration and we do feel that. These considerations are crucial when you couple our sensors to the electromagnetic field which is really responsible for the brain activity in our body; those change substantially in Einstein's view given that stationary matter (in Newton's theory) is accelerating all the time causing for electromagnetic radiation and therefore nontrivial neural activity. In our work above, regarding the evolution of the profile field, we considered a first order differential equation which means that it couples directly to the electrical currents in the brain. Regarding our geometry of our psychological space, we now conjecture that it is **locally**, meaning associated to a **classical** part of the brain, where classicality is a product of the spirit which is impossible to describe in any mathematical way, is given by

$$g_{\mu\nu}(x)dx^{\mu} \otimes dx^{\nu} + \Omega^2(\overline{\alpha}, x)\left(\delta_{\alpha_j \alpha_k} d\alpha_j \otimes d\alpha_k + \delta_{jk} dt_j \otimes dt_k\right).$$

On top of this, you have to consider the $2N$ dimensional complex profile bundle. Indeed, the point behind coupling the profile operators to our state of motion is that we can now write down interaction terms coupling possibly distinct matter currents quadratically to our profile operators[13] by means of the following intertwiners

$$\mathbf{Z}^{\mathbf{t},\mathbf{u}\,ab\,(B,l)(B',m)}_{\mathbf{r},\mathbf{s}\;(A,j)(A',k);} := \delta^t_j \delta^u_k \delta^l_r \delta^m_s \sigma^{aB'B} \sigma^b_{A'A}$$

where $\sigma^{aB'B} = \sigma^a_{C'C}\overline{\omega}^{B'C'}\omega^{BC}$. So, this allows for brain currents and their derivatives not only to guide our profile, but also to interact with it in a non-linear fashion. For example, one may consider the Lorentz tensor $g(\tilde{e}^a(x), \nabla^C_{J(x)}\tilde{e}^b(x))$ coupling to $A^{jA}_{lB}(x)\overline{A}^{kA'}_{mB'}(x)$ in the equation of motion $\nabla^{C,N}_{J(x)}A = \ldots$. There is one exceptional question or issue, which one may call the primary question and it is, "what do I think". Indeed, in a conversation, it is not enough to simply adjust your points of view on several issues, but you must also know what issue to speak about regarding what has been said before. You also have to think, to select the relevant sentence of all possible things you might be able to say at that very moment in time! Note that I claim something very different from AI which can learn how to think by digesting loads of text and seeing correlations between them; humans, on the other hand seem to bypass this issue being well capable of reasoning about things with very little or no input indeed.

The reader must reflect further here, given that language contains a certain duality; indeed, given an issue (X, P) one can wonder "what is my profile on that issue", which naively results in a pair (X^*, P^*) where X^* is the white perspective on that matter, meaning I give you a definite profile, and P^* is the black perspective meaning I want to change any profile on that matter. This is a legitimate question which also has the white-black perspective build in. So, there is a distinction between the questions "what state is your country in?" and "how do you perceive the state of your country?"; indeed, if you would anwer the second question from a from a non-white perspective, then there is no way you can answer the first one in the way we have anticipated before. Indeed, you can also say, I look at it from a mixed point of view or I have no point

[13]Here, we stress again that not every part of the brain has the same effective complexity, or number of degrees of freedom albeit the fundamental prescription at level zero is entirely democratic and does not divide the brain into cells, neurons and so on. Indeed, it is the mind which does that in a totally mysterious way and therefore the total bundle is dependent upon the localization in brain. This is most conveniently modelled by neural networks associated to the mental level where the nodes are associated to the "brain entities" and the lines connecting the nodes are associated to electrical circuits. Each line can be thought of as carrying an issue and a profile operator and the nodes really live in the direct sum of those lines regarding the profile operators and in the direct product regarding the Hilbert spaces on which these issues are represented. This is similar to a spin network in Loop Quantum Gravity.

of view on the matter, it is complex. This game of embedded questions is in principle infinite and the dynamics should be as such as to stimulate whiteness at a certain level. So, we must add N new variables q_j to our description above of the geometry taking trinary values $0, 1, 2$ where 0 means "no point of view, its complex", 1 a mixed point of view and 2 a definite profile. A comment of a more technical nature is in place here: the profile space of (X, P) is a six dimensional real manifold so X^* cannot just be a linear operator which severely complicates our algebraic point of view. In this book, we shall concern ourselves with primary issues meaning (a) they do not refer to one and another and (b) anyone has a distinct point of view upon them; as explained here, this is a huge simplification of reality - even worse, most computers are simply white and give totally nonsensical answers to questions they have not been trained for. Another kind of duality, which is compatible with our framework, regards the question (X^*, P^*) whether "how you see some issue (X, P) changing?". In that case, the white perspective X^* is given by iP and $P^* = iX$; hence, one notices that $X^{**} = -X$, $P^{**} = -P$ meaning that "your vision upon the change of your vision of the change of a certain issue" is the same as the opposite vision on that issue. This is an interesting consequence of our language which goes beyond mere word play. As a final example of issues which do refer to one and another regarding the spacetime symmetries, one may consider the components of the energy momentum vector of a particle, here the white realities attached to each of them boost in one and another upon changing your own motion. So, albeit the operators do not change, a point of view we also took in our work upon the operational formulation of quantum field theory, and albeit the decision is white, the wavefunction itself must transform under a different representation as the one considered here. Indeed, we have only considered questions so far which do not "rotate" into one and another upon changing your reference frame, but the extension is fairly easy to make: denote by $(M^{ab})^c{}_d = \eta^{ac}\delta^b_d - \eta^{bc}\delta^a_d$ then transforming the white momentum operators X^b of your brain currents as $\Lambda^a{}_b X^b$ and the associated antihermitean "position operators" P^c as $((\Lambda^{-1}))_b{}^c P^b$ clearly preserves the algebra (note that we raise and lower indices here with respect to the Euclidean metric), given that

$$(\Lambda^{-1})_b{}^c \Lambda^a{}_d \left[P^b, X^d\right] = (\Lambda^{-1})^{bc}\Lambda^{ab}1 = \delta^{ac}1$$

Then, a Lorentz transformation is given by

$$\widehat{\Lambda} := e^{-2\alpha_{j0}M^{j0}\otimes\sigma^3 + \alpha_{jk}M^{jk}\otimes 1_2}$$

and the reader verifies that

$$\widehat{\Lambda}^T \omega_4 \widehat{\Lambda} = \omega_4$$

as well as

$$\widehat{\Lambda}^\dagger 1_4 \otimes i\sigma^1 \widehat{\Lambda} = 1_4 \otimes i\sigma^1$$

as it should. This last property implies that all associated charges are Hermitian as they should. One notices furthermore that the Lorentz transformations leave the A indices invariant and therefore it is possible to just consider the components A^{i0}_{j0} of the profile matrices which transform as a matrix in the defining representation of the Lorentz group; hence, it is possible to couple those with the classical currents $g(\tilde{e}_a(x), \nabla^C_{J(x)} \tilde{e}^b(x))$.

As a final comment, we must mention that the mental world is inherently, at its core, definite and classical; for example the consciousness parameters[14] t_j and the profile matrices A are classical whereas the choice projectors $\pi_{j,A}$ constitute a finite dimensional quantum system of $2N$ states. Regarding definitenes, we clearly argued, by means of our first duality where you question the white reality of a profile attached to an issue, that there has to be a certain level at which we are all white otherwise nothing of value can be communicated in this world. This makes the dynamics of the profile and choice variables trivial at that level. At lower levels of embedded issues, nature aspires definiteness so that the entire dynamics for one issue, which has 9 free real variables, has a 3 dimensional submanifold as attractor. That the world is inherently classical has been stressed many times by the founding fathers of quantum theory and it seems to be somewhat of a forgotten lesson for those who apire an exclusive quantum view on the world. In that regard are the brain currents guiding the equations of motion of the profile matrices as well as the choice projectors classical between two conscious observations. These classical currents must of course be related to their quantum counterparts as determined by the last observation of the latter; (undergoing a classical dynamics which is the "classical limit" of the quantum dynamics) but I shall not engage in a precise description here, that is way beyond our current level of understanding. So keep this in mind when I discuss these issues further in the next chapter.

[14]They are also crucial for determining when a measurement of some quantal issue should occur.

Chapter 3

Deeper exploration and consequences.

Now that we have discussed in some detail the appropriate language of issues, thoughts and feelings, let me now come to the more interesting part, which is the dynamics. This shall be subject of this chapter; the next one will deal with further nontrivial insights regarding the mechanism of social interactions which are even more complicated as I have outlined so far. Note that we shall draw nontrivial conclusions from our mere language only and those should be subject of further investigation.

3.1 General discussion.

In this part, we shall further engage in the kinematical setup explained in the previous chapter meaning we shall try to formulate some constraints upon the dynamics; ultimately, the goal is that you should be able to program this theory on a computer such that the different persona in your programme engage in a meaningful conversation. This what I call psychology and I can assure you that the standards are way higher as those of accredited psychologists. This work is a result of the reflection of someone who studied Jung and Freud's works at the age of 13 and later went on to study exact sciences, more in particular physics and mathematics. Jung and Freud's writings are muddled, mystical and lack any grounding in a more fundamental way of reflection about the world. Moreover, there is no clear separation between morality, sociology and psychology and one experiences as well a profound lack of understanding regarding the biophysical underpinning of their "science". I thought of Freud's ideas as banal, way too simple to be even considered; this was just story telling, there was no process of falsification, I mean this was "not even wrong" to state Wolfgang Pauli. Jung was far more interesting what concerned his observations; for example, he would find out several examples of the same symbols, paintings, artwork in different cultures which lived on distinct continents and never had any contact with each other! This suggests that there are many things we have in common which go beyond our perceptions and even our history. Randomness of the Darwinian process would suggest a wild variety of different traits, but that did not appear to be the case. He gave a place in our psyche which should "explain" this phenomenon, which is our collective unconsciousness. Now Jung did propagate a lot of ideas which are, in my view total nonsense. One of them is that the goal of life is to discover and become your true self. It seems obvious to me that you are always your true self even if you tell lies to others or hide your ideas where you would prefer them to be in the open. What the grandmaster suggested is that we should engage in our unconsciousness; a totally ridiculous thing to do. My unconsciousnss regards all fine processes in my brain or even in a separate spiritual world which are simply not communicated to me on a level I am unaware of. I don't care about such things as any decent scientist should! Only mystics and mentally troubled people (including many professional mental healthcare workers), who have even not the slightest understanding of the miraculous ways our physical world operates, indulge themselves in that kind of "armchair" philosophy. In this book, I make a serious effort to be precise, so you can agree or disagree with me; but, at least, we can discuss about something! This is not to say that their descriptive approach is not worthwhile studying but one is left with very little if no understanding at all as well as with a myriad of epistemological adventures which belong to Alice in Wonderland. In other words, the approach is not scientific, just as botany and anthropology are not. The aim of this part, is a modest attempt to fill in that gap; to provide for very accurate definitions and to *explain* why things are the way they are from very simple principles. In other words, we enter the area of *predictive* psychology based upon very few observations which are usually not behavioristic in nature.

The limitations set upon our kinematical framework, as explained in detail in the previous chapter imply that we shall study mentality at a level of poor "intelligence" albeit I shall, as promised previously discuss a bit of how logical principles embedded into the dynamics might lead to emergent rationality; this is for now the best we can do. An approach to higher intelligence will require new *principles* of language formation, something which is still beyond our grasp at this moment in time. Further ideas regarding this topic will follow at the end of this book but are by no means complete nor at the stage where proper quantitative, but nevertheless qualitative, investigation becomes feasible. It is my philosophy that any person in society deserves an optimal satisfaction as long as gratuitous murder and world domination do not belong to the personal desirata; indeed, this might be part of the ultimate goal of societal life and could very well be encrypted in the dynamics. It is my hope that at least the viewpoints put forwards in this book will constitute a ground for reflection. Discussions about morality and ideology, in my opinion, belong to the lofty saloons where big men can enjoy cocky woman and Cuban cigars.

Up till now, it must be clear for the reader that I completely negate the delusion that one can know the intend behind someones actions, that it is possible to know someones emotions and certainly that it is pointless to contradict a person speaking openly about his or her intentions or emotions. There is no way to know these things and therefore it is pointless to discuss it from a scientific point of view. People should just stop thinking in this way regarding societal interactions which is the well known foundation for religious murder. Long live Copenhagen quantum theory in this regard, that its pragmatism may serve as a lesson for peaceful and respectful communication. On the other hand, a person yearns for epistemology, for an explanation why we are the way we are and where our thoughts originate from even if this subject is dead from the scientific point of view. That is, an irrational urge for an explanation behind human rationality is a firm part of our being and it needs to be dealt with too. Privately please and not by general policy makers! Historically, the church fulfilled that part and nowadays meditation centers as well as private psychologists, as intelligent conversation partners, are there to fill in that part of our lives if mandatory. In this regard, total privacy as well as absence of any reporting should be guaranteed. The psychologist is no doctor and in case of serious worries about a client should do everything in his or her power to send him off to a medical doctor; by no means should he directly contact a physician himself. This book is not about learning how to be a wicked conversational partner but about the basic physico-spiritual laws behind low intelligence psychic interactions. I will explain why these laws are the way they are and discuss the basic observational ramifications.

3.2 Dynamics of questions.

In this chapter, we shall put forwards some further principles any suitable theory behind basal psychological interactions should satisfy. Intelligent conversations usually require something as creativity and insight and it appears to be difficult to find out a theory about that. Indeed, current AI is limited to finding statistical distributions associated to standard answers to certain types of questions by feeding the system with a lot of text. Once you would ask it something which is weakly correlated to the texts it has devoured, the probability of getting garbage is pretty large. This is not how the human mind works given that AI reads many more books as humans do; we also know how to deal with conflicting information and critically make up our own mind, I presume AI is nowhere close to that level. Up till now, we have dealt with positions and dichotomies regarding issues, but we have not suggested any basic theory behind the very nature of the formulation of those issues. Here, I believe that computer scientists have added a valuable point of view by means of binary codation of data as well as commands (questions, actions). Binary numbers are 1 and 0, whereas words are of the form $1001011\ldots$, sequences which are shaped in time, where at each instant, a letter is chosen. Quantum mechanically, we consider quibits

$$\cos(\theta)|0\rangle + e^{i\beta}\sin(\theta)|1\rangle$$

indicating the probabilities for $|0\rangle, |1\rangle$ to be chosen as well as the interference between both. This means that at each instant, both 0 and 1 are allowed for and that interference between 0 and 1 is possible with measurement giving 0 with probability $\cos(\theta)^2$ and 1 with probability $\sin(\theta)^2$. In standard probability theory, one would only dispose of two positive real numbers which sum up to one and not dispose of an angle β which is "forgotten" but plays a dynamical role for sure. Indeed, β can be seen as mystery, an unknown factor in our ways of communication which can only be measured if we know exactly how to replicate the state

$$\Psi = \cos(\theta)|0\rangle + e^{i\beta}\sin(\theta)|1\rangle.$$

This is approximately true in simple experiments in physics with an infinite number of degrees of freedom where the circumstances are so rough that the details of the state do not really matter in the outcome of the experiment; for example, variations on tiny length scales do not matter if the experiment probes for the behavior on scales far larger than those. Indeed, the behavior of humans in the desert does not really differ from one and another whereas interactions with a beautiful companion of the "opposite" sex might differ substantially. This unknown factor indicates also that different realities co-exist at the same time; in the binary system above, two distinct questions do suffice.

In physics, we call such a simple quantum-system a quibit: it is the fundamental ingredient behind quantum computing, where a system can only be in two quantum states. Putting N quibits after one and another, we have the potentiality to form 2^N words of length N with $2^N - 1$ real components of mystery. So, the degree of disorder in such system is $N + 1$ which is nothing but $\log_2(2^{N+1})$ which is the Shannon-Von Neumann entropy associated to this system. Indeed, it is meaningful to regard disorder in this way as a word is equivalent to one message no matter how long it is. However, the complexity of the message usually increases with the length of the word or the number of words and therefore N could equally stand for that. So in a way, the higher the disorder, the more complex it becomes and this is also how we experience society. So, a language is therefore always embedded in

$$W := \oplus_{j=1}^{S\,\infty} \otimes_j V$$

where V is the one quibit space, $\otimes_j V$ is the j-quibit space and $\oplus^S$ means that we sum up over words of different length in any order[1]. It is reasonable to assume that at any instant in time, a person has at least one element out of quantal word space W in mind. In case this is not so, then the person is totally dead; otherwise, depending upon the complexity of the quantal word, it is gradually more (un)consciously alive. Usually, what we call a dead person, is still alive in a way; it is just so that the spirit of the body is totally dead, meaning no quantal words are formed anymore at the highest operational level of the person, but the atoms and molecules making up the person are certainly still alive. This is

[1]Note that W also contains the empty sentence.

the most accurate definition possible of being dead or alive. Notice that alive does not imply conscious, so this goes beyond the usual "*je pense donc j'existe*" if thought is being restricted to conscious acts.

In this regard, the language formation process has to be interpreted as a process where more complex text $T \in D$, where

$$D := \oplus_{j=0}^{\infty} W$$

is a possibly infinite ordered collection of sentences, can be formulated. In this book, we mainly study the dynamics of V which we define as the lowest level of complexity possible; speculation about higher language formation and principles valid therein shall be postponed for the future. Here, a comment is in place, I am talking about language formation and not language recognition, something which Chatgpt is very good at; the latter is no miracle indeed, if you feed enough text (to learn it how to build sentences) to the system as well as the complete Oxford dictionary, where you define an equivalence between words and indicate which ones are more posh than others, then you can effectively learn it how to reformulate a text in a posh language. Here, we are interested if one could describe the origin of language as arising from a simple dynamical process, a (Darwinian) evolution. If this were not possible, then we have to conclude that our speech is in the hands of the creator, just as we create the ability of computers to recognize patterns, that it is a gift which cannot be understood. Likewise, with the development of language, comes the development of attributes α_j to words, sentences and text. So, in a dynamical picture of evolution where not the entire Platonic space of ideas has been encoded upfront (in principle of course) , we must conclude that the manifold as well as the vector bundle grows in dimension through the intervention of intelligence and not measurement (which sticks to questions already known); so, if such an evolution could at all be described in a mathematical language, then this needs to be of a historical origin which goes way beyond our own lifespan and needs to be passed on to our siblings who are totally unconscious of it; to be a bit provocative here, even elementary particles should have some innate property of collectiveness build in, leading to atom and molecule formation as they project down often enough on the pure energy atom state. Indeed, why should an electron be in a stationary arbit around the nucleus? It could "free" itself from the tyranny of the photon field by spontanous localization and then slowely drifting away by being in a superposition of higher energy states which are farther removed from the nucleus.

Most scientists, including myself, are interested in finding biological markers for our mental capacities, which, as mentioned previously, is only part of the explanation; but it is for sure an imortant topic to study to what extend our physical constitution "lifts" towards the spirit, meaning that we dissect the person as much as possible and see how far our reductionist point of view on the world carries. It could simply be, in a way, that spirits attached to N binary composite entities cannot give "meaning" to the full space of 2^N classical words;

in either complexity or disorder does not simply add up, in our definition it is sub-additive; indeed, not every N letter word has a meaningful interpretation such as **cdkz** does not make any sense in english. Therefore, the complexity of an N-bit spirit is less or equal to the sum of the complexities of the individual ones; and in practice it is much-much less as we know that a gas in equilibrium forgets about all the small details of the colliding atoms and can be effectively described in terms of 3 intensive (T,p,μ) and 3 extensive (S,V,N) variables. The problem with collective spirits usually is that its complexity might be less than the one of its "members" due to destructive interference processes, a well known phenomenon in societal life where the community is usually much less refined than its most complex individuals. Complex life forms require basic laws of nature which offer room for stability on sufficiently long time-scales; only gravitation and electromagnetism, which, in a way, make life posssible, are also in position to destroy the universe in the long run. Therefore, some scientists say, that the mere existence of humans, with their complex form of interaction, who create societies whose ingenuity may oscillate in time and not even have a mean positive growth factor, is equivalent to evidence that, when it really becomes necessary, the human endeavour is a divine one (cosmologists think our universe is a fix and physicists are entertaining the anthropic principle in these days). I, on the other hand, still want to advocate a kind of Darwinian universe where spirits with extreme complexity come and go periodically; given our current poor understanding of such issues, I proclaim that happy indifference is the best way to live with this uncertainty. If the Gods play it as such, then they are for sure compassionate with me; otherwise, in absence of their existence, I am for sure more devilish than I know of.

Given a lack of a natural bio-physical understanding at this point, albeit recent research has correlated brain activity to certain "presumed conditions of the mind", we shall confine ourselves in the sequal in discussing the interactions between the vectors determining the probabilities of which choice projector $\pi_{A,j}$ to apply (for any j) on one side and the perspective dichotomy matrices on the other. More direct interactions between matter currents and those matrices-vectors must exist but these issues are far too complex to consider at this point. We shall discuss this issue in detail in the next chapter.

A practical question is how to find out the natural values α_j corresponding to white perception of issues? In physics, we are pretty lucky that the gravitational field (as well as the external electromagnetic field) is weak and as good as time independent so that a metal bar does not change in length in our perception. The natural measure stick therefore is one where meters are expressed in fractions of a metal bar and where the kilogram is defined in a similar way. Again, we are lucky here; in principle Einstein's theory about the relationship between spacetime geometry and the stress energy tensor of matter, assuming the independence of the gravitational motion of an object regarding its internal constitution, except for its rest-mass[2] leads to an infinite mass renormalization and a slight warping of spacetime around that object. In quasi-static gravitational fields, this renormalized mass is almost constant which allows again for the introduction of a unit of mass. Likewise, do atomic clocks determine a natural unit of time and it is a miracle that in those units, which are associated to physical processes, the local speed of light is constant in all directions of space! Regarding our mental variables, we are by far not that lucky; first of all, quantities such as length and time are easily associated to real numbers, but how about feelings or perceptions? Even if they could be modelled by a real number (which we assumed so far), what would be the natural unit, the divine reference frame? We can only guess and it would for sure be helpful if one could measure the physical brain energy consumed by a mental thought!

Given a finite set of N issues, as well as natural flat inertial coordinates in psychological space associated to rigid local space-time measure sticks (since we are searching for a physico-spiritual correspondence), we shall now look for technical generalizations of this idea. Up till now, we have assumed that the psychological variables covered the entire real line, which is easy from a technical point of view since it provides for a unique representation of our Lie algebra; in case the domain would be a finite interval, then we have ambiguities originating from the boundary condions and technically, it is impossible for one of the white-black operators to have a finite spectrum. The psychological space (we just concentrate on the α_j variables here) at hand is empirically (as most scales in psychological tests are) determined as an N-dimensional convex space with the barycenter as origin. A convex space of dimension N is a part of $\mathbb{R}^N$ such that the line element connecting any two points belonging to it, also belongs to it. So, in one dimension, there is exactly one line element whereas in two dimensions we have a polygon. A convex space is bounded by subspaces of lower dimension. Those of dimension zero, in either points, are called extremal elements; that is, they cannot be the midpoint of any nontrivial line segment within the body. A piecewise flat simplicial manifold is a space which is formed by means of gluing convex spaces together along the boundaries. The flat space metric is given by

$$s^2(x_1, x_2) = \sum_{i=1}^{N} (x_1^i - x_2^i)^2.$$

It is important here to ensure consistency of this procedure by ensuring that axes with a different dimension (unit) cannot rotate into one and another and that scaling always has to occur with respect to space-time units. This means, that if we take rigid objects determining mass and length, then scaling of the length or time by λ induces a scaling of the mass by λ^{-1} (holding $c, \hbar$ fixed) which means that a metric of the type

$$s^2(p_1, p_2) = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

[2]Actually he started from the assumption that the motion of such particle should not depend upon its mass.

will transform as

$$s^2(p_1, p_2) = \lambda^2(x_1 - x_2)^2 + \lambda^{-2}(y_1 - y_2)^2$$

in case x denotes length and y mass. In general relativity, such scalings also renormalize Newton's constant by λ^2 so that on very small scales (small λ) masses blow up, but G goes to zero. In our setup, it does not make sense to consider scale invariance of the white operators X by a factor of λ and P by the inverse of λ since such a transformation coincides precisely with a boost around the z axis; there is no need to treat the z-axis different from any of the other spatial directions. At first sight, you might object and say that meter and kilogram are mere conventions and we should be able to redefine them at will; you are free to do that in the material world and the mental world will automatically follow the new conventions and rescale its appreciations (which of course coincide with measurement) appropriately.

As mentioned previously, an important part of the dynamics of an individual's profile and choice regards its interaction with others, even elementary particles seem to have this trait at a very basic level. So, the way communities organize themselves must be the consquence of a balance equation between personal desirata of the spirit on one hand and the will to socialize on the other. If you want to convince others about your profile and choice (as well as the particular state of another (related) issue in case you take the conservative viewpoint there, but that appears to be of secondary importance[3]) regarding a certain issue in order to reach more harmony, then you will have to be extremely wicked and cautious in order to succeed. This appears to be another balance equation which is that the urge to change others in their way of approaching an issue, comes with great care and effort. I believe our innate profile is full of balance equations as such and it would definitely be interesting to find a more basic description behind those.

In general, each mental characteristic must manifest itself in physical reality by means of actions; the trouble is that most of our actions are an effective product of many distinct "traits" and that it is generally impossible for an outside observer to disentangle those (albeit they would like to believe they can). Mathematically, actions form a closed system, a mathematical group which must translate as a projection of our brain state. It is maybe useful to comment upon why we choose to work with the real number system in quantifying states of issues, emotions and so on. The idea is again an operational one of to unite and divide, that is plus and quotient; if we choose one nontrivial unit, then the addition leads to the natural numbers whereas division leads to the positive rational numbers. Introducing the neutral element as well as an antagonist or the

[3]For example, in an attempt to convince others to be progressive regarding the state of their country and focus on its change instead of on their fixed perception, you might convince them that it is reasonable to rebel and eventually reach a consensus on where to go in the long run. It is usually of primary importance to make them rebellious, the rest are details to be discussed later on.

opposite includes the negative rational numbers. Closing those in the difference metric gives one the real numbers. The fact that division and addition seperately do not depend upon the order of its arguments are called the principles of commutativity and associativity and mathematicians have played around with non-commutative and non-associative systems such as the octonions which may express a higher awareness. Given that we think elementary particles are rather silly and simple, the real number system is more as sufficient for these purposes and different dimensions are assumed to commute. However, elementary particles in the quantum world have shown to add a slight complication to this idea be somewhat more complex being that your current manifestation does not commute with the current impetus (change of manifestation). In a way, we are forced into the Einsteinian view (regarding the generality of the basic laws) as too strict theories, with little or no internal symmetries, who attempt to predict someones profile and conservative choices without allowing for any liberty usually run into conflict with reality. For example, too strict constraints and balance equations could lead to a humanity where everyone is the same: a society without great leaders or scientists. As a final comment of a technical nature, we discuss the particular origination of the metric, as well on spacetime as on the appreciation-consciousness space, from a scalar product. The latter is completely determined by the requirement that the act of projection preserves the addition on the smallest scales; that is, the projection of a sum of two quantities is given by the sum of the projections. It is an expression of the fact that God loves pieceful recognition at the shortest possible scales. In that vein, chaotic or fractal geometries are not considered.

3.3 Dynamics of the choice projectors.

In our discussion in the previous chapter, we related the symmetries of the local profile matrices to the spacetime action of the local Lorentz group. I believe that the mental attraction or repulsion between two minds is not in the first place dominated by the way we look at things (dichotomy) but by the (upper or lower) choice we make regarding those issues. The combination of the dichotomy and the profile choice is what we call the decision; the reader must understand that two distinct profiles can make the same decision, even a realistic (either self adjoint) one, but they may differ in upper-lower choice. In other words, the angle (choice) from which you sell your decision is more important than the decision itself and can lead to repulsion either attraction. For example, realistic profiles on the lower side are given by

$$A = \begin{pmatrix} \frac{-i(ca+1+ida)}{b} & c+id \\ a & ib \end{pmatrix}$$

assuming $b \neq 0$. This is in my experience a true fact of life, often it happens that two persons say the same thing but from different angles and one person gets accepted whereas the other one rejected. To initiate the discussion, notice that in the theory of Dirac particles, all local symmetries of the scalar product

$$\overline{\psi}(x)\psi(x)$$

are given by $SO(3,1) \times_T U(1) \times U(1)$ where T stands for twisted. Indeed, one $U(1)$ is associated to the 4×4 identity matrix, and another one is associated to γ^0, but this generator anti-commutes with all boost matrices $\mathcal{J}^{0j}$ so that the action gets a twist. The twist has not been accounted for yet in physics, but the remaining part has; the effective local symmetry group of the Dirac particle being $SO(3,1) \times U(1)$ where the connection associated to the first group is delivered automatically by the vierbein and the $U(1)$ part is the usual electromagnetic 4 vector field. In this vein, one can always choose a gauge where the Dirac field consists out of one real spacetime field only. The choice variables for one issue form a complex two vector satisfying

$$C(x) = \left(\begin{array}{c} \Phi(x) \\ \Psi(x) \end{array} \right)$$

and the only thing that we require is that

$$C^\dagger(x)C(x) = 1.$$

The full symmetry of this scalar product is of course $U(1) \times SU(2)$ which is precisely the same as the internal symmetry group of the electroweak interactions. In contrast to the case of the profile operators, it does not make sense to associate the $SU(2)$ part with rotations in some eigenspace given that a rotation around 90 degrees would turn an upper choice into a lower choice, which is of course utter nonsense. So, the whole group is an internal group; but just as happens in the electroweak interactions, where one has a proper definition of an electron and neutrino (breaking the $SU(2)$ gauge invariance), likewise do we have here a proper distinction between upper and lower choices. This suggests for a pretty identical application of the Higgs mechanism by adding upper and lower $SU(2)$ singlets (as a replacement for the right handed electrons and neutrino's) to the theory and coupling those to the doublet and Higgs spinor in order to create different masses for the upper and lower profiles. I leave it up to the reader to decide whether he insists upon this implementation of the Higgs mechanism to be classical or quantum and therefore looking upon the $C(x)$ spinor as a classical or quantum constrained entity satisfying $C(x)^\dagger C(x) = 1$. From a classical point of view, which is somewhat more unusual, one can understand the symmetry breaking[4] at several levels: (a) the physical Higgs field H should

[4]Actually, there does not exist any agreement in the literature upon what it means for a dynamical symmetry to be broken classically; some simply say that if a particular solution breaks this symmetry, meaning that the orbit is not invariant under this symmetry, then the symmetry is spontaneously broken. This is not a very useful point of view, another definition would be that you define the class of a configuration as all configurations which can be reached by means of a physical (exterior) operation. One then says that the symmetry is spontaneously broken if such class is not invariant under the symmetry transformations. Of course, a moot point here is how to define preciesly what you mean by a physical operation; usually, this is thought of as being associated to an observer, but how to change the orbit of planets around the sun (which are not circular)?

have suitable falloff conditions towards spatial infinity such that there exists no realistic operation which can turn one ground state value v for the Higgs into another one (boundary conditions) (b) the physical Higgs field H is much smaller in absolute value as v is (in case the universe is spatially compact) so that you limit the possible space of solutions. There are some distinctions with the electroweak theory however, some of which are certain and others maybe uncertain meaning we don't have enough data here to make the decision. To illustrate those, let us start by making the folowing observations:

- lower profiles regarding an issue flock more more together in large groups with high density (that is another reason why I called lower profiles in the canonical dichotomy black, since they cause for "spiritual black holes"),

- upper profiles are more solitary amongst one and another and attract each other more on larger distances

- lower and upper profiles repel one and another on short distance scales to the effect that the conversation shifts in topic where reconsilliation can be achieved

- lower people act faster regarding this particular issue as upper profiles do; acting to change something requires less time and effort than actions wanting to sustain a definite point of view.

These mere observations would suggest, from a traditional point of view, that the interaction fields have as source term minus the charge density of the choice field, so that alike charged particles attract one and another; more in particular, the unbroken $U(1)$ part must lead to universal attraction between both types, meaning they have charges of the same sign; whereas the broken $SU(2)$ part would lead to a dominant short range repulsion between the two opposites. So, the spiritual world seems to be the opposite in that regard of the physical world, possibly creating instabilities, whereas the latter has a steady lowest energy state. Another remark is that one would like to couple the choice field to the profile field as well as to the psychic reality (the wave function). Regarding the latter, it is clear that the choice field has no canonical action (since it is no group for instance) on the mental state and therefore cannot couple to it directly; we have to use the coupling to the profile field instead. But there is still another possibility, we can use our unitary representation of $SO(1,2)$ which is a non-unitary representation of $SU(2)$ as well as $SO(1,3) \sim SL(2,\mathbb{C})$ to provide for a nonunitary action of the $su(2)$ gauge field strength on the mental state of the universe. Indeed, for our $U(1) \times SU(2)$ gauge field $A^a_\mu(x)$ where a runs from 0 to 3, we could take the operator

$$F^a_{\mu\nu}(x) J_a(x)$$

where $J^0(x) = 1$ and the $J^j(x)$ have been defined previously and the generalization thereof to multiple issues is canonical. Note that those operators are spacetime dependent; indeed the X_j, P_k operators, pertaining to distinct issues, were also thought of as belonging to a single mind localized at a spacetime point which we assumed to be somewhat smeared out so that one has effectively a countable number of minds only. The mind was assumed to evolve according to the classical currents $J(x)$, so that our issue operators (but not the wavefunction itself) are effectively dragged along those currents. Note that we really use two different times here: on one hand you have the psychological time t_p which is associated to the wavefunction and the latter is supposed to collapse in this time whereas the time t_j we aspire here is the "personal mental time" which is associated to the classical body of the j' th observer. So, beware, this is not traditional quantum field theory! Of course, given an initial hypersurface Σ of constant time t_p, we can all set our times t_j such that $t_j = t_p$ and the position of our body is given by $\vec{x}_j$. Hence, for later times t_j, t_p there is a unique mapping from all spacetime points y reached by an observer, as long as two observers do not occupy the same point, something which we exclude, to the index j of the observer. So, we really should have denoted X^j_k, P^j_k where j denotes the observer and k the issue at hand. Regarding the profile matrices, choice fields and gauge fields, which do not depend upon the psychological variables, we already did that since those depended upon the spacetime location y and the profile and choice fields were attached to the observer: they do not evolve according to a partial differential equation, but merely obey an ODE with respect to the personal currents $J(x)$. The gauge fields on the other hand do obey a partial differential equation which causes for propagation of profiles and choices. To make these ideas concrete, one must reflect that a local gauge transformation correspondingly affects psychic reality; indeed, if the probabilities for an upper-lower choice shift, then also reality shifts nontrivially! This is an extension of what is usually meant by spontaneous symmetry breaking, that the state on which the theory acts does not possess the symmetries of the dynamics and, moreover, that the questions we ask are not gauge invariant themselves. Since the generators J^2, J^3 are anti-Hermitean, we must again import an operator T, like in section 4.2 which commutes with J^1 but anti-commutes with J^2, J^3 (there S did the same thing but then with respect to J^3 instead of J^1), so that we can construct scalar products of the kind

$$\langle\Psi|TF^a_{\mu\nu}(x)J_a(x)|\Psi\rangle, \quad \langle\Psi|TF^{\mu\nu\,b}(x)F^a_{\mu\nu}(x)J_b(x)J_a(x)|\Psi\rangle$$

where the first term can be coupled to something anti-symmetric such as

$$J^{[\mu}(x)\nabla_{J(x)}J^{\nu]}(x).$$

Under a gauge transformation $U(x) = e^{i\alpha^a \frac{\sigma_a}{2}}$ which acts upon the wavefunction as $\widehat{U}(x) := e^{i\alpha^a J_a(x)}$ one has that $|\Psi\rangle \to \widehat{U}(x)|\Psi\rangle$ and $(F^a_{\mu\nu}(x)\frac{\sigma_a}{2}) \to U(x)(F^a_{\mu\nu}(x)\frac{\sigma_a}{2})U^\dagger(x)$ where the latter yields for infinitesimal α^b that $F^a_{\mu\nu}(x) \to (\delta^a_c + f_{cb}{}^a\alpha^b)F^c_{\mu\nu}(x)$ whereas $\langle\Psi|TJ_a(x)|\Psi\rangle$ transforms as

$$(\delta^d_a - f_{ae}{}^d\alpha^e)\langle\Psi|TJ_d(x)|\Psi\rangle$$

so that both combined give

$$(\delta^a_c + f_{cb}{}^a \alpha^b)(\delta^d_a - f_{ae}{}^d \alpha^e) = \delta^d_c - f_{cb}{}^d \alpha^b + f_{cb}{}^d \alpha^b = \delta^d_c$$

as it should. Note that ideally, we would give explicit formulae for T, S but it appears that this requires further study of the particular (non-unitary) representation of those operators on Hilbert space and we shall leave such investigations for the future. It is my mere intention here to show that everything is consistent and that three different kind of groups are represented on the wave function by means of the same operators. Finally, in the above, it is understood that $\Psi(\alpha^j_k)$ and that $J_a(x)$ acts according to X^j_k, P^j_k for one particular j and that the scalar product is taken over all α^k_l, thus over all issues and all minds (observers). It must be understood here that we work of course in the unitary gauge and that gauge transformations are never performed; they are just an aspect of the dynamics and not of the interpretation thereof. Given that $\widehat{U}(x)$ is non unitary, we have to renormalize and therefore divide the interaction terms through

$$\langle \Psi | T | \Psi \rangle$$

which we assume to be nonzero.

The above view opens the door for psychic interactions given that the state is entangled which, in a way, are very real. Sometimes people are attracted towards one and another without any good reason or prior communication. Concerning the choice field, we shall give a brief qualitative view here and resort to the so-called dipole "Coulomb" approximation, ignoring mass terms and Yukawa type corrections to the potential due to the broken $SU(2)$ part. Regarding the situation of N spatially separated persons and one issue only, we obtain that the spatially integrated densities

$$\Phi_i := \frac{\int_{\mathcal{B}_i} \Phi(x) \sqrt{h}(x) d^3\vec{x}}{\sqrt{\int_{\mathcal{B}_i} \sqrt{h}(x) d^3\vec{x}}}$$

with upper-lower components integrated over the spatial bodies of the person and with $\Phi(x)$ of slow variation over the body. Then, one could consider corrections to the "perspective mass" of the i'th individual, due to interactions, as follows

$$M_i = a_i ||\Phi_i||^2 + a_{ij} \sum_{j \neq i} |\langle \Phi_i | \Phi_j \rangle| + b_{ij} \sum_{j \neq i} ||\Phi_i|| ||\Phi_j||$$

is the most general formula possible where the a_{ij}, b_{ij}, c_{ij} are coupling functions depending upon other physico-spiritual entities as well as an average distance between the bodies using the length scales set by the coupling constants of the theory. In a way, those are needed to include the last term which does not depend upon Φ_i and $b_{ij} > |a_{ij}| > 0$ given that otherwise M_i can always become negative which is forbidden. The coupling functions vanish in the limit for distances r_{ij}experience dictates that sexuality plays an important role in the interactions. One may consider what kind of other issues would mix with the dynamical law for this particular single issue. In the next chapter, I shall discuss some issues which I believe to be a foundational importance, in a way resembling the holy trinity in religion, meaning that they interfere with any issue and clothe our communication. One such variable is sexuality, modeling it by means of a binary variable S_i where $S_i = -1$ if and only if the subject is male and $+1$ if it is female, then a simple expansion gives

$$a_{ij}(r_{ij}, S_i, S_j) = \frac{\widetilde{a}_{ij} + \widehat{a}_{ij} S_i S_j + \ldots}{r_{ij}}$$

and likewise so for b_{ij}. Resorting terms gives

$$M_i = a_i ||\Phi_i||^2 + \sum_{i \neq j} \frac{\widetilde{a}_{ij} |\langle \Phi_i | \Phi_j \rangle| + \widetilde{b}_{ij} ||\Phi_i|| ||\Phi_j||}{r_{ij}} +$$

$$S_i \sum_{i \neq j} S_j \frac{\widehat{a}_{ij} |\langle \Phi_i | \Phi_j \rangle| + \widehat{b}_{ij} ||\Phi_i|| ||\Phi_j||}{r_{ij}}$$

leading to the conclusion that $\widetilde{b}_{ij} > 0$ (the potential energy of upper-lower communication is positive, leading to repulsion) and $\widetilde{a}_{ij} < -\widetilde{b}_{ij}$ since identical choices should overall attract and therefore lower the energy. There are corrections to this depending upon the sexuality; in this regard, we take the viewpoint that interactions between opposite sexes with identical choices have negative contribution (causing for more attraction) meaning that

$$-(\widetilde{a}_{ij} + \widetilde{b}_{ij}) > \widehat{a}_{ij} + \widehat{b}_{ij} > 0.$$

Furthermore, we assume that opposite sexes with the opposite choice leads to less repulsion (and maybe even attraction), leading to $\widehat{b}_{ij} > 0$ ($\widehat{b}_{ij} > \widetilde{b}_{ij}$). This formula then suggests the following observations:

- interactions between upper-upper (lower - lower) choices result in overall decrease of the mental energy of each individual leading to attraction and a feeling of lightness (this effect is stronger between opposite sexes as between the same sexes)

- interactions between upper-lower choices lead to repulsion in the case of opposite sexes (increase of individual mass, a heavy feeling), but still might cause for attraction between the opposite sexes.

3.4 Further symmetries.

There exist plenty of issues which we can think about, and wonder about our profile. Next, we may consider taking an action (which contains as well information regarding an issue and possibly your profile thereupon) of expressing yourself. Now, there exist several possibilities here; either this new issue (consideration) refers to your previous thought, or it is only tangential to it. In case it refers to your previous thought and profile thereupon, would you also express your profile and if you would express a profile, would it be the same as the one you just considered? For example, I can wonder about punching someone on his face and think I definetly have a straight answer in mind (white profile) regarding this issue, but I could just communicate that I was thinking about it and, in case I decide to express my profile, I might utter that it is complicated, that there there are pro's and con's. We shall now concentrate on this very last possibility, that you faithfully express the issue you were considering but you may lie a bit about your profile. Actually, what I claim really matters is not the conceiling of your true thoughts, everyone does that to some extend, but the way you alter your expressions accordingly when your thoughts change. To be precise, we have an action profile matrix q_a and a thought profile matrix q_t, the latter which can undergo a change by means of an *action* q_i and we must ask how this change affects the action profile matrix. For example,

$$q_a \to q_i q_a,\ q_t \to q_i q_t$$

defines straight types, meaning the action responds in the same way as the mind does, and

$$q_a \to q_a q_i^\dagger,\ q_t \to q_i q_t$$

defines the maximally twisted types, meaning the action is just the opposite. Mixed types are those who twist themselves to some degree, meaning for example that

$$(\beta_\lambda(e^q))(q_a) = e^{(1-\lambda)q} q_a e^{\lambda q^\dagger}$$

where $\lambda \in [0,1]$ and the reader notices that

$$\beta_\lambda(e^q e^w) \neq \beta_\lambda(e^q)\beta_\lambda(e^w)$$

due to non-commutativity of q, w except in the cases $\lambda = 0, 1$. Mathematically, the reader should get used to the terminology that β_0, β_1 are called the vector and conjugate vector representations, $(\beta_{\frac{1}{2}})^2$ is the usual conjugate representation which is equivalent to a Lorentz transformation in the defining representation. It must be said that it is possible to just consider those types as actions of a change e^q on action profiles not necessarily referring to a corresponding change in the thought profile by means of the vector representation. There exist two distinct natural conjugations on the profile operators q, which are the complex conjugation $\overline{q}$ and the charge conjugation $q^c := \sigma_2 \overline{q} \sigma_2$ and

$$q^\dagger = -q^c$$

in case q has vanishing trace. Now, one may wonder to what extend they should relate to symmetries of interactions between two distinct spiritual beings regarding this particular issue. We define w to be self-dual in case $w^c = w$ (or $\overline{w} = w$) or anti self-dual in case $w^c = -w$ (or $\overline{w} = -w$). The first condition means that

w has no charge whereas the second one says that its charge conjugate is minus itself. We say that w and w^c transform accordingly if and only if

$$(\beta^c_\lambda(e^q))w^c = ((\beta_\lambda(e^q))w)^c$$

and likewise so for the complex conjugate. We can now consider a pair of action profiles, located at nearby spatial locations and consider the joint profile as a single profile by means of the following

$$w \otimes v^c \to q = wv^c; \text{ with action } (\beta_\lambda(e^x) \otimes \beta^c_\mu(e^y))(w \otimes v^c) \to (\beta_\lambda(e^x)w)(\beta^c_\mu(e^y)v^c)$$

as well as

$$w \otimes v \to q = wv; \text{ with action } (\beta_\lambda(e^x) \otimes \beta_\mu(e^y))(w \otimes v) \to (\beta_\lambda(e^x)w)(\beta_\mu(e^y)v)$$

and

$$w \otimes \overline{v} \to q = w\overline{v}; \text{ with action } (\beta_\lambda(e^x) \otimes \overline{\beta_\mu}(e^y))(w \otimes \overline{v}) \to (\beta_\lambda(e^x)w)(\overline{\beta_\mu}(e^y)\overline{v}).$$

In some exceptional cases, these projections do define actions themselves; for example

$$(\beta_\lambda(e^x)w)\,(\beta^c_{1-\lambda}(e^{-(x^\dagger)^c})v^c) = e^{\lambda x}\,(wv^c)\,e^{-\lambda x} := \gamma(e^{\lambda x})(wv^c)$$

and the reader should notice that $(x^\dagger)^c = -x$ since x must be traceless. Similar results hold for the other two choices:

$$(\beta_\lambda(e^x)w)\,(\beta_{1-\lambda}(e^{-(x^\dagger)})v) = e^{\lambda x}\,(wv)\,e^{-\lambda x} := \gamma(e^{\lambda x})(wv)$$

and

$$(\beta_\lambda(e^x)w)\,(\overline{\beta_{1-\lambda}}(e^{-\overline{x^\dagger}}\overline{v}) = e^{\lambda x}\,(w\overline{v})\,e^{-\lambda x} := \gamma(e^{\lambda x})(w\overline{v}).$$

One notices that $\lambda = \frac{1}{2}$ is special and in all cases, we would first look at the associated transformations e^x that preserve duality meaning respectively that $x^\dagger = -x^c$ (which is identically satisfied) $x^\dagger = -x$ and finally $x^\dagger = -\overline{x}$. One notices furthermore that such "alligned actions" on action profiles naturally lead to invariants (conservation laws) such as are given by $\mathrm{Tr}(wv^c), \det(wv^c)$ in the first case. They are for sure useful in everyday conversations where people are adaptive to one and another. It remains to be seen how these mathematical symmetries should further reflect in the dynamics.

Chapter 4

Psychic symbols and social interactions.

This chapter is by far the most outlandish one in this book; we have argued so far that there are very close parallelisms between the physical and mental world with one huge exception which was that mental energy appears to be negative and that therefore, the spirit has an unstable ground state, craving for action and physical energy consumption. Ultimately, the body runs out of energy by means of the mind-matter correspondence and tames the spirit in its tendency to consume. Indeed, to further elaborate on this, a human eats, drinks, thinks, moves around by itself very much in contrast to, say, a (steam) engine where the burning of fuel forces the cylinders to compress. If you would not couple an engine to a heat bath, it wouldn't do anything at all and remain at rest. The mind therefore is distinguished by a will to live, to be active; I don't claim that in the future highly advanced artificial intelligence would not be able to look for its own energy resources to be active and therefore also to have an effective will to live, even if not programmed to do so, given that I see no rational basis for the claim that such aspects of life should be limited to organic structures only. But what I do claim is that such a thing would require a change upon the traditional viewpoint of the lowest energy state in physics. It is for the moment, as far as I know, an open question as to where the electrical signals in the brain come from; for example, in a computer, a vibrating crystal is responsible for keeping track of time even if the power has been switched off. Maybe, our brain also contains such vibrating structures with a certain lifetime, sending impulses to the heart to pump and to the lungs to breathe. Even if that were the case, then it would still not explain why we feed ourselves or even why we know such a thing as hunger or appetite in order to survive. We are "programmed" as such and it costs energy to maintain all those functions; physical systems don't feed themselves, they simply undergo and survive most comfortably in the common lowest energy state possible. They harmonize in this sense, whereas the spirit does not. When I first wrote the text below, I was very much attracted to the idea of mental energy centers in the physical body, correlated to our state of mind and behavior, which closely resemble the chakra's in the Indian literature. I will without any shame use this terminology and speak about it in a realist sense beyond mere philosophy even if I cannot pinpoint as such a physiological grounding at this moment in time. Aside from that, I also tend to think that there must have been an (evolutionary?) mechanism in our psyche based upon mental images and concepts, which I shall call archetypes. As usual, I will try to be as mathematical as possible as this opens new perspectives to old ideas.

I shall use words here which are mostly used by mystics, such as astral eye, which basically refers to the divine light in you which you can actually see. Many schizophrenic people see shadows and hear things which most people don't; instead of calling it a desease or disorder, which explains nothing, I will take a more scientific point of view here and try to explain why we would see such things in the first place! From the point of view of physics, this is entirely possible, we are blind and deaf as hell; we only see a tiny bandwith of the electromagnetic spectrum and our ears likewise have a limited range. My tolerance here towards such a viewpoint stems from the fact that I have once in my life experienced such a thing for a couple of months myself, something which lawmakers and psychiatrists call psychosis. I can confess to you that my experience was nothing like what DSM V describes; these images are very real, I could see the most complex three dimensional figures in my mind spnning and oscillating at random in the utmost detail. My perception was much more sharp as it now is, you start to see connections everywhere, nothing is random any longer and likewise is this so for the voices which definetly seem to come from outside of your brain. I have no mercy, neither any affiliation with those butchers of the mind who look only at the most superficial and irrelevant things. I know of people who claim they can see aura's attached to physical bodies; it is not so that they want to be interesting or heard and who would I be to judge that this is delusional. Even if I would say it is, that is still no explanation of why they see such things in the first place and it is a downright insult to those cultures in the world who do recognize the existence of such a thing. As explained in part 1, there is definetly something connecting us all which goes beyond our observations, so let us carry this idea a bit further and maybe develop some theory. Other mental "reference subjects" which I shall use are the belly and the heart, where the former refers to both food and gut feeling whereas the latter is the source of life and compassion in the symbolic sense. What I want to suggest here is that those physical and metaphysical meanings are not formed by accident and deeply ingrained into the dynamics which shapes humanity as we know it. Indeed, the association of life with mercy is deeply embedded in any religion and we all recognize this as valuable and something to aspire. Finally, I will invoque such terms as marriage and sexuality as being central to the human endeavour. As always, our fundamental dichotomy remains the upper-lower choice one can take; let us now further introduce what I want to speak about.

Since we have rationally identified mental interactions with gauge theories, we must take the concept of psychic or mental radiation seriously. Indeed, sometimes you are in awe (or just the opposite) for a certain person without knowing why; he or she seems to radiate something quite mysterious and it is not that there is any rational ground for it regarding our basic senses such as smell and eyesight. Indeed, the girl in question may even not be physically very appealing or having a sweaty odeur, she is nevertheless breathtaking for a completely unknown reason. She radiates! I believe that we subconsciously detect such thing all the time and that we are drown by such mechanisms to alikes; you just knew there was a click from the very beginning. You did not have to talk to her, you did not have to experience her naked body, you just felt that she was fine for you. I think there is no rational ground to dismiss this as an illusion, quite the contrary! Another ramification of our findings so far is that upper-lower fields destroy the positivity of mass (so the mind anti-gravitates) whereas the gauge bosons obey it and therefore gravitate. Hence, people with a huge amount of (negative) psychic energy within themselves have the experience of being lighter in the head (floating in the air). This is also very real, I feel light in my head when thinking deeply, but after a while the body protests due to the enormous amount of fuel I am burning and I have to take a nap and some food. In the subsequent discussion, special attention will be paid to the Switchoriem as he was the reference person of the free theory without facts, logic and so on which all favour an upper profile in the end as evidence builds up. I will further define the astral (or third) eye here as being able to see this psychic radiation, allowing one to probe the "soul" of another person (of course from your perspective, so there is little or no objective value to that). The mental belly reflects then how someone feels in a given context; these are two distinct things, you can be drawn to a person by her beauty but still it might not feel all right. Last but not least, you have the heart which shows mercy or empathy; a heart can be so called warm or cold and likewise so for the belly and eye. These three symbols or functions are not issues, there is no profile neither (upper-lower) choice in them, but they constitute mere (quantum) variables which are not only functions of the psychic variables but also act upon them as to define a closed algebra[1]; just as consciousness was a (classical) variable and not an issue. Another few comments are in place here, first of all physicist's don't speak about issues but about variables. I illustrated this by means of our eyes who seem to have no freedom to take a profile, neither choice: for them, everything can effectively be described at the level of the conservative variable operator

[1]By this physicists usually mean that some function of the dynamical variables $F(z)$ defines an action on any other function $g(z)$ of the dynamical variables by means of, for example, $F(z) \star g(z) - g(z) \star F(z)$ where $\star$ is the Moyal product (which amounts to the usual definition of the Poisson bracket by means of $\{f(z), g(z)\} = \lim_{\hbar \to 0} \frac{1}{i\hbar}(f(z) \star g(z) - g(z) \star f(z))$.). The requirement that several functions $F_i(z)$ constitute a Lie algebra amounts then to the condition that $F_i(z) \star F_j(z) - F_j(z) \star F_i(z) = \sum_k c_{ijk} F_k(z)$ with c_{ijk} a complex number. One could even strenghten this for $\hbar \neq 0$ by demanding that $F_i(z) \star F_j(z) = \sum_k d_{ijk} F_k(z) + d_{ij} 1$ where 1 denotes the canonical central extension of your algebra. In any way, in a system with the operation a Lie algebra, one can always look of course for representations, which possess a natural product underlying the Lie bracket, such that the resulting algebra is a mere central extension.

(unless you turn blind or so). Second, so far we have assumed that all our conservative/progressive issue operators were commuting and that you could mix them. This is not true in general either, in nature, it is very well possible to construct variables (conservative issues) which do not commute and do not constitute our fundamental dichotomy either; moreover it often does not make sense to mix arbitrary issues as that amounts to comparing apples with pears. The really daring thing I propose here is that the way you primarily interact with people by preferring either your perception of the heart, the eye, or the belly, will have ramifications on your physical constitution; the way you look, whether or not you get fat during a marriage and so on. I believe that distinct cultures also have different traditions of approaching the way they prefer to interact with others in the sense to what element of the above trinity they find the most important one; that is a second conjecture if you want.

A parameter which is important in the way people interact, and can be seen as a white appreciation of the issue "to what degree am I energized mentally?" (so there is no need to enlarge our language here, we just have to include one extra issue), regards the mental energy they use (radiate?). If a person mentally engaged in a conversation speaks to someone who uses way less energy, then either the other person can upgrade and engage in more activity or, on the other hand, try to downgrade the speaker either by shifting the topic of the conversation or serving some food which lowers mental activity. As said, the astral eye can see the psychic person but still then, it remains to determine wether this information results in an attraction or repulsion as the two extreme opposites; the former being called the hot eye and the latter the cold one meaning that (minus) the average energy of the eye is high or extremely low. This coincides with our previous philosophy which was that the mind loves to increase (minus) its energy and is therefore attracted to anything which amounts to this effect and repulsed towards anything which causes for the opposite. Life is not that simple and we involve in a much more complicated way with one and another mixing the eye, belly and heart. It might be worthwhile to quantify this more as it appears to me that nature has forseen that, in engaging with one and another, there usually is a minimal "temperature" associated to the trinity (which in physics would amount to a choice of density matrix, a kinematical and not dynamical constraint); this implies that, for example, even if the expectation values of the individual energies attached to the eye, heart and belly are close to zero, we still engage ourselves with a minimal amount of psychic energy leading to an overall satisfaction in engaging with others even if not very gratifying on several points (so, we use a different "Hamiltonian" as just the three separate individual "Hamiltonians"). I say, usually the case, because some people are cold as hell and still thrive in society at a formal high level; very dangerous I must add. A third conjecture I make is that this overall temperature (an effective parameter describing the more detailed and complicated upper-lower interactions) constitutes the main foundation of societal interactions which goes way beyond things such as intellect and money. For example, you may have a deep friendship with a person who does not resonate with you on an intellectual level, whereas you might want to restrict the number of contact hours with a person with whom you intellectually thrive but otherwise are rather cold of. Let me also mention that the association of the belly to "gut feeling" is by no means a gratuitious play of language, given that eating and drinking together while talking leads in general to a better feeling (and therefore more patience) regarding the person/conversation. Indeed, family gatherings over dinner or meetings between partners in the restaurant are aspects of fostering community life in many cultures. I claim even more than this which is that your psyche (upper-lower choice, profile, white reality parameters, consciousness parameters) is correlated to the kind of food you like and which in a way is the best for you.

In a way, the trinity expresses how we see, feel and relate; three cornerstones of human interactions. But we already had four cornerstones of upper-lower interactions, those were given by the generators of the classical $U(1) \times SU(2)$ gauge theory (the profile operators did not define a new gauge field as explained in part 1). Now comes the real beef, we have discussed in part 1 upon how we can associate the Pauli matrices with (anti) hermitean operators on the Hilbert space of square integrable functions in the white variables so that classical variables can couple in a nontrivial way to a quantum system; what we proclaim now is (you may call that conjecture four) that the trinity coincides with the $SO(1,2)$ part and its associated local charges (which are globally conserved)[2]. This implies that overall, in the entire society, global empathy, spiritual perception and feeling are conserved quantities; they do not evolve in time. At the level of a single individual, which we shall discuss first in full detail, this implies that the correct evolution operator in the psychic variables is given by a multiple of the identity operator if one takes quantum corrrections into acount (classically the Hamiltonian vanishes)! Pretty boring indeed, but a reflection of the fact that without social engagements or any exterior world, our conceptions of what some of us think what is and what is not (note that the notions of back and white are collective ones, since any individual can choose its own profile) do not alter. If we were only to interact with "simple" matter, then there is little or no engagement and the reality, as seen by some of us, remains the same. In a way, this expresses conservation of three distinct types of energies; this is a pretty damning constraint upon humanity which I believe to have been experimentally verified already over history. Humanity does not change fundamentally, we still go to war for the most silly reasons, most of us have very limited empathy and are mostly interested in telling about themselves, enlightened minds keep on having difficulties with authorities, life does not feel any different now as it did in the middle ages and so on. The only way humanity progresses is by means of paradigm shifts; meaning an old lunatic now has become sane, but at the same

[2]Note that the algebra holds at a local level of charges for local gauge symmetries, for example as is the case for the $su(2)$ gauge transformations in loop quantum gravity by means of the Ashtekar variables or in non abelian gauge theory by means of the time component of the Noether charges. For global symmetries, the situation is not far worse given that integrated charges will only deviate from the correct algebra by means of boundary terms.

time a new category of lunatics is born providing for a countereffect. Never ever has society reached the only valid conclusion, after so many generations, which is that lunatics don't exist, only enlightened spirits think as such and there are not too many of them. Concretely, the heart corresponds to $H \sim -\sigma^1$, the belly to $B \sim i\sigma^2$ and the eye to $E \sim i\sigma^3$. How those operators were connected to the other Pauli matrices and what suggestive relationships exist between them has been discussed in part 1. The heart is nothing but the Schitchoriem energy plus $\frac{1}{2}$, which is related to time not only by means of the Schrodinger equation but also by means of its very definition as a Hermitean quadratic form assoicated to the identity matrix! The eye preserves white and white and is therefore associated to E whereas the belly B mixes them; the curious thing is that for any operator O, which is a linear combination of X, P holds that

$$[B, [E, O]] = i\,[H, O]$$

and more in general

$$[B, [E, V]] - [E, [B, V]] = 2i\,[H, V]$$

for any operator V. The spiritual heart is therefore associated to the "awareness-impetus" or conservative-progressive. Hence, it is meaninful to consider thermal states with respect to the heart or Schwitchoriem Hamiltonian and calculate expectation values of all other operators, such as

$$\frac{\mathrm{Tr}(Ee^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} = \epsilon$$

where β is the inverse temperature of the state and ϵ the mean energy corresponding to the eye. Regarding the spiritual belly operator, $i\sigma^2 \sim B$, we notice the following conjugation

$$([B, O])^c := \left[iE, [B, O]^\dagger\right] = \left[iE, \left[B, -O^\dagger\right]\right] = \left[B, \left[iE, O^\dagger\right]\right] = [B, O]$$

for O a real linear combination of X, P and the superscript c denotes the charge conjugate by means of (i) the eye operator. This is completely equivalent to

$$\left(\sigma^2 v\right)^c := -\sigma^3\overline{(\sigma^2 v)} = \sigma^3\sigma^2\overline{v} = -\sigma^2\sigma^3\overline{v} = \sigma^2 v$$

where $v = (aX, bP)^T$ with $a, b \in \mathbb{R}$ and $\overline{v} = (\overline{a}X^\dagger, \overline{b}P^\dagger)^T$. So this means that the conjugate action of iE on the adjoint action of B equals the adjoint action of B and therefore charge conjugation intertwines between both actions. A similar result holds for the heart operator. The heart operator is positive definite and has a discrete spectrum of the form $n + \frac{1}{2}$ where $n \in \mathbb{N}$ meaning it is always activated. B, E on the other hand have a continuous spectrum which covers the entire real line; for example, the eigenstates associated to E are given by

$$(-is\frac{d}{ds} - \frac{i}{2})\Psi_\alpha(s) = \alpha\Psi_\alpha(s)$$

resulting in

$$\Psi_\alpha(s) = \frac{1}{\sqrt{2}} e^{\frac{i}{4}(2\alpha+i)\ln(s^2)}$$

and the reader may verify that all orthogonality properties are satisied meaning

$$\int ds\, \overline{\Psi_\beta(s)}\Psi_\alpha(s) = \delta(\alpha - \beta).$$

Regarding the belly

$$B = e^{-\frac{i\pi}{4}[H,\cdot]}E$$

which is a unitary transformation and hence preserves the spectral decomposition. So therefore, either the belly or the eye can be off, meaning having zero eigenvalue. It is furthermore possible to consider states which are invariant under $e^{i\frac{\pi}{4}H}$ up to a unitary factor; these regard superpositions of the form

$$\Phi_k := \sum_{n=0}^{\infty} a_{k,n}|8n+k\rangle$$

where $k \in \mathbb{N}$. In this case,

$$e^{i\frac{\pi}{4}H}\Phi_k = e^{i\frac{\pi(2k+1)}{8}}\Phi_k$$

so that

$$\langle \Phi_k|E|\Phi_k\rangle = \langle \Phi_k|B|\Phi_k\rangle.$$

In particular do we have that for thermal density matrices

$$\frac{\mathrm{Tr}(Ee^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} = \frac{\mathrm{Tr}(Be^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$$

so that both have the same energy and

$$\frac{\mathrm{Tr}(He^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} = -\frac{d}{d\beta}\ln\left(\mathrm{Tr}\, e^{-\beta H}\right) = \frac{\beta}{2}\coth\left(\frac{\beta}{2}\right)$$

as a small computation reveals. Another interesting observation is that $H - B = -P^2 \geq 0$ and therefore $H \geq B$; by means of our rotation, we likewise obtain that $H \geq E$ meaning the heart is dominant over the eye and belly. As mentioned in part 1, when thinking about an issue, we use the dichotomy and upper-lower choice to get an operator $Z = aX + bP$ and we can only measure its energy defined by

$$Z^\dagger Z = |a|^2X^2 - |b|^2P^2 + (\overline{a}b - \overline{b}a)(XP + \frac{1}{2}) - \frac{1}{2}(\overline{a}b + \overline{b}a)1 =$$

$$(|a|^2 - |b|^2)B + (|a|^2 + |b|^2)H + i(\overline{a}b - \overline{b}a)E - \frac{1}{2}(\overline{a}b + \overline{b}a)1.$$

Hence, any decision operator canonically defines the way we communicate with one and another: a remarkable conclusion! For example, Schwitchoriems communicate by the heart, white people by $H+B$ and black people by $H-B$. Also note that the heart is the only pure (non mixed) choice regarding the trinity you can make. To communicate with the eye, you need $b = i\lambda a$ with $\lambda \in \mathbb{R}$, this results in

$$|a|^2(-2\lambda E + (1-\lambda^2)B + (1+\lambda^2)H).$$

Now, before we come to the fascinating issue of mental superposition between two partners, where they forget about anything whatsoever and are consumed by a blissfull feeling, something which happens when you are quietly sitting with your wife in the couch and think about nothing, time passes quickly and you just experience "temperature". To make the analogy with Wagner's Tristan und Isolde when she is going to serve him on the ship the love potion which will make him forget about who he is and join in a blissfull union. First, we shall treat distinctions between several nationalities which I have observed in the past and which approximately seem to hold. To be precise, the Belgians and the Dutch communicate maximally by means of the belly, meaning they are either white or black and there is no grey zone; the result is that those people either have an opinion about everything or they revolt the system. Indeed, the Dutch, which are black, are known for their outspoken opinions even if there is no rational, compelling reason to be as such: this leads to a vibrant debate culture with lots of specific suggestions being made; the downside is that this leads to a society where everything is classified and subject to social conventions and the Dutch are as such indeed, leading to a serious embedding of psychiatric institutions to cure those who are different. Belgians are somewhat more white-black mixed leading to less rules but more conflicts and social anxiety. Both cultures have the tradition of heavy food, beer and a complete absence of spirituality, something which resides in the eye. The Catholic church in Belgium is almost dead and the Lutheran church in the Netherlands is not one of spiritual beauty and contemplation, but one of strict adherence to rules and debates about biblical interpretations. They just don't live through a religious ceremony, and I even guess they do not understand what this is supposed to mean. The Polish and the Italians mainly go by the eye, they are very spiritual and kind hearted people; the Polish also seem to involve the belly to a higher degree as the Italians do. Indeed, their food is also certainly more healthy as the one in Belgium and Holland, but very basic and certainly not as light and delicious as Italians cook. Polish also drink beer, but by far not as much as the Belgians and the Dutch and they consume more spiritus which, as the name says it, stimulates the eye and not as much the belly. Italians on the other hand drink more wine and liquor which are drinks of the heart and are somewhat a bit more spiritual as the Polish. Indeed, Italians and Polish are known for story telling, not debating; this results in somewhat a less dynamical atmosphere as the one in Belgium and Holland, but opens the avenue for long and thoughtful conversations and loads of creativity. Indeed, Italy has long been the cultural centre of the world and food consumption there merely accompanies a long conversation and social gathering, just as this is the case in Poland, whereas Belgians and certainly the Dutch are "functional eaters" spending little time at the table. The French and Swiss in my opinion almost equally share the belly and eye and the heart, leading to healthy, solid food with a good culture of wine and a bit of beer and a nice spirituality: those operators are given by $s \leq r \leq (\sqrt{2}+1)s$ or $r \leq s \leq (\sqrt{2}+1)r$ such that $\frac{r^2-s^2}{2rs} = \sin(\theta - \psi)$ and $a = re^{\theta}$, $b = se^{i\psi}$ leading to

$$(r^2+s^2)H + (r^2-s^2)B + (r^2-s^2)E$$

and the reader verifies that the ratio $\frac{r^2-s^2}{r^2+s^2}$ is optimal at $r = (\sqrt{2}+1)s$ resulting in $\frac{2(1+\sqrt{2})}{2(2+\sqrt{2})}$ leading to $H \geq \pm\frac{(1+\sqrt{2})}{(2+\sqrt{2})}(B+E) \sim \pm 0.707(B+E)$. It must be clear here that I am speaking of issues for which no logical settlement has been established yet; but there is more than that, in conversations people will always have the tendency to select the issue as such that such communication mode is justified. For example, when talking to a Dutch person about a mathematical theorem, he will simply give an answer and that's it. Italians, on the other hand, might start discussing the underlying assumptions, the beauty or the ugliness of the result, and novel ideas regarding counterexamples if some assumptions are dropped or generalizations thereof. In my opinion, as far as experience with both cultures reaches, this is indeed the case.

4.1 Spiritual bounds.

Now, we come to the discussion of joint states or spiritual marriages if one wants to. Many of us have had the strange feeling of synchronicity by which I mean that highly correlated events occur without an obvious causal explanation from known mechanisms. Now, science tends to dismiss those as pure coincidences, but I think the occurances are just too high the be explained as such as is the case for the speed of evolution of the human species, suggesting for something way beyond random selection. Now, of course, I am realistic enough to set bounds to which this an be realized in practise, just like Schrodinger's cat will probably never materialize. So, channeling of conscious thoughts does seem to require too much information for such mechanisms to carry; but I am speaking of phenomena people often experience, like for instance me calling my ex wife or vice versa and she saying, "I was just thinking about you and thought you maybe you felt like this". Or, people travelling around the globe going to places where they have never been before and meeting people as if it were predistined to be as such, as if they knew these persons for their entire life. I am fed up with the priests of modern science who wish to impose upon you (even by law) that this is an illusion of some kind; that we are all classical separated individuals who can be detached from society with razor blade precision. Some of them even try to ridicule the idea by so called designing "objective" tests without realizing that, by this very act, they fall within the same category as all those people, which they call crackpots themselves, denying quantum mechanics because they never have seen a Schrodinger's cat. I hear you already say, surely mister Noldus, I am sane of mind, I also have noticed those things but I do not see why they could not be ascribed to a classical mechanism. I can only say, why bother? I mean, ultimately, each mystery of quantum theory may be ascribed to a classical underlying mechanism. But the point is that if this were the case, then our view upon what a particle really is and how it behaves energetically should be radically overthrown! For example, you can explain the Schrodinger equation as being the result of a kind of self-field interaction of the particle, causing it to spontaneously accelerate and self-interfere; nobody has ever constructed such equations but it may very well be possible. Likewise could entanglment be explained by the opening of wormholes which are stable for a while; but this requires negative (mental?) energy, so is a particle mentally active? Does it microscopically change our very perspective upon spacetime; that we are blind to such things. In the previous section, we explained that the full wave function lives in an infinite tensor product

$$\mathcal{H}_T := \otimes_{i=1}^{\infty} \mathcal{H}_i$$

where $\mathcal{H}_i$ denotes the Hilbert space of square integrable functions with respect to the flat metric attached to the mental issues defining spirit i. These Hilbert spaces all have a representation on spacetime which is evolving towards the future. Hence, we already have that spirits are entangled, but here we shall go one step further by considering entangled operators or so called "contracts", as I prefer to call them, where individual observers take expectation values of the individual heart, belly and eye operators coupled to other eye, heart and belly operators after projecting down the communal state by means of their own mental energy profile operator (and possibly the profile itself if the latter is definite). This is the weak form of contracts, where people still think for themselves but care about the feelings of others. The stronger form is when you effectively do not project down by means of an individual operator, but society projects down on a complex entanglement of individual operators, so that you effectively stop thinking and feel the resulting state from other product operators attached to the heart, belly and eye respectively. They continue to live in a complicated superposition of product states attached to common values of the society of which they are not aware at the level of (maybe unconcious) thought, which presupposes individuality, but merely experience a certain temperature regarding an interaction operator which they freely choose (in either they feel the response of others regarding things they care about). The strongest form of collectiveness happens when you feel those things precisely in the way society wants you to feel them, that is when the individual heart, belly and eye operators are canonically defined by means of the communal operator used to perform the reduction of the communal wavefunction. This viewpoint is the mathematical realization of the dichotomy of individualism versus collectiveness at four possible levels (the fourth one being that you only consider your own isolated feelings and not how they interact with others); as mentioned previously, an isolated heart always feels (luke) warm but individual hearts in a collective gathering can be either very warm or cruel (negative temperature) as happens in bad marriages or oppressive regimes. Let us therefore refer to the isolated heart as the bored heart; there is a lot to say for it, but ultimately we all crave for variation and not necessarily in the positive way (more empathy, or hotter heart) but also in the negative way where people enjoy being cruel towards one and another and have destructive tendencies.

Let me stress that this is a rather unusual extension of quantum theory which always pressuposes that each observer must independently ask him or herself an isolated question and that different observers are asking questions at slightly different times. There are two issues here, if two distinct local observers would ask two different questions at the same time then there is only one collapse of the wavefunction and albeit the order in which the questions are asked does not matter, there is only given one answer which is a communal one, in either the product of both eigenvalues. This is the reflection of the fact that in quantum theory, there is only one global observer really who controls the entire universe, so in order to correct that you would have to change the usual interpretation in the sense that both observers should be aware of the result of their individual questions and this is indeed a well defined notion in relativistic (but not Euclidian) Quantum Field Theory due to causality of the local observables. In our approach to quantum theory, we refined this issue even more: we did not care about coordinates, so when a measurement apparatus measures the so called position of a particle, it does not produce any number, the latter is an interpretation a local human gives regarding the change of state of the measurment apparatus. There are really two different processes here which are usually identified as one and all details are swept under the carpet which causes for a terrible confusion. The first thing is that in first instance, nobody cares about the eigenvalue of some global product operator (which is a meaningless quantity), but they only deal with the projection itself which changes the state of each individual measurement apparatus and this is the only thing which is "felt" by the apparatus in some way. This is a radical change of perspective on quantum theory where observables pertaining to the relation between the outside world and inner world of the observers, a position Bohr very much defended, are not confined any longer to one local observer only and therefore one may forget about the eigenvalue (that only serves God). The real measured eigenvalue by the local observers regards some operator pertaining to the inside world of the observer who is measuring himself accordingly without further changing the outside world. In this vein is the meaurement apparatus "aware" of something hitting it by observing the currents of its constituting variables. We, as humans, come second in line, we observe photons scattering on the changed measurement apparatus and we presume that the measurement apparatus is faithful or static towards us in the sense that it did not alter its appearance between the moment it responded to the interaction with the particle and us interacting with the masurement apparatus by means of photons. Likewise, the observation our eyes make is one of local electrical currents which are coupled to the photon field and not the photon field itself; we just assume the photons project down accordingly and interpret the feeling (which we express as a number) attached to the state of our eyes, after all eyes in the universe have measured photons at the same time, as corresponding to an objective property of the photon. The measurement apparatus does exactly the same thing when so called measuring the position of a particle hitting it, I really measures "unusual" internal currents due to the impact of the particle arriving there, it only "knows" about the particle through perception of its own state and comparing it with its bias of what is it and what is the rest! So, an eye is getting classical at least every nanosecond and its state is what we ascribe to light with all it colours attached. For the mathematicians under us, you would say that this can be described within category theory where true observation happens on the level of objects and the impact of it reflects on the level of functions. Indeed, I claim that we cannot observe the outside world really but we can only know the response of our own body regarding the interactions with the outside world. We just have to assume that all photon measurement apparati are the same and producing commensurable results otherwise there would be no possibility for us to deduce any laws. I did not discuss energy-momentum observations of photons as yet in this book, but the reader may very well take a simplist approach and forget about the electrical currents inside the eye and just do Fourier analysis as usual of the photons as defined with regards to the local observers as if one were projecting down the photon field in first instance. This is something we do all the time in science to the extend that most even fail to comprehend that they are doing it. The observer is always swept under the carpet and people wrongly attach meaning to the values some abstract global operator attaches to the interaction of the observer with the outside world. As said, this is not what we can measure directly, we are only aware of our own body and ascribe some aspects of it as due to some unknown interaction with the environment, we are just guessing basically. The fact that we are guessing reflects in distinct opinions regarding what is internal or holistic: indeed, stomach pain could be thought of as extraeneous due to deamons pinching needles there. In the middle ages, this could have been the standard view, whereas nowadays we think of either bacteria or too much acid being responsible for the sensation. In the first case, it is still an "exterior" cause whereas no doctor would say that the universe is leaking extra acids into you stomach by means of wormholes. He would just say that you produce yourself these acids without even being able to understand the mechanism of how this happens: it is just a convention. This is what I would call a mental inertia principle, that we presume something happens when we experience something internally which we do not ascribe to ourselves, but this differs from person to person! Psychiatrists have the illusion that their idea of the interior-exterior world is the true objective one and that they can decide whether your senses are appropriate or not given that they presume their observation of the outside world and interpretation theirof to be the holy one; meanwhile, he or she is just having the same trust in this very principle. The reader must understand here that there is no truth in all of this: the reason why we value the doctors opinion regarding the acids in the stomach is because he prescribes calcium based medication or zantac and this helps. So, he has free will and can act to cure the evil. But nothing prevents you to deny that this is all just a matter of biochemistry, but that there is a deeper underlying reality in which the calcium serves as an offer to the spirits so that they stop putting acids in you. The reason why western society does not uphold such view anymore is because it is redundant in many ways; now, unlike zantac, psychiatric drugs have no effect whatsoever but just put you asleep so that the "issue" does not occur. It is very much like putting someone in prison in very many different meanings.

In this section, we go another step further, as mentioned previously, an observer in a community stops asking individualized questions (all questions themselves are entangled), it does not engage into independent thought any longer but merely feels, just like we feel temperature which is also not an observable quantity, the impact of society upon its emotions without adding anything to it. Indeed, when being indivualized, you just take expectation values, which is a form of higher awarenes, of your individual heart, belly and eye operators with respect to an (approximate) eigenstate of your decision operator; in a way, this is how you feel about yourself when dealing with your thoughts without thinking about others. Everyone, who is individualist, feels he is a compassionate person. In the previous edition of this book, I described the joint situation, which I want to discuss, as being the result of a measurement done by a higher joint spirit who sees us as quantum; I changed my mind of presenting it in that way because, albeit such a point of view is holistic in nature, there is nothing higher about this spirit at all in the sense explained in part 1 since you can always break free out of it at a higher level and return to individualism (consciously or unconsciously) at a lower level of issues. For example, in a two person society, the relevant operator may be

$$H \otimes (aE + bH + cB) + B \otimes (dH + eE)$$

where we have done the canonical decomposition with respect to the first individual. In case the two person spiritual state is $|\Psi\rangle$ then I define the "feeling" of the heart of the first observer as being given by

$$\langle\Psi|H \otimes (aE + bH + cB)|\Psi\rangle$$

and that may very well be a negative number indeed. Note that at this point, as mentioned before, you are not dealing with this issue in your mind at that level, it is not that it is mixed with other issues, which is a very different thing, but it is just not even definite. It pertains to questions about how others relate to your individual decisions regarding the decisions they have taken themselves. As I said, you basically stop thinking about your current profile on these matters (consciously or unconsciously) but you just feel the communal response at a higher level. Now, there is another higher awareness regarding this feeling in the communal spirit and that pertains to the issue of wether you like this feeling or not; for example, you have people who crave on complete adversity towards others. As mentioned in part one, we have excluded so far these higher questions referring to more basic questions (of possibly different observers) since in general, you cannot ask for a decision and heart at the same time unless you are a Schwitchoriem where all those notions coincide. So the best you can do is ask for an expectation value regarding those issues. In that way is individualism the safest strategy to go through life, you are inherently peaceful, mercy is the dominant force whereas in society you might become a killer such as happened, for example, to the Germans in the 1930 ties when peaceful people got in the grip of fascism and started to haunt Jews, something they would never done on an individual basis. People, who have a bad feeling when immersed in the collective "reality" might want to withdraw and isolate themselves becoming peaceful again; usually, this is seen as a danger to society as, especially prominent, members wish to cancel their engagement. In that way is the curch of immense importance since it urges us to be, in the first place, one with God and Jesus and they are warm, constructive in nature and hence offer a, perhaps imaginary, reinforcement of an individualist kind of attitude towards others. Indeed, religious life is a solitary one, far removed from the hustle and bustle of society. The importance of this cannot be stressed enough for example by means of the Polish people, were religion was the state enemey during communist times and people remained friendly overall due to the immersion in the beauty (spiritual eye) of the creation. Poles have finally stood up against communism and fascism during for example, the Warsaw uprisal. It makes them into an extremely resilient, creative and warm nation where I was happy to reside for a while. I will just give away some further examples of compound operators and try to formulate some principles behind them; not all compound operators are realized, actually very few of them, and one ultimately has to come up with some selection principle.

Before we proceed to that, let us introduce the notion of sex conjugation (another dichotomy, woman versus man) interchanging men and woman as well as their profiles/choices and all that: a Hermitian operator S which acts on $\mathcal{H}_T$ and for which the state of the universe is neutral (that is has eigenvalue close to zero). Here, it must be taken into account that there exist different dominant notions of rationality (logic) between the different sexes who aspire precisely the opposite. This has nothing to do with white and black as we all aspire to become white eventually and alikes still attract. Let me explain what I mean by means of some examples: men who aspire classical logic, which has been the driving seat of progress in humanity, study sciences, engineering ... whereas female logic usually leads to specialization into the social sciences, psychology, nursing, psychiatry and even medicine (which operates according to intuitionistic logic). The men who are in the middle between classical and intuitionistic logic usually study economics or law, whereas the intuitionistic ones choose for medicine, psychiatry and psychology. Indeed, doctors, psyhiatrists and psychologists are wizards from a classical point of view; this reflects in the fact that, especially psychiatry, changes of theory or point of view every couple of decades, whereas physicists and mathematicians never ever change their mind, they just refine and generalize their knowledge. Indeed, classical logic is stable, God given as to speak, whereas intuitionistic logic is temporal and suggestive instead of having a precise insight. Medical doctors also classify somewhat in this category but to a lesser extend; they would argue that it is the very nature of their field that it is as such, but this is total nonsense according to me. The great progress in medicine has not come from trained doctors, but from physicists, engineers and chemists (hard core boys), who have constructed microscopes, RX and MRI apparati and offered a deeper insight into the working and development of pharamaceutical drugs. Woman who want to break free out of their traditional role study precisely mathematics, physics and engineering, resonating very much with those men at least on the intellectual side. Remember that I have said that such resonance is secondary to the more primitive way of approaching relationships regarding the upper-lower perspectives. For example, you may meet intellectuals of the same kind who take different sides and profiles on things which are unknown; with a slight abuse of language, you may say that in this case the lower perspective is the more rational one (but that is a matter of taste). Such distinctions may lead to severe clashes between equally qualified collegues and cause for personal frictions whereas you might sympathize with someone who has a different kind of logic but who makes the same choice as you do. Ultimately, where intuitionistic logic and classical logic meet one and another, fruitful results start to emerge. Therefore, since men and woman procreate, there is always a mixture of different rationalities making room for real feelings such as given by the heart, eye and belly; things which are not completely rational but seem to stabilize society. In that sense might a civilization which is entirely based upon one choice of logic be wiped out in a very short while. Therefore, any civilization which is low on spiritual investment and thrives primarily on logic (of some kind) as the primary force, is bound to lead to psychiatric patients, psychopaths seizing power and huge divorce rates. This is why science and religion should coexist.

In order to find a principle limiting the possible joint operators which occur in the world, I constructed the notion of a marriage contract which constitutes the very basis by which your decisions couple to one and another: indeed, in a marriage it is important that you take commensurable decisions even though you might dislike the profile your partner attaches to it. Point is that you rarely discuss such profile and just deal with the practicality of doing similar things. So even though there may be a repulsion regarding the choice field, there are paradoxically enough warm feelings attached to the partnership which leads to marriages where both partners function together very well but dislike one and another. In analogy with the single individual where your decision determined your way of dealing with the trinity, likewise will we consider here that the way a couple deals with a certain issue (we shall restrict here to $N = 1$) determines how the couple projects itself towards society and how they feel themselves in the union. The big difference here is that, when people engage, they loose their own opinion and take a common position which is an entangled one. This reflects that when a couple speaks out, they never reveal their own thoughts but present a compromise in which different individual opinions live together in superposition. A further point is that you want to express yourself in the union as faithful as possible regarding your own values and that you want your partner to appreciate your decision even if he or she dislikes the side from which you approach it. In case of a mixture of the belly and heart, the marital contract MC could take the form

$$MC = \left(\begin{array}{cc} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{array} \right)$$

which agrees for $\theta = \pi$ with the heart $-\sigma^1$ and $\theta = \frac{\pi}{2}$ with σ^2 which is the belly. One would expect such theta angle to be a dynamical variable varying from culture to culture: in Belgium for example, people seem to love others primarily based on whether their partner feels good in their company or not (the marriage contract is one of the belly). In Poland, on the contrary, it seems (I am no expert in this) important that your heart is connected to the spiritual beauty of your partner. This is of course a direct consequence of the fact that Poles usually express themselves by means of the eye and Belgians by means of the belly as discussed before. Before we discuss this in somewhat more detail, notice that

$$\left[\ln(\sqrt{-P^2 + \epsilon^2}), XP\right] = \frac{-P^2}{(-P^2 + \epsilon^2)} \sim 1$$

is an approximate Heisenberg conjugate to the operator $XP + a1$. It is not an exact one which is logical due to fringe effects. Therefore, a "Polish marriage" is given by the operator, given that $(i\sigma^1)(\sigma^3) = \sigma^2$,

$$a(X,P)^\dagger(X,P) \otimes (X,P)^\dagger \sigma^2 (X,P) b(X,P)^\dagger \sigma^2 (X,P) \otimes (X,P)^\dagger (X,P)$$

$$= -4aH \otimes E - 4bE \otimes H.$$

I will leave further exploration of those ideas for a differentt book of mine on psychology which basically contains the same material as in this book but with further comments and ramifications.

Chapter 5

More on psychology and the kitchen.

In the past chapter, we have discussed how spirituality reflects itself in the kitchen. Now, I proclaim that the right kind of food can actually help a lot in staying healthy physically as well as mentally. Beneath, I provide for a list of ingredients which I experienced to be especially helpful in that regard.

- Baked mushrooms with fat cheese
- Camomille, earl green and plain english tea
- Gin and tonic with fever tree tonic
- Chocolate cake
- Cheeses: French or Danish blue (Roquefort), Gruyere and Raclette, Chaumes and ripe brie (not to be put in the refrigerator), in general all other creamy or milky cheeses.
- Polish or german fish dishes such as mackarel in sour cream with onion and pickeled cucumber
- Olives (preferably black) in garlic and sunripe tomato sauce; Spanish chorizo with russian salad and manchego ham.
- Wine; cabarnet sauvignon, shiraz, chianti, merlot based wines, champaign, Toro (red Spanish wine - bull's blood: highly recommended)
- Greek moussaka
- Turkish or Greek mezze
- Cherry cola (or pepsi) as well coca cola as Fentimans or any type of cranberry juice

- Strongbow (the original one) or Guiness
- Lamb chops (baked with little salt, some lime and a tiny amount of garlic in greasy butter)
- lukewarm Bolognese sauce with garlic french bread dipped in Swiss raclette
- Hot peppers, Jalapenos, english cottage cheese, goat cheese, (mexican) salsa sauce; perhaps poored into a delicious dish of nachos
- vegetables: chicory, beetroot, ginger, brussels sprouts, parsnip, sunripe tomatous
- Pototoes; always in the form of chips or as second choice croquette fried in italian or greek olive oil
- no rice, noodles or any kind of pastery
- chop choy with chinese duck
- Polish beetroot soup barcz or jurek (sourcabbage with cream, eggs, sous-sages,...)
- Cheeses: preferrably goat cheeses (feta, goat cheese with herbs), english cottage cheeses and cheddar (medium).
- Grilled or smoked fishes: mackarel, salmon, haddock, mussels, crevettes with garlic and olive oil or soya
- Raw fishes: salmon, traut with wasabi or ginger and soya.
- no baked fishes; steamed fishes are allowed but have no value.‘
- Raw steamed beef with raw eggs, pickles, pickled cucumber and french fries with mayonaise furnished with sunripe tomatous.
- Beers: Bombardier, Peroni, Polish beers such as Zywiec, Warka and Czech beer such as Pilsner, Dutch beer such as Bavaria, Belgian beers in general.

With these foods, you do not need to watch your calory intake, just eat as much until you are satisfied and that is all.

Chapter 6

Final thoughts.

I shall be very brief here: the quest of this book is a noble one, to engage in a truely profound understanding of the very basic characteristics of human communication. Alas, the practise is ugly as hell and higher thoughts creep in the dynamics such as “der Wille zur macht”. I can only affirm that Nietschze was right on the ball in his “between good and evil” and “Also sprach Zarathustra”. To say it bluntly, surely you are not going to tell me that justice does not comprehend that psychiatry and psychology are total bullshit?! Everyone sees this at first glance! There are no mental deseases, just expelled and rejected spirits because they simply are the way they are.

Indeed, it appears to me that things such as ego, vanity, jaleousy and “der Wille zur macht” constitute the main source of humanities problems. I deem the very noble cause of trying eliminate those and improve upon the situation is unfortunately a lost one. The thing we have learned in this book from a purely mathematical point of view is that the safest and by far easiest solution is to force people into individualism, to abandon the social workplace as it stands now and promote work from home. Even the current form of schooling should in my mind be discussed given that specialised interactive education packages exist which do allow one to obtain a good education without the need for going to class. The very best scientists in the past have provided for the very possibility for such a world to exist: factory labour is largely not required any longer given that most processes have been automated by machines. Office work can easily been done on a distance and the gifted researcher might have no business at a university office; he or she can interact by means of the internet with alike minds. Conferences are in my opinion crucial for academia, but then in a novel sense, where things are discussed thoroughly and the gathering is not overwhelmed by means of nonsensical lectures. The only places where people should flock together and socialize is in the sports club, the church and maybe, once a week, in a bar or good restaurant. Of course, some elementary public duties need to be carried out, such as the community workers picking up your garbage or the bus or train driver to operate his vehicle; but all the rest can largely be done from home. In this way should politics be completely revised, social pillars should seize to exist and practicality should be the new ideology. In other words, life should focus again on family, a couple of friends and God; maybe not in the theological sense but in the spiritual one which I have outlined in this book. Work should be meaningful and encourage innovation and creativity, there should be more artists, scientists and musicians. People would, I believe, be better of in this way and be mostly under control. Indeed, police and justice which constitute in a way complicated social professions, would have far less impact and prominence. The duty to go to church once a week and to engage in a spiritual life should be written into the constitution. In this way, wise people will govern the system and not technocrats or so called specialists who are currently dangerous as hell. As long as real science does not understand those very basic and bad emotions which lead to the utmost forms of cruelty, I believe this solution to be the human optimum. In that way can I tell you that Jezus wanted too much, it is not realistic to proceed as such; maybe it can be in the future. This is all I have to say about this.

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