# Proceedings on qualitative and quantitative psychology: restrictions on higher senses.

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# Contents

**Some words upfront.**

This second small book, or cahier, provides the reader with a deepening of the understanding of the basics of the theory laid out in the previous one. In particular, we shall theorize here about the possibility of broadening ones senses as well as ones intelligence. Indeed, we have posited a duality between elementary thought processes and mechanical actions on one side, and one between mechanical actions and senses on the other completing the triangle. But what now if our thought processes also become actions corresponding to a higher material reality? For example, the black-white theory in the previous book dealt with spiritual communications of a kind resulting in lower level actions of moving towards or away from one and another or even biologically, growing or getting smaller. The theory, as formulated there mathematically, supposed that those spiritual interactions could only travel on the space of lower senses given that they constitute the reference frame as to speak. From the scientific point of view, this is entirely logical due to the duality between those spiritual charges and the lower senses of rotating into three space or feeling with ones hands or smelling with the nose. The action-sense duality goes even further than that given that the same tissues such as the arm or nose are capable of both. In another book of mine, "Foundations of science with applications in sociology and medicine", I have described the appropriate setting for thoughts and lower level processes to intertwine in a more generous, but rather still restrictive, way. This involves the possibility for black-white processes to propagate in dimensions beyond our four axis; that is in the tangent bundle. The appropriate geometry herefore has been provided by Ehresmann long time ago and is presented in that reference. It might therefore be that the arm and nose are also capable, independently, of black-white interactions beyond the standard setting but that the persona only sees the shapes in mostly $2+1$ and sometimes $3+1$ dimensions, imaginary axis as to speak. Therefore, the precise nature of the topic of this cahier is wether it is possible to break these dualities at first and later on restore them at some higher level, starting at the psychic one where more as four axis, three of space and one of time, may be developed. In the previous book, a slight hint towards this possibility from the kinematical point of view has been uncovered; in these proceedings we elaborate this theory further on.

The two little books presented here should sparkle interest in foundational questions about the structure of life and language itself. There is a deep connection with affinity for colours and food of a certain kind or colour. The electromagnetic spectrum, in a way, deeply connects with the way we behave and interact as living beings. Moreover, it provides us with several strategies of how to remain healthy and cure one and another. Another hope of mine, is that sociology will enter physics given that basic quantum field theory can be explained in terms of primary sociological processes of birth and rebirth as well as asking the right questions in order to remain or to depart from one another. In some sense, every system is a dialogue with mechanical answers provided to any type of question. The intelligence mainly resides in asking for the right questions in order to stimulate the best possible answers.

# Chapter 1

# Introduction.

In the previous proceedings, we came to the conclusion that the most simple theory describing observed psychic phenomena on the first two levels of observation, that is level zero of our eyes, hands and other external bodily parts, and level one of astral projections reflecting basic thought processes, that is a four parameter strategy projection within the space of alternatives chiefly defined by actions on level zero where chiefly refers to the fact that this definition is a bit blurry and sensitive to local interpretations of higher language, display a remarkable algebraic structure. Indeed, thought processes, which are defined as sitting at the first level and primitive action processes as well as sensations, which by definition sit at level zero, are both described mathematically by means of four real numbers and require a level of manipulation which is only present in the real quaternion algebra. Indeed, the central element in the quaternion algebra can serve for awareness and impetus on level zero as well as level one whereas the momentum and rotation hide in the vectorial parts. More specifically, the quaternions are generated by $e_i; i : 1 \ldots 3$ as well as the identity 1 where $e_i e_j = \epsilon_{ijk} e_k - \delta_{ij} 1$ implying that

$$e_i e_j + e_j e_i = -2\delta_{ij} 1$$

which is the defining property of a Clifford algebra. Here, the volume form or totally antisymmetric symbol $\epsilon_{ijk}$ equals one when $ijk = 123$ or an even permutation theirof, $-1$ when it regards an odd permutation and zero otherwise. One can take complex multiples of these units $e_i, 1$ or real ones; in the first case, we speak about the complex quaternions whereas in the latter about the real ones. On the real quaternion algebra, we can define a notion of conjugation $\overline{q}$ where $q = a + a^i e_i$ and the former is given by

$$\overline{q} = a - a^i e_i$$

leading to $\overline{q}q = q\overline{q} = a^2 + (a^i)^2$ a positive quantity. The reader should verify that

$$\overline{qw} = \overline{w}\overline{q}$$

as well as

$$\overline{\overline{q}} = q.$$

Given that particles often carry a charge, which is displayed by a Hermitian operator on a Hilbert space, one has the notion of a charge conjugate $\star q$ which reads

$$q^{\star} = a + a^1 e_1 - a^2 e_2 + a_3 e_3$$

where $ie_2$ flips of sign. The correspondence between the three dimensional rotations generated by $e_i$ and the three boosts in Minkowski spacetime comes indeed from complexifying the quaternions, in either by also considering $ie_i$. One verifies that

$$(qw)^{\star} = w^{\star} q^{\star}$$

and $(q^{\star})^{\star} = q$ as well as that

$$\overline{z} := \overline{b + ib^j e_j} = b - ib^j e_j$$

and therefore

$$\overline{z}z = z\overline{z} = b^2 - (b^j)^2$$

a metric of Lorentzian signature. Therefore, mechanical labour is given by complex quaternions where the real part corresponds to a rotational vector or spin and the second to a translation or momentum given by the boosts. Usually, constraints hold between the translation and rotational aspect, for example, a spin one massless particle has, amongst others, obeys $b^2 - (b^j)^2 = 0 = \overline{z}z$ and $0 = a^2 - (a^j)^2$ which is properly expressed as $\overline{q^2} = -q^2$. In standard physics, we moreover argue that a rotating particle cannot be seen by means of our standard senses so that only purely imaginary quaternions make up for spacetime; in either, our mind cannot swerve into rotational space but only into some coherent assemblation of momentum or boost space. In case one would wish to allow for such a thing, the reader notices that any scalar observable defined in this quaternion picture needs to remain invariant under the unique action of the Lorentz group which preserves as well the real as complex parts separately. The latter is given by

$$(\alpha(e^q))(q') = e^q q' e^{\overline{q}}.$$

In the Einsteinian world, this is the only transformation possible given that it just reduces for vectorial $q$ to the standard defining action of the $q'^a$. In the spinor world, more is possible; there we have a left handed and right handed one. The imaginary quaternions do not constitute an algebra as is the case for the real quaternions given that the product of two imaginary quaternions is a real quaternion. Usually, this is expressed as saying that the rotational part resides in the tangent bundle over four dimensional real spacetime $\mathcal{M}$ or some spinor bundle over it. In our black-white theory, we have only a complex two dimensional vector where the correct norm is given by the Euclidean one and an interpretation in terms of an inherent *psychic* energy-momentum instead of a *mechanic* energy momentum. The latter depends upon the first spacetime derivatives of the complex Klein Gordon two vector, and the reader is advised to see the previous proceedings for more explanation, and belongs to the initial four dimensional contrained data. Therefore, both the black and white theory as well as the theory of matter are described by complex, constrained, quaternion fields over space-time.

Naturally, the psychic energy momentum vector has a Euclidean norm and therefore corresponds to the real part whereas the mechanic momentum corresponds to the complex part associated to the boosts which is logical given that it is that vector which must obey the causality demands of the Minkowskian metric. Regarding the matter quaternions, the real part corresponds to spin whereas the complex part to energy momentum which is also the case in standard quantum theory given that the $ie_j$ constitute Hermitian operators in the Pauli representation. As such, we have a complex action quaternion $q_a$ and a thought quaternion $q_t$, the latter which can undergo a change $q_i$ which is ought to be the same for the action process as well. For example,

$$q_a \to q_i q_a,\; q_t \to q_i q_t$$

for the straight types, meaning the action responds in the same way as the level one mind does, and

$$q_a \to q_a q_i^\dagger,\; q_t \to q_i q_t$$

for the maximally twisted types meaning the action is just the opposite. Here, $q^\dagger$ is defined as the composition of the complex and quaternion conjugate $\overline{q}$. In a way, thoughts are assumed to be without friction which reflects itself in the left multiplication by means of the "group" quaternion; pure and diametrically opposite types are also conservative regarding this aspect. Mixed types, those who twist themselves, lose energy which results into a kind of symmetry breaking, the easiest type of such breaking being

$$(\beta_\lambda(e^q))(q') = e^{(1-\lambda)q} q' e^{\lambda q^\dagger}$$

where $\lambda \in [0,1]$ and the reader verifies that

$$\beta_\lambda(e^q e^w) \neq \beta_\lambda(e^q)\beta_\lambda(e^w)$$

meaning that the subsequent action of thought process does not result in the subsequent action of physical processes. There is an historical twist building up which is also something this author discovered in his book on "Generally covariant quantum theory". Mathematically, the reader should get used to the terminology that $\beta_0, \beta_1$ are called the vector and covector representations, $(\beta_{\frac{1}{2}})^2$ is the usual conjugate representation which is equivalent to a Lorentz transformation in the defining representation over half of the angle. More generally, one could consider $\beta_{\mu_1} \circ \beta_{\mu_2} \circ \ldots \circ \beta_{\mu_n} = (\beta_{\frac{1}{2}})^2$, all applied to the same argument, where

$$\sum_{i=1}^{n} \mu_i = 1, n-1$$

as well as $\mu_i \geq 0$ for all $i$. This implies that $n = 2$ and the most general situation is $\lambda, 1 - \lambda$. In case $\lambda = \frac{1}{2}$ we can construct multiple mappings such as

$$\beta_{\frac{1}{2}} \circ \beta_{\frac{1}{2}} \to \beta_{\frac{1}{2}} \otimes \beta^c_{\frac{1}{2}}, q = ww^c \to w \otimes w$$

as well as well as

$$\beta_{\frac{1}{2}} \circ \beta_{\frac{1}{2}} \to \beta_{\frac{1}{2}} \otimes \beta_{\frac{1}{2}}, q = ww \to w \otimes w$$

and

$$q_i \circ \overline{q}_i \to q_i \otimes \overline{q}_i, q = w\overline{w} \to w \otimes w$$

in case $q$ is a scalar. Other possibilities do exist. Regarding the first two, they coincide in case $w$ is self-dual meaning $w^c = w$ or anti self-dual in case $w^c = -w$ where $c$ is the composition of the charge $q^\star$ and complex conjugation. The first condition means that $w$ has no charge whereas the second one has given that its charge conjugate is minus itself. In both cases, we would first look at those transformations that preserve duality and are given by $e^q$ with $q^\dagger = q^c$ which distinguishes $wv^c$ from $w\overline{v}$. The doubling trick really goes as follows in the second case

$$\beta_{\frac{1}{2}}(q) \otimes \beta_{\frac{1}{2}}(q) \triangle (w \otimes w) := (\beta_{\frac{1}{2}}(q) \triangle w) \otimes (\beta_{\frac{1}{2}}(q) \triangle w) \to_{\text{paste}} (\beta_{\frac{1}{2}}(q))^2 \triangle w$$

$$\to_{\text{doublure}} (\beta_{\frac{1}{2}}(q))^2(w) \otimes (\beta_{\frac{1}{2}}(q))^2(w).$$

This shows that pairs $w - w$ can behave as right/left handed given by $ww, \overline{ww}$ or right-left handed Cooper pairs $w\overline{w}$ which constitute spinless bosons where each participant is oppositely twisted in nature $(\lambda, 1 - \lambda)$ such that both of them together in the tensor product formalism transform as a neutral element or switchoriem, being a conventional matter "element" of bosonic nature (integer spin). The second configuration is stable if and only if $\lambda = \frac{1}{2}$ otherwise such conjugated pairs will scatter away from one and another. The third option is always stable for Lorentz transformations but not so for the complex dilaton sector whereas considering $w^\dagger w$, under the left multiplication, would only provide for invariance under the rotation and $U(1)$ subgroups. Charge conjugated pairs, such as occuring in the first transformation law, are always unstable but instantaneously in equilibrium with the environment if the dilaton sector is dynamically broken. This is in agreement with the result that people of equal polarization do not radiate out psychic energy when remaining together; in this reasoning, it has been used that the mechanic momentum of the psychic field is determined by the psychic one using "shape equivalence".

What are the psychological or spiritual ramifications of these findings? First of all, the whole product-tensor product equivalence only works when the complex energy-momentum vectors are equal meaning that only primitive equals at level one, possibly with different but opposite tastes, *glue* together in a way which equals the tastes on that level of an *individual* switchoriem. This is often observed in nature, where first two partners combine in a way to be of equal energy and then bind to level out with the large-scale environment. So, this very special equilibrium point is a stable one; in case it cannot be reached, both partners will quickly leave one and another or the environment respectively. There is something even more special going on, namely that this can only occur in pairs and that groups with an odd number of participants are necessarily stable at a higher, meaning weaker, platform. One can of course also implement gluing of such distinct pairs with different transformation laws. One such example is that of children forming a closed chain, holding hands, at a scouts camp; every child, labelled by the index $i$, can be modelled as $w_i \otimes v_i$ where $w_i$ represents the left arm and $v_i$ the right one. Both arms can move independently but we call the body stable if and only if $v_i = w_i^c$ in which case the two axis connects both arms. To connect the children, a gluing occurs between $v_{i-1}$ and $w_i$ and cyclically between $v_M$ and $w_1$ something which we express by using cyclic quaternion invariants defined by means of

$$i(vw) = i(wv)$$

and $i(qvq^{-1}) = i(v)$. Lacing then occurs by means of considering products of the kind

$$i_1(v_1 w_2) \ldots i_M(v_M w_1)$$

and the motions which can be performed are infinitesimal rotations in the plane perpendicular to the two axis (that is: the two axis is the rotation axis). Finite rotations require a spatial deformation of the chain and or wrist/arm positions. The reader may therefore posit that $v_i, w_{i+1}$ must form a conjugated pair and in order for *substantiated* bodily stability, meaning left and right arm move in the same way, it is required that

$$(1 - \lambda_i) q_i^c = -(1 - \lambda_{i+1}) q_{i+1}$$

and

$$\lambda_i ((q_i)^\dagger)^c = (1 - \lambda_i) q_i.$$

Also $q_i = q_{i+1}$ otherwise, mental equilibrium does not set in, hence $\lambda_i$ cannot depend upon $i$ and therefore

$$q^c = -q = q^\dagger, \, \lambda = \frac{1}{2}.$$

This shows that $q$ must be of the form $a^2 e_2 + ib1$ where as usual the complex $U(1)$ sector gives a deflection angle illustrating that the left arm of $i$ shrinks in the opposite direction of the right arm of $i + 1$. These deflection angles are the same given that we assumed both spirits to be the same; if they were not, instabilities would occure; these results are pretty elementary and every scouts leader knows them.

The quaternion formalism offers plenty of more possibilities as the vector formalism does; indeed, additional representations on the action quaternion are provided by

$$q_a \to q_i q_a q_i^\dagger, \, q_a \to q_i q_a \overline{q_i}, q_a \to q_a \overline{q_i}$$

giving rise to distinct Lorentz invariant charges such as

$$S(q_a\widetilde{q_a}), S(q_a), q_a\overline{q_a}$$

where $\widetilde{q}$ denotes the complex conjugation of $q$ and $S$ the scalar part. That is $S(z+z^ie_i)=z$ and the reader is invited to verify which possibilities correspond to the appropriate transformation laws. Likewise, quaternion currents and bi-currents may be developed such as

$$j(q_a)^b := \widetilde{q_a}e^bq_a$$

under the second transformation law rotating as

$$\Lambda(q_i)^b_c q_i j(q_a)^c q_i^\dagger$$

and so on. Plenty of other varieties exist by combining transformation laws of different kinds. We leave these possibilities for further exploration.

# Chapter 2

# Spatio-temporal distribution of black-white polarization vectors within the body.

In this short chapter, we reason why the astral projections of spirits associated to bodily parts of persona are usually more black dominant as those associated to the outside bodily appearance defined by the epidermal region. Given that the main central spirit of the entire body consists out of $N$ spirits of former persona, who group in different subsocieties associated to their primitive astral characteristics, it is rather normal that conflicts will appear about the governance of the lower spirits in the body. The weakest groups (which are of course more black dominant or less energetic) will always prefer to remain safely inside the body given that the fights at the boundary are the wickedest ones and require the most energy. This is the natural state of being and very often, the person looks healthy from the outside but has a poor heart or brain from the inside. It is the former which fails first; however, when failure of the heart is too prominent (due to blackness or lack of energy) the brain and other organs start to fail whereas the epidermus has to remain in pretty good shape. Therefore, "cancer of the skin" is the most leathal desease from the natural point of view. In case skin poisoning occurs, the better spirits will withdraw inside, taking over the heart and lungs as quickly as possible; usually, in the long run, such person will suffer from dementia or organ failure. Most likely, one kidney is sacrified first or the liver gets in a poor state. Skin poisoning, can in principle be benificial for white in case it is not too overt, that plenty of healthy regions still exist and that lung and heart fortificstion are mild. In that case, balancing all along, the battle at the main level to expell black spirits, usually called deamons in the popular language, which are genetically imprinted, can begin.

People with no skin poisoning but with heart problems usually are bi-polar in the way that the second strongest group, is too far removed from the first one in the black-white plane; these people are not necessarily bad but are like a balance with a huge angular momentum. They could cure out by milk intake and sleeping on satin sheets with a pillow on a marble platform in a big stone room. The room must be dark away from the person but filled with candlelight nearby. The skin needs to be fed and cleansed regularly with milky products and seaweed whereas a strict lactose diet is prescribed with additional intake of high doses of sugar liquids. No meat, no vegetables and just a fair amount of smoked and grilled fish (a volonte) added with one potato. The philosophy is that the huge room favours whiteness whereas the dark character in a distance allows for black spirits to escape. The light nearby should force them to escape just as the milk intake does. The fish and potato are largely neutral however whereas the sugar reinforces the most white subgroup. The cleansing of the skin is necessary to allow for the white spirits to remain inside and finalize the battle. It is, unfortunately, only so that most people understand this at the face of immanent death.

Of course, those who want to live in darkness and have a somewhat smaller heart as white people do need to be disclosed from the sun and reside in the blackest environments possible; those are cellars made out of brick floors and walls with some moisturizing plants hanging on them to forsee in the necessary amount of air as well as a bit of candlelight to feed the eyes. They need to eat neutral to remain alive as otherwise the blood would blacken wich leads to a lack of oxigen affinity of the blood needed to substantiate basic functions. Indeed, the core of a human, the skeleton is white and it needs to remain so as rottening bones are the primary cause of destruction for ultra black people. In this way, nature has ensured itself that persona need a minimum amount of light inside in order to survive., However, given that black rejects white on the lowest level-one of psychic interactions, the muscle tissue is of the *quantal* Switchoriem type which is percieved as the color red; fat around the muscles indicates for a white appetite and indeed muscles strenghten greatly when drinking milk or eating eggs which are both white products. Muscles of people who are more black from the inside can look greenish, which is more black as white is. Cows look white from the outside and have huge muscles and love the stand on the green grass and in the sun. Moreover, they excrete white milk in abundance; henceforth, they are pretty white animals from the inside if not in their brain. It is usually so that extremely pretty people from the outside are also white from the inside as there is no point for a body spirit to prolifirate itself that much from the epidermus and muscle side if other, equally important organic and brain functions, need whitening. People who are ugly from the outside according to local community standards are mostly very black inside if not poisoned under the skin too much.

The reason why the skin needs to be white far enough from the equator as well as the poles is due to the amount of sunshine resulting in a green, blue, yellow, white environment as well as due to oxigen being the necessary substance of life.

Black or brown people who live in a neighborhood of the equator are living in an evironment of brown, red and deep purple. Blue only originates from water associated to rivers and oceans, the former which are highly polluted due to brownish and black substances present on the land. Those people do not withstand sunshine too well, their color brown or yellow being a “schutz” against the sun causing substantiated dampening mainly for black colours and somewhat less for white ones. People further away to the poles can be more white due to more reduced sunlight; one would suggest an increased amount of blackness is also possible but this possibility is negated by the increased amount of oxygen in the air. Actually, the decreased amount of oxygen in land towards central Africa causes people to have the blackest skin there. Negro’s are definetly more energetic in Africa as they are in central europe whereas for Marrocans or Egyptians things do not matter much given that their slightly brownish skin is more yellowish as white is, compensating therefore for the adverse effect in oxygen intake. So, although they weaken slightly physically compared to their natural state in Egypt, they become more psychic here and regarding their physical strength, are still very much on pair with the white inhabitants. Therefore, immigration of negro’s in Europe should occur first in the Alpes, Pyrenees located in Austria, Switzerland, Italy, France and Spain, whereas the Spanish desert may serve very well too given that fresh water is available.

# Chapter 3

# About new senses and motoric abilities.

The topic, which I shall explain in this chapter is rather controversial in nature in a various degree for exact scientists, more in particular for physicists something which I will substantially explain later on. Indeed, the physicist is conservative here for reasons going beyond the comprehension of psychologists and psychiatrists; this attitude is justified given that we are speaking about observed effects which are rather weak from the motoric side. Can you destroy an artery with the mind? Absolutely not on time scales which are comparable to a substantiated feeling of anguish. Can you cause an erect penis with a person, merely thinking about him on a distance? Yes, you can, but this is either the consequence of tender whispering or manipulation of penile intruders which are under your control, such as is the case with snakiem snakiem injections or crémes. The first method being known as white magic, the latter as black magic. So, it is possible to interact, even in different ways, on a distance by means of the mind; however, given that our most basic, level zero, senses suggest a four dimensional world, do these extra weak "motoric" abilities of the mind beyond our most basic motoric tools such as hands, fingers, feet and legs and certainly beyond the naive reach of our basic body, travel in extra dimensions? Is our astral spirit therefore extended in dimensions which are beyond our standard vision and is a sufficient strength of this extension a required condition to "see" them in a way? This is an almost philosophical question as long as we do not know how to substantially materialize, meaning localize, using those other dimensions, on spatial distances far from us in our level zero universe. Indeed, if I could be sitting here at my desk in La Herradura and be at the same time in Herentals physically kicking with level zero "real" feet my mother on her dirty mouth. The appearances I could make are at most of the visual kind or of a slight caressing one so that it might equally be feasable that I am sending a delicate, highly personal, coded signal to that spot so that she only experiences it. This is, in a way, reasonable given that she leaves a spiritual imprint in my spiritual memory so that I could code the signal accordingly. However, this codification, and the complexity theirof suggests my spirit to be much more vigilent as my persona is at level zero of speech and bodily interactions. This is a debate without solution in a way and depends upon our certitude of the personal reality of it; a good way to test this theory would constitute in a real transformation into a vampire bat. In that case, I would say these extra dimensions are real given that such transformations cannot be explained from the four dimensional perspective. The physicist is however still much more intelligent as this as the number $4 = 3 + 1$ is really special regarding the nature of interactions and has been confirmed to be sufficient in low-level intelligence situations.

The main reasons as to why physicists stick to a $3 + 1$ view on spacetime is that the presence of one time direction gives birth to the definition of a causal relation; this means that you can, upon a global binary choice, say when an event preceeds another one, excluding configurations with circular time dimensions where a traveler may return to its past by means of traveling towards the future. Another ramification of this is that the energy momentum current of matter in quantum theory can be measured independently, which is the so-called commutation property, at spatially separated events, those are defined as not being causally related. It is a debate however whether all observable measurable quantities "commute" at spacelike separated events; I have repeatedly argued they are *not* due to the inherent lack of localization properties for relativistic particles. In the first case, assuming that they do commute, no signals can be send acausally because it is not clear who is sending something, whereas in the latter signals can be send. However, it must be emphasized that those causality violating effects are very small so that time travel appears to be impossible. Another reason for choosing $3+1$ can be found in physical theories themselves, light, as we know it, would be very different; in two spatial dimensions, one would just dispose of a spatial electrical field and a scalar. Light in the vacuum would just have one electric polarization and no magnetic component. In more as three spatial dimensions, light would produce many more fields as just the electric and magnetic one. Electromagnetic duality is only valid in $3 + 1$ dimensions. This symmetry is remnicent of the symmetry between boost and rotation parameters in $3 + 1$ dimensions where the boosts refer to rotations between space and time and the rotations are between two spatial axis. The symmetry is inherently present in the complex quaternions whose real part constitutes the largest division algebra possible over the real numbers; so number theoretical aspects join symmetries between space and time as well as symmetries in the theory of light. Moreover, as argued before, the real quaternions are associated to quibit quantum states which constitute the quantal prototype carriers for yes no answer-type questions. The complex quaternions are then needed to unify state and impetus as explained previously. Also, Einstein offered, beyond the arguments given up till now, his own insights; gravitational theories with a conformal symmetry, meaning a local rescaling of your measure sticks, can only exist starting at three spatial dimensions. As it turns out, general relativity has shown to favour $3 + 1$ in terms of simplicity of stable black hole solutions; they all behave as classical thermodynamic systems something which is not true in higher spatial dimensions. So, our conclusion appears to be that space-time exotism is precluded for low intelligence point particle systems. There is, however, an exception to this debate; once one starts to allow for irreducible gheist manipulations with a certain finite geometry, such as those of a closed or open strings, then geometrical consistency constraints force more than one time dimension as well as minimally five spatial dimensions as has been shown by this author lately [1]. Standard string considerations already hinted towards more spatial dimensions, however, the latter quantization is not geometrical in nature and thereby avoids the issue with multiple times. The funny thing is that $2 + 5$ also provides one with the boost - spatial rotation duality given that one has $(2 \times 5) = 10$ boost parameters and $(5 \times 4)/2 = 10$ rotation parameters out of a total of $(7 \times 6)/2 = 21$ parameters leaving only the time-time rotation to be considered. However, this would suggest a world image over a real 7 dimensional number system whereas the only other real division algebra is given by the non-associative octonions (whereas quaternions are associative meaning that the way of multiplying more as two factors fully reduces to the binary multiplication). Octonions have seven non-commutative directions and one identity element which would suggest a conformal and electromagnetic symmetry to put away with them. In a way, it is reassuring that strings are the most complex geometric manifestations possible compatible with the idea that aritmetic can properly discribe the universe. Unlike, $3 + 1$, one has not the quibit real quaternion correspondance, with strings, four colours are needed which suggests they are described by a tensor product of *two* quibits. Moreover, the Hodge duality on the spatial sector is lost meaning that the number of rotation parameters equals the number of spatial dimensions which is equivalent to saying that labour can be minimally performed in any spatial direction. In the stringy world, one has 5 real axis and 10 spatial rotations which suggests a different kind of symmetry, which is that the number of time dimensions times the number of space dimensions must equal the rotational symmetry as well as boost symmetry. It is just so that this more complex principle coincides with the easier one in $3 + 1$ but not in $5 + 2$. All of this is dandy fine, but the reader may guess that those extra dimensions are special given that we cannot see them with our eyes and neither can manipulate them with our hands. In the same vein do we experience one time at level zero albeit one may argue that psychic signals obey synchronicity also which requires an abandonment of strict causality. However, those whispers which appear to go beyond our standard $3 + 1$ notions of space and time are always heard in $3 + 1$ space-time in a way. Physicists are therefore wise enough to maintain the notion that the geometry of $5 + 2$ spacetime needs to be of the product form $\mathcal{M} \times \mathcal{N}$ where the former is a standard $3 + 1$ universe and the latter a $2 + 1$ compact and closed universe in which the string can live and vibrate so that its extensions are invisible from the $\mathcal{M}$ point of view. Roger Penrose has argued that standard $5 \times 2$ relativity does not allow for a product *geometry* and would cause the hosepipes $\mathcal{N}$ to shrink to a point so that an infinity or singularity occurs. The string effectively dies out there. It is possible however, to keep the $\mathcal{N}$ geometry fixed and still allow for dynamical interplay between between both sectors. This is achieved by Ehresmann who proposes a geometry more subtle as Riemannian geometry is, which is fine given that the string can only leak from the hosepipe into the bulk by means of gravitational interactions which are very small except at extremely high energies. These extra dimensions have to be small moreover otherwise electromagnetism would look very different in the bulk as one would need to explain why extensions of the string in the extra dimensions are not too large as to destroy the $1/r$ form of the Coulomb potential at intermediate scales in the $3+1$ dimensional bulk. The nice thing however, is that the compactified geometry replaces the ad-hoc charges and coupling constants in the theory so that a more geometrical picture becomes feasable. So, the reader should learn that lots of pitfalls exist in such extended setting and that nature needs to take precautionary measures as not to destroy our eyes and hands as well as our sense of progression of time or impetus.

Up till now, we have discussed that one irreducible spirit at one site in spacetime leads to the following coincidences: the number of motoric degrees of freedom is $2^N$ complex, the psychic ones are given by $2^N$ complex whereas spacetime itself is given by $2^N$ real where $N = 2$. This only holds for the number of spacetime dimensions equal to four and fails for any other number unless extra colors or letters are added to the alphabet such as is the case for $N = 3$ employing the real octonions. So, in way, quibits or the irreducible quantal binary systems are one to one with the world of our individual senses where stabilization also suggests the number two as argued above by means of the gluing procedure. The latter two, however, is universal in nature and holds for any real or complex Clifford algebra. This strongly supports the product theory for spacetime with an Ehresmann geometry instead of a Riemannian one. Indeed, evaporating a body spiritually from our 3+1 spacetime and putting it in the bulk whatever the latter is would require energies beyond the sum of the eigenmass and the gravitational energies, the former which is positive whereas the latter is negative. Concretely this amount is given by

$$M10^{17} - M67.000.000$$

Joule, where $M$ is the mass in kilograms which is a hugely positive number $67.000.000 = 10 \times 6.700.000$ where the number 10 is the gravitational acceleration in standard SI units and 6.700.000 the earth radius in meters. This is equivalent to letting a lightbulb of 60 Watt burn for $10^{15}$ seconds which is rougly equivalent to a billion years. Even if that would work out, the hosepipe would immediately collapse under such huge energies otherwise Newton's law would fail appreciably. So, the answer is, vampires do not exist and psychic interactions traveling through the hosepipe will never be able to push or pull a small sugar cube of Tirlemont.

The issue now is that when you couple $N$ spirits, the minimal spiritual language granted to the joint spirit is given by the tensorproduct of the languages. So, if each of them can only ask one yes or no question corresponding to a quibit, then the space at hand is given by

$$\otimes_N \mathbb{C}^2 = \mathbb{C}^{2N}.$$

It may be that the joint spirit has a richer language as this but up till now, there is no experiment even pointing in that direction. I have explained these details in two previous books called "Foundations of science with applications in sociology and medicine" as well as "Generally covariant quantum theory with applications." Given that extra questions come with extra senses as well as motoric abilities which largely obey causal relations and therefore mainly live in a real quaternion bulk spacetime, one may wonder wether the hosepipe $\mathcal{N}$ may depend upon the local number of gheists. Within this author's viewpoint upon standard quantum field theory, the local degrees of freedom at an interaction vertex, meaning a point, were given by a four vector and a spin vector which is in a way a spin vector, constrained by several physical principles. The interaction vertex required a so called intertwiner, which is a mathematical object in terms of the spin indices of the incoming and outgoing particles, invariant under local spin rotations. The last requirement simply means that interaction does not distinguish any direction in space-time. Now, such intertwiners do not come cheaply and only very specific types are allowed for which ensures that our theories are well defined. In case $N$ strings do collide, there is not enough room in $\mathcal{N}$ to accomodate for a gluing satisfying all the conditions. For example, colliding two strings in all freedom, would result in a hosepipe $\mathcal{N} \times \mathcal{N}$ but further restrictions lowering the dimension to at least $1+3$ is possible. In general, there are no topological restrictions on string scattering so that vertices are equipped with the Fock fibre or hosepipe

$$\mathcal{F} := \oplus_{n=0}^{\infty} \otimes_{\gamma}^{n} \mathcal{N}$$

where the subscript $\gamma$ indicates that some restrictions may hold on the tensor-product which reveal the so-called statistics of the "hosepipe beings". The sum is taken over all multifold occupancies which expresses the potentiality for an infinite number of gheists to remain at that same event. The statistics has to do with the distinguishable character of the gheists or their respective individualization skills. The bulk is to remain $3+1$ dimensional as argued before which points to the idea that true synchronicity is a weak phenomenon and that causal rules govern in primary order the universe. Magic is limited and rules hold everywhere in primary instance.

# Chapter 4

# Sociological models.

It may be clear from the discussion in this proceedings as well as the previous one that a society cannot be too diverse; that is, assuming that a complex psychic quaternion is fully determined in a way by its real part by means of a pychic energy momentum versus mechanic enery momentum correspondence principle associated to shape equivalence, we arrive at the conclusion that the black-white polarization two vectors in the complex two plane must not differ radically. Also, switchoriems should in a way have the necessary mobility regarding their work place towards evironments in which they obtain their preferred colour. This is to ensure that they experience turmoil once in a while causing for some excitement leaving the ratio a bit behind. There are several scales to be considered here: (a) a mundial one (b) a continental scale (c) a scale on the level of the country and (d) on the province or region (e) the city you live in and finally (f) your street and neighbours. A unified world on the largest scale will necessarily imply a central council at the level of (b) such that all continents are equally gauged and henceforth coincide with the mundial one. From the logistic point of view, the gauge at the level of countries should not differ too much either, however, cultural distinctions due to geological and hydrological distinctions are unavoidable. Nature has forseen in those fluctuations and we should not be scared to meet them with the appropriate dignity. At the level of provinces however, I would suggest a threefold differentiation regarding the national gauge or benchmark; the absolute black versus absolute white quarterplane has to be divided into three sectors each of an angle around 30 degrees. Each province may be associated to such pie which should be chosen in a way commensurable with nature, local culture and food supplies. At the level of cities and villages, the whole provincial pie needs to be cut into three equidistant annuli, so that the variety remains the same (in terms of angles) but the strengths vary appreciably. This suggests that national representation needs to occur at the level of the province council and not the town or city councils. In this way, everything gets gauged around the diagonal in the absolute black - white plane with a deficit angle of 15 degrees on both sides. Now, cut out a deficit angle of 5 percent on both sides and dub those workers of the local switchoriem type as mobile ones, within the same province due to a need for psychological differentiation from the benchmark. The other two pies of 10 percent could go and work in an environment in which they are switchoriems. Everyone needs the right balance between work and marriage and some marriages are of the brainy type (associated to the astral eye) whereas others are of the belly or pinal gland. It is my personal conviction that a global agency on the level of the province, state and continent needs to exist in order to locate families in the appropriate place, taking into account their professional desires as well as "marriage vows".

The more people live together harmoniously, the better the extra senses may develop and the higher state of being is allowed for. If the city of Antwerp, say, chooses for the pinal gland, then its inhabitants need to work elsewhere and the workers must originate from a more hearthy place. Indeed, black or switchoriem types relative to the city benchmark ought to work there as it stimulates them to be brainy and positive. Necessarily, the buildings need to be switchoriem to white; their inhabitants need broader environments to intellectually optimize, a place where the heart in the collective spirit is mainly absent. This suggests that the astral gland may only live in the day and the communal heart flourishes naturally in the evening due to the architecture and a suitable social nightlife in some parts of the city. This requires public transport facilities until 3 am. This mandates clean cities with art decau buildings in the French or English Victorian style of which the old Brussels used to be a good example. People who are black on the benchmark of the province containing the city should live in smaller villages and cozy towns. Superwhite people should work and live in suburbs such as Kapellen, near Antwerp in villa's or small castles. This model should bring relative peace and satisfaction; obviously,one can discuss the construction of provincial airports as well as limited national and international mobility.

Models with more social tension may also be constructed; this is not bad per se as it depends upon how one wishes society to evolve. However, with the right kind of "dopage" with rebels, the above which should be in a way a golden mean would certainly acquire a more spicy flavour ideally suited for stress and back testing. A serious question which would emerge is that of "freedom in tourism"; how long can one stay at a place where one is too far out of tune? Should there be a cap and a floor on such people being present? If yes, how to ensure this in the most democratic way possible without the need for explicit intervention? It is not good in society to be too cautious regarding the accuracy of the practical working of its social services; however, a certain standard should be assured and professionalisation of its core values is always a good way to ensure its success.

# Chapter 5

# The psychological arrow of time.

Prior to embarking upon more profound investigations into the nature of consciousness, it is salient to have a deep understanding as to where our feeling of spatial holism emerges from if the mechanical world is fundamentally one of space and time without any further ingredients added to it. It is impossible for me to present those issues in full detail for the psychologist here as this would be equivalent to a full term course for physicists in quantum field theory. However, I have written a summary of that theory and I shall present it here prior to embarking into a more detailed investigation. The following two sections summarize the main formalistic ideas as well as concepts and the third one discusses the issue of the psychological arrow of time.

## 5.1 Basic reflections upon the laws of physics.

In this section, I see nature as a communist reflects upon society; the foundational quantity of everything is contained in an action signifying "work" or "rabota". That is, consider $\phi(\gamma(s), p(\gamma(s))) \in \mathbf{B}$ where $\gamma : [a, b] \to \mathcal{M}$ is a curve joining an event $x$ to an event $y$ in spacetime $\mathcal{M}$ in affine parametrization with respect to a Lorentzian or Riemannian metric where, moreover, $p$ is a field on that line associated with the physical quantity of "momentum". We do not really know yet what momentum is but it represents a kind of weight or importance given to that motion, $p$ must not be proportional to $\dot{\gamma}$ as weight might sometimes be disfavourable to the current motion. Given that rednecks love calculus, $\mathbf{B}$ is a division algebra over the real numbers with standard operations $+, .$, that is $\mathbb{R}, \mathbb{C}$ or $\mathbb{Q}$, the real quaternions, disregarding the non-associative octonions which have been discussed in chapter three regarding string like interactions.

A *frictionless* theory is a dreamworld as no waste is produced. Mathematically, this translates as follows: there exists an involution † and operation $\star$ such that $\phi(\gamma(b-s), p(\gamma(b-s))) \star \phi(\gamma(s), p(\gamma(s)) = 1_\star$ and $\phi(\gamma(b-s), p(\gamma(b-s))) = \phi(\gamma(s), p(\gamma(s)))^\dagger$. It is worthwile to comment upon these; the first one means that reversing the process is arithmetically equivalent to taking the inverse whereas the second one says that the inverse has a metrical significance. This last stance is useful as inverting two processes must preserve the distance between them. No discussion about this is allowed for.

As a consequence, the constant curve $\gamma_e(s) = x = y$ satisfies

$$\phi(\gamma_e(s), p(\gamma_e(s)))^2 = 1_\star$$

which for $\star = +$ and $\mathbf{B} = \mathbb{R}$ gives $\phi(\gamma(b-s), p(\gamma(b-s)) = -\phi(\gamma(s), p(\gamma(s))$ and $\phi(\gamma_e(s), p(\gamma_e(s))) = 0$. These simple observations give rise to the notion of work *and* classical physics. For, $\mathbf{B} = \mathbb{C}$, we have that

$$\phi(\gamma(b-s), p(\gamma(b-s)) = \overline{\phi(\gamma(s), p(\gamma(s))}$$

and $|\phi(\gamma(s), p(\gamma(s))|^2 = 1$ what leads to the $U(1)$ Fourier waves in quantum theory.

## The classical theory.

The idea is to write down a first order differential equation for the quantity of labour. Reparametrization invariance forces $\frac{d}{ds}\phi(\gamma(s), p(\gamma(s))$ to be proportional to $\frac{d}{ds}\gamma(s)$. Furthermore, the reversion property implies

$$\frac{d}{ds}\phi(\gamma(s), p(\gamma(s)) = \frac{d}{ds}\gamma(s) \cdot \mathbf{F}(\gamma(s), p(\gamma(s)))$$

which is the old Newtonian expression with $\mathbf{F}$ having the meaning of force. Note that this is an expression local in spacetime as the change of work does not depend upon its previous history. Theories with radiative effects do *not* satisfy this expression but nevertheless a more complicated one. To complete the dynamics, Newton supposed that $p(\gamma(s))$ must maximally stimulate the direction in which the particle is moving and that, therefore

$$p(\gamma(s)) = m\dot{\gamma}(s)$$

where $m > 0$ and expresses the weight attached to persistance of motion, called *physical mass*. Another observation was of an Einsteinian nature, namely that the change of work should reflect the change in an inherent physical property of the particle and not depend upon external forces at all. This would mean that, in a way, a particle is free infinitesimally; the lowest order, in the derivatives of the worldline, of such type of invariant is given by the momentum squared

$$h(p(\gamma(s)), p(\gamma(s)))$$

which suggests something like

$$\frac{d}{ds}\left(\frac{m}{2}\left(\frac{d}{ds}\gamma(s)\right)^2\right) = \frac{d}{ds}\phi(\gamma(s), \frac{d}{ds}\gamma(s))$$

and bestowes $\phi(\gamma(s), \frac{d}{ds}\gamma(s))$ with the dimension of mass which it should given that the notion of force must be associated to something intrinsic which is change of momentum

$$\mathbf{F}(\gamma(s), p(\gamma(s))) := \frac{d}{ds}p(\gamma(s)).$$

This is the simplest idea possible, given that the kinetic term is the lowest order invariant and $m$ can be thought of as some material based constant. This leads to

$$\frac{m}{2}\left(\frac{d}{ds}\gamma(b)\right)^2 - \frac{m}{2}\left(\frac{d}{ds}\gamma(a)\right)^2 = \phi(\gamma(b), p(\gamma(b))) - \phi(\gamma(a), p(\gamma(a)))$$

and in a way generalizes a concerved quantity given that $\phi$ depends upon the entire path and not just the endpoints in general.

One could make higher derivative theories also in this way and allow for Newtonian laws with third order derivatives. These naturally appear in the context of backreactions in electromagnetism for example and allow for "unphysical" solutions with causality going backwards in time. For example, an electron would accelerate prior to turning on a lightbulb. Note also that the interpretation of $\gamma$ as the physical path of the particle natually emerges given that Newton's law fixes it entirely given the two "initial data" of position and momentum.

## Quantum theory.

Now we derive quantum theory, as well as the probability interpretation, in the same vein. One notices that the *obvious*, but not only, candidate for an equation of motion is given by

$$\hbar\frac{d}{ds}\phi(\gamma(s), p(\gamma(s))) = ig(p(\gamma(s)), \dot{\gamma}(s))\phi(\gamma(s), p(\gamma(s)))$$

where $p$ is the energy momentum vector and $\dot{\gamma}(s)$ dimensionless. Notice that $\hbar$ is needed for dimensional reasons to get a nontrivial theory given that $\phi$ must be dimensionless as any physical quantity is a real and not a unitary number. On flat spacetimes $\phi(\gamma(s), p(\gamma(s)))$ is topological as it just depends upon the homotopy class or winding number. That is,

$$\phi(\gamma(b), p(\gamma(b))) = e^{ip.(y-x)}$$

which is the standard Fourier wave in $y$ with base point, or origin $x$. Given that $e^{ip.(y-x)}$ provides for a trivial unitary mapping between $e^{ip.(z-y)}$ and $e^{ip.(z-x)}$, the waves are identical up to a momentum dependent constant multiplicative $U(1)$ factor. In traditional relativistic quantum theory, this is precisely the implication of the translation symmetry in Minkowski quantum field theory. To arrive at the so called quantum field "propagator" for a quantum field on Minkowski, we notice that the total "propagator"

$$D(x,y) = \alpha \int_{\mathbb{R}^4} d^4p\theta(p^0)\delta(g(p,p) - m^2)\phi(\gamma(b), p)$$

is the expression we are looking after. Here, the delta function inserts the condition that the four momentum must square to the mass squared, in either that the rest energy squared of the particle is equal to $(mc^2)^2$ which is the well known Einstein formula. We have put $c = 1$ here, something which physicists call natural units. $\theta(x) = 1$ if and only if $x > 0$ and zero otherwise; henceforth, the condition $\theta(p^0)$ refers to the positivity of the energy and chooses for the positive root. In contrast to classical physics, the solution $\phi$ depends upon *three* kinds of data, being $x, y, p$ instead of $x, p(x)$; indeed, in $\phi(x, w, p)$, as well $w$ as $p$ are uncertain, which is kind of logical given that the momentum is *not* necessarily the maximal forward one but is dragged over the curve as to indicate the initial direction of preference. In order to recuperate the classical bi-functional way of thinking, we have to integrate over $p$ on mass-shell. It utters nothing but the Heisenberg uncertainty principle that if the positions $x, y$ are known sharply, then the momentum is totally uncertain apart from the fact that it needs to be forwards pointing in time and have mass energy $mc^2$. Actually, what this integration says is that all preferences are taken into account democratically unless some higher intelligence, due to a spiritual interaction, desires differently. This is actually a classical Bayesian way of thinking except that the weights or probabilities are here given by complex numbers. In this vein, quantum theory is one of reality $w$ and desire $p$ which do not need to coincide, as happens in the classical case. The theory may get more psychological than this by having more complex "momenta" and Einsteinian constraints as happens in (an appropriate version of) string theory. $x$ is here interpreted as the point of birth of a particle and $y$ as the point of dissapearance, death or annihilation. Actually, this is all there is to free quantum field theory; we dispose of no Hamiltonian operator but the so called "Wightman function" from which the "Feynman propagator" can be uniquely defined.

To arrive at the full expression in curved spacetime, we notice that

$$D(x,y) \in \mathbb{R}$$

if $x$ is spacelike to $y$, written by $x \sim y$, and, given that $D(x,y) = \overline{D(y,x)}$ in general, we arrive at the conclusion that for spacelike separated events the creation and annihilation processes at $x$ and $y$ can be swapped without altering the "propagator". This is the proper mathematical expression of the so-called Bose-Einstein statistics, a desirable property given that in a free theory the dynamics between birth and death without life is a balanced one meaning the modulus of the "amplitude" $D(x, y)$ must remain the same. Given that swapping birth and death twice is, in general, an identity operation; there are two options $D(x, y) = \pm D(y, x)$ for $x \sim y$. The plus sign is, as mentioned previously, the Bose property and the minus sign the Fermi property. In the last case, birth and death without life are distinguished whereas in the former they are not. Given that at one event, nothing or merely one birth or death event can occur, at most one Fermi particle in a particular state can exist whereas in the Bose world particles can accumulate arbitrarily. Living beings therefore have to mainly satisfy the Fermi property; in light of Bose statistics, only geodesic lines give rise to the general property that dragging measure sticks along them preserves all local metrical properties. More complicated options do exist as explicitated by the "Schroedinger" equation

$$\frac{d}{ds}\phi(x, y = \exp_x(w); w, k, s) =$$

$$(ig(w(s), k(s)) - \kappa\sqrt{h(w(s), w(s))} - \frac{2}{L^2\sigma^3(x, \exp_x(ws))}\sigma_{\alpha'}(x, \exp_x(ws))w^{\alpha'}(s))\phi(x, y = \exp_x(w); w, k, s)$$

where $s \in [0, 1]$ and $w, k \in T_x\mathcal{M}$, $h$ is a Riemannian metric delivering an energy $\mu$ and $L$ is a huge mass smoothening out the lightcone. Finally,

$$\frac{D}{ds}k(s) = \frac{D}{ds}w(s) = 0.$$

Notice that violation of unitarity occurs by means of $\kappa, L$: they represent irreducible *imaginary* friction terms, meaning that every process has a "cost" which cannot be undone -no perfect "Carnot cycle". It endows spacetime with a kind of effective granularity in the metrics determined by $h$ and $\sigma^2$. The solution to the equation reads

$$\phi(x, y = \exp_x(w); w, k) := \phi(x, y = \exp_x(w); w, k, 1) = e^{ik.w}e^{-\kappa\int_0^1\sqrt{h(w(s),w(s))}ds - \frac{1}{L^2\sigma^2(x,\exp_x(w))}}$$

which produces a generalized wave given by

$$\phi(x, y; k) := \sum_{w\in T_x\mathcal{M}:\exp_x(w)=y}\phi(x, y = \exp_x(w); w, k).$$

This calls for the following definition of the propagator

$$D(x, y) = \alpha\sum_{w\in T_x\mathcal{M}:\exp_x(w)=y}\int_{\mathbb{R}^4}d^4p\theta(p^0)$$

$$e^{-\mu h(p(0),p(0)) - \mu h(p(1),p(1)) - \mu(1-\theta(g(w,w)))(h(R_{w(0)}p(0),R_{w(0)}p(0)) - h(R_{-w(1)}p(1),R_{-w(1)}p(1)))}$$

$$\delta(g(p, p) - m^2)\phi(x, y; w, p)$$

where the $\mu$ terms express that the creation and annihilation processes come at a cost.

Under reasonable conditions, this regularized propagator is smooth everywhere and has exponential falloff behavior towards infinity. The Feynman propagator gets the following universal prescription:

$$\Delta_{F,\mu,\kappa,L}(x,y) = \sum_{w:\exp_x(w)=y \text{ and w is in the future lightcone of x}} D_{\mu,\kappa,L}(x,w) + \sum_{w':\exp_y(w')=x \text{ and w' is in the future lightcone of y}} D_{\mu,\kappa,L}(y,w') + \sum_{w:\exp_x(w)=y \text{ and w is spacelike at x}} D_{\mu,\kappa,L}(x,w)$$

meaning that all "information" has to travel towards the future which constitutes clearly the right function to study interactions with.

This theory has been worked out in full detail in [1, 2]. It turns out that all Feynman, or scattering, diagrams are finite in all known interaction theories for particles of spin less than $\frac{5}{2}$. Moreover, they are suitably bounded and show "exponential falloff behavior" even on Riemannian spaces with negative sectional curvature in case the friction parameter $\kappa$ is large enough. All theories need a modification of the standard Dyson expression for diagrams with a large number of internal vertices in order to procure analytical results and make the whole power series well defined. The interested reader is referred to these books for an entire elaboration of this theory. We now study the same theory from a different but more conventional perspective.

## 5.2 Haute Weinbergian cuisine.

Whereas this previous section procured extremely deep connections between different branches of physics from an elementary point of view, this section is somewhat more traditional but no less profound. It is just so that in the end, the same formalism is recovered in all known cases but maybe a different avenue is opened up. The latter, however, might be completely isomorphic to the one of the previous section however. This section is a lot more technical as all the previous ones and should encourage the reader to take up a book on geometry and operator algebra in order to understand its meaning. If not so, the reader should understand deeply that physicists understand the same material from many different points of view and that this richness in understanding has plenty of philosphical implications which could not be raised neither understood without a proper study of the consequences of this basic language. Effectively, a slight militant attitude towards the exact sciences in this chapter is entirely justified given that we have provided the reader with abundant evidence that appearantly complex issues which seemingly require loads of experimentation can be understood from very simple deductions from "die sprache selbst". The deeper messaage of this series it to get rid of the hopeless mubling in the current *university* curricula regarding the social sciences; these practices belong in my view at some college and not a prestigious institution of God.

## Classical physics revisited.

Consider a particle moving in a bundle $\mathcal{E}$ over a Lorentzian spacetime $(\mathcal{M}, g)$ where the fibres are equipped with a metric field and the associated connection preserves the total metric (which is usually a product metric). Regard the wordline as an immersion $\gamma : \mathbb{R} \to \mathcal{E}$ and the momentum as its the push forward of $\partial_t$ with equals

$$\frac{D}{dt} := \nabla_{\frac{d}{dt}\gamma(t)}$$

where $\nabla$ is the bundle connection. Given that we shall only work with functions $f : \mathcal{E} \to \mathbb{R}$, the latter expression can be taken for $(\partial_t)_\star$ as an ordinary vectorfield instead of a general derivative operator. To every curve $\gamma$ and function $f$ we can attach a function $\gamma_f : \mathbb{R} \to \mathbb{R} : t \to f(\gamma(t))$. We can now define a $C^\infty(\mathbb{R})$ algebra of operators $\mathbb{L}$ on the function space $f : \mathcal{E} \to \mathbb{R}$ mappping them to functions from $\mathbb{R}$ to $\mathbb{R}$. Concretely

$$[(\gamma_f)(g)](t) := f(\gamma(t))g(\gamma(t))$$

and

$$[p_\gamma f](t) := \frac{d}{dt} f(\gamma(t)).$$

We have moreover,

$$\gamma_f(g + h) = \gamma_f(g) + \gamma_f(h)$$

and

$$[(\partial_t)(\gamma_f g)](t) := [(\partial_t)_\star f](t)g(\gamma(t)) + f(\gamma(t))[(\partial_t)_\star(g)](t).$$

This suggests to extend the definition of the momentum in this way to functions $\mathbb{R} \to \mathbb{R}$. The same comment holds for $\gamma_f$. In this vein,

$$[\gamma_g \gamma_f h](t) = g(\gamma(t))f(\gamma(t))h(\gamma(t))$$

and

$$[p_\gamma \gamma_f h](t) := \partial_t(f(\gamma(t))h(\gamma(t)))$$

as well as

$$[\gamma_f p_\gamma h](t) := f(\gamma(t))\partial_t h(\gamma(t)).$$

Finally,

$$[p_\gamma p_\gamma h](t) = (\partial_t)^2 h(\gamma(t))$$

which induces a real algebra generated by

$$\gamma_g, p_\gamma$$

where $\gamma$ varies over all immersions. This algebra is represented by means of linear operators on the function algebra

$$\mathcal{B} := C^\infty(\mathbb{R}) \otimes C^\infty(\mathcal{E})$$

which may be given the structure of an Hilbert algebra in the usual $L^2$ sense by introducing an einbein on the "time line" $\mathbb{R}$. Concretely

$$[\gamma_f, \gamma_h]\,(g) = 0 = [p_\gamma, p_\gamma]\,(g), [p_\gamma, \gamma_f]\,(g) = p_\gamma(f)\gamma_\star(g) = \gamma_{p_\gamma(f)}(g)$$

where $\gamma_\star$ is the pull back defined by the immersion $\gamma$. Here, the commutation relations employ the full $\mathcal{B}$ action but are understood to apply on $f, g, h \in C^\infty(\mathcal{E})$ and result in an element of $C^\infty(\mathbb{R})$.

Covariant dynamics requires dynamics without potential energy terms; therefore, any force has to be implemented in the momentum what explains the bundle $\mathcal{E}$. Moreover, according to Einstein himself, every force, including the gravitational one, can be gauged away in some point so that locally and physically every particle is a free one meaning that the correct equation is the geodesic bundle equation. Therefore, the classical Hamiltonian is a constraint and moreover, commuting it with a vector leaves a covector if it were an invariant energy so that

$$[\mathcal{H}(\gamma_f, p_\gamma), p_\gamma]$$

cannot represent $\frac{D}{dt}p_\gamma$ unless we would make an extra metric contraction. Actually, the whole Hamiltonian edifice is kind of meaningless as we shall see now. Indeed, taking $\mathcal{H}(\gamma_f, p_\gamma)$ to be $p_\gamma$ with equations of motion given by

$$[\frac{D}{dt}\triangle\gamma_f](g) := [p_\gamma, \gamma_f](g) = \gamma_{p_\gamma(f)}(g)$$

and

$$[\frac{D}{dt}(\partial_t)_\star](g) = [\frac{D}{dt}\triangle p_\gamma](dg) := [p_\gamma, p_\gamma](dg) = [p_\gamma, p_\gamma](g) = 0$$

where

$$[\frac{D}{dt}\triangle\zeta](g) = [\frac{D}{dt}, \zeta](g).$$

There is nothing more to say really apart from the constraint $g(p_\gamma, p_\gamma) = \frac{m^2c^4}{\hbar^2}$ which is the mass energy relation. This is all what is allowed in classical physics of point particles really and we now proceed to quantum theory. Notice that the dynamical content is completely implied by the commutator algebra which constitutes a total unision between dynamics and kinematics. Physically, this is entirely trivial and completely justified given that the momentum just corresponds to the energy in a rest frame. Note also the presence of $\hbar$ in the latter formula which is there for dimensional reasons; alas, it does not do anything else apart from setting a time scale given that the covariant derivative does not depend upon it.

## Relativistic Quantum Theory.

As we have shown in the previous section, the Poisson Bracket really is a commutator and the Hamiltonian formulation is rather void given that the total free momentum, constrained by the quantum mechanical mass formula is the only real quantity of interest. Unlike in classical physics, quantum mechanics cannot use an external time in a sense given that a particle is not specified anymore by a worldline but by a wave. In a way, it is the complex dual of the classical situation where "worldlines" correspond to functions $\psi : \mathcal{E} \to \mathbb{C}$ which are $C^\infty$. The operators $\gamma_f$ and $p_\gamma$ are replaced then by $x_f$ and $i\nabla_V$ where $V$ is a real vectorfield over $\mathcal{E}$ and $f$ is a real valued function over $\mathcal{E}$. Here, $[x_f](g)(x) = f(x)g(x)$ and

$$P(V)(g) := i\nabla_V(g) = iV(g).$$

They obey the algebra

$$[x_f, x_h] = 0, [i\nabla_V, i\nabla_W] = -R(V,W)(\cdot) - \nabla_{[V,W]}$$

and finally

$$[i\nabla_V, x_f] = x_{iV(f)}.$$

The momentum commutation relations have been put in this exotic form because the covariant derivative can work on vectorfields and higher objects too. The $i$ is just there to ensure that the momentum operator is real given that the commutator of two real operators is imaginary. The situation here is very different as one cannot just pick a Hamiltonian linear in the momenta given that one would as thus preselect a nondynamical arrow of time. Hence our only choice is given by

$$H = \sum_{i,j=1}^{n} \eta^{ij} \nabla_{E_i} \nabla_{E_j}$$

where the $E_i$ correspond to loical vielbeins and $\eta^{ij}$ is the inverse of the standard flat metric. In order for this to work $\nabla$ must be extended to the spin connection to digest local boost transformations. Furthermore, one has

$$H = m^2$$

as constraint. It is clear one has no Heisenberg type dynamics here as the vectorfields really are spacetime vectorfields; hence, the entire theory is encapsulated by the constraint and the geometry of the bundle $\mathcal{E}$. It has been shown by Ashtekar and Magnon that this theory only works out fine in stationary spacetimes with Minkowski as the prime example due to the existence of scalar products on leafs of a foliation for which the latter is preserved in "time".

In 2011, I wrote a book about an operational approach to quantum theory with local vacua delineating a Fock-Hilbert bundle $\otimes_{x \in \mathcal{M}} \mathcal{H}_x$ over the spacetime manifold $\mathcal{M}$. However, the approach was troublesome and muddled with two "fundamental errors" of mine, not due to a lack of mathematical precision, but being the consequence of a poor understanding of what curved spacetime really signifies. This error found a natural solution in [1] written on generally covariant quantum theory from the point of view of the Feynman series.

Concretely, we assumed $\mathcal{H}_x$ to be constructed by means of a cyclic quasi-free vacuum state $|0\rangle_x$ and multiparticle states showing Bose or Fermi statistics constructed in the Fock way. The dynamical object was a unitary bi-field $U(x,y)$ mapping $\mathcal{H}_y \to \mathcal{H}_x$ and obeying a Schroedinger like differential equation

$$\frac{d}{dt}U(t,s) = iHU(t,s)$$

but then with the times $t,s$ replaced by $x,y$. The two errors in the book originated from the mathematical implementation of this idea I conceived; first of all $U(t,s) = U(t)U^{\dagger}(s)$ and moreover the only covariant first order differential operator *homogeneous* in the spacetime coordinates is given by the covariant Dirac operator $D$. The first condition is equivalent to a "cohomology" condition

$$U(x,y)U(y,z)U(z,x) = 1$$

which turns out to hold in Minkowski or any maximally symmetric spacetime only and reflects the absence of local gravitational degrees of freedom. Consequently, the only solution I was able to find of my field equations was free quantum field theory on Minkowski in a way I shall explain later. The Dirac operator gives all sorts of trouble meaning we have to replace the complex numbers by an appropriate Clifford algebra of signature $(1,3)$ or $(3,1)$. This gives rise to negative probabilities and huge problems with the spectral theorem even for finite dimensional Clifford bi-modules. The approach was clearly dead as it stood which I realised later on.

## Taking bi-fields seriously.

As pointed out in [1], the idea of a Hilbert bundle *is* adequate, but the correct differential equation for $U(y,x)$ needs to run over geodesics connecting $x$ with $y$ in a fully reparametrization invariant way. The obvious candidate is given by

$$\frac{d}{ds}U(\gamma(s),x) = i\dot{\gamma}(s)^a P_a U(\gamma(s),x)$$

where $\gamma(s)$ is the unique geodesic connecting $x$ with $y$ and $P_a$ equals the free momentum generator, given by the expression

$$P_a = \sum_{\text{particles j, internal degrees}\,\sigma_j} \int \frac{d^3k}{\sqrt{k_0}} k_a a^{\dagger}_{k;j,\sigma_j} a_{k,j,\sigma_j}$$

at the point $z$ with respect to the dragged vierbein in $x$ along the geodesic. The coincidence limit is fixed by $U(x,x) = 1$; this suggests one to enlarge the notation to $U(y,x;e_a(y),e_b(x))$ as well as a unitary action $T(\Lambda, e_b(z))$ of the orthochronous Lorentz group $\Lambda \in O^+(1,3)$ on $U(z,x;{e'}_b(z),e_a(x))$ by means of conjugation $TUT^{\dagger}$. All this has been explained in [2]; in order for $T(\Lambda(s), e_a(\gamma(x))$ to shift through $\frac{d}{ds}$ we need a Lorentz covariant derivative and, henceforth, an antihermitian connection $L_{\mu}(z, e_b(z))$ such that

$$\left(\frac{d}{ds} + \dot{\gamma}^{\mu}(s)L_{\mu}(\gamma(s),\Lambda_a^b(s)e_b(\gamma(s)))\right) T(\Lambda(s),e_b(\gamma(s)))U(\gamma(s),x;e_b(\gamma(s)),e_a(x))T^{\dagger}(\Lambda(s),e_b(\gamma(s)))$$

$$-T(\Lambda(s), e_b(\gamma(s)))U(\gamma(s), x; e_b(\gamma(s)), e_a(x))T^\dagger(\Lambda(s), e_b(\gamma(s)))\dot{\gamma}^\mu(s)L_\mu(\gamma(s), \Lambda_a^b(s)e_b(\gamma(s))) =$$

$$T(\Lambda(s), e_b(\gamma(s)))\left[\left(\frac{d}{ds} + \dot{\gamma}^\mu(s)L_\mu(\gamma(s), e_b(\gamma(s)))\right)U(\gamma(s), x; e_b(\gamma(s)), e_a(x))\right]T^\dagger(\Lambda, e_b(\gamma(s)))$$

$$-T(\Lambda, e_b(\gamma(s)))U(\gamma(s), x; e_b(\gamma(s)), e_a(x))\dot{\gamma}^\mu(s)L_\mu(\gamma(s), e_b(\gamma(s)))T^\dagger(\Lambda(s), e_b(\gamma(s))).$$

In case we dispose of multiple geodesics connecting $x$ with $y$, we just multiply the corresponding unitary operators in the *same* vierbein at $y$, the order of which does not matter given that all $P_a$ commute and because the action of the Lorentz group acts by boosting the momenta. Therefore, we can just sum up the momenta which can accomodate for topology change of Minkowski into a flat spacelike cylinder giving rise to the correct field picture.

There is however a small caveat here in case multiple geodesics connect $x$ and $y$ in the sense that the gauge field might acquire a nontrivial significance due to multivaluedness of $e_a(y)$, where the latter is the dragged vielbein from $x$ to $y$. Hence, it is better to replace the argument $y$ by a tangent vector $V$ in $T\mathcal{M}_x$ and take the $x$ perspective where $\exp_x(v) = y$. In that case, we set $L_\mu(sv, \Lambda_a^b(s)e_b(\exp_x(sv)))$ to zero in case

$$\frac{D}{ds}e_b(\exp_x(sv)) = 0$$

for $s = 0 \ldots 1$. In other words, the vielbein in the warped point in that direction must be the dragged one; this makes both formalisms entirely equivalent what the free theory is concerned. Notice that by construction, $U(y, x) = U^\dagger(x, y)$ due to the minus sign caused by flipping $\dot{\gamma}^\mu(s)$. Given that the connection $L_\mu(v, e_b(\exp_x(v)))$ is a new object defined on

$$T\mathcal{M}_x(v) \times V\mathcal{M}_g(\exp_x(v))$$

where $V\mathcal{M}_g(\exp_x(v))$ is the nonlinear space of $g$ vierbeins over $\mathcal{M}$, which is equivalent to the group manifold $O^+(1,3)$ regarded as a homogeneous space with a hyperbolic Cartan metric of signature $(3,3)$; it might be opportune to make it more dynamical and invent a new type of non abelian Yang-Mills theory over $T\mathcal{M}_x$. This author tried this also in 2011 but failed to recognize the bundle perspective as well was stuck with Clifford modules for replacements of Hilbert spaces. The easiest thing is to see $L_\mu$ as $L_{\partial_{v^j}\exp_x(v)}(v, e_b(\exp_x(v)))$ where $e_b$ varies independently and refers to $y = \exp_x(v)$ and subsequently write out a Yang-Mills equation of the kind

$$\left(D_v L_{\partial_{v^j}\exp_x(v)}(v, e_b(\exp_x(v)))\right)_{kl} = \left(\partial_{v^{[k}} - L_{[k}\right)L_{l]} = (dL)_{kl} - (L \wedge L)_{kl} = 0$$

where $d$ is the Hodge operator on flat tangent space. Life could be more exciting as to pick out the zero solution in parallel transport gauge and we leave this new piece of physics for further examination of the bored ones.

So far, we have determined only our quantum connection; now, we develop bi-fields which are nothing but the warps of coincidence fields meaning

$$\Phi(y,x) : \left( \prod_{v \in T\mathcal{M}_x : \exp_x(v) = y} U(y = \exp_x(v), x; (\exp_x(v))_\star e_a(x), e_a(x)) \right) \Phi(x,x)$$

$$\left( \prod_{v \in T\mathcal{M}_x : \exp_x(v) = y} U(y = \exp_x(v), x; (\exp_x(v))_\star e_a(x), e_a(x)) \right)^\dagger .$$

Here,

$$\Phi(x,x) = \sum_{\text{particles j with internal quantum numbers } \sigma_j} \int_{\mathbb{R}^3} \frac{d^3k}{k_0} (v_{\sigma_j} \sqrt{k_0} a^\dagger_{k,j,\sigma_j} + \overline{v}_{\sigma_j} \sqrt{k_0} a_{k,j,\sigma_j})$$

where $v_{\sigma_j}$ is an internal field vector associated to the internal particle degrees of freedom. They are needed to obtain different physical behaviour, $\frac{d^3k}{k_0}$ is the on shell relativistic measure in Fourier space on Minkowski and finally, $\sqrt{k_0} a_{k,j,\sigma_j}$ is relativistic normalization of the creation annihilation algebra. I leave it as an elementary excercise to find out principles determining $v_{\sigma_j}$. So $\Phi(x,x)$ is the proper democratic relativistic expression taking into all matter degrees of freedom in the universe.

### Interaction theory.

So far, we have delineated the free theory from an operational bi-field formalism which reduces in Minkowski to a single field formalism due to the remarkable "cohomology" property

$$U(x,y)U(y,z)U(z,x) = 1$$

where we have dropped the vielbeins and assumed dragging allalong which is logical given that dragging is trivial and hence consistent along closed paths due to the vanishing of the Riemann tensor. The trick now is to work directly into an interaction picture and forget about a closed bi-field equation. That is, we write down spacetime interaction densities of the kind

$$i\lambda \int_{\mathcal{M}} \sqrt{g(y)} \Phi(y,x)\Phi(y,z)\Phi(y,p)\Phi(y,q).$$

This is an obvious excercise leading to a completely equivalent formalism as in [1, 2].

## 5.3 The Psychological arrow of time from level-zero processes.

We shall provide here the argument from the Weinbergian point of view; however, the entire reasoning easily copies almost at verbatim to the Wightman approach to quantum theory. In our regularization procedures, we have introduced a timelike vectorfield in order to Wick rotate the spacetime metric into a Riemannian one and have left the procedure open and to the discretion of the reader. It is however, possible to write down a highly coupled system, given that locally, for fixed $x$, a bi-field $\Phi(x, v)$ exists on $T\mathcal{M}_x$ and henceforth an operational energy-momentum operator $T_{ab}(x, v)$ with local Lorentz indices $a, b$ referring to the vielbein $e^a_\mu$. In contrast to the spacetime symmetry group which is expressed by means of general linear tranformations, the Lorentz transformations are restrictive enough to separate a energy-momentum vector, associated to the boosts, from a "hydrodynamical" pressure part, associated to the rotations. This was the main idea behind the Weinbergian construction and henceforth we may consider the scalar currents

$$V^\mu(x) := \langle\psi|P^a(x)|\psi\rangle e^\mu_a(x)$$

where $|\psi\rangle$ is the state of the universe and $V^\mu(x)$ is assumed to be a timelike or null vector. This not automatically so given that only

$$P_a P^a = m^2$$

and therefore

$$\eta^{ab}\langle\psi|P_a P_b|\psi\rangle = m^2 > 0.$$

This does not fix however the sign of

$$\eta^{ab}\langle\psi|P_a|\psi\rangle\langle\psi|P_b|\psi\rangle$$

which is the expression we really need. Therefore, we make an assumption here and those matters can be further discussed. Henceforth, given an orginal spactial three surface referring to an instantanious now, we have just provided for a dynamical spiritual clock associated to the gheistly projection on spacetime. This automatically determines the later "nows" and henceforth provides us with a flow of time. It is indeed the insistance on asymptotic freedom at the shortest scales which turns out to provide for the psychological arrow of time. I have proclaimed this idea in 2011 but failed at some small technical issues; this rectifies that situation.

There is another interesting issue to be remarked here; although we just defined a psychological arrow of time from level zero processes, one could wonder what the gravitational backreaction is of level one psychic processes, given that the notion of negative energy, and therefore mass, is ssociated to either the gauge bosons or the psychic black-white particles showing up in our theory. I proclaim that the level one black-white matter *destroys* the positivity of mass whereas the gauge bosons violate it and therefore gravitate. The reason why I say this is because people with a huge amount of psychic energy within themselves have the experience of an upwards pressure in the head. This happens only when he is a Switchoriem who always feels light and energized *mentally* because he is not losing any psychic, in the sense of astral, energy. This effect keeps him or her shielded from black-white interactions so that only gravity remains as a mode of communication; whereas level-zero gravity attracts the "bodies", the astral "bodies" repel one and another which explains why spirits ascend to "heaven" in the long run given that black-white communication is impossible to substantiate unless they conquer a new body (astrally or even as a person). Hence, the Switchoriem is a rather emotional person and values money much more as a mode of communication in opposition to the more advanced psychic interaction modes of the astral eye and belly. The reason for this is his permanently hot or cold heart (heart eigenstates come with eigenvalue $\pm 1$) blocking any kind of empathy and even sympathy with another person who prefers a lukewarm heart on average. He is doomed to extreme individualism and physical death eventually due to a bad physical constitution or by being cold bloodedly killed due to disagreement amongst themselves. This brings me back to the issue that in principle no spiritual heart is necessary to substantiate a long life on earth given that the spiritual heart uses the most amount of spiritual gazoline. However, I can assure the reader the problems are even more severe there as the higher level Switchoriem is even more a cold blooded logical killer and the black-white extremes at that level might be preferred. Indeed, those who are passionate regarding intellectual supremacy are the biggest killers. It appears therefore that some branch cut may appear at the level of the Switchoriem and rotating the pies of angle $\frac{\pi}{4}$ around the axes at angles $\frac{\pi}{8}, \frac{3\pi}{8}$ respectively when going up to a next to a level. Given that those two axes remain invariant under these processes, I proclaim them to be nature's optimum from the psychic point of view.

In this vein, nature appears to disfavor a Switchoriem couple although they are in peace with nature. After a while they get ice cold with respect to one and another and engage in a divorce because they are both close to being a Switchoriem individually or they fight in court the shit out of one and another because they differ too much. In any case, such marriages are extremely difficult and results in visits to the psychologist as well as psychiatrist. On the other hand positive or white marriages ($\frac{3\pi}{8}$) and negative or black marriages ($\frac{\pi}{8}$) between two compatible persons are very joyful or dark on one hand but this is a positive characteristic for both if the interpersonal black-white accordances are strong enough, but they are not very much accepted from the societial point of view given that they disrupt the global peace too much. Having an almost equal polarization means the eyes are almost one and then the choice between the heart and the belly follow. Being in the environment, however, they constitute themselves a white or black micro-society which stands out because both start radiating; this evoques negative sentiments with people of the opposite kind but for different reasons. White radiating people evoque the sentiment of jaleousy with others whereas black radiating people erupt disgust. Indeed, the greedy eye is still the first cause for disrupting peace whereas equally beautiful people have a harmony in which the heart and belly can fluctuate. In this sense, the psychic eye only flashes when people are much more or less pretty. The same holds for the belly chacra which means that you are disgusted of the other person; indeed, you do not like a warm belly but it makes you weaker towards to other person and brings peace. The eye is violent in that way given that it scares black people away when being white whereas the belly chacra is much softer and it installs a disgusting weak peace. This explains also why black people poison white ones by means of food, to switch on the belly charca which is commensurable with swithing of the eye and at the same time preventing white people to grow. In a two couple family, where each individual couple is arranged according to the eye, a harmony exists in the family at level-zero of basic actions and observations leading to a very rational and cold hearted atmosphere at level one and zero when both couples are together, as seen from a *large distance* by exterior observers. On short distances, trouble may arise due to the black jaleousy and the white disgust: there are two necessary conditions to circumvent that problem. One of them is that partner exchange is natural given that there needs to be a strong pairwise bound at level zero, meaning the one of elementary conversations and actions. In other words, the black or Switchoriem woman in the black couple has to be horny (by means of the eye and belly chacra's which are activated) of the white or Switchoriem male (who has the same chacra's activated) in the white couple and the other way around for the black to Switchoriem male regarding the white to Switchoriem woman. The horny character comes usually from the black side whereas the chacra's flip on in both ways. The *dynamics* of nature has that constraint, that opposite hearts and sexes cause for a hot eye and belly whereas this does not occur with equal sexes making agreements between white and black men much more difficult. We have implemented that constraint ad hoc before by *demanding* that white men correspond by heat conjugation to a black woman and the other way around (heat from eye and belly chacra radiation). Indeed, heart conjugation interchanged impetus and awareness given that the latter is a real quantity and the former an imaginary one for a black man. To finally ultra-stabilize such scenario, intellectual agreement at level two is more than welcome; I forsee that such higher coupling between couples may become fashionable in the (perhaps not too) near future. All societial difficulties are indeed relegated to the one of the kitchen and not of the belly charca which is often confused with the kitchen. They need to consume Switchoriem food at the dinner table in four and eat their own food in two in the bedroom. I have discussed the appropriate food intake in [2].

This viewpoint is naturally substantiated from our algebraic viewpoint where the basic infinitesimal actions at level zero are provided by the $su(2)$ Lie algebra consisting of three anti-Hermitian operators $S, WB, Z$ satisfying $S(WB) = Z$, $(WB)Z = S$, $S^2 = (WB)^2 = Z^2 = -1$. Here $iZ$ gets the interpretion of the spritual *eye* given that it preserves black and white operators and therefore they serve as a number operator for black and white profiles $-iS$ and $-i(WB)$ have different interpretations associated to the heart and belly. The spiritual heart must be associated to the awareness-impetus relation and is therefore given by the $-i(WB)$ operator whereas $-iS$ must be then related to the belly chacra. There is only one thing for sure in nature, it is birth and rebirth so the heart operator must be almost a physical symmetry of nature, it is a kinematical requirement. Given that equal physical hearts give opposite spiritual hearts, being a spiritual heart opposite is a good way to constitute exclusive couples given that, as a couple, you are heart neutral meaning that at short distance you loose a gigantic amount of spiritual energy. The spiritual eye, on the other hand, sets the benchmark for psychic interactions and it is a kind of inertia principle to demand that it is almost entirely conserved on a global scale. People who have equal spiritual eye polarization find each other beautiful *physically* given that they loose no spiritual energy whatsoever when being on small scales together. A physicist would utter that without gravitation $WB, Z$ are exactly conserved whereas $S$ must be a small gravitational correction because the eye needs to be in piece for the entire universe and the hearts as well. However, in physics, it would be unlikely that life would form in the first place given that gravition is reponsible for clutting low intelligence spirits together who would otherwise repel. In that vein is gravitation a kind of whip mandatory to become more intelligent as beauty and the heart do not cause for such things. Indeed, rationality, or going over to higher spiritual levels is the only way in the long run to overcome the wild heart as well as the differences in strength and beauty. On the other hand, the whip cannot be too forceful which is why people say that inertia must almost hold exactly. The belly cannot be conserved at all given that we would float exactly in the middle between life and death as well as beauty and uglyness. This whip would be too hard and cause for a complete gravitational disaster in terms of spiritual black hole due to the negative mass squared of spiritual energy. Hence, it would be meaningful to ask on long time scales the spiritual belly is of the Switchoriem type

$$\mathrm{Tr}(Se^{-\beta H}) = \epsilon \mathrm{Tr}(e^{-\beta H})$$

where $\beta$ is the temperature associated to a time scale, $H$ the total dynamics of the system and $\epsilon$ a very small energy which would naturally correspond to $\frac{1}{\beta}$. This is so for reasons of harmony at level zero assured by procreation (which requires sex conjugation on average given that birth of a single individual breaks it) as well as harmony at level one which requires white-black or heart conjugation. No spiritual person has a heart for him or herself on average given that

$$(\alpha, \beta)^T (WB)(\alpha, \beta) = 0$$

for all $\alpha, \beta$. White people, on the other hand, always have a white spiritual eye whereas black people always have a dark one; Switchoriems have on average no definite colour in the eye given that they do not reside in an eigenstate of the spiritual eye operator. They are the ones that schmuck themselves up by means of beauty products whereas the extreme cases accept their own form of spiritual beauty. Regarding the spiritual belly operator, $-iS$, we conclude that pure black or white profiles do not lose on average charge either given that the opposite reality condition holds

$$((-iS)v)^c = (-iZ)((-iS)v)^\star = (-iZ)(-iS)v^\star = (iS)(-iZ)v^\star = -(-iS)v$$

for $v^c = v$. This is however sufficient to ensure that both forms of the Heisenberg algebra remain invariant

$$(X,P)^T(WB)(X,P) = (X,P)^\dagger(-iS)(X,P) = -1.$$

For mixed profiles, the belly chacra is almost always on whereas the Belgian marriage shuts it off completely. Therefore, in Belgium the characteristics of each couple pertain to the spiritual eye. The heart is always on given that it is associated to the identity operator on Hilbert space and couples who are able to shut off the belly and eye operator are by far the most energetic ones at that level of spirituality. This leaves us with the interpretation of the identity operator in the two times two matrix algebra: it is associated to the *observational* eye, which is nothing but our true "physical" eyes. The *act* of observation however does not commute with the activation of the spiritual belly and eye. They are all in a mixed state when observation happens. Therefore, when shutting down observational skills, your eye and belly are lukewarm and nothing is seen. These couples give warm feelings in the belly and look neutral towards the public: this is the Roman way of dealing with the first level of spirituality. There is a general peaceful atmosphere where jealousy between couples due to the eye is surpressed. At the individual level, there is a total blance between all three (as is the case for all marriages); the expectation value of any individual chacra vanishes when taking scalar product of the nature

$$\langle v|A \otimes 1|v\rangle = 0 = \langle v|1 \otimes A|v\rangle.$$

Such couples do radiate towards one and another but moderately when being together in the joint spirit of being together physically but in the individual spirit. It is not noticable towards others that they are joint or not. The Roman marriage is not the most efficient one from the energetic perspective but it is a very happy given that the belly and pinal gland operate moderately at the unconscious level; in any case, it does *not* invite both participants to go to a higher level and such societies are mostly stagnant or stationary. Societies based upon the vanishing of the belly operator are best governed by communists who give an equal amount of money to everybody as well as Switchoriem clothes covering up for beauty and enhancing the looks of ugly people. Serious entrepreneurs and intellectuals should remain absent in such system except at a level where their identity is unknown to the people. From the energetic perspective, a *permanently* radiating pinal gland is has the largest cost, followed by the belly and finally the heart (which is always in an eigenstate). Gravitionally, this implies that Polish people ascend the quickest from earth spiritually given that they use the most energy as a couple, followed by the Belgians and the Italians who go the slowliest to heaven.

Here, we have distinguished the spiritual eye by means of the physical eyes as an observator and the pinal gland as an emittor: obervation (possibly of oneself) always happens with the spiritual eye through the eyes whereas emission occurs by means of gland in the forehead. Given that the gland needs to function at a minimal level in order to observe through the physical eyes, a defunct gland leads to all other two chacra's being shut down on average and having no observational skills whatsoever. The Polish people can shut off the gland by means of higher charges due to a higher ethic of relationship formation whereas plenty of Belgians need psychiatry due to an ethic (the belly) which leaves the gland open in social interactions (but with an average zero charge loss). They do not posses the necessary ethic to be pleasant at level one in social interactions. Regarding the astral eye and the three chacra's, we have been speaking about self-observation; a person can observe its own astral body (oeroburos) by means of the physical eye whereas it is rather obvious that the pinal gland regarding an observable activation of the three chacra's, needs to be the most efficient one from the energetic perspective since otherwise observation through the physical eye would not be possible. From the physionomical perspective this is rather obvious given that the pinal gland is closely located near the physical eyes as well as resides in the brain. The $(X,P)^\dagger(-iZ)(X,P)$ operator measures spiritual *charge* whereas the astral gland is represented by an operator $C$ such that the action of $Z$ on the pair $(X,P)$ is given by $[-iC,X]$ or $[-iC,P]$. We shall interpret this sentence in two different ways later on leading to $C=(X,P)^\dagger(-iZ)(X,P)$ effectively. The spiritual eye comes with the lowest cost followed by the pinal gland whereas the belly which is the most primitive of all. In that way, the Roman, Dutch, Spanish, Swiss and Austrian *couples* use plenty of energy in relationship investement whereas the Belgians the least and Polish are intermediate. They are usually horny as hell regarding their partner whereas the Italians are in peace. The Belgians are in a permanent war with their partner and have the highest divorce rate. It could be said that given the Belgians save the most on spiritual energy, they must be the smartest ones and use this for higher skills. Unfortunately, this is not the case, the cold belly leads to very astute people which requires rationality at a much higher level to obtain the necessary warmth and this not within their reach. They suffer from delusions of grandeur given that they are uncapable of dealing in a responsible way with the pinal gland as well as their spiritual eyes. They value money as a means for marriage as it is otherwise totally empty. Not even the Romans are so greedy what the first level of spirituality is concered whereas they are still the most intellectual ones together with the British. Only old Egyptian pharao's can easily deal with this level of communication. The Swiss come first in this regard, they switch off the belly and pinal gland chacra's whereas the spiritual eyes are almost totally closed. This used to be the old French recipe too but this efficiency at level one is nowhere obtained in the rest of the world. The Swiss are closely followed by the Italians and Spanish, next come the Austrians together with the Dutch. This explains why, dynamically, in an environment where the Switchoriem is a neutral element, only this type is capable of having spiritual self-adoration *exactly* all the time due to the principle of inertia, given that the Switchoriem vacuum remains invariant. Indeed, the Switchoriem is described by the annihilation and creation algebra which reveals that there is no observation by means of the spiritual eyes. Generically, he will have his belly chacra (the penis or vagina) on in a varying non-trivial lukewarm way to feel comfortable with himself and is lukewarm on average regarding his own beauty. Now, we *proclaim* the existence of a Hermitian sex operator $C$, a marital contract of some kind, where the sex *conjugation* $\widehat{C}$ is defined in the usual way by means of the Lie bracket

$$\widehat{C}(X) = [C, X]$$

mapping Hermitian operators to anti-hermitian operators given that change of sexual quantity is a kind of impetus. A *dynamical aspect* of the theory is that $\widehat{C}(X, P) = C(X, P)^{\star}$ preserving purity, meaning a black male is mapped to a black female, as well as process-state duality meaning a black female observes states where black men *undergo* processes. They see change whereas black males only see subsequent images. Therefore, given that a white male person should correspond to an anti-Hermitian operator if black people are described by a Hermitian one, the black female must be described by $i$ times the male impetus operator. Although this is energetically the worst solution, it is the best whip to get to a higher spiritual level given that people must learn to settle for their differences at that level. Belgians, being truely at level one, must undergo this whip whereas Romans, Duth, Swiss and Spanish people torture one and another on a higher plane of awareness. So, in the Belgian vein, $C$ *must* be given by the belly operator $-iS$ given that it is the only operator preserving the Heisenberg algebra as well as reversing the reality condition.

Humanity does not always follow level-one nature however and wishes to proceed to a higher level by means of neutral conduct at level zero and one as much as possible and henceforth enjoy different marital *contracts* which are of the hot Polish nature but then at a higher plane: the highest one is given by shutting down the spiritual eye (Roman system) as well as the belly and pinal gland (as much as possible) which is the Swiss system. In case of a mixture of the belly and heart this gives marital contract $MC$ of the form

$$MC = \begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix}$$

which agree for $\theta = \frac{\pi}{2}$ with $-i(WB)$ whereas $\theta = 0$ with $-iS$, the belly chacra, which expresses a possibly disgusting peace due to too large discrepancies in the astral eye causing jealousy and abnormally low sexuality between Belgian partners involved in a *straight* marriage contract. One would expect such theta angle to be a dynamical variable varying from culture to culture. Where I come from we have that

$$[MC, WB] = \pm 2iZ$$

which fixes $C = \pm iS$ being the belly operator, the natural contract at level-one as to speak. Another law would be

$$[MC, WB] = \pm 2iS$$

which provides for $C = \mp iZ$ being the eye operator itself, a contract made upon physical (level zero) and mental beauty (at levels one and two at least). Being in the nucleus of an operator $A$ means that you do not eat neither cure out by means of $A$ at that level. The Belgians poison their food to block the belly chacra given that it is hard to find an exact belly conjugate partner allowing for couples of almost opposite polarity to marry one and another. This is a cruel solution to get to a higher intellectual level given that the lack of emotional skill causes difficulties of a more primitive nature blocking for intellectual progress. In what follows, we shall make a slightly, seemingly innocent, wrong choice of convention by means of working with the Hermitian conjugate instead of just ignoring the reality conditions and proceeding in the complex fashion. Indeed, the heart operator $-i(WB)$ was first associated to the quadratic form

$$(X,P)^T(WB)(X,P) = -1$$

which is a matter of complex symplectic geometry. When taking the Hermitian convention and using the reality conditions, we get

$$(X,P)^\dagger(iWB)(X,P) = i(XP + PX) = i(2XP + 1) = D$$

being a totally different Hermitian operator. This is clearly the wrong way of proceeding what the interpretation is concerned as a spiritual heart provides for spiritual life and therefore should commute with all other spiritual operators. We nevertheless adapt the Hermitian convention here to show to the reader that the *mathematical* aspects remain the same and that nature is described by means of complex geometry and not Hermitian geometry. Henceforth, we leave all interpretational aspects to the discretion of the reader.

We now proceed by making some elementary spiritual computations; defining

$$(X,P)^\dagger(WB)(X,P) = XP + PX = 2XP + 1 = -iD$$

and

$$[XP, H] = \left[XP, X^2 - P^2\right] = 2X^2 + 2P^2 = 2(X,P)^\dagger(-iZ)(X,P) = 2E$$

we have that all Hermitian quantities are described by a $su(2)$ Lie algebra (with charge 4). Indeed,

$$[D,H] = 4iE,\ [D,E] = 4iH,\ [E,H] = \left[X^2 + P^2, X^2 - P^2\right] = 4iD$$

where $XP$ can be interpreted as the complex black number operator given that

$$[XP, X] = X$$

for the creation operator of a black quantum whereas

$$[XP, P] = -P$$

for the annihilation operator $P$ of a black quantum. The black vacuum is defined by $P|0_b\rangle = 0$ and the correct scalar product for one black quanta is provided by the involution $b$ such that $X^b = P$ and therefore

$$\langle 0_b|X^b X|0_b\rangle_b = 1.$$

The latter involution can only be inner when

$$X^b = A^{-1}X^\dagger A^\dagger = A^{-1}XA^\dagger$$

where $A$ is given by the natural Hermitian, functional[1], implementation of the belly chacra operator $-iS$ so that

$$\langle\psi|\phi\rangle_b = \langle\psi|A|\phi\rangle.$$

Regarding the Hermitian operator $B$ functionally corresponding to $-i(WB)$, one has that $A - iB + X^2$ commutes with the number operator $XP$

$$\left[A - iB + X^2, XP\right] = P^2 + X^2 - P^2 + X^2 - 2X^2 = 0$$

and moreover, for the operator $C$ functionally corresponding to $-iZ$

$$[C, XP] = 0$$

and regarding

$$\left[-\sqrt{(P+\epsilon)^{-1}}, XP\right] = P(P+\epsilon)^{-1} \sim 1$$

is the Heisenberg conjugate to the number operator $XP$. It is not an exact one which is logical due to fringe effects. Therefore, a Polish marriage is given by the operator

$$(X,P)^\dagger(X,P)\otimes(X,P)^\dagger(-iZ)(X,P)-(X,P)^\dagger(-iZ)(X,P)\otimes(X,P)^\dagger(X,P) = H\otimes E-E\otimes H$$

and the reader notices that

$$(-iZ)_2((X,P)\otimes(-iZ)(X,P) - (-iZ)(X,P)\otimes(X,P)) = 0$$

where

$$Z_2 = Z\otimes 1 + 1\otimes Z.$$

This means that $C_2$ leaves the kernel of that operator invariant or vice versa, which is the entire basis of cohomology theory at the operator level. Indeed, a particular black white profile corresponds to an eigenvector $|\phi\rangle_\theta$ of the operator

$$A(\theta) = (\cos(\theta), \sin(\theta))^T(X,P) = \cos(\theta)X + \sin(\theta)P,\ A(\theta)^\dagger = A(2\pi - \theta)$$

and

$$A(\theta)^\dagger A(\theta) = \cos(\theta)^2X^2 - \sin(\theta)^2P^2 - \cos(\theta)\sin(\theta)1 = X^2 - \sin^2(\theta)E - \cos(\theta)\sin(\theta)1$$

whereas

$$A(\theta)^2 = -i\cos(\theta)\sin(\theta)D + X^2 - \sin^2(\theta)H.$$

[1]Here functional means that the mapping $-iS \to [B,\cdot]$ is a Lie algebra homormorphism. As stated before, this mapping does provide one with the correct ontology whereas the respective quadratic forms $(X,P)^\dagger(-iS)(X,P)$ do not given that the latter expression equals the identity operator and therefore corresponds to the heart.[2]

Also,

$$A(\theta)A^\dagger(\theta) = \cos(\theta)^2 X^2 - \sin(\theta)^2 P^2 + \cos(\theta)\sin(\theta)1 = A^\dagger(\theta)A(\theta) + \sin(2\theta)1$$

so that the operator $A(\theta)$ also provides for a creation annihilation pair of a unit $\sin(2\theta)$ implying that

$$A(\theta)A^\dagger(\theta)|\psi\rangle_\theta = \sin(2\theta)|\psi\rangle_\theta + \lambda_\theta A^\dagger(\theta)|\psi\rangle_\theta$$

for an eigenvector $|\psi\rangle_\theta$ meaning

$$A_\theta|\phi\rangle_\theta = \lambda_\theta|\phi\rangle_\theta.$$

We now see that

$$C_2(|\psi\rangle_\theta \otimes C|\psi\rangle_\theta - C|\psi\rangle_\theta \otimes |\psi\rangle_\theta) = 0$$

where, given the operators

$$FM(\theta) := ((\cos(\theta),\sin(\theta))^T \otimes (\cos(\theta),\sin(\theta))^T)((X,P)\otimes(-iZ)(X,P) - (-iZ)(X,P)\otimes(X,P))$$

and

$$BM(\theta) := ((\cos(\theta),\sin(\theta))^T \otimes (\cos(\theta),\sin(\theta))^T)((X,P)\otimes(-iZ)(X,P) + (-iZ)(X,P)\otimes(X,P))$$

and noticing that

$$(\cos(\theta),\sin(\theta))^T(-iZ)(X,P) = [C, A_\theta]$$

one has that

$$\left[C_2^t, A(\theta)\otimes A(\theta)\right] = A(\theta)\otimes[C, A(\theta)] - [C, A(\theta)]\otimes A(\theta) = FM(\theta)$$

and

$$[C_2, A(\theta)\otimes A(\theta)] = BM(\theta)$$

where

$$C_2^t = 1\otimes C - C\otimes 1.$$

Given that we already established that

$$C_2^t C_2 = 1\otimes C^2 - C^2\otimes 1 = C_2 C_2^t$$

due to the fact that it is *not* mandatory that $C^2 \neq 1$. Indeed, this is also the case in ordinary quantum theory where the eigenvalues of the generator of any rotation operator around some axis are quantized by half integer numbers. This result is well known to be true for all irreducible unitary representations and emerge from the demand of a continuous group representation of $SU(2)$. Now the representation we have here is one on the level of the *adjoint* action and usual operators are given by expression

$$U(\theta) = e^{i\theta C}.$$

The action which we alluded to is given by

$$X \to U(\theta) X U(\theta)^{\dagger} = e^{i\theta [C, \cdot]} X$$

and our operators all give the right result here. Now in standard particle theory, we moreover demand that the action of the $U(\theta)$ on the Hilbert space at hand is a *group* action meaning $U(2\pi) = 1$. This is how the quantization of spin arises in quantum field theory and this is so for very good reasons. Indeed, it stems from the old Schroedinger school where one speaks about spinor space and spacetime vectors can naturally be seen as squares of spinor and a conjugate spinor. Henceforth, the interesting representations of the $GL(2, \mathbb{C})$ algebra, which coincides with the one of $su(2)$ are those where the geometric action on the spinor space is crucially used. More in particular, the interesting quantum representation arise from the natural correspondance between the left matric action of $GL(2, \mathbb{C})$ on $\mathbb{C}^2$ and the corresponding action on flat space-time itself. This is an instrumentalist point of view where the notion of spin is correlated to real rotations in space of your measurement apparatus. In principle there is no need for this and what we posit is that this does not need to be so at the mental level. There is no a priori geometrical correspondance between black space and spacetime given that we have no geometrical interpretation of black space to start with. Everything which happens here is abstract so that there is no reason to demand for quantization of the spectrum of the $C$ operator. However, if we were to relate the operator $C$ to sex duality, then it is rather normal to demand that $C^2 = 1$ and that therefore we work, regarding $C$ is a tensorproduct of the defining representation $su(2)$ only. We shall not insist however on this in the sequal as sexuality may be quantal too which is certainly the case for shemales and hermaphrodites. We obtain, by means of the Jacobi identity that

$$[C_2, FM(\theta)] = \left[C_2^t, BM(\theta)\right] = 0$$

given that

$$[C, A(\theta)] = A^{\dagger}(\theta).$$

One has that

$$[C_2, A(\theta) \otimes A(\theta) - A(2\pi - \theta) \otimes A(2\pi - \theta)] = 0$$

and likewise

$$\left[C_2^t, A(\theta) \otimes A(\theta) + A(2\pi - \theta) \otimes A(2\pi - \theta)\right] = 0$$

so that we obtain that

$$BM(\theta) = BM(2\pi - \theta),\ FM(\theta) = -FM(2\pi - \theta).$$

One can now propose twisted marriages based upon the operator

$$A(\theta) \otimes A(2\pi - \theta)$$

leading to

$$BT(\theta) = [C_2, A(\theta) \otimes A(2\pi - \theta)]$$

as well as

$$FT(\theta) = \left[C_2^t, A(\theta) \otimes A(2\pi - \theta)\right]$$

implying that

$$\left[C_2^t, BT(\theta)\right] = [C_2, FT(\theta)] = 0.$$

At the level of a single entity, this implies that

$$CA_\theta|\phi\rangle = (\cos(\theta), \sin(\theta))^T (X, -P)|\phi\rangle + A_\theta C|\phi\rangle = A(2\pi - \theta)|\phi\rangle + A_\theta C|\phi\rangle$$

which suggests to take eigenstates $|\phi_\lambda\rangle$ of the eye operator

$$C|\phi_{\lambda,e}\rangle = \lambda|\phi_{\lambda,e}\rangle$$

so that

$$CA_\theta|\phi_{\lambda,e}\rangle = A(2\pi - \theta)|\phi_{\lambda,e}\rangle + \lambda A_\theta|\phi_{\lambda,e}\rangle.$$

At the two person level then, we have that

$$C_2 FM(\theta)|\phi_{\lambda,e}\rangle \otimes |\psi_{\mu,e}\rangle = (\lambda + \mu)FM(\theta)|\phi_{\lambda,e}\rangle \otimes |\psi_{\mu,e}\rangle$$

a well as

$$C_2 FT(\theta)|\phi_{\lambda,e}\rangle \otimes |\psi_{\mu,e}\rangle = (\lambda + \mu)FT(\theta)|\phi_{\lambda,e}\rangle \otimes |\psi_{\mu,e}\rangle$$

which vanishes in case $\lambda + \mu = 0$ being, in extreme cases a black-white marriage which does *not* involve partner fidelity given that both behave independently and likewise so for the twisted marriage. This means that the pinal gland remains closed for such bindings and the belly glows. An alike Polish marriage is one where $\lambda \sim \mu$ which opens the pinal gland of both when the respective partners consume it. Such marriages give peace as the third eye sees everything but they evoque disgust or jealousy with others. So such a Polish marriage is energetically the least favourable one an really puts an extremely strong binding energy between both partners. In case both partners are extremely white, they have a very strong cosy home but outside they need to individualize where the woman becomes mostly much more black to evoque the impression of neutrality. Another possiblity would be to engage in quantum contracts at that point which makes them look at other partners. For Poles who value beauty, social interactions with other couples are a difficult exercise. However, such couples are individually extremely energetic whereas the opposite Polish marriage are a strain. Indeed, the opening of the eye and its associated energetic cost is far outnumbered by the energetic gain due to white-white interactions. The Belgian couples do have a poor understanding of one and another, this could improve by looking for a suitable mixture of both the belly and astral gland operator $C_2$. In the following computations, we shall assume that $A^2 = C^2 = 1$. However, the conclusion remains universally valid. Commuting it with the operator $B_2$ provides

$$[aA_2 + cC_2, A_2] = 2i(aC_2 - cA_2)$$

and

$$(aA_2{+}cC_2)(aC_2{-}cA_2) = -i(a^2{+}c^2)B_2{+}(a^2{-}c^2)(A{\otimes}C{+}C{\otimes}A){-}ac(A{\otimes}A{-}C{\otimes}C)$$

which reduces in case $a^2 = c^2 = \frac{1}{2}$ to

$$(aA_2 + cC_2)(aC_2 - cA_2) = -iB_2 - \pm\frac{1}{4}(A \otimes A - C \otimes C)$$

where the $\pm$ stands for equality or opposite character. This suggests one to look for eigenstates of $A_2 \pm C_2$ which are in the kernel of $A \otimes A - C \otimes C$ which requires the condition

$$(A \pm C)|\psi\rangle = 0$$

on the individual states. This is possible because

$$[A_2 \pm C_2, A \otimes A - C \otimes C] = 2i(B \otimes (A \pm C) + (A \pm C) \otimes B)$$

so that

$$\lambda(A \otimes A - C \otimes C)|\psi\rangle = (A \otimes A - C \otimes C)(A_2 \pm C_2)|\psi\rangle =$$

$$-2i(B \otimes (A \pm C) + (A \pm C) \otimes B)|\psi\rangle + (A_2 \pm C_2)(A \otimes A - C \otimes C)|\psi\rangle = 0$$

in case one finds a representation where both $(A \otimes A - C \otimes C)|\psi\rangle = 0$ hold and $|\psi\rangle$ can be written in terms of one individual states in the kernel of $A \pm C$ where the last thing is certainly possible. We shall not further investigate this consistency condition any further here. Assuming the latter hold we have that

$$B_2\frac{1}{\sqrt{2}}(A_2 \pm C_2)|\psi\rangle = \mp\sqrt{2}i(C_2 \mp A_2)|\psi\rangle + \frac{1}{\sqrt{2}}(A_2 \pm C_2)B_2|\psi\rangle$$

meaning the if $|\psi\rangle$ belongs to the kernal of $(A_2 \pm C_2)$ then $B_2|\psi\rangle$ must be in the kernel too. Therefore, we have a new marriage contract based upon

$$|\psi\rangle \in \text{Ker}(A_2 \pm C_2) \text{ and } B_2\text{Ker}(A_2 \pm C_2) \subseteq \text{Ker}(A_2 \pm C_2)$$

so that it becomes possible to pick out definite $B_2$ states too. Maybe it is possible to construct a human such that one can shape pairs satisfying

$$(A_2 \pm C_2)|\psi\rangle = B_2|\psi\rangle = 0.$$

Given the Lie algebra of the generators $A_2, B_2, C_2$ this imposes further that

$$A_2|\psi\rangle = B_2|\psi\rangle = C_2|\psi\rangle = 0$$

being the most symmetrical sitution possible. This is a French marriage based upon liberté ($B_2$, coloured blue), fraternité ($A_2$, coloured white) and finally egalité ($B_2$, coloured red). However, the marriage contract here does not work by means of conjugation by means of a sex operator which is neverteless the case for the standard Polish fermionic *twisted* contract with a partners having an alike spiritual eye. The Belgians prefer a fermionic *straight* contract by means of the belly operator, just as the Italians do by means of the *astral eye* operator. The reader may enjoy playing further with the original algebra and notice that

$$\frac{1}{\sqrt{2}}(A_2 \pm C_2)(C_2 \mp A_2)|\psi\rangle = -iB_2|\psi\rangle$$

so that

$$\frac{1}{\sqrt{2}}(A_2 \pm C_2)(\mp\sqrt{2}i(C_2 \mp A_2)|\psi\rangle + \frac{1}{\sqrt{2}}(A_2 \pm C_2)B_2|\psi\rangle) = \mp\sqrt{2}B_2|\psi\rangle + \frac{1}{2}(A_2 \pm C_2)^2 B_2|\psi\rangle$$

which is a consistency condition upon our stronger theory and suggests that the latter most likely holds. The reader may enjoy computing expectations of operators different from they eye operator in the Polish marriage and a small computation reveals that

$$((X,P)\otimes(-iZ)(X,P)-(-iZ)(X,P)\otimes(X,P))^\dagger(-iS)_2((X,P)\otimes(-iZ)(X,P)-(-iZ)(X,P)\otimes(X,P)) = 0$$

and the reader is invited to study further consequences.

We have already largely discussed a duality in the white-black benchmark related to the different sexes; that is $iP_j$ is the black female operator corresponding to the male impetus $P_j$. As is well known in quantum theory, this defines a notion of temperature or heat and therefore black woman are hot and energized in the presence of white males. They proclaim chaos whereas the men a democracy which causes democracies governed by a woman to be kind of chaotic in a totalitarian way. On the other hand, the white female benchmark corresponds to $iX_j$ and therefore their temperature is completely arbitrary and variable. These are the hot-warm types who make the black man, who is extremely rigid, completely nuts. When those woman aspire order, they become completely uncertain in gestures, changing between a blush and a totally face, resulting in a lack of comprehension from the male side. The British Queen and PM are white as hell in this sense and play silly games all the time when they want order and law whereas white men just kill to get it. Black woman can easily be distinguished facially from those white ones in the presence of white men as the latter cocky playgirls possess the art of confounding chaos with devilish "unease" to progress in a definite direction. In this vein, there is a clear psychological and astral confusion between males and females: white males and females are democratic - rigid in a certain - uneasy way and they fortify one and another leading to violent discussions and sex. Henceforth, some gap in strength and polarization between males and females in a marriage is needed in order to feel on one hands fortified psychically whereas on the other, conversational aspects need to remain within boundaries. So, banking, as stands up till now, is a black business at the retail level and becomes more white when doing commercial and private banking, with different benchmarks from the European and British side.

But what about poor people needing white banking?

**Summary.**

Our theory is initially just defined by means of a creation - annihilation pair $a, a^\dagger$ where the involution $\dagger$ indicates creation of life; as said before creation of life in a life person means going over to a higer level and death at this one. Also, the vacuum state $|0\rangle$ belongs to the theory. Geometrically, this is expressed by means of

$$(X,P)^T(WB)(X,P) = -1$$

where everything remains at the complex level. The symmetry group of this geometry is the two dimensional Symplectic group which is equivalent to a complexification of $SU(2)$ which is the same as $GL(2,\mathbb{C})$ or the universal cover of the Lorentz group in four real dimensions. To realize the appropriate complexification we have to make one direction imaginary as the associated symplectic form is of the ultrahyperbolic reality $(--++)$ with two time and space directions. Now, our observations break this symmetry and the appropriate reduction therefore constitutes in imposing Hermiticity conditions but complexifying in one direction. More in particular, we have introduced the quadratic forms

$$(X,P)^\dagger S(X,P),\ (X,P)^\dagger(-iZ)(X,P), (X,P)^T(-i(WB))(X,P), (X,P)^\dagger(X,P)$$

and imposed the reality condition

$$(X,P)^c = (iZ)(X,P)^\dagger$$

hereby fixing the $z$-axis in space assuring that all quadratic forms are complex as well as real in nature and that all adjoints vanish effectively. This condition implies that

$$(X,P)^\dagger S(X,P) = (X,P)^T(-i(WB))(X,P) = -1$$

effectively reducing the entire algebra to $su(2)$. To get the appropriate complexified $su(2)$ we either have to opt for the real or complex conventions

$$(X,P)^T(-iS)(X,P), (X,P)^T(-i(WB))(X,P), (X,P)^T(-iZ)(X,P), (X,P)^T(X,P)$$

which generate the quaternion algebra with the charge operator

$$(X,P)^T(X,P)$$

associated to the identity as rotation around the $z$-axis. This is kind of funny as having spiritual charge can make you rotate around your physical length axis. Indeed, Switchoriems tend to walk straight through and never turn around almost. Something very curious happens here *mathematically* given that the identity operator on the Hilbert space is associated to the heart $-i(WB)$. From the philosophical point, this was meant to be as impetus kills awareness or life kills death. Therefore, the other three generators

$$(X,P)^T(-iS)(X,P),\ (X,P)^T(-iZ)(X,P),\ (X,P)^T(X,P)$$

determine a Lie-algebra which *must* be equal to $su(2)$. There are two curious facts here: one is that the heart is associated to *time* and therefore commutes with all operations in space (and time is a positive quantity by defintion). The second curiosity is that the harmonic oscillator Hamiltonian is associated to the identity operator in two dimensions and to rotations around the three axis. Hence, this one must be associated to the gravitational $z$-axis given that the height of an object must remain fixed in a gravitational field. The belly and eye operators merely express rotations around the spatial $x, y$ axes which are quite cumbersome given that they require you to move in the $z$ direction. The rotations around this axis still constitute a $U(1)$ symmetry of nature which explains why we can easily rotate on our feet. Some of these aspects were revealed during a telepatic discussion with Prof Norbert Van den Bergh. Insisting upon the Hermitian character of black, by mere choice, imposes that $P^\dagger = -P$, $X^\dagger = X$. Given that this Hermiticity of blackness was arbitrary, we define a sex dual, interchanging black and white by means of $C_1(X,P) = (iP, iX)$ or $C_2(X,P) = (iP, iX)$ which constitute the only two options. Indeed, $-iS$ switches the reality conditions but preserves both expressions of the Heisenberg algebra. Woman, in the same vein, correspond to imaginary momenta in the male psychic Fourier transformation which reflects itself all the time into human behaviour. Males think that females proceed in an imaginary way whereas females feel the same about males namely that the male reality is opposite their own. The reality condition distinguishes awareness from impetus and the fact that $i(WB)$ respects it but destoys $-iS$. One may opt for *different* choices of sex duality but in this one male impetus is related to female awareness with a little tricky imaginary unit creeping in. In a way, this has always been as such as a switchoriem woman makes a switchoriem man's heart tick by means of the belly conjugate, given that impetus really *is* the male heart. This is why black men are interested in black woman from the point of the belly, because they help them to better comprehend themselves which removes beauty and uglyness as well as individualism and keeps them away from mental starvation. The $i$ really makes a huge distinction as the white male, corresponding to 1, is repulsive for the black man at the psychic level. In a way, a black woman has plenty of characteristics of a white male but she is less liberal in the sense that she chooses definetly for small or large measures in her behavioural aspects. She never balances around the Switchoriem; depending upon the particular state in behavioural space of the black male, this provides for some elasticity if the female chooses for the highest weight in her behavioral traits where her husband resides. If this is not so, then opposition occurs and she can kill his heart. White males, then, make black woman crazy as they are oscilatory, a condition dubbed as cyclothemia by black psychiatrists. Indeed, white males have no preference at all whereas black woman do. White females then do not have a preference either but the are not complex cyclic in the psychic way; they are cyclothemic in the real way which results in an infinite positive amplitude at *everything* they do and they can literally do everything. They are extremists which go with full force whereas the white male is much more modest. Indeed, white woman destroy everything which stands in their way whereas white men are much better negotiators. For this very reason, black men often use white females against white dominant (but not complete white) males, by concquering their minds for another goal with money and hereby causing for a spiritual discrepancy breaking the heart. The pure white male cannot be destroyed in this way and literally slices a white female's throat if she were to oppose him because that testifies of evil behaviour. If you cannot stand the biggest liberitarian, then you have malicious intentions regarding the "Wille Zur Macht".

# Chapter 6

# Some further musings.

In the previous book on psychology [3] I have explained that the correct dynamical theory is one which is mathematically very well known in theoretical physics, it is namely of the electro-weak kind with bosonic charges instead of fermionic ones but then, the gauge bosons mediating the interactions between black and white have negative free energy. One might also posit that the black charge carriers have negative free energy and the $U(1) \times SU(2)$ sector is perfectly fine so that inertial energy would equal gravitational energy given that spiritual charges anti-gravitate in the sense that they carry negative mass. In that vein, given that opposite charges attract one and another at zero-level of physical interactions in the electroweak theory, and electrons certainly do attract protons and vice versa, one might see the first level of spirituality as a protection mechanism against such destructive behaviour. Indeed, quantum mechanics was invented as the primary mechanical cause for the stability of the atom which is classically unstable and would ultimately destroy the electron and proton due to radiative effects. The would collide and annihilate one and another. In that vein is the belly chacra a warning to procreate with people of the opposite spiritual charge given that they are also physically opposite to one and another. Spiritually black people are usually much smaller and less physically strong as spiritually white people are but white men usually want to take care of black woman and the other way around. Such marriage is not even possible physically as the tiny pussy is way too small for the big penis which requires "cosmetic" surgery on the male penis to make it suitable for the thight pussy. The belly chacra tames the pussy and penis given that such protective marriages are rather sexy from both sides. Indeed, the man can easily dominate under standard circumstances whereas the female feels safe. The same happens in quantum theory, it is the joint electron proton spirit who has to ask suitable questions in order to protect both; the questions asked are the best possible given that the electron remains safely on a distance from the proton. These are the so called stationary states which show that the spirituality of an electron-proton marriage is such that the marriage is stable. Electrons usually repel one and another but nature has invented a tiny contract, which is the Polish-Italian marriage, their spin needs to oppose in order to join. This is also in real life, white woman have a hard physical relationship with white men for example, not seldom a quarrel starts and physicality in terms of hardcore sex is mandatory to lower the energy levels. Their marriage could be a Polish-Italian one where the astral gland brings peace and understanding given that they are both stubborn and pig headed. However, alikes also have very strong *physical* ways to bind but only so when a neutral element joins them. This is the strong nuclear theory in theoretical physics where the presence of a neutron is sufficient to settle for the repulsion between two protons and likewise so for their anti-particles albeit such binding has never been observed. From the psychological point of view this appears to be natural given that a positive attitude is mandatory to negotiate by means of the neutral lawyer. By duality however, the neutron will be repelled at the level-one of psychic interaction by both white partners. There is another issue to be discussed here which is the fact that we chosen for so called parastatistics where black charges add up arbitrarily within a person where the interchange operator between persons in a marriage is of the fermionic kind which only allows for exclusive two partner contracts. Let me remind you that in atomic physics the contract between all electrons is a fermionic one but this is not so with the nucleus due to the presence of neutrons. Atoms themselves are usually considered to make bosonic contracts when they bind together which is incompatible with the existence of separate electron contracts of one atom and the nucleus of another. In this regard and using observational evidence, I have opted for a bosonic instead of fermionic theory which indicates that those charges reside at least at the level of atoms.

Indeed, atoms belonging to distinct persons are not willing to bind given that it would destroy the body; this adds an extra fermionic charge to the creation and annihilation operators regarding the total bodily spirit of the persona. The interaction bosons however do not feel this distinction which is precisely the reason why such issues were left behind for now. The marriage contract is of a higher spiritual level just as is the case for the contract between an electron and a proton. Indeed, alike persons circle around one and another physically which is the spiritual contract regarding the level zero interactions. However, they attract one and another by means of the open astral gland and silent astral eye in a Polish-Italian marriage. Problematic are enoounters with people of alike black-white polarization but different energies.

# Chapter 7

# Some mathematical concepts.

We now define those spaces which are important in the standard formulation of quantum mechanics, the so-called Hilbert spaces $\mathcal{H}$. Those concern a generalization of standard Euclidean geometry and are defined by a so-called scalar product or sesquilinear form $\langle v|w\rangle$ where $v, w \in \mathcal{H}$ which is a module over the complex numbers. For real Hilbert spaces, the scalar product between $v$ and $w$ equals the product of the oriented length of the projection of $v$ on $w$ times the length of $w$ and is therefore a real number. This quantity satisfies:

$$\begin{aligned} \langle v|w\rangle &= \langle w|v\rangle \\ \langle v|aw+bu\rangle &= a\langle v|w\rangle + b\langle v|u\rangle \\ \langle v|v\rangle &\geq 0 \,\text{and equality holds if and only if}\, v=0. \end{aligned}$$

The axioms for a sequilinear form are exactly the same except that $\langle v|w\rangle = \overline{\langle w|v\rangle}$ where the bar denotes the complex conjugate defined by $\overline{a+bi} = a - bi$. For example, $\mathbb{C}$ is a Hilbert space with scalar product $\overline{v}w$; to complete the definition, the reader must understand that a scalar product is a higher structure than a metric $d$. More specifically, we first define a norm $||v||$ by means of

$$||v|| = \sqrt{\langle v|v\rangle}.$$

To understand the properties it satisfies, we need the following result

$$|\langle v|w\rangle| \leq ||v||||w||$$

which reads that the absolute value of the product of the oriented length of the projection of $v$ on $w$ with the length of $w$ is smaller or equal than the length of $v$ times the length of $w$ which is intuitively obvious. The formal proof goes as follows

$$0 \leq ||v+\lambda w||^2 = ||v||^2 + |\lambda|^2\,||w||^2 + 2\mathrm{Re}\left(\overline{\lambda}\langle w|v\rangle\right)$$

where $\mathrm{Re}(a+ib)=a$ in either the real part of a complex number. The reader should verify that the real part of a complex number $z$ can be written as $\frac{1}{2}(z+\overline{z})$ while the imaginary part reads $-i\frac{1}{2}(z-\overline{z})$. Here, the modulus of a complex number is defined by

$$|z|=\sqrt{z\overline{z}}=\sqrt{a^2+b^2}$$

and satisfies

$$|z+z'|^2=|z|^2+|z'|^2+(z\overline{z'}+\overline{z}z')$$

where the last term equals up to a factor two

$$aa'+bb'$$

the absolute value of which is smaller or equal to $|a|\,|a'|+|b|\,|b'|$ the square of which

$$a^2a'^2+b^2b'^2+2\,|a|\,|a'|\,|b|\,|b'|\leq\left(a^2+b^2\right)\left(a'^2+b'^2\right)=|z|^2\,|z'|^2\,.$$

Therefore,

$$|z+z'|^2\leq(|z|+|z'|)^2$$

and thus

$$|z+z'|\leq|z|+|z'|$$

which is the so-called triangle inequality. Therefore, we obtain again a metric on the complex numbers by

$$d(z,z')=|z-z'|$$

and the reader notices that the norm is just a higher dimensional analogue of the modulus. Coming back to our original proof, the reader may see that we can choose $\lambda$ such that

$$\mathrm{Re}\left(\overline{\lambda}\langle w|v\rangle\right)=-\,|\lambda|\,|\langle v|w\rangle|$$

and therefore, we obtain that

$$0\leq||v||^2+|\lambda|^2\,||w||^2-2\,|\lambda|\,|\langle v|w\rangle|$$

which is a polynomial equation of second degree in the positive variable $|\lambda|$. The reader should know that for at most one zero to exist we need that the discriminant is smaller or equal than zero

$$0\geq4\,|\langle v|w\rangle|^2-4||v||^2||w||^2$$

which gives precisely our result and equality holds only if $w=-\lambda v$. Hence,

$$||v+w||\leq||v||+||w||$$

which defines again a metric topology by means of

$$d(v,w)=||v-w||.$$

A Hilbert space is therefore a module over $\mathbb{C}$ equipped with a sesquilinear form such that the associated metric space is complete. The completeness assumption is extremely important for the theory of linear operators but before we come to that, let us understand a bit better the geometry defined by the sesquilinear form with regard to a so-called orthonormal basis. Two nonzero vectors $v, w$ are called orthogonal to one and another if and only if $\langle v|w\rangle = 0$ and we say that a vector $v$ is normalized if and only if $||v|| = 1$. Again, by means of the axiom of choice, we can show that there exists a basis $(e_i)_{i\in I}$ such that $\langle e_i|e_j\rangle = \delta_{ij}$ where $\delta_{ij}$ is the Kronecker symbol which equals 0 if $i \neq j$ and one otherwise. In a finite dimensional Hilbert space, where $v = \sum_{i=1}^{n} v^i e_i$ this implies that

$$\langle v|w\rangle = \sum_{i,j=1}^{n} \overline{v^i} w^j \delta_{ij}$$

a generalization of the standard inproduct in three dimensional Euclidean geometry. From now on, we shall adopt the Einstein convention and simply ignore the summation sign when dealing with expressions such as $\overline{v^i} w^j \delta_{ij}$. $\delta_{ij}$ is our first example of a tensor which we shall define in full generality later on.

In quantum mechanics, the following two operations are important, one of which equals the minimal join $\otimes$ called the direct product in the literature. Given two Hilbert spaces $\mathcal{H}_i$, the tensor product $\mathcal{H}_1 \otimes \mathcal{H}_2$ is again a Hilbert space defined in the following way: one starts from vectors $v_1 \otimes v_2$ where $v_i \in \mathcal{H}_i$ and the latter is just a different notation for $(v_1, v_2)$. Next, one takes finite formal sums $\sum_{i=1}^{n} z_i v^i \otimes w^i$ of these elements and considers the following equivalence relation

$$\begin{aligned} z(v \otimes w) &\equiv (zv) \otimes w \equiv v \otimes (zw) \\ v \otimes w_1 + v \otimes w_2 &\equiv v \otimes (w_1 + w_2) \end{aligned}$$

We define $\mathcal{H}$ as the linear space of these equivalence classes and now turn it into a Hilbert space. The scalar product is fixed by the definition

$$\langle v_1 \otimes w_1|v_2 \otimes w_2\rangle := \langle v_1|v_2\rangle\langle w_1|w_2\rangle$$

and we simply take the completion of $\mathcal{H}$ in the associated metric topology. Likewise, one can define the direct sum $\mathcal{H}_1 \oplus \mathcal{H}_2$ but this time the equivalence relation is defined by

$$\begin{aligned} z(v \oplus w) &\equiv (zv) \oplus (zw) \\ v_1 \oplus w_1 + v_2 \oplus w_2 &\equiv (v_1 + v_2) \oplus (w_1 + w_2) \end{aligned}$$

resulting in the scalar product

$$\langle v_1 \oplus w_1|v_2 \oplus w_2\rangle := \langle v_1|v_2\rangle + \langle w_1|w_2\rangle.$$

The reader should check that a basis for $\mathcal{H}_1 \otimes \mathcal{H}_2$ is given by $v_i \otimes w_j$ where $v_i$ constitutes a basis for $\mathcal{H}_1$ and $w_j$ for $\mathcal{H}_2$. Likewise, a basis for $\mathcal{H}_1 \oplus \mathcal{H}_2$ is given by $v_i \oplus 0, 0 \oplus w_j$.

Now, we come to the theory of linear operators which constitute the natural functions $A : \mathcal{H}_1 \to \mathcal{H}_2$ between Hilbert spaces. They are natural in the sense that

$$A(zv + w) = zA(v) + A(w)$$

which implies $A(0) = 0$. It is evident that this property implies that $A$ is completely determined by its action on a basis $(e_i)_{i \in I}$ which we write down as

$$Ae_i = A_i^j f_j$$

where $(f_j)_{j \in J}$ constitutes a basis of $\mathcal{H}_2$ and the reader should keep in mind the Einstein convention. Therefore,

$$A(v^i e_i) = (A_i^j v^i) f_j$$

which, for finite dimensional Hilbert spaces, boils down to an ordinary matrix multiplication where the $j$ index is the so called row index and the $i$ pertains to the columns. This constitutes the second example of a tensor and you may notice that the indices are placed differently this time - we have one upper and lower index. Hence, the point of view of operators is better than the one of matrices in the sense that the former is basis independent while the latter is not; sometimes, it is convenient to work in the latter picture while making computations but usually, the reader will understand that the former is much more economic to work in. Regarding the matrix picture, we have the notion of a basis transformation which is *not* a linear operator since it acts trivially on the vectors given that it just changes the basis representation. Such changes of basis are however denoted in the same way:

$$e'_i = \sum_{k \in I} O_i^k e_k$$

and

$$f'_j = \sum_{k \in J} V_j^k f_k$$

where the $O$ and $V$ are invertible in the sense that there exist $O^{-1}, V^{-1}$ such that

$$(O^{-1})_k^i O_l^k = \delta_{il} = O_k^i (O^{-1})_l^k$$

and likewise for $V$. We have then that

$$A_i'^j f'_j = Ae'_i = O_i^k Ae_k = A_k^l O_i^k f_l = (V^{-1})_l^j A_k^l O_i^k f'_l$$

implying that

$$A_i'^j = (V^{-1})_l^j A_k^l O_i^k.$$

In the next section, we shall further work on the theory of linear operators especially in the context of the spectral theorem but let us come back for a moment to the previous section. Here, given a topological space, we introduced the formal vector space over $\mathbb{Z}_3$ generated by all closed, connected, $k$-dimensional subspaces, an object which is infinite dimensional. Then, we put the equivalence relation on them

$$S_1^k \equiv S_2^k$$

if and only if there exists a connected $k+1$ surface $T^{k+1}$ such that $\partial T^{k+1} = S_1^k - S_2^k$. This reduces the infinite dimensional vector space to a finite dimensional one over $\mathbb{Z}$. To construct $H_k$, we divide out by the module of exact $k$ surfaces, that is those which can be written as the boundary of $k+1$ surface (those are automatically closed since the operator $\partial$ satisfies $\partial^2 = 0$). This brings us to the notion of a quotient space: let $M, N$ by $K$ modules where $N \subseteq M$, then the quotient module

$$\frac{M}{N}$$

has as elements the equivalence classes in $M$ determined by the equivalence relation

$$v \equiv w \text{ if and only if } v - w \in N.$$

There is a long category theoretical treatment of homology and cohomology to which we shall come back later in the context of differentiable manifolds where the cohomology is determined by the exterior derivative $d$ satisfying again $d^2 = 0$ and $d$ and $\partial$ are isomorphic in a sense.

## 7.1 Operator theory.

We shall directly treat the infinite dimensional case, leaving the finite dimensional one as an easy subcase. However, since I shall not give any proof of any theorem which follows, the reader might be capable of formulating his or her own proof in the finite dimensional case. The latter is much easier to do given that infinite dimensions come with many subtleties not present in the finite dimensional case and consult a book on functional analysis for the more formal proofs. In standard quantum mechanics, the Hilbert space is time independent, something which we shall sharply criticize in the next chapter given that it implies the theory is not generally covariant. All mathematical results of interest pertain to operators $A : \mathcal{H} \to \mathcal{H}$ and the reader shall understand after a while why those results are not open for generalization.

Before we lift off, let us study some topologies on the Hilbert space $\mathcal{H}$ as well as on the space of operators. On $\mathcal{H}$, we have already studied the norm topology determined as before, we now come to the weak topology: this one is generated by so-called linear functionals which are linear maps $\omega$ from $\mathcal{H}$ to $\mathbb{C}$. The space of linear functionals is again a vector space and usually we restrict to those functionals which are continuous in the norm topology. The latter form again a vector space called the topological dual $\mathcal{H}^\star$: now, one has the result that a functional is continuous in the norm topology if and only if

$$|\omega(v)| \leq C||v||$$

for some $C > 0$. Let us give a formal proof of this result: obviously, the inequality implies continuity by linearity of $\omega$. Conversely, take a linear functional and assume that there exists a sequence of linear independent, normalized vectors $v_n$ such that $\omega(v_n) \to \infty$ in the limit for $n$ to $\infty$. Then, by taking a subsequence, one can assume that $\omega(v_n) > n^2$ and the sequence of vectors $w_k = \sum_{n=0}^{k} \frac{1}{n^2} v_n$ converges to $w = \sum_{n=0}^{\infty} \frac{1}{n^2} v_n$ whose norm is finite and $\omega(w_k) \to \infty$ which is in contradiction to the continuity.

Given this result, one can show that any bounded functional in $\mathcal{H}^\star$ is of the form

$$\omega(w) = \langle v|w\rangle$$

with $||v|| < \infty$ and the reader should try to prove this result for him or herself. Geometrically, this is obvious given that $\omega$ is fully determined by its null hyperplane $W = \{w|\omega(w) = 0\}$ as well as the action on its normal vector $\frac{v}{||v||}$. This motivates the following definition: we define the open sets

$$\mathcal{O}_{\epsilon;v_1,\ldots,v_n}(w) = \{w' \,|\, |\langle w - w'|v_i\rangle| < \epsilon \,\text{for}\, i = 1\ldots n\}$$

which constitute a basis for the so-called *weak* or $\star$-topology.

The weak topology is weaker than the norm topology meaning that every open in the weak topology is also open in the norm topology. The proof is evident by means of the inequality

$$|\langle w - w'|v_i\rangle| \leq ||w - w'||||v_i||$$

and therefore if

$$||w - w'|| < \frac{\epsilon}{\max_{i=1}^{n} ||v_i||}$$

then $w' \in \mathcal{O}_{\epsilon;v_1,\ldots,v_n}(w)$. Another criterion for compactness of a metric space like $\mathcal{H}$ is that $K$ is compact if and only if every sequence $(x_i)_{i\in I}$ in $K$ has a subsequence $(x_{i_j})_{j\in J}$ converging to a limit point in $K$. We will now prove the equivalence with the old definition: if $K$ is compact and $(x_i)_{i\in I}$ is a sequence, then consider a cover $C^n$ by means of $\frac{1}{n}$-open balls. For each $n$, by the fact that $K$ can be covered by means of a finite number of open balls of radius $\frac{1}{n}$, we have a sequence with a cardinal number of $I$ elements contained in a decreasing sequence of open sets $\mathcal{O}_{n+1} \subseteq \mathcal{O}_n$ of diameter less than $\frac{2}{n}$. Hence, we find a subsequence and accumulation point in $K$. To prove the converse, we shall assume that $K$ has a countable dense subset of points; therefore, any open cover of $K$ by means of open neighborhoods can be replaced by a countable one $\mathcal{O}_n$. Suppose there does not exist a finite subcover, then we can find a subsequence of points $x_m \notin \cup_{n=1}^{m} \mathcal{O}_n$ which must have an accumulation point $x \in \mathcal{O}_k$ for some $k$. But then, some $x_m \in \mathcal{O}_k$ for $m$ arbitrarily large in contradiction to the assumption.

In the sequel, we shall only work with separable Hilbert spaces, meaning Hilbert spaces having a countable basis, corresponding to the lowest kind of infinity. The reader can easily show that for such spaces, the closed unit ball is not compact in the norm topology but it is compact in the weak topology, where the above theorem also holds for the weak topology in this case given that it has a countable basis. We now come to some definition of some norms one can define on the linear space of all operators and we start by the most important one:

$$||A||_{\sup} = \sup_{||v||=1} ||Av||$$

which is the so called supremum norm. In case it is finite, we call the operator bounded and a whole theory of bounded operators exists poured in the abstract framework of $C^{\star}$-algebra's. We shall not deal with this subcase here as most physical operators are unbounded. Two other topologies are of interest, the strong and weak $\star$ topology. The former is defined by means of the open neighborhoods

$$\mathcal{O}_{\epsilon;v_1,\ldots v_n}(A) = \{B| \; ||(B-A)v_k|| < \epsilon \, \mathrm{for} \, k = 1 \ldots n\}$$

while the latter has as open neighborhoods

$$\mathcal{O}_{\epsilon;v_1,\ldots v_n,w_1,\ldots,w_n}(A) = \{B| \; |\langle (B-A)v_k|w_k\rangle| < \epsilon \, \mathrm{for} \, k = 1 \ldots n\}.$$

The reader may show that both topologies are Hausdorff and that the weak-$\star$ topology is weaker than the strong one. Also, we leave it as an exercise for the reader to show that all three topologies coincide for operators on finite dimensional Hilbert spaces.

We are now in a position to develop the theory of interest: I shall refrain from giving the physically important examples, since we need the general theory of the next section for that. So, in general, operators have a domain $\mathcal{D} \subset \mathcal{H}$ which we shall assume to be a dense subspace in the sequel; the adjoint $A^{\dagger}$ of $A$ is defined by means of the following procedure. Consider the subspace $\mathcal{D}^{\star}$ of vectors $v$ such that

$$|\langle v|Aw\rangle| < C(v)||w||$$

for all $w \in \mathcal{D}$. Then, the functional $w \to \langle v|Aw\rangle$ can be continuously extended to $\mathcal{H}$. Therefore, there exists a vector $z$ such that

$$\langle v|Aw\rangle = \langle z|w\rangle$$

and we define $A^{\dagger}v = z$ which can be easily seen to be a linear operator. Therefore, the domain of $A^{\dagger}$ equals $\mathcal{D}^{\star}$. The following cases are of special interest:

- $A = A^{\dagger}$ and $\mathcal{D} = \mathcal{D}^{\star} = \mathcal{H}$ in which case the operator is called self-adjoint,

- $AA^{\dagger} = A^{\dagger}A$ and $\mathcal{D} = \mathcal{D}^{\star} = \mathcal{H}$ in which case the operator is called normal,

- $UU^\dagger = U^\dagger U = 1$ and $\mathcal{D} = \mathcal{D}^\star = \mathcal{H}$ in which case the operator is called unitary,

- $P^2 = P = P^\dagger$ and $\mathcal{D} = \mathcal{D}^\star = \mathcal{H}$ in which case the operator is called a Hermitian projector.

As we shall see later on, self-adjoint or Hermitian operators are related to the unitary ones which play an important role in the standard formulation of quantum mechanics which is not suited at all for cosmology as we shall study in the next chapter. We shall work towards two different theorems: one which says that a suitable class of operators may be extended to Hermitian operators, where an extension of an operator is a novel operator defined on a larger domain coinciding with the original operator on its domain. The second result states that any normal operator may be decomposed into orthogonal projection operators in the weak $\star$ topology.

Let us first treat the extension of a so-called partial isometry $V$ with a domain $\mathcal{D}$ which is not required to be dense; the latter satisfies the property that

$$\langle V(v)|V(w)\rangle = \langle v|w\rangle$$

for all $v, w \in \mathcal{D}$. Hence, by continuity, we can extend $V$ to the closure $\overline{\mathcal{D}}$ of $\mathcal{D}$ which results in a linear homeomorphism between $\overline{\mathcal{D}}$ and $\overline{\mathrm{Im}(V)}$ where $\mathrm{Im}(V) = \{Vw | w \in \mathcal{D}\}$ is the image of $V$. Now, only in case the orthogonal complement $\mathcal{D}^\perp = \{w | \langle w|v\rangle = 0 \,\forall v \in \mathcal{D}\}$ has the same dimension as the orthogonal complement of the image $(\mathrm{Im}(V))^\perp$ can we obtain an extension by means of a unitary operator $W : \overline{\mathcal{D}}^\perp \to (\mathrm{Im}(V))^\perp$ resulting in a unitary operator $U = V \oplus W : \mathcal{H} \to \mathcal{H}$ which is a unitary extension of $U$. Now, the reader notices that for any subspace $W$, $W^\perp$ is closed in the weak and therefore also in the norm topology; moreover, $W^{\perp\perp} := (W^\perp)^\perp$ is the weak closure of $W$.

Now, Von Neumann was aware of the Cayley transform between Hermitian and unitary operators in finite dimensional Hilbert spaces: more precisely, a self-adjoint operator $A$ gets mapped to

$$U = (A - i1)(A + i1)^{-1}$$

where $(A \pm i1)$ is invertible in finite dimensions given that the equation $Av = \mp iv$ has no solution since otherwise

$$\mp i||v||^2 = \langle v|Av\rangle = \langle Av|v\rangle = \pm i||v||^2$$

implying $v = 0$. Moreover, $(A + i1)$ commutes with $(A - i1)$ implying that $U$ is unitary. He wondered what conditions should apply on $A$ for $U$ to be a partial isometry, so that one could extend the latter to a unitary operator defining a Hermitian one by means of the inverse Cayley transformation:

$$A = -i(U + 1)(U - 1)^{-1}.$$

The operator $A \pm i1$ should be injective by the same argument as before which suggests the condition that $\mathcal{D} \subseteq \mathcal{D}^\star$ and $A = A^\dagger$ on $\mathcal{D}$ which is the defining condition for $A$ to be a symmetric operator. This does, in contrast to the finite dimensional case not imply that $A \pm i1$ is surjective. Therefore,

$$U : \mathrm{Im}(A + i1) \to \mathrm{Im}(A - i1)$$

and we now have to do three things : (a) verify that it is a partial isometry indeed (b) close $\mathrm{Im}(A \pm i1)$ and finally (c) verify whether $\mathrm{Im}(A + i1)^\perp$ and $\mathrm{Im}(A - i1)^\perp$ have the same dimension. To prove (a) we notice that

$$\langle U(A + i1)v | U(A + i1)w \rangle = \langle (A - i1)v | (A - i1)w \rangle =$$

$$\langle Av | Aw \rangle + i\langle v | Aw \rangle - i\langle Av | w \rangle + \langle v | w \rangle$$

and this last expression equals by symmetry of $A$

$$\langle Av | Aw \rangle + \langle v | w \rangle = \langle (A + i1)v | (A + i1)w \rangle$$

for all $v, w \in \mathcal{D}$. Usually, in the literature, one closes the operator $A$ prior to making the Cayley transform albeit this is not really necessary; $U$ trivially extends to an operator

$$U : \overline{\mathrm{Im}(A + i1)} \to \overline{\mathrm{Im}(A - i1)}$$

and now we need (c) for $U$ to be extensible to $\mathcal{H}$. This last condition can be phrased somewhat more conveniently by means of

$$\mathrm{Im}(A \pm i1)^\perp = \mathrm{Ker}(A^\dagger \mp i1).$$

Indeed

$$\langle w | (A \pm i1)v \rangle = 0$$

for all $v \in \mathcal{D}$ is equivalent to $w \in \mathcal{D}^\star$ and

$$\langle (A^\dagger \mp i1)w | v \rangle = 0.$$

This is true if and only if $(A^\dagger \mp i1)w = 0$ since $\mathcal{D}$ is dense in $\mathcal{H}$; by definition $\mathrm{Ker}(B) = \{w | Bw = 0\}$.

This finishes our first major result: symmetric, densely defined operators have self-adjoint extensions if and only if the dimensions of $\mathrm{Ker}(A^\dagger \mp i1)$ are equal to one and another. Now, we come to the second major result for normal operators $A$ showing the following: there exists a projection valued measure $dP(z)$ on the complex plane such that in a weak $\star$ sense

$$A = \int_{\mathbb{C}} z \, dP(z).$$

Here, we meet for the first time an integral, something I presume you have studied in high school; to make precise that what I am going to say, I shall have to introduce some measure theory and theory of distributions as well. But before we come to that, we see that if we want to achieve such result, we need to study the invertibility properties of the operator $(A - z1)$. In particular, we need to look at those cases where $(A - z1)$ is not invertible; logically, there are three possibilities:

- $(A - z1)$ is not injective nor surjective; then $z$ is said to belong to the discrete spectrum,

- $(A - z1)$ is not injective, but surjective; in that case $z$ belongs to the residual spectrum,

- $(A - z1)$ is injective, but not surjective; in that case $z$ belongs to the continuous spectrum.

For normal operators, we have the result that the residual spectrum is empty. Note that if $A$ is normal, then $A_z = A - z1$ is likewise normal; also, we have that $A$ is injective if and only if $A^\dagger$ is which can be proven by means of $Av = 0$ if and only if $A^\dagger v = 0$. However, the fact that $A$ is surjective does not necessarily imply that $A^\dagger$ is. So suppose that $z$ belongs to the residual spectrum, then

$$\langle v|A_z w\rangle = 0$$

for all $w$ implies that $v = 0$. But then $\mathrm{Ker}(A_z^\dagger) = \mathrm{Ker}(A_z) = 0$ which is a contradiction. Therefore, the residual spectrum is empty. In case $z$ belongs to the discrete spectrum, there exists a unique Hermitian projection operator $P_z$ on $\mathrm{Ker}(A_z)$. $P_z$ commutes with $A$, $AP_z = P_z A = zP_z$ since $\langle v|AP_z w\rangle = z\langle v|P_z w\rangle = \langle \overline{z}P_z v|w\rangle = \langle A^\dagger P_z v|w\rangle = \langle v|P_z Aw\rangle$, and therefore also with $A^\dagger$ given that $P_z$ is Hermitian; moreover, if $z \neq z'$ belongs to the discrete spectrum, then $P_z P_{z'} = 0$ as follows from

$$zP_z P_{z'} = AP_z P'_z = z' P_z P_{z'}.$$

This looks already very much like the result we want to obtain; it is clear that on separable Hilbert spaces, the discrete spectrum is discrete meaning that it consists of at most a countable number of elements. We give the example of a bounded linear operator, for which one can prove that the spectrum is compact, given by $Ae_n = \frac{1}{n}e_n$ where $n > 0$ and $e_m$ an orthonormal basis. The discrete spectrum is given by $\{\frac{1}{n}|n \in \mathbb{N}_0\}$ and 0 belongs to the continuous spectrum, given that for example $\sum_{n=1}^{\infty} \frac{1}{n}e_n$ does not belong to the image of $A$. So, the continuous spectrum can have "measure zero" and does therefore not contribute to the spectral decomposition in this case.

The continuous spectrum is obviously empty for operators on finite dimensional Hilbert spaces and the reader should be able, at this point, to prove the spectral theorem by means of the fundamental theorem of algebra. Indeed, here the complex numbers become important since they garantuee a non-empty spectrum - a statement which we shall prove shortly - which should allow the reader to complete the proof by induction on the dimension of $\mathcal{H}$. Here, some notation becomes important: given a unit vector $v$, denote by

$$P = vv^\dagger$$

the operator defined by $Pw = v\langle v|w\rangle$. Then $P$ is a rank-one Hermitian projection operator and in case $AP = zP$ we obtain that $Av = zv$ meaning $v$ is a so-called eigenvector and $z$ belongs to the discrete spectrum $\sigma_d(A)$. Therefore, in finite dimensions, our statement becomes

$$A = \sum_{z \in \sigma_d(A)} z P_z$$

where $\sum_{z\in\sigma_d(A)} P_z = 1$. Before coming to the continuous spectrum in general, let us first show that the spectrum is non-empty in the finite dimensional case; the infinite-dimensional proof relies on methods in complex analysis, which we shall only study in a while and are therefore out of reach for the moment. I warned you I was not going to prove every statement in full detail but merely outline the general ideas, which is after all the most important thing to do. In finite dimensions, one introduces the so called determinant $\det(A)$ of a matrix, not operator, $A^i_j$ as follows:

$$\det(A) = \epsilon_{i_1 \ldots i_n} A^{i_1}_1 \ldots A^{i_n}_n$$

where again, Einstein summation has been taken into account. Here, we meet our second tensor $\epsilon_{i_1\ldots i_n}$ which transforms as a density. To understand this tensor, the reader must know something about the permutation group $S_n$; the latter consists of all bijections $\rho, \tau : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$. The reader should check that $S_n$ equipped with the standard composition $\rho\tau$ forms a non-commutative group indeed. A transposition is a bijection which swaps two indices $i, j$ and we denote by $(ij)$; clearly, every nontrivial permutation can be written as a product of transpositions. We now show that every product of transpositions equivalent to the identity must contain an even number of them. For this, we use the following rules: (a) $(i, j)(k, l) = (k, l)(i, j)$ if all four indices are distinct (b) $(jk)(ij) = (ij)(ki) = (ki)(kj)$ if $i, j, k$ are all distinct an finally (c) $(ij)^2 = 1$. By means of these operations, we can rewrite our product of transpositions as

$$(i_{2n} i_{2n-1}) \ldots (i_{2k+2} i_{2k+1})(1 i_{2k-1})(1 i_{2k-3}) \ldots (1, i_1)$$

where all $i_k$ are different from 1. Since in the product $(1i_{2k-1})(1i_{2k-3}) \ldots (1, i_3)$ $i_1$ can only get to 1 we must have another $(1, i_1)$ in this product which can again be shuffled trough so that it arrives in place of $(1, i_3)$. In this way $k$ is even and the original product can be reduced to $(i_{2n} i_{2n-1}) \ldots (i_{2k+2} i_{2k+1})$ which does not contain 1; therefore, in a similar way, one arrives to the fact that $n$ is even which concludes the proof. Given two distinct products $p_i$ for a permutation $\rho$, then $p_1 p_2^{-1}$ is a product for 1 and therefore even, which implies that both $p_i$ contain an even or odd number of transpositions. This motivates the following definition:

$$\text{sign}(\rho) = 1$$

if $\rho$ is written as an even product of transpositions and minus one otherwise. Now, $\epsilon_{i_1 \ldots i_n}$ is defined by $\epsilon_{i_1 \ldots i_n} = \text{sign}(\rho)\epsilon_{i_{\rho(1)} \ldots i_{\rho(n)}}$ and $\epsilon_{12 \ldots n} = 1$. $\epsilon_{i_1 \ldots i_n}$ is called the totally antisymmetric symbol and vanishes if any two indices are equal to one and another. Under a basis transformation $O^i_j$, the symbol transforms as

$$\epsilon'_{j_1 \ldots j_n} = \epsilon_{i_1 \ldots i_n} (O^{-1})^{i_1}_{j_1} \ldots (O^{-1})^{i_n}_{j_n} = \det(O^{-1})\epsilon_{j_1 \ldots j_n}.$$

Therefore, the $\epsilon$ symbol is basis independent if and only if one only considers transformations $O$ with $\det(O^{-1}) = 1$.

The reader should prove that the determinant enjoys the following properties:

- $\det(AB) = \det(A)\det(B)$,
- $\det(1) = 1$,
- $\det(A^\dagger) = \overline{\det(A)}$.

From this, it follows that $\det(A^{-1}) = (\det(A))^{-1}$ and therefore, $A$ is invertible if and only if $\det(A) = 0$. Another formula for the determinant is given by

$$\det(A) = \sum_{\rho \in S_n} \text{sign}(\rho) A_1^{\rho(1)} \ldots A_n^{\rho(n)}$$

which may be helpful in the above. This brings us back to the existence of elements in the spectrum, called eigenvalues. $z \in \sigma_d(A)$ if and only if $A_z$ is not injective nor surjective if and only if

$$\det(A_z) = 0.$$

The latter equation is of the form $P(z) = 0$ where $P(z)$ is a complex polynomial of degree $n$ in the variable $z$. The fundamental theorem tells us that it can be written as

$$\prod_{i=1}^{n} (b_i - z) = 0$$

and therefore we have $n$ roots corresponding to $z = b_i$ some of which may be equal to one and another. This finishes the proof that the spectrum is nonempty.

We now investigate the continuous spectrum in case $A$ is bounded. For $z$ in the continuous spectrum, we have the following: there exists a sequence of unit vectors $v_n$ such that

$$||A_z v_n|| \to 0$$

in the limit for $n$ to $\infty$. So, elements in the continuous spectrum give rise to approximate eigenvectors. We have that $\text{Im}(A_z)^{\perp}$ is zero since $A_z$ is injective and therefore $\text{Im}(A_z)$ is dense in $\mathcal{H}$. Suppose, on the contrary that

$$\inf_{||v||=1} ||A_z v|| \geq C$$

then $\text{Im}(A_z)$ is closed given that if $||A_z(v_n) - A_z(v_m)||$ is a Cauchy sequence then $||v_n - v_m||$ is with limit vector $v$ and $A_z v$ is the limit point of $A_z v_n$ by continuity which shows $A_z$ is surjective in contradiction to the fact that it belongs to the continuous spectrum. We have likewise that if $z \neq z'$ then

$$\lim_{n,m\to\infty} \langle v_n | w_m \rangle = 0$$

where $(v_n)_{n\in\mathbb{N}}$ corresponds to $A_z$ and $(w_n)_{n\in\mathbb{N}}$ to $A_{z'}$ which resembles the property of Hermitian projection operators in the discrete spectrum.

We now come to the construction of the spectral measure: given a measurable subset $\mathcal{O} \subseteq \mathbb{C}$, we define $P_{\mathcal{O}}$ to be the smallest Hermitian projection operator with the property that if $z \in \sigma(A) \cap \mathcal{O}$ and $(v_n)_{n\in\mathbb{N}}$ a sequence of (approximate) eigenvectors for $z$, then $||P_{\mathcal{O}}(v_n) - v_n|| \to 0$ in the limit for $n \to \infty$. Here, a measurable subset $A$ is constructed in the following way:

- any open set is measurable,
- the complement of a measurable set is measurable,
- the infinite union of measurable sets is measurable.

We shall come back to these foundational statements of measure theory later on but let us continue with the main argument. From all the above, it is clear that

$$P_{\mathcal{O}} P_{\mathcal{V}} = P_{\mathcal{O}\cap\mathcal{V}}$$

as the reader may wish to verify. Given a countable partition $(B_n)_{n\in\mathbb{N}}$ of $\mathbb{C}$ by means of measurable sets[1] we can take partial sums

$$A_{(B_n)_{n\in\mathbb{N}}} = \sum_{n=0}^{\infty} z_n P_{B_n}$$

where $z_n \in B_n$. As usual, the integral is defined by refining the partition and the remainder of the proof consists in showing that these sums converge in the weak-$\star$ topology to the aforementioned integral as well as $A$. The first statement is a delicate technical exercise, while the latter result involves again the axiom of choice. We omit the proofs of these statements as they contain very little novel ideas.

The reader should try to show that the spectrum of a self-adjoint operator is

[1] A partition satisfies the properties that $B_n \cap B_m = \emptyset$ for $n \neq m$ and $\cup_{n=0}^{\infty} B_n = \mathbb{C}$.

a subset of $\mathbb{R}$ and for a unitary operator a subset of the unit circle $|z| = 1$. Regarding the standard formulation of quantum mechanics, one has the important Stone-Von Neumann theorem whose proof relies on the spectral theorem but which we shall not outline here. In standard quantum theory, one has a fixed Hilbert space and the time evolution is given by given by unitary operators $U(t)$ with the property that $U(t + t') = U(t)U(t')$ and $U(0) = 1$. In case the mapping $t \to U(t)$ is continuous with respect to the strong operator topology and standard metric topology on $\mathbb{R}$, there exists a unique Hermitian operator such that

$$U(t) = e^{iHt}$$

where $e$ is a generalization to operators of the usual exponential map you know from high school. More generally, given any continuous map $f : \mathbb{C} \to \mathbb{C}$ and normal operator $A$, we have that

$$f(A) := \int_{\mathbb{C}} f(z)dP(z)$$

where we use the spectral decomposition

$$A = \int_{\mathbb{C}} zdP(z).$$

There is a rich literature on these issues and the reader in invited to learn more about operator theory albeit it does not constitute a suitable language for quantum mechanics in the presence of a gravitational field. Before we turn our head towards general analysis, let me mention two interesting generalizations of the setting so far without giving any proof.

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