Elements 1: Some Integrals

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Abstract. In this note we give some integrals.

Key words: Number π , Gamma function, Definite integration.

The number π is defined by: $\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.141592...$, this note presents a collection of integrals involving the mathematical constant π :

$$\int_1^{\infty} \tan^{-1}\left(x^2-\sqrt{x^4-1}\right)dx = \int_1^{\infty}\tan^{-1}\left(\frac{1}{x^2+\sqrt{x^4-1}}\right)dx = \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} - \frac{\pi}{4} \qquad (1)$$

$$\int_1^{\infty}\left(\frac{\pi}{4} - \tan^{-1}\sqrt{\frac{x^2-1}{x^2+1}}\right)dx = \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} - \frac{\pi}{4} \qquad (2)$$

$$\int_0^{\pi/4}\sqrt{\frac{1+\tan^2 x}{1-\tan^2 x}}\,dx = \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} \qquad (3)$$

$$\int_{1/\sqrt{2}}^{1}\sin^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)dx = \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} - \frac{\pi}{2\sqrt{2}} \qquad (4)$$

$$\int_{1/\sqrt{2}}^{1}\cos^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)dx = \frac{\pi}{2} - \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} \qquad (5)$$

$$\int_1^{\infty}\left(\frac{\pi}{4} - \sin^{-1}\sqrt{\frac{x^2-1}{2x^2}}\right)dx = \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} - \frac{\pi}{4} \qquad (6)$$

$$\int_{1/\sqrt{2}}^{1}\sin^{-1}\sqrt{\frac{2\left(1-x^2\right)}{1+\sqrt{1-4x^2+4x^4}}}\,dx = \frac{\left(\Gamma(1/4)\right)^2}{4\sqrt{2\pi}} - \frac{\pi}{2\sqrt{2}} \qquad (7)$$

$$\int_0^{2-\sqrt{2}} \frac{\sqrt{2x^2-2x-1+\sqrt{1+4x}}}{x}\,dx = \frac{\pi}{2\sqrt{2}} + 2 - \sqrt{2} - \sqrt{2}\ln\left(\sqrt{2}+1\right) \tag{8}$$

$$\int_2^{2+\sqrt{2}} \frac{\sqrt{2x^2-2x-1-\sqrt{1+4x}}}{x}\,dx = 2 + \sqrt{2} - \frac{\pi}{2\sqrt{2}} - \sqrt{2}\ln\left(\sqrt{2}+1\right) \tag{9}$$

$$\int_{\left(\sqrt{2}+1\right)/2}^{2} \frac{\sqrt{1+2x-2x^2+\sqrt{1+4x}}}{x}\,dx = \frac{\pi}{2\sqrt{2}} - 1 - \frac{1}{\sqrt{2}} + \sqrt{2}\ln\left(\sqrt{2}+1\right) \tag{10}$$

$$\int_0^{\left(\sqrt{2}-1\right)/2} \frac{\sqrt{1-2x-2x^2-\sqrt{1-4x}}}{x}\,dx = \frac{\pi}{2\sqrt{2}} + 1 - \frac{1}{\sqrt{2}} - \sqrt{2}\ln\left(\sqrt{2}+1\right) \tag{11}$$

$$\int_0^{\infty} \left(1 - \sqrt[4]{\frac{x^2}{1+x^2}}\right) dx = \frac{\left(\Gamma(3/4)\right)^2}{\sqrt{2\pi}} \tag{12}$$

$$\int_0^{\infty} \left(1 - \sqrt{\tanh x}\right) \cosh x\,dx = \frac{\left(\Gamma(3/4)\right)^2}{\sqrt{2\pi}} \tag{13}$$

$$\int_0^{\pi/2} \left(1 - \sqrt{\sin x}\right) \frac{1}{\cos^2 x}\,dx = \frac{\left(\Gamma(3/4)\right)^2}{\sqrt{2\pi}} \tag{14}$$

$$\int_0^{1} \frac{1-x^2}{1+x^2+\sqrt{1+6x^2-3x^4}}\,dx = \frac{\left(\Gamma(1/3)\right)^3}{4\pi\sqrt{3}\sqrt[3]{2}} - \frac{\sqrt{3}\left(\Gamma(2/3)\right)^3}{2\pi\sqrt[3]{4}} \tag{15}$$

$$\int_0^{\left(\sqrt{2}+1\right)/2} \sqrt[4]{\frac{1+2x-2x^2+\sqrt{1+4x}}{2x^2}}\,dx = \frac{\pi}{4\sqrt{2}} - \frac{1}{4\sqrt{2}}\ln\left(\frac{2+\sqrt{2}}{2-\sqrt{2}}\right) + \frac{\left(\Gamma(1/4)\right)^2}{8\sqrt{\pi}} + \frac{\sqrt{2}+1}{2} \tag{16}$$

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