Kaluza hypothesis as a key to the "hidden" geometry of quantum electrodynamics

We cannot solve our problems with the same thinking we used when we created them. Albert Einstein

Victor Paromov, PhD
Email: vikus68@yahoo.com
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Abstract

This letter attempts to show that the Kaluza hypothesis of the five-dimensional (5D) spacetime (in Klein's interpretation and absence of the cylinder condition) is sufficient to explain the quantum electrodynamics (QED). It is a short version of the article "Fractal Structure of the Spacetime, the Fundamentally Broken Symmetry" (http://vixra.org/abs/1806.0181) written as an attempt to start a discussion in the physics community. To author's regret, it has been rejected by Nature Physics on 9/24/2018 and by arxiv.org (gr-qc) on 10/23/18 in spite of the endorsement from Dr. S. M. Kopeikin.

Introduction

It is a common belief that the original Kaluza hypothesis [1] adding a fifth extra [forth spatial] dimension to the Einsteinian spacetime model cannot lead to a self-consistent quantum theory of electromagnetism, even in its Klein's interpretation [2]. Indeed, the Kaluza-Klein (KK) extensions [3] of the General Relativity (GR) theory remain essentially classical field theories and are not quantized naturally. Conversion of any given KK theory into a self-consistent quantum field theory like the quantum electrodynamics (QED) seems highly problematic. Moreover, taking to the account the experimental evidence [4] of the Bell's inequalities' break [5] at the "quantum scale", the classical field theoretical approach may not be applied to particles' interactions in general.

Thus, it is likely that the classical KK approach to the unification of electromagnetism with gravity based on the Kaluza hypothesis cannot lead to a self-consistent quantum field theory. However, the classical KK approach is not the only possible way of building a quantum field theory based on the geometry of spacetime. The argument below is not a formal proof, it is an attempt of constructing of a non-classical field theory (an abelian gauge field theory) mathematically similar to the QED, however, founded solely on the spacetime geometry (i.e. the Kaluza's model of 5D spacetime in Klein's interpretation), in the best traditions of the GR.

Results

Let us start from the original Kaluza's description of the 5D spacetime disregarding the cylinder condition [1]. Briefly, each world point of the 4D (Einsteinian or ordinary) spacetime "contains" an extra spatial dimension represented by a microscopic 1D circle (S^1) of a subatomic size (according to the Klein's interpretation [2]). An additional geometrical condition for the 5D spacetime model is that each extradimensional S^1 is uniform and a maximal section.

Let us assume that any elementary electric charge (e.g. electron) induces 5D spacetime curvature, just like any massive object induces 4D spacetime curvature. It is suggested that the 5D spacetime geometry is the one and only origin of the particle movement (the Einsteinian understanding of force). The three main differences of the charge-induced curvature compared to the mass-induced curvature should be taken to the account: 1) the effect of mass appears in the 4D spacetime only, whereas electric charges influence both the 5D spacetime (directly) and the 4D spacetime (indirectly); 2) the latter effect is dual, i.e. one type of charges induces positive geometric 5D spacetime curvature, and another type induces negative curvature; 3) due to the compact size of the fifth dimension [2] the 5D spacetime geometry cannot be directly accessed by the observer (as any elementary 5D object can only be accessed as a 4D projection, a 4D world point). Let us call this phenomenon the undetectability condition.

Let us describe the movement of a point-like charged test particle (e.g. electron). For the observer, the particle movement in the 5D spacetime is inaccessible due to the undetectability condition, and hence, it should be somehow translated either into the movement in 4D spacetime or into the movement in 3D space and absolute time. The latter description is preferable for simplicity reasons. Notably, as the fifth dimension size is presumably microscopic [2], one cannot describe the charge-induced curved local 5D spacetime with a classical field (e.g. by a KK GR extension) due to the fifth dimension (i.e. the fourth spatial dimension) inevitably collapsing into a point. However, instead of describing the actual 5D spacetime curvature in $\mathbf{R}^{4+1}$ (like all the KK models do [3]) one could describe the 4D-space curvature (corresponding to the 5D spacetime curvature with absolute time) as an additional parameter of the ordinary 3D space (corresponding to the 4D spacetime with absolute time). Thus, the 4D-space curvature should be described by a scalar field in $\mathbf{R}^3$ (assuming the global 3D space is $\mathbf{R}^3$, for simplicity). In addition, it is assumed that the particles always move by the geodesics in the 4D space (the geodesic condition). If all possible 4D (spatial) geodesics can be translated into 3D geodesics, one would be able to describe the 4D-space curvature as a field in $\mathbf{R}^3$, the simplified global 3D space.

Thus, if the extradimensional curvature governs the charged particle movement, the former can be described by a scalar field in $\mathbf{R}^3$ with the two conditions: 1) scalar 4D-space curvature is described by a scalar field, and 2) all possible 4D-space geodesics are properly translated into 3D-space geodesics. As there are two types of electric charges, it is logical to assume that one type induces positive geometrical curvature, and another type induces negative geometrical curvature.

First, let us consider the case of the positive 4D spatial curvature (time curvature is disregarded). If an elementary charge induces a stable curvature of the flat 4D space, this local space can be modeled by a small size hypersphere S^4. Although the global 4D space cannot be modeled by a hypersphere (presumably, the global 4D space topology is $S^3 \times S^1$), the local 4D space perfectly can, if its size does not exceed the extra dimension's diameter. In reality, this curved space may have some curvature gradients and shape imperfections disregarded for simplicity reasons. Let us assume this local space as an ideal hypersphere embedded in flat $\mathbf{R}^5$. Next, one needs to find a proper transition from this local S^4 geometry to the global (simplified) geometry, $\mathbf{R^3}$. First, S^4 can be mapped to S^3, the intersection of S^4 with hyperplane $\mathbf{R^4}$ containing the center of S^4. The latter condition assures that S^3 is the geodesic of S^4 and is predetermined by the Kaluza model's additional condition (see above). Then, the S^4 original scalar curvature lost in the transition can be described as a scalar field in each point of S^3. The important property of this local S^3 is that it is isometric to the global (real) space, which is also assumed as S^3. Thus, one can substitute the local space with the global space preserving the geodesic condition. Then, the extradimensional spatial scalar curvature can be described by some scalar field in S^3 manifold.

Next, one can use the stereographic projection and translate S^3 into $\mathbf{RP^3}$ providing the scalar field description in the ordinary (simplified) space, $\mathbf{R^3}$. However, this scalar field depends on the inner parameter (curvature) of the 4D space, which cannot be described with a real field due to the undetectability condition. Therefore, one needs to implement a complex scalar field. Ideally, S^3 should be translated into a complex manifold that 1) accounts for the periodicity (circle symmetry) of the extra spatial dimension, 2) preserves the geodesic condition, and 3) is isometrically embeddable in $\mathbf{R^3}$ space. That can be done, if one replaces S^3 with a unit sphere $S(\mathbf{C^2})$ in the complex coordinate space $\mathbf{C^2}$ and uses the principal Hopf bundle [6] over the complex projective space: $U(1) \rightarrow S(\mathbf{C^2}) \rightarrow \mathbf{CP^1}$. The projection map: $S(\mathbf{C^2}) \rightarrow \mathbf{CP^1}$ gives a Riemannian submersion with totally geodesic fibers isometric to U(1). This Hopf bundle is a generalization of the geometrical fibration: $S^1 \rightarrow S^3 \rightarrow S^2$. As the Hopf fibration is known to assign a great circle of S^3 to each point on S^2, it maps all the geodesics of S^3 onto S^2, which in turn is projected to planar $\mathbf{CP^1}$, hence preserving the geodesic condition. Thus, $\mathbf{CP^1}$ gives the simplest possible type of the scalar field that can be used to describe the local extradimensional curvature (i.e. the original S^4 scalar curvature) in global space, $\mathbf{R^3}$ (flat 3D space).

A similar construction can be used with the hyperbolic pseudosphere H^4, in case the elementary charge is assumed inducing negatively curved local 4D space. With similar reasoning, the H^4 curvature can be described by a scalar field in $\mathbf{CP^1}$ with an opposite sign compared to the case above. Briefly, one can take the intersection of H^4 and $\mathbf{R^4}$, which again gives S^3. The H^4 original curvature lost in the transition can be described by a scalar field in each point of this S^3. Assuming the negative extradimensional curvature counteracts the positive curvature, the second type scalar field should have an opposite sign compared to the case above (positive curvature). The Hopf fibration: $U(1) \rightarrow S(\mathbf{C^2}) \rightarrow \mathbf{CP^1}$ again translates S^3 (replaced by $S(\mathbf{C^2})$) into the complex projective space preserving the geodesic condition. Finally, one obtains the "negative" type complex scalar field (with an opposite sign) describing the opposite elementary charge action in the global (simplified) 3D space, $\mathbf{R^3}$.

Thus, the extradimensional curvature assumed governing the electromagnetic field can be described with a complex scalar field having two counteracting components, the "positive" and the "negative" fields. This two-component complex scalar field has a Fubini-Study metric (which is an Einsteinian metric), and an action:

$$S = \int d^4x (\partial\mu\phi^*)(\partial^\mu\phi) - V(|\phi|) \qquad (1)$$

where ϕ acts as the "positively" charged field, ϕ^* acts as the "negatively" charged field, and $V(|\phi|)$ is the complex scalar field potential.

Further derivation of the equations of motion is rather trivial. By the construction, action (1) has a global symmetry under the group U(1), i.e. $\phi \rightarrow e^{i\alpha}\phi$, which can be translated to a local symmetry by introducing a gauge field with the gauge covariant derivative $D\mu\phi \rightarrow e^{i\alpha(x)}D\mu\phi$, where e is the elementary electric charge. One can find the gauge transformation-invariant form of the above-stated action, add the gauge field kinetic term defined by the transformation group U(1), $F_{\mu\nu}F^{\mu\nu}$ (where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$) and write the equations of motion with respect to the gauge field. Thus, the scalar field dynamics can be given by the Lagrangian density:

$$L = (D\mu\phi^*)(D^\mu\phi) - V(|\phi|) - 1/4\ F_{\mu\nu}F^{\mu\nu} \qquad (2)$$

The equation (2) is similar to the QED Lagrangian, except it describes spinless charged scalar fields, not spin ½ leptons. The spin introduction is explained below.

A point-like test particle's movement in the global 4D space is governed by the geometry of local space, S^4. The main alteration type of this local space is the extradimensional curvature. In the 3D space, this curvature is inaccessible by the observer and is observed only indirectly, as the electrostatic field. This geometrical alteration is already taken care of by the equation (2). However, assuming particles are in constant motion in local S^4, one should take to the account an additional geometric alteration, torsion. The motion in S^4 can be approximated as having the two components: a "visible" movement in the ordinary (simplified) 3D space, $\mathbf{R^3}$ and a "hidden" movement along the special extra coordinate. For the observer, the latter appears as a constant "invisible" spin along the microscopic S^1. This spin is the origin of the 4D space torsion indirectly observed in the 3D space as the magnetic field. This extradimensional torsion requires one to introduce an additional correction for the scalar field description given by the equation (2).

Although the spin can have infinite possible directions in local 4D space (in S^4), for the observer, all those directions are reduced to just two, clockwise and counterclockwise (or "up" and "down"), due to the separate accessibility of the "visible" movement in $\mathbf{R^3}$ and the "hidden" spin along the microscopic S^1. As there are two parts of the complex scalar field and two possible spin directions, one must introduce certain corrections to equation (2) accounting for the proper spin direction and making the right commutation. The proper corrections are made by the Dirac matrices and replacing the field ϕ to the bispinor field ψ. Then, the Lagrangian (2) takes the form:

$$L = \bar{\psi}(i\gamma^\mu D_\mu)\psi - V(|\phi|) - 1/4\ F_{\mu\nu}F^{\mu\nu} \qquad (3)$$

where ψ is a bispinor field, i.e. electron-positron field; $\bar{\psi} \equiv \psi^\dagger\gamma^0$ is the Dirac adjoint; and γ^μ are the Dirac matrices. Notably, the gauge field potential $V(|\phi|)$ is now depends on both 4D-space curvature and 4D-space torsion, and its minimum $V(|\phi|)_{\min}$ occurs at $|\phi| \neq 0$ in the presence of electric charge, due to the initial assumption that the charge is the origin of the extradimensional geometrical alterations. Hence, the gauge field behaves as a massive field, and its mass is proportional to the lepton's ground state mass-energy m, i.e. conventional electron's mass. Thus, the potential can be expressed as $V(|\phi|) = \bar{\psi}m\psi$. After the proper replacement, equation (3) takes the final form of the QED Lagrangian:

$$L = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - 1/4\ F_{\mu\nu}F^{\mu\nu} \qquad (4)$$

Discussion

The above-presented reasoning shows that the 5D spacetime geometry hypothesized by Kaluza in 1921 [1] in the Klein's interpretation [2] is sufficient for constructing a group U(1) abelian gauge theory mathematically identical to the QED. The theory can be directly quantized in the same way as the QED (e.g. using the S-matrix technique). Notably, the quantum nature of the resulting theory is predetermined by the special properties of the fifth [forth spatial] dimension (microscopic size, closeness, and simple connectedness). The undetectability condition explains the necessity to use a complex scalar field, not a real field for the descriptions of elementary charges' dynamics. Due to this condition, parameters related to the extra-dimensional geometry, i.e. the electric charge of the bispinor field, the Compton wavelength, and consequently the Planck constant, cannot be determined by the theory and should be added *ad hoc* in accordance with the experimental values.

The unification with gravity comes quite naturally with the $V(|\phi|)$, the complex scalar field potential presently known as the Higg's field. The latter can be interpreted as a geometric alteration of the 4D (ordinary or Einsteinian) spacetime, the "visible" part of the 5D spacetime. Assuming that alterations of the 5D spacetime are directly induced by the electric charges, the Higg's field represents the "induced mass", i.e. the 4D spacetime geometrical alterations induced by the charges indirectly. Assuming the charges are in constant motion along the round extra coordinate, their dynamics (for the observer) always has a certain "hidden" periodicity. Thus, an elementary charge must have certain attributes of a wave, which, however, can only be described by complex-valued parameters due to the undetectability condition.

Considering these facts, it seems reasonable to conclude that the original Kaluza hypothesis (in Klein's interpretation and absence of the cylinder condition) is fully sufficient to explain the electromagnetism. In general, this means that the 5D spacetime geometry may be sufficient to describe the electromagnetism, as the 4D spacetime geometry sufficiently explains the gravity. Moreover, in case the Kaluza spacetime model is extended with three additional more compact extra spatial dimensions, it seems possible that a similar approach can lead to the nuclear forces' descriptions solely based on the spacetime geometry.

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