Astrophysical Applications of Yilmaz Gravity Theory

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Abstract

Hüseyin Yilmaz proposed the form of a theory of gravitation (Yilmaz 1958, 1971) that has later been shown to present only minor conceptual change for Einstein's General Relativity. The primary effect of the change is to modify terms of second order in the gravitational potential or its derivatives. Since most of the weak field tests that have been taken as confirmation of General Relativity are of first order, the Yilmaz theory continues to pass all of these tests, but there are some interesting effects of the higher order terms that arise in the Yilmaz theory. These corrections move the metric singularity back to the location of a point particle source. This eliminates the black hole event horizon and permits the existence of intrinsic magnetic moments for stellar mass black hole candidates and supermassive AGN. It is shown here that the same second order corrections also eliminate the need for cosmological "dark energy". Additional considerations dicussed here show that the Yilmaz theory correctly encompasses all of the major observational tests that must be satisfied by an acceptable relativistic gravity theory.

Keywords: Event Horizon, Black Holes, Dark Energy, Exponential Metric

1. Introduction

Although the Yilmaz gravity theory is well established in the literature (Yilmaz 1958 ... 1997), it is not widely known. Many of its major points of contact with astrophysics will be presented here. In addition to previously unpublished results, some older points will be placed in appendixes and footnotes, but made available here for the sake of completeness. Though the aim is to provide a synopsis of the contacts between the Yilmaz gravity theory and observational results, there is a wider context of physics that is at stake here. One aspect of this consists of the philosophical question of how we should regard a theory that agrees with observational constraints, but is not yet part of mainstream theoretical physics. Another is the question of how much consideration we should give to some of the problems that arise from features that are part of the accepted gravity theory, Einstein's General Relativity.

The question of how quantum mechanics and general relativity might be reconciled has recently been sharpened by considering what happens to a freely falling particle of matter approaching an event horizon. The possibility that it might meet a radiative "firewall" has recently become a very active research topic (e.g., Almheiri, Marolf, Polchinski & Sully 2013, Chowdhury & Puhm 2013, Abramowicz, Kluzniak & Lasota 2013, Anastopoulos & Savvidou, 2014, Hawking 2014, Polchinski, 2016, Conklin, Holdom & Ren 2018). Apparently there are both theoretical and observational concerns with the concept of an event horizon, however, the necessity for event horizons seems not to have been questioned. They have so far been accepted without proof. Although there are many astronomical objects that are known to be compact and massive enough to be black holes, if event horizons exist, none have been shown to possess this essential attribute of a black hole. As shown below, event horizons do not exist in the Yilmaz theory.

Einstein developed general relativity with the aim of explaining gravitational phenomena as manifestations of spacetime geometry alone. In his field equations he included all forms of energy as sources of gravitation and curvature but expressly rejected a separate gravitational field as a source of energy. Instead of having separate gravitational potentials, the metric coefficients of general relativity take the dual roles of potentials and descriptors of spacetime geometry. One of the problems that this presents for quantum theory is that the covariant derivatives of the metric tensor are identically zero. Potentials that exist separately from the metric may provide a path to a quantum theory of gravity.

Since the field energy densities are of second order in the derivatives of the potentials, they can be introduced as source terms in the field equations of general relativity without affecting the foundational weak-field tests of the theory, but they might decisively influence outcomes in tests in strong gravitational fields. Although others (Wald 1984; Yu 1992) had demonstrated that Einstein's quadrupole gravitational radiation formula could not be consistently and generally derived from conventional general relativity, Lo (1995) showed that consistency could be achieved by the inclusion of a gravitational field stress-energy tensor as a source term in Einstein's field equations.

One of the aims of this article is to show that the inclusion of a second order field stress-energy tensor can also eliminate event horizons and the need for "dark energy". The Yilmaz corrections for general relativity (Yilmaz 1958, 1971, 1977, 1980, 1982, 1992) provide definite forms for the stress-energy tensor of the gravitational field (see Appendix B). The theory provides for gravitational radiation and is known to pass all of the observational tests so far required of gravity theories.

The intent here is to first focus on the places where gravity theory most profoundly affects current astrophysics: in event horizons, dark energy, compact objects and gravitational waves. The readers who have further interest in the details of the theory can find many of them in Appendix B. Informative discussions of various aspects of the theory have also been provided by Alley (1995), Menzel (1976), Mizobuchi (1985) and Yilmaz (1975).

2. Weak-field Tests of Gravity Theories

The weak-field tests that must be passed by a viable gravity theory can be encompassed by spherical coordinates (ct, r, θ, Φ) and a diagonal metric of the isotropic form

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda}(dr^2 + r^2 d\theta^2 + (r \sin\theta \; d\Phi)^2) \tag{1}$$

where ν and λ are functions of spatial coordinates and (sometimes) time. c is the free-space speed of light, the metric coefficients are $g_{00} = e^{\nu}, g_{11} = -e^{\lambda}, g_{22} = -r^2 e^{\lambda}, g_{33} = -e^{\lambda}(r \sin\theta)^2$. In the Yilmaz theory, the isotropic form with all spatial dimensions affected by gravity in the same way is considered to be necessary for consistency with the Hughes-Drever experiments (Hughes, Robinson & Beltran-Lopez 1960 and Drever 1961) that demonstrated the isotropy of inertia.

The geodesics of particles and photons in this metric can be described by the generalized energy-momentum equations

$$g^{ij} p_i p_j = g_{ij} p^i p^j = m_o^2 c^2 \tag{2}$$

where m_0 is particle rest mass (zero for photons). For particles in an equatorial plane, $\theta = 0$, $\sin\theta = 1$, $p_0 = E/c$, where E is the particle energy, and p_Φ is the angular momentum, this first form of Eq. 2 becomes.

$$e^{-\nu} E^2/c^2 - e^{-\lambda}[p_r^2 + p_\Phi^2/r^2] = m_0^2 c^2 \tag{3}$$

The Einstein field equations and static solutions for $\nu(r)$ and $\lambda(r)$ for a single, central field source mass, M, can be found in many standard works on relativity theory, (e.g. Tolman 1934, 1987, Weinberg 1972). In both the Einstein and Yilmaz theories, Eq. 3 easily accounts for all of the classical weak-field tests of gravity theory that have been performed in the solar system. These include the perihelion shift of the planet Mercury, for which the orbital axis precesses by 43 seconds of arc per century, the deflection of star light that grazes the sun, and the time delay of radar echoes from Venus when their path grazes the sun. (For photons of light or radar, $m_0 = 0$ in Eq. 3.)

2.1. The Yilmaz Metric g_{00}

We can begin to explore the Yilmaz metric for a static gravitational field by considering the behavior of test particles in the field. If a particle of mass m is displaced quasistatically by $d\vec{r}$ by a force $\vec{F}$, the work done is $\vec{F} \cdot d\vec{r} = c^2 dm$. This merely recognizes the Einstein relationship between energy and mass ($E = mc^2$). If the force is due to a gravitational field for which the potential is U; and rendered dimensionless by division by c^2, such that $\phi = U/c^2$, then the gravitational force on m is given by $\vec{F} = -mc^2 \vec{\nabla}\phi$. The change of potential associated with a quasistatic displacement of the particle is the work done by an opposing external force, thus

$$c^2 dm = mc^2 \vec{\nabla}\phi \cdot \vec{dr} = mc^2 d\phi \tag{4}$$

which integrates to $m = m_a e^{(\phi - \phi(a))}$ where the mass would be m_a with the particle at rest at position $\vec{r}_a$. If the potential would be zero at this point we can write (Martinis & Perkovic 2009)

$$m = m_0 e^{\phi} \tag{5}$$

We can consider a similar process from the standpoint of the the energy-momentum equation. It is capable of encompassing a state of geodesic motion in which a particle is at rest, at least temporarily, with no three-momentum, $\vec{p} = 0$ under the influence of a gravitational force alone. No opposing force needs to be considered here. This might occur, for example at the apex of motion of a projectile that travels radially outward from the surface of a central mass, M. In this case, Eq. 2 becomes

$$g^{00} E^2/c^2 = g^{00} m^2 c^2 = m_0^2 c^2 \tag{6}$$

Substituting for m from Eq.5 leads immediately to $g^{00} e^{2\phi} = 1$. Noting that $g_{00} = 1/g^{00}$ for a diagonal metric we have

$$\mathbf{g_{00} = e^{2\phi}} \tag{7}$$

While Eq. 7 is a general result, Eq.5 must be modified if a particle is in motion in a gravitational field. The end result is that the mass given by Eq. 5 needs to be increased by the Lorentz factor $1/\sqrt{1 - v^2/c^2}$, where v is the proper speed of the particle at its current location.[1] Thus the more general result is[2]

$$m = m_0 e^{\phi}/\sqrt{1 - v^2/c^2} \tag{8}$$

2.2. Spatial Dependence of the Static Metric

The spatial dependence of the Yilmaz metric can be determined by imposition of a "harmonic coordinate" condition. This condition is expressed as (Weinberg 1972, p. 163)[3]

$$\partial_j(\sqrt{-g}\; g^{kj}) = 0 \tag{9}$$

where (g) is the determinant of the metric. In rectangular coordinates and the metric form of Eq. 1, this leads to

$$\partial_0(3\lambda - \nu)/2 + \nabla(\lambda + \nu)/2 = 0 \tag{10}$$

[1] This is most easily shown from Eq. 2 in cartesian coordinates where $p^1 = m_0 dx/d\tau, p^2 = m_0 dy/d\tau, p^3 = m_0 dz/d\tau$, the proper speed is $v = (-g_{11}dx^2 - g_{22}dy^2 - g_{33}dz^2)^{1/2}/\sqrt{g_{00}}dt$ and $dt/d\tau = \gamma/\sqrt{g_{00}}$.

[2] The same sorts of mass changes are encompassed by the solutions from General Relativity since e^{ϕ} is the equivalent of $\sqrt{g_{00}}$. All that is necessary is to determine the General Relativity form for the Einsteinian gravitational potential, ϕ_{GR}.

[3] This condition has physical significance for light and other waves that might exist in the space described by the metric. A plane wave of the form $\psi = e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$ should satisfy a generalized d'Alembertian equation, $\Box\psi = (1/\sqrt{-g})\partial_i(\sqrt{-g}\; g^{ij}\partial_j\psi) = 0$. This will generate nonzero terms of the form $\mathbf{k}\cdot\partial_i(\sqrt{-g}\; g^{ij}\nabla\psi)$ and make the speed of light depend on its direction of travel unless $\partial_i(\sqrt{-g}\; g^{ij}) = 0$.

For static fields the time derivatives vanish, leaving the requirement that $\nabla(\lambda + \nu) = 0$ Thus $\lambda + \nu = constant$. Recalling that $\nu = 2\phi$, at places where the gravitational potential ϕ would be zero we take the constant to be zero as a boundary condition, leaving $\lambda = -2\phi$. The static Yilmaz metric is then[4]

$$ds^2 = e^{2\phi}c^2dt^2 - e^{-2\phi}(dr^2 + r^2d\theta^2 + (r\sin\theta\ d\Phi)^2) \tag{11}$$

For the case of the metric in the space exterior to a central spherical mass, M, centered on $r = 0$, the Yilmaz potential is simply the Newtonian gravitational potential, $\phi = -GM/c^2r$ and, we repeat

$$g_{00} = e^{2\phi} \approx 1 - 2GM/c^2r + 2(GM/c^2r)^2 + ... \tag{12}$$

This solution has a point particle singularity at $r = 0$, but it is of little practical consequence. The Kretschmann invariant is zero rather than divergent at $r = 0$. There is no curvature singularity there. In contrast, the event horizon was discovered in the solution of the Einstein field equations of general relativity. For the metric of the spacetime beyond a central point mass and the spatially isotropic metric of Eq. 1, the solution of the Einstein field equations yields

$$g_{00} = e^{\nu} = \frac{(1 - GM/2c^2r)^2}{(1 + GM/2c^2r)^2} \approx 1 - 2(GM/2c^2r) + 2(GM/2c^2r)^2 + ... \tag{13}$$

and

$$e^{\lambda} = (1 + GM/2c^2r)^4 \approx 1 + 2(GM/c^r) + 2(3/4))GM/c^2r)^2 + ... \tag{14}$$

It should be noted that in the Einstein theory, ν is not a gravitational potential. Since Eq. 7 depends only upon special relativity and the definition of a potential, it is apparent that the potential for the solution from General Relativity is given by

$$e^{2\phi_{GR}} = \frac{(1 - GM/2c^2r)^2}{(1 + GM/2c^2r)^2} \tag{15}$$

from which we see that the gravitational potential ϕ_{GR} for the isotropic metric would be

$$\begin{aligned} \phi_{GR} &= ln((1 - GM/2c^2r)/(1 + GM/2c^2r)) \\ &\approx -GM/c^2r - (GM/c^2r)^3/12 - (GM/c^2r)^5/80\ \ . . . \end{aligned} \tag{16}$$

This is a Newtonian potential at lowest order, but diverges at the location of an event horizon that occurs for $r = GM/2c^2$.[5] Further, the proper acceleration of a freely falling body diverges as the event horizon is approached according to

$$a = F/m = -c^2\nabla\phi_{GR} = \frac{-(GM/r^2)}{(1 - (GM/2c^2r)^2)} \tag{17}$$

[4]Yilmaz usually used different conventions for the relationship between gravitational forces and potentials and used the negative of the ϕ used here.

[5]This event horizon radius is smaller by a factor of four than that obtained in the usual Schwarzschild coordinates. The fact that the size of the event horizon is coordinate dependent has been taken to suggest that it does not represent something physically real.

Lorentz invariance of the spectral distribution of virtual photons in the quantum plenum requires their energy density to be a cubic function of frequency (Boyer 1969). It has also been shown that an accelerating particle experiences a headwind of thermal photons with a temperature proportional to the magnitude of the acceleration (Fulling 1973, Davies 1975, Unruh 1976, Matsas & Vanzella 2002). Thus a particle approaching the event horizon would be incinerated as the acceleration and radiation temperature increase without limit. This divergence is the essence of the "firewall problem" at the event horizon.

2.3. PPN Constraints

General Relativity has been thoroughly confirmed where it has been tested in the weak gravitational fields within our solar system where $GM/c^2r = 2 \times 10^{-6}$ at the photosphere of the sun and $GM/c^2r = 7 \times 10^{-10}$ at earth's surface. But surely there are reasons for caution when extrapolating by six orders of magnitude to reach an event horizon where $GM/c^2r = 2$.

Recapitulating the weak-field results, the metric can be written as a parameterized post-Newtonian (PPN) expansion to terms of second order, using the tags, α, β, γ and δ as

$$g_{00} = e^{\nu} \approx 1 - \alpha 2\phi + \beta 2\phi^2 \tag{18}$$

$$-g_{ii} = e^{\lambda} \approx 1 + \gamma 2\phi + \delta 2\phi^2 \tag{19}$$

Expanding the metric coefficients of Eqs. 13 and 14, we find $\alpha = \beta = \gamma = 1$ and $\delta = 3/4$. The Yilmaz solutions of Eq. 12 differ only by having $\delta = 1$. $\alpha = 1$ is needed to account for gravitational red shifts for earth and sun, $\beta = 1$ is needed to account for results from laser ranging of the moon and photon redshifts in weak fields. $\gamma = 1$ is needed for the deflection of light rays passing the limb of the sun and the Venus radar echo delay. The fourth parameter δ cannot be determined by the classical weak-field experiments alone, but Yilmaz (Yilmaz 1977, 1980) has made a case for $\delta = 1 \pm 0.005$ from two additional experiments. These are the Hughes-Drever experiments on the isotropy of space (Hughes, Robinson & Beltran-Lopez 1960 and Drever 1961) and the neutron phase shift experiment (Colella, Overhauser & Warner 1975). In addition to the need for second order terms to account for gravitational radiation (Lo 1995), this Yilmaz analysis indicates that General Relativity needs corrections in terms of second order in ϕ.

2.4. Gravitational redshifts

It is well known that frequency shifts occur for photons in gravitational fields. If the frequency would be ν_0 at a location where $\phi = 0$, then the frequency at other locations is given by

$$\nu = \nu_0 \; {g_{00}}^{-1/2} \tag{20}$$

and according to Eq. 7, this would be

$$\nu = \nu_0 \; e^{-\phi} \tag{21}$$

Eq. 21 can be be shown to be an EXACT requirement of special relativity and the principle of equivalence (See Appendix A). The photon red shift result of Eq. 21 was stated by Einstein in a 1907 paper (available as translated by H.M. Schwartz, Am. J. Phys, 45, 899, 1977). Although Einstein had first arrived at a first order approximation, he noted that "**in all strictness**" this first order result must be replaced by the exponential form of Eq. 21. For a time after 1907, Einstein maintained that the metric coefficients must be strictly exponential functions in order to conform to the requirements of special relativity, but his final development of general relativity satisfied the requirement only to terms of first order.

2.5. Relativistic Accretion Disks

There is an interesting astrophysical application of Eq. 3 consisting of the description of particles in accretion disks in x-ray binary star systems. Consider a particle in orbit around a central mass, M, for which $\phi = -GM/c^2r$. Assuming that angular momentum, p_Φ, is conserved, and rearranging Eq. 3 we obtain.

$$e^{4\phi}(p_r/m_0c)^2 = (E/m_0c^2)^2 - e^{2\phi}(1 + a^2\phi^2e^{2\phi}) \tag{22}$$

Here $a = p_\Phi c/(GMm_0)$ is now a dimensionless conserved angular momentum parameter. Eq. 22 is similar to the energy equation of classical mechanics with the last terms at the right taking the role of an effective potential $U(r)$ for the radial motion; i.e. $U(r) = e^{2\phi}(1+a^2\phi^2e^{2\phi})$. Bound orbits can occur for suitably low energies. Circular orbits can occur for $p_r = 0$. Their radii can be located by setting the derivative of the effective potential with respect to ϕ to zero. Circular orbits occur for $dU/dr = 0$, for which we find (Robertson 1999)

$$a^2 = -e^{-2\phi}/(\phi + 2\phi^2) \tag{23}$$

with particle energies of

$$E = m_0c^2 \exp(\phi)\sqrt{\frac{1+\phi}{1+2\phi}} \tag{24}$$

Orbits are stable if the second derivatives are positive at turning points. They are unstable otherwise. There is an innermost (marginally) stable orbit that can be found by setting the first two derivatives of the effective potential with respect to ϕ to zero. This produces coupled equations which have the solution, $\phi = -(3 - \sqrt{5})/4 \approx$ -0.191 for which $a^2 = 12.4$. For this innermost marginally stable circular orbit, the particle energy is reduced below its rest mass value, m_0c^2 by 5.5% in spite of its motion at $\sim$ 35% of light speed. This is a result of the reduction of mass that occurs for a particle that is lower in the gravity field of the central mass for $\phi < 0$, in accord with Eq. 8.

In relativistic astrophysical accretion disks, such as those found in low mass x-ray binary systems, orbiting fluid particles lose energy due to friction while fluid viscosity transports angular momentum outward. This allows the fluid to spiral inward to a central compact star or black hole candidate. In the process, accreting plasma can become hot enough to allow energy to be radiated away as soft x-rays. Apparently 5.5% of the rest mass energy can be converted to radiation for particles that would reach an innermost marginally stable orbit at $r = 5.24GM/c^2$. [6] This provides luminosities far in excess of what can be obtained from nuclear interactions. Nuclear processes typically result in energy losses of less than about one percent. Note also that it would take an extremely compact object to exist inside the innermost marginally stable orbit. A mass as large as that of the sun would have a radius of only 7.8 km if it were all contained inside its innermost stable orbit. Thus it is not surprising that the x-ray binary systems have very compact neutron stars or black hole candidates as their central massive objects.

According to Eq. 24, it would appear to require infinite energy for a particle to exist in a circular orbit for $\phi = -1/2$. But there are simply no stable orbits with $\phi < -0.191$. As can be shown by working through the solutions for photon trajectories (take $m_0 = 0$ in Eq. 3), there is an unstable circular orbit for photons for $\phi = -1/2$ and $r = 2GM/c^2$. [7] For particles with nonzero rest mass, there are no stable elliptical orbits that pass inside the photon orbit.

2.6. Black hole candidates

In addition to accommodating relativisitic accretion disks, it should be noted that it might be possible for objects to become so compact that $\phi \ll -2$. In this circumstance, photons emitted at the surface would be extremely red shifted as observed distantly and very little, if any, luminosity would be observed for the object. A correct treatment of realistic compact objects of astrophysical interest will require consideration of their internal structures and trapped radiation fields, especially in cases involving an active collapse process. Nevertheless, the external luminosity differences between a very large redshift, z, and the $z = \infty$ of a black hole might be small and subtle. Other differences, such as the presence of magnetic fields, might betray the lack of an event horizon. Evidence has been presented (Robertson & Leiter 2002) for the existence of intrinsic magnetic moments in stellar mass black hole candidates.

It has been proposed that trapped radiation pressure is capable of slowing the rate of gravitational collapse of black hole candidates to such an extent that

[6] The innermost marginally stable orbit of the Schwarzschild metric of General Relativity has a radius of $6GM/c^2$ and the energy lost in an accretion disk would be 5.7% of the accreting particles rest mass energy. In the isotropic metric of General Relativity, the marginally stable orbit radius is $4.96GM/c^2$ and the accretion disk energy loss is 5.7%. General Relativity thus encompasses mass changes similar to those of Eqs. 8 and 24.

[7] It is meaningless happenstance that the photon orbit here has $\phi = -1/2$ which I corresponds to the event horizon in Schwarzschild coordinates. In isotropic coordinates for General Relativity, $g_{00} = (1 - GM/2c^2r)^2/(1 + GM/2c^2r)^2$ and the event horizon occurs for $r = GM/2c^2$.

it would take many Hubble times for a collapsing star to radiate its mass away. It has been shown to be possible (Robertson & Leiter 2003) to maintain an Eddington balance between gravity and radiation pressure even at high redshifts, however, it is not clear that this condition would always hold in active gravitational collapse. Eddington balanced objects of this kind have been designated as ECO (Mitra 2000, 2002, 2006) or MECO when strongly magnetic (Robertson & Leiter 2003, 2004, 2006). Additional observational evidence for magnetic moments in AGN has also been found (Schild, Leiter & Robertson 2006, 2008) and the MECO model has been extended to encompass active galactic nuclei. ECO and MECO models have been shown to be capable of accounting for the luminosity constraints that have been found for black hole candidates without delving very far into the details of their internal structures.

The previous models of ECO and MECO have been based on the Schwarzschild metric, which allows extreme redshifts of surface luminosity without requiring extreme compactness; i.e. $\phi_{GR} \sim -1/2, -2$. To achieve the same necessary redshifts in the exponential metric would require a much greater degree of compactness such that $\phi \sim -10, -20$ might be required. This is so far beyond the $\phi \sim -0.2$ of a neutron star that it is very clear that a MECO model based on an exponential metric would require a very exotic and extremely hot material such as a quark-gluon plasma at its core. Except for this requirement of an extremely exotic core material, the MECO model needs only minor revisions for compatibility with the exterior Yilmaz exponential metric (Robertson 2016).

3. The Field Equations of the Yilmaz Metric

Eq. 7 depends only upon the definition of a potential and the correctness of special relativity. The problem is that this expression for g_{00} is NOT part of the solution for the metric of any configuration of gravity sources within the context of General Relativity. The solutions for a central mass for the Einstein field equations begin to differ in terms of second order in GM/c^2r and eventually terminate in an event horizon that occurs before $r \to 0$. This occurrence of an event horizon is considered by many to be a mere coordinate singularity that can be removed by a transformation to different coordinates, but as noted above, there is a genuine divergence of a physical quantity, the norm of the acceleration vector of a freely falling particle, at an event horizon. This is an important matter that apparently cannot just be ignored in view of the firewall problem that it presents.

Eq. 7 can be incorporated into the solution of the Einstein field equations if they are modified by the addition of a second order source term to the right hand side of the Einstein field equations:

$$G_i^j = -(8\pi G/c^4)T_i^j \tag{25}$$

where G_i^j is the Einstein tensor (hopefully, not to be confused with the Newtonian gravitational constant of the right member) and T_i^j is mass-energy tensor of matter. Einstein intended that T_i^j include all sources of mass-energy except any energy of the gravitational field itself. As amended by Yilmaz, the field equations are

$$G_i^j = -(8\pi G/c^4)(T_i^j + t_i^j) \tag{26}$$

where t_i^j is the stress-energy density of the gravitational field. This clean separation of matter and field effects can be maintained as long as there are separate particle and fields, but the situation is much more complicated in a matter plus field continuum. This will be discussed later, but for now we imagine this separation of matter and field to be possible.

In the case of the field of a central particle mass centered at the origin of coordinates, the external g-field in the Newtonian gravity limit would be $\mathbf{g} = -c^2\nabla\phi$ and, by analogy with the energy of the electric field for an electric charge, the gravitational field energy density would be $t_0^0 = E_g = -g^2/(8\pi G)$. For $r > 0$, $T_0^0 = 0$. Note that t_0^0 is a negative source term as an anti-gravity character is needed to keep gravity from being self-generating (Lo 1995). The corresponding Yilmaz equation for G_0^0 for the space outside the central particle is

$$\begin{aligned} G_0^0 = e^{-2\phi}[(2/r^2)\partial_r(r^2\partial_r\phi) + (\partial_r\phi)^2] = \\ -(8\pi G/c^4)[0 - g^2/(8\pi G)] \end{aligned} \tag{27}$$

If the second degree terms are ignored for the moment, we would have

$$(1/r^2)\partial_r(r^2\partial_r\phi) = 0 \tag{28}$$

for which the solution is, of course, the Newtonian potential

$$\phi = -GM/c^2 r \tag{29}$$

If this be the case, we find that the field energy density would be given by $g^2/(8\pi G) = e^{-2\phi}c^4(\nabla\phi)^2/(8\pi G)$, which is exactly what is required to satisfy Eq. 27 exactly. Here ∇ is given by its metric form equivalent, $\nabla \to e^{\phi}\partial_r$. The metric is thus given exactly by Eq. 11. It should be noted that the other field equations for G_1^1, G_2^2 and G_3^3 have similar second degree terms that require the other components of t_i^j to be $t_2^2 = t_3^3 = t_0^0 = -t_1^1$ for a complete solution.

This addition of E_g is all that is required to make the metric of Eq 11 become a solution of the Einstein gravitational field equations. The failure to include it has led to the Hilbert modification of the original Schwarzschild (Schwarzschild 1916) solution with its gravitational time dilation singularity and event horizon at $r = GM/2c^2$, for an isotropic metric.[8] With $\phi = -GM/c^2r$, $g_{00} \to 0$ as $r \to 0$, but this is just an indicator of the inadequacy of a classical point particle model. With the simple addition of a gravitational field as an energy source, the Yilmaz theory eliminates event horizons. The lack of the negative t_0^0 is the reason that curvature collapses g_{00} to zero before $r \to 0$ is reached, thus generating the event horizon condition of the unmodified Einstein theory.

[8]But Schwarzschild's original solution had neither, see translation of original article by Antoci and Loinger, arxiv.org/abs/physics/9905030

4. Gravitational waves

As another example of how the inclusion of the field stress-energy terms can improve our understanding, consider a topic of considerable recent interest; viz, gravitational waves. We imagine these to be disturbances of spacetime that propagate essentially in a Minkowskian interstellar vacuum. In this case, the Einstein equations would be equivalent to those obtained by setting the Ricci tensor components equal to zero. Consider a metric that might represent gravitational waves propagating along a z-axis with small perturbations of opposite amplitudes in the x and y directions, i.e.;

$$ds^2 = c^2dt^2 - e^{4\phi}dx^2 - e^{-4\phi}dy^2 - dz^2 \tag{30}$$

Where $\phi = \phi(ct \pm z)$ represents a wavelike distortion of what would otherwise be Minkowskian spacetime propagating along the z-axis direction. Denoting partial derivatives with respect to z with primes and time derivatives with dots, the Ricci tensor components for the metric are:

$$\begin{gathered} R_{11} = -2e^{4\phi}(\ddot{\phi} - \phi'') \qquad R_{22} = 2e^{-4\phi}(\ddot{\phi} - \phi'') \\ R_{00} = 8(\dot{\phi})^2, \quad R_{33} = 8(\phi')^2, \quad R_{30} = -8(\dot{\phi})(\phi') \end{gathered} \tag{31}$$

If set equal to zero, R_{11} and R_{22} obviously yield the wave equation for waves of arbitrary amplitude propagating along the z-axis. Unfortunately, these waves are NOT solutions of the Einstein field equations because none of R_{00}, R_{30} or R_{33} are exactly zero, as required by Einstein. In fact R_{00}, R_{30} and R_{33} look suspiciously like terms that represent the energy that would propagate with ordinary waves in space[9]. If we insist that $R_{ik} = 0$ because we are in matter free space, we are effectively saying that gravitational waves of the kind considered here are not solutions of the Einstein field equations. They would be solutions only for infinitesimal amplitudes and first order. It would make a lot more sense to say that there are energy densities associated with these waves that should be included as source terms in the right members of the Einstein field equations. Given the form of the metric of Eq. 9, and recognizing that the function, ϕ, is a time dependent gravitation potential, it should be apparent that the squared derivatives of ϕ might have something to do with the energy that propagates with the gravitational waves. All five equations can be satisfied in this way without restricting the waves to be of infinitesimal amplitude. This is another case in which we can gain consistency by regarding gravity fields as real entities that can contribute as sources for the Einstein tensor.

5. Spherically symmetric mass distributions

Some additional topics of interest for relativistic astrophysics concern the interior structures of neutron stars and black hole candidates and the metric of

[9]The Einstein equations, in the form $G_i^j = -(8\pi G/c^4)T_i^j$ can be rewritten as $R_{ik} = -(8\pi G/c^4)(T_{ik} - g_{ik}T/2)$, where T_{ik} is the stress-energy tensor that represents matter and/or all other energy densities and T is its trace.

cosmology. For stars and black hole candidates, the matter sources of the field equations might be regarded as continuous. The cosmological metric that will be discussed below will be used to describe a cosmos consisting of a fairly uniform distribution of discrete galaxy "dust" particles. In each of these circumstances, the field equations for a completely spherically symmetric mass distribution will be those obtained from Eq.s B.1 - B.4 of Appendix B. For the metric form of Eq. 1, Eqs. B.4, B.5 and B.6 yield:

$$t^0_0 = -t^1_1 = (c^4/8\pi G)[e^{-\nu}3\dot{\lambda}^2/4 + e^{-\lambda}(\lambda'\nu'/2 + \lambda'^2/4)] \tag{32}$$

and

$$t^2_2 = t^3_3 = (c^4/8\pi G)[-e^{-\nu}3\dot{\lambda}^2/4 + e^{-\lambda}(\lambda'\nu'/2 + \lambda'^2/4)]. \tag{33}$$

Here dots represent partial derivatives with respect to time, and primes represent partial derivatives with respect to the radial coordinate, r. With these terms for the right member of Eq. B.1, the field equations become

$$G^0_0 \rightarrow e^{-\lambda}[(1/r^2)\partial_r(r^2\partial_r\lambda) + \lambda'(\lambda' + \nu')/2] = -(8\pi G/c^4)T^0_0 \tag{34}$$

$$G^1_1 \rightarrow -e^{-\nu}(\ddot{\lambda} + \dot{\lambda}(3\dot{\lambda} - \dot{\nu})/2) + e^{-\lambda}(\lambda' + \nu')/r = -(8\pi G/c^4)T^1_1 \tag{35}$$

$$\begin{aligned} G^2_2 \rightarrow \quad & - e^{-\nu}(\ddot{\lambda} + \dot{\lambda}(3\dot{\lambda} - \dot{\nu})/2) + (1/2)e^{-\lambda}[\lambda'' + \nu'' \\ & +(\lambda' + \nu')/r + (\lambda' + \nu')^2/2] = -(8\pi G/c^4)T^2_2 \end{aligned} \tag{36}$$

The G^3_3 equation repeats the one for G^2_2 with $T^3_3 = T^2_2$.

6. Cosmological Red Shifts

In this section we will obtain a relation between redshift and luminosity distance for SNe 1a for a model universe consisting of an expanding, isotropic, spherically symmetric cosmic dust comprised of galaxy sized dust particles. We will use the metric form of Eq. 1 as applied by an observer located at $r = 0$ at the present time, $t = 0$. There is no "universal" time for the universe in this approach, only the local time of an observer at the origin of coordinates. But there is time dependence that excludes the use of the metric Eq. 11. For these present conditions, $\lambda = \nu = 0$ at the observer's location and the observer's local spacetime is Minkowskian. Photons emitted at earlier (negative) times and at large distances, r, will be detected as redshifted by $1 + z = e^{-\nu/2}$. Photons emitted at earlier times into a particular solid angle will be spread over a larger area as the universe expands while they are in transit. As a result, they will appear to have come from a more distant source[10]. The luminosity distance will be enlarged and given by $d_L = (1+z)r$. The approach taken here was motivated by that of Mizobuchi (1985) but differs significantly in details.

[10]In conventional FRW cosmology, the measured photon flux is diluted by two factors of $(1 + z)$. The individual photons redshift by a factor (1 + z), and the photons hit the detector less frequently, due to time dilation. In the present approach, one of these factors is taken into account in $\nu \neq 0$.

For this model universe, only T_0^0 is nonzero. $T_0^0 = \sigma u_0 u^0 = \sigma c^2$, where σ is the average mass density of "dust particles" in the universe. These equations can be considerably simplified by application of the harmonic coordinate condition $\partial_j(\sqrt{-g}\; g^{jk}) = 0$. As noted previously, this condition assures that the speed of light be the same in all directions in space. This provides two relations

$$\partial_0 e^{(3\lambda-\nu)/2} = 0 \tag{37}$$

$$\partial_r e^{(\lambda+\nu)/2} = 0 \tag{38}$$

Defining two new functions

$$\phi_0 = (3\lambda - \nu)/8, \quad \phi_1 = (\lambda + \nu)/8 \tag{39}$$

it is apparent from Eqs. 37 & 38 that we must have $\phi_0 = \phi_0(r)$, independent of time, and $\phi_1 = \phi_1(t)$, must be independent of r. In terms of these functions,

$$\lambda = 2\phi_0 + 2\phi_1 \qquad \nu = 6\phi_1 - 2\phi_0 \tag{40}$$

Equations 34 - 36 can then be rewritten as

$$(1/r^2)\partial_r(r^2\partial_r\phi_0) = -(4\pi G/c^2)\sigma e^{(2\phi_0+2\phi_1)} \tag{41}$$

and

$$\ddot{\phi}_1 = 0. \tag{42}$$

For a consistent set of equations with ϕ_0 independent of time, we must choose a condition that allows σ to vary with time in a way that removes the time dependence of the right member of Eq. 41. From Eq. 42 we see that ϕ_1 will vary linearly with time. By choosing our observation point to be located at $r = 0$, $t = 0$, where the present value of mass-energy density would be σ_0, we can take

$$\sigma = \sigma_0 e^{-2\phi_1} \tag{43}$$

where σ_0 is a constant. Taking $\phi_1 = C_1 t$, with C_1 an integration constant, satisfies Eq. 42 and provides for a cosmos with a matter density that decreases with time; i.e., an expanding universe. Substituting for σ in Eq.41, and defining

$$R_0 = \sqrt{c^2/(4\pi G\sigma_0)}, \quad x = r/R_0, \quad T = ct/R_0 \tag{44}$$

Eq. 41 becomes

$$d^2\phi_0/dx^2 + (2/x)d\phi_0/dx = -e^{2\phi_0} \tag{45}$$

A low order solution can be obtained by expanding the exponential function of the right member. Assuming that $\phi_0 = \Sigma a_n x^{n+2}$, we find,

$$\phi_0 = -x^2/6 + x^4/60 - x^6/687 + x^8/3565\ \ldots \tag{46}$$

This fits well to $x \sim 1$, but numerical solutions are needed for larger values of x. The solutions of Eqs. 42 and 45 allow us to determine the metric functions

$$\lambda = 2C_1 T + 2\phi_0 \tag{47}$$

$$\nu = 6C_1T - 2\phi_0 \tag{48}$$

C_1 is proportional to the Hubble constant as will be seen by considering the gravitational red shift that would be expected for a photon emitted at some previous time and detected now at our location $x = 0,\ T = 0$. A null geodesic photon path taken from r to zero and time T in the past to the present will have $ds^2 = 0$. Thus, $e^{\nu/2}cdt = -e^{\lambda/2}dr$, where the negative sign is taken because r decreases as t increases from the time of emission to our detecting it at the present time. Substituting the solutions for λ and ν into this last relation and rearranging, we obtain

$$\int_T^0 e^{2C_1T}dT = (1 - e^{2C_1T})/2C_1 = -\int_x^0 e^{2\phi_0}dx \tag{49}$$

For large values of x, the integral on the right must be evaluated numerically, but it is instructive to first consider the expansions to lowest orders, for which we obtain

$$T + C_1T^2 = -x + x^3/9 \tag{50}$$

To lowest order, we have $T = -x$, or $t = -r/c$, as expected. The redshift of a photon, to lowest order, would be

$$z = e^{-\nu/2} - 1 \approx 3C_1x = 3C_1\sqrt{4\pi G\sigma_0}\ r/c \tag{51}$$

it is now apparent that the Hubble constant, H_0, is given by

$$H_0 = 3C_1\sqrt{4\pi G\sigma_0} \tag{52}$$

By numerically solving Eq. 45 for ϕ_0 and integrating numerically, the integral on the right side of Eq. 49 is found to have the limiting value of -2.1405 for very large x. For the corresponding time, $T \to -\infty$, we find from Eq. 49 that $1/(2C_1) = 2.1405$, or $C_1 = 0.2336$. This is a self-consistent choice for C_1 that leaves only one free parameter, σ_0, to be chosen to fit the redshift-luminosity data. With the value of C_1 now determined, the appropriate time, T, for any x, can be found from

$$T = [1/(2C_1)]\ln[1 - 2C_1\int_0^x e^{2\phi_0}dx] \tag{53}$$

Once T is calculated for given x, the values of $\phi_1,\ \nu,\ \lambda,\ z$ and $d_L = r(1+z)$ can be computed and tablulated for each x. Numerical solution data for $C_1 = 0.2336$ and $\sigma_0 = 1.06 \times 10^{-29} g\ cm^{-3}$ are given in Table 1.

This choice for σ_0 was based on a Hubble constant obtained by a least squares fit to 166 data points for $z \leq 0.1$ from the data of the Supernova Cosmology Project (Amanullah et al 2010, Suzuki et al. 2012). This fit yielded $H_0 = 64.5 \pm 0.7\ km\ s^{-1}\ Mpc^{-1}$. Eq. 52 was then used to calculate $\sigma_0 = 1.06 \times 10^{-29} g\ cm^{-3}$ This choice for σ provides a very good fit to the supernova redshift data over the whole range of observed redshifts as shown in Figure 1.

Table 1: Parameters of redshift and distance calculations

x	$\phi_0(x)$	$\int_0^x e^{2\phi_0} dx$	T	z	[a] $d_L(Mpc)$)
$C_1 = 0.2336$					
0.000	0.000	0.000	0.000	0.000	0.000
0.100	-0.002	0.100	-0.102	0.073	350
0.200	-0.007	0.199	-0.209	0.150	750
0.300	-0.015	0.297	-0.320	0.233	1206
0.400	-0.026	0.393	-0.434	0.321	1722
0.500	-0.041	0.487	-0.552	0.414	2304
0.600	-0.058	0.577	-0.673	0.512	2958
0.700	-0.078	0.665	-0.796	0.616	3687
0.800	-0.100	0.748	-0.921	0.725	4498
0.900	-0.125	0.828	-1.047	0.838	5394
1.000	-0.152	0.904	-1.175	0.957	6380
1.100	-0.180	0.976	-1.303	1.080	7459
1.200	-0.211	1.043	-1.431	1.208	8637
1.300	-0.242	1.107	-1.559	1.339	9915
1.400	-0.275	1.167	-1.686	1.475	11296
1.500	-0.309	1.222	-1.812	1.614	12783
1.600	-0.344	1.275	-1.937	1.756	14377
1.700	-0.379	1.323	-2.061	1.901	16079
1.800	-0.415	1.368	-2.182	2.049	17890
1.900	-0.451	1.410	-2.302	2.198	19810
2.000	-0.487	1.450	-2.420	2.350	21840
3.000	-0.845	1.718	-3.473	3.896	47880
4.000	-1.164	1.853	-4.299	5.353	82840
5.000	-1.434	1.928	-4.945	6.625	124280
6.000	-1.662	1.973	-5.459	7.704	170250
7.000	-1.856	2.003	-5.878	8.618	219480
8.000	-2.023	2.024	-6.228	9.399	271200
9.000	-2.168	2.039	-6.525	10.075	324900
10.000	-2.296	2.050	-6.783	10.670	380400

[a] These values calculated for $\sigma_0 = 1.06 \times 10^{-29}\ g\ cm^{-3}$ are shown by the solid line on Fig. 1.

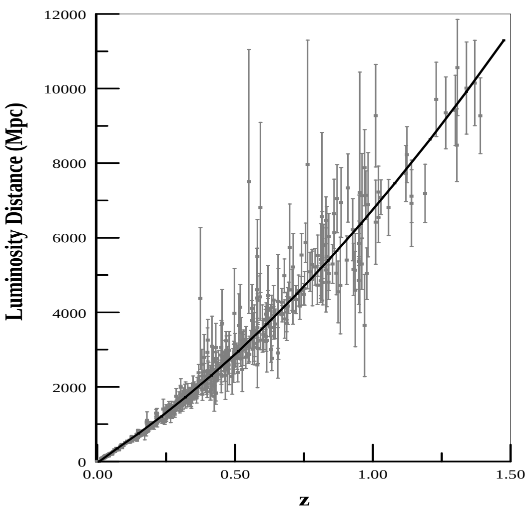

Figure 1: **Figure 1.**, Luminosity distance vs redshift for SNe 1a. Data from The Supernova Cosmology Project, (Amanullah et al 2010, Suzuki et al. 2012). The fitted curve is determined by only one free parameter, $\sigma_0 = 1.06 \times 10^{-29} g\ cm^{-3}$ that was obtained from the Hubble Constant fitted for $z \leq 0.1$ (see text).

The value of σ_0 is very close to the critical density that would be obtained for a Friedmann-Robertson-Walker (FRW) metric and the same Hubble constant.

Accommodating the data for the large values of z shown in Fig. 1 within the framework of General Relativity requires the introduction of Einstein's cosmological constant with non-zero value. It represents the apparent existence of a "dark energy" that is supposedly driving an acceleration of the expansion of the universe. In addition, the existence of a substantial amount of non-baryonic "dark matter" seems to be required. These ideas have been quantified in terms of contributions to the critical density required for a universe with an average locally flat space-time. Letting Ω_m be the fraction of the critical density contributed by matter, including both baryonic and dark matter, and Ω_Λ the fraction contributed by "dark energy", and assuming that $\Omega_m + \Omega_\Lambda = 1$, the "concordance values" found for these parameters are $\Omega_\Lambda = 0.729$ and $\Omega_m = 0.271$ (Suzuki et al. 2012).

It is difficult to see how any theoretical curve fitted to the hard won, but noisy, observational data shown in Fig. 1 might be used to draw any sweeping conclusions about the composition of the universe, yet that is exactly what has been done. A theoretical relation that fits the high redshift data of Fig. 1 no better than the line on Fig. 1 has been used as the basis for claiming that the stuff of the universe is 72.9% dark energy. The baryonic matter contribution to Ω_m is believed to be no larger than about 0.05 - 0.10, with the visible part of it being only about one fifth of this. The non-baryonic contribution to Ω_m is the dominant part.

The requirement for $\Omega_m + \Omega_\Lambda = 1$ arose from the wish to have a flat space-time within the context of General Relativity and a FRW metric. In the Yilmaz theory, the values of the metric coefficients are determined locally. The potentials can be set to zero as boundary conditions at the location of an observer and in the absence of gravitating bodies, they will remain zero in the surrounding space and the space-time would be locally Minkowskian. Time is returned to its special relativistic status of being relative to the observer. There is no universal time here. No particular requirement is imposed on the density of mass-energy that would necessitate the existence of non-baryonic dark matter. The Yilmaz theory is indifferent to its existence. If it exists, then it must contribute to the mass-energy density, but finding it is an empirical matter rather than an exercise in faith in a gravity theory.

Three other attempts have been made to apply the Yilmaz theory to cosmology. Yilmaz (1958) developed a metric for a static universe. Increasing evidence of the inadequacy of this approach led him to the extensions in his 1971 theory. Mizobuchi (1985) applied the 1971 theory to a cosmological model consisting of a perfect fluid. This was not a central point of an otherwise very informative article, but it appears to have been based on the erroneous inclusion of a factor of $\sqrt{-g}$ in T^ν_μ, where $(-g)$ is the determinant of the metric. T^ν_μ should have been taken to be just the diagonal matter tensor of a perfect fluid, $T^\nu_\mu \to (\rho c^2, -P, -P, -P)$. The approach taken here was motivated by that of Mizobuchi (1985) but correcting the error leads to significantly different results.

A third attempt to apply the Yilmaz theory to cosmology was provided by

Ibison (2006). Ibison assumed the correctness of the flat-space FRW metric $ds^2 = dt^2 - a(t)^2(dr^2 = r^2 d\theta^2 + r^2 sin^2\theta d\Phi^2)$ and found a coordinate transformation to the form of Eq. 1. This transformation was shown to satisfy the harmonic coordinate condition, but only at the expense of leaving λ and ν dependent only on time with no position dependence. Ibison's transformation, $dt = a(\zeta)^3 d\zeta$, produces a result that is equivalent to setting $\phi_0 = 0$ and is incapable of fitting the the SNe 1a redshift data. Fig. 1 shows that the redshift data is nicely encompassed by the Yilmaz theory and the metric of Eq. 1.

The Cosmological Principle asserts that the universe is spatially homogenous and isotropic, but it does not demand strict adherence to the FRW metric. The FRW metric mathematically ensures a translational invariance that would leave the universe with the same appearance to all observers at the same "cosmic time", but that is not the only way to obtain consistency with the principle. Form invariance of Eq. 1, the requirement that $\phi_\alpha^\beta = 0$ hold at the observer's location and the requirement that $\rho_0 = constant$ satisfy the requirements. In this case, however, a "cosmic time" would have no meaning.

It should be possible to extend this discussion of cosmology back to the early universe, but that would necessitate the inclusion of the internal pressures of both matter and radiation. Eqs. 34, 35 and 36 become much more complex when pressures are not negligible. It might possibly require the abandonment of the harmonic coordinate condition. Nevertheless, it would seem to be preferable to approach the study of the early universe in the same way, from the standpoint of an observer at the origin in a locally Minkoswkian spacetime. This would permit the existence of gravitational redshifts of photons just as has been found here for the Yilmaz theory. In contrast, the FRW metric with $g_{00} \equiv 0$ completely precludes the possibility of occurrence of the gravitational redshift. In addition, no cosmological translational invariance has been imposed by the Yilmaz theory. We may hope that we are not privileged observers in some way, but we do not impose that as a mathematical requirement. It is left to be verified by other means if possible.

As will be seen in the discussion of neutron stars, the right member of the field equations for a perfect fluid is $(T_i^j + t_i^j) = (\rho c^2, -p, -p, -p)$, where ρc^2 and p are symbolic place holders for both mass-energy densities and material and gravitational stress-energy contributions. It might be possible to decompose these such that $\rho c^2 = \sigma c^2 + t_0^0$ and $p = p_k + t_i^i$ for $i = 1, 2, 3$, where p_k is the kinetic contribution to pressure and σ excludes the field energy density within the space occupied by a fluid particle. Then in Eqs. 34, 35 and 36, we could take $T_0^0 = \sigma c^2$, and for other components, $T_i^i = -p_k$. It then might be possible to retain the use of the harmonic coordinate condition. If so, this would lead to equations for ϕ_0 and ϕ_1 that are fairly simple in appearance. Eqs 35 and 36, would be identical and it would be readily apparent that the effect of the kinetic pressure would be to cause an acceleration in the rate of expansion; i.e., $\ddot{\phi}_1 > 0$. But this remains as a research agenda for someone else.

7. Neutron Stars

Although neutron stars are very compact objects, with typically $1.4 M_\odot$ and ~ 10 km radius, their surface gravitational potentials are only $GM/c^2 r \sim 0.21$. Gravitational field energy density terms of second order in potentials this small are not of great importance. The limit on neutron star mass is set by the support that can be provided by neutron degeneracy pressure subject also to a limit that it should not exceed the pressure of a fully relativistic gas with $p \sim \rho c^2/3$ or a causality limit with sound speed less than light speed with $\partial p/\partial \rho < c^2$.

Pressure is a non-gravitational contact force between fluid elements, yet in the case of a neutron star, the presence of pressure depends entirely on the existence of the gravitational field. It is not clear just how to separate pressure from a gravitational energy density and this has been the source of some confusion. In a 1995 arXiv paper with the title "Yilmaz Cancels Newton" (Misner 1995, 1999), Misner claimed that the Yilmaz theory could not correctly include pressure. That was immediately followed in the same journal issue by a similarly delayed refutation by Yilmaz and Alley (Yilmaz & Alley 1999). Misner made an error that will be obvious here, but Yilmaz and Alley did not not clearly expose it.

In the time independent equations that should be applicable to a neutron star, the Yilmaz equations for the metric form, Eq. 1, differ from Eqs. 34 - 36 by the omission of time derivatives. In this case, neither T_1^1 nor T_2^2 are zero.

$$G_0^0 \to \; e^{-\lambda}[(1/r^2)\partial_r(r^2\partial_r\lambda) + \lambda'(\lambda'+\nu')/2] = -(8\pi G/c^4)T_0^0 \tag{54}$$

$$G_1^1 \to \quad e^{-\lambda}(\lambda'+\nu')/r = -(8\pi G/c^4)T_1^1 \tag{55}$$

$$\begin{aligned} G_2^2 \to \quad & (1/2)e^{-\lambda}[\lambda'' + \nu'' + (\lambda'+\nu')/r \\ & +(\lambda'+\nu')^2/2] = -(8\pi G/c^4)T_2^2 \end{aligned} \tag{56}$$

For this spherically symmetric case, $T_3^3 = T_2^2$ and the G_3^3 equation again repeats the one for G_2^2.

Misner assumed that $T_i^j = [(\rho + p/c^2)u_i u^j - p\delta_i^j]$, where p represents pressure and u_i is the four-velocity of a fluid parcel. Here ρ is mass-energy density[11] and in this static case Misner would have $T_1^1 = T_2^2 = T_3^3 = -p$. But his entire argument was also based on the assumption that $\nu = -\lambda$, as shown clearly in his Eq. 3.1. This leads immediately to $p \equiv 0$ in Eqs. 55 & 56, which is essentially why he stated that Yilmaz cancelled Newton. Misner's entire analysis based on $\nu = -\lambda$ is simply wrong. It requires both independent metric coefficients ν and λ to correctly describe the interior of a gravitating object.

But there is more to consider here. First, if $\nu \neq -\lambda$, the harmonic coordinate condition cannot apply[12]. Next, as shown by Yilmaz and Alley, Misner's

[11] ρ necessarily includes the gravitational field energy density in the space occupied by a fluid particle, whereas σ does not: $\rho c^2 = \sigma c^2 + t_0^0$.

[12] The failure of the harmonic coordinate condition led Robertson (1999) to abandon the field equations and instead to calculate the metric components from a superposition of potentials

assumption that $T_i^j = [(\rho + p/c^2)u_i u^j - p\delta_i^j]$ is also wrong. The reason for this is that the Yilmaz theory requires that T_i^j must satisfy the Freud identity requirement $\bar{\partial}_j(\sqrt{-K}\ T_i^j) = 0$ (appendix B), but Misner's assumed expression does not.[13]

On the other hand, since the covariant derivative of G_i^j vanishes, it is apparent from Eq. B.1 that the Yilmaz theory must require that $(T_i^j + t_i^j); j = 0$. This is known as the Bianchi requirement. Since Misner's assumed expression for T_i^j does satisfy this covariant derivative requirement, Alley and Yilmaz proposed that it is $(T_i^j + t_i^j)$ that is given by $[(\rho + p/c^2)u_i u^j - p\delta_i^j]$. They went on to show that this is the case through second order terms. If this holds generally, it is necessary that T_i^j be given by $T_i^j = [(\rho + p/c^2)u_i u^j - p\delta_i^j - t_i^j]$ and it is still necessary for T_i^j to satisfy the Freud requirement. Straightforward but very tedious and lengthy tensor algebra shows that it does.

The end result is that the symbol ρ in the right member of the field equations represents both the mass-energy density and the energy density of the gravitational field in the space occupied by a fluid element. Similarly, the symbol p, represents not only mechanical pressure, but also a stress-energy component of the gravitational field. ρ and p are symbolic place holders for all forms of energies, including those of the gravitational field. It is only in the absence of matter terms that it is necessary to include a term such as $g^2/(8\pi G)$ to represent the field energy density in the right member of the field equations.

Substituting $T_i^j = [(\rho + p/c^2)u_i u^j - p\delta_i^j - t_i^j]$ into Eqs. 34 - 36 produces field equations for the interior of a neutron star that are, of course, exactly the same as those of conventional general relativity. These are:

$$e^{-\lambda}[(1/r^2)\partial_r(r^2\partial_r\lambda) + \lambda'^2/4] = -8\pi G\rho/c^2 \tag{57}$$

$$e^{-\lambda}[(\lambda' + \nu')/r + \lambda'\nu'/2 + \lambda'^2/4] = (8\pi G/c^4)p \tag{58}$$

$$\begin{gathered}(1/2)e^{-\lambda}[\lambda'' + \nu'' + (\lambda' + \nu')/r \\ +(\lambda' + \nu')^2/2 + \nu'^2/2] = (8\pi G/c^4)p\end{gathered} \tag{59}$$

$$p' = -\nu'(p + \rho c^2)/2 \tag{60}$$

Eq. 60 can be deduced from the two preceding ones, but it also results from both the Freud and Bianchi identity requirements. The fact that the Freud identity is satisfied shows that the set of equations rightfully belongs to the Yilmaz theory. Further, $T_i^j = (\rho + p/c^2)u_i u^j - p\delta_i^j$ cannot belong to the Einstein theory, for if it did, t_i^j would not be contained within it.

The way that the Yilmaz solution of these equations differs from that of conventional general relativity consists of the boundary conditions that apply.

of shells of matter inside the star. The results erroneously suggested that neutron stars might have masses as large as $10M_\odot$ because no pressure limits were imposed.

[13]In spherical coordinates, it is necessary to take the derivatives of the Freud requirement as covariant derivatives relative to the local Minkowskian background to avoid problems with pseudotensors. (Yilmaz 1992), see Appendix B.

Table 2: Calculated Neutron Star Properties

Metric	$^a\rho(0)$	$^bP(0)$	$p(0)$	$\phi(R)$	^cz_s	$R(km)$	$M(M_\odot)$
Yilmaz	12.1	20.90	0.192	0.269	0.308	7.70	1.40
	15.9	47.7	1/3	0.394	0.483	6.79	1.81
Eq. C.1-C.4	const 2.8	8.4	1/3	0.417	0.516	8.33	2.34
GR-Iso	12.1	20.90	0.192	0.264	0.304	7.86	1.40
	15.9	47.7	1/3	0.380	0.469	7.11	1.82
	const		1/3	0.400	0.500		
GR-Sch	12.1	20.90	0.192	0.206	0.304	10.07	1.40
	15.9	47.7	1/3	0.268	0.469	10.07	1.82

$^a10^{14}$ g cm^{-3} $^b10^{34}$ erg cm^{-3} ^{c} surface red shift

The Yilmaz theory requires a match to the metric of Eq. 11 at the star surface rather than the metric of Eqs. 13 & 14. Since these metrics first show a minor difference in second order terms one would not expect major differences and as shown in Table 2, this is the case. Thus the previous conclusions that have been reached concerning maximum neutron star mass are essentially unaffected. A maximum mass of about $3M_\odot$ (Rhoades & Ruffini 1974, Kalogera & Baym 1996) is still a firm limit.

Results of numerical solutions of the field equations (57- 60) for Yilmaz boundary conditions and a realistic equation of state (Wiringa, Fiks & Fabrocini 1988) are shown in Table 2. The details of the numerical solution are given below. A maximum neutron star mass of about $1.8M_\odot$ for a limiting relativistic core pressure of $p = \rho c^2/3$ is shown there. $p(0) = (1/3)\rho c^2$ should correspond to a maximum pressure because the core would be fully relativistic under these conditions and no longer cool enough to permit the neglect of radiation.

At the outset of numerical solutions of Eqs. 57 - 60, the values of λ and ν are not known at either the surface or the center of the star. Nevertheless, we can avoid having to do trial and error solutions starting from some guess of either of these at the surface by defining new variables as $y = \lambda - \lambda(0)$, $w = \nu - \nu(0)$, a new radial coordinate as $x = r\sqrt{4\pi G\rho(0)/(c^2 e^{\lambda(0)})}$, a dimensionless pressure, $P = p/(\rho(0)c^2)$, and dimensionless density, $z = \rho/\rho(0)$. With primes now denoting derivatives with respect to x, Eqs. 57, 58 & 60 become

$$(1/x^2)\partial_x(x^2\partial_x y) + y'^2/4 = -2ze^y \tag{61}$$

$$w'/x + y'/x + y'w'/2 + y'^2/4 = 2Pe^y \tag{62}$$

$$P' = -w'(P + z)/2 \tag{63}$$

Initial conditions for the solution of these equations are $y = y' = 0$, $w = w' = 0$, $z = 1$ at x=0. At the outer surface of the star at $r = R \rightarrow\ x = X$, the boundary conditions are $P = 0$, which serves to determine X, $\nu = -Xw'$ and $\lambda = -\nu$. The condition $\nu = -Xw'$ follows from matching the external exponential metric and its derivative[14] for which $\nu = -2GM/(c^2 r) = -x\nu'$.

What is needed for the solution of these equations is a neutron equation of state and an initial trial central pressure $P(0)$. For the latter, it will be assumed here that $P(0) = 1/3$ corresponds to the maximum realistic pressure because the core would be fully relativistic under these conditions and no longer cool enough to permit the neglect of radiation. Trial and error was used to determine a pressure that would correspond to a canonical $1.4M_\odot$ neutron star.

The AV14+UVII neutron model equation of state of Wiringa, Fiks, and Fabrocini was used here (Wiringa, Fiks & Fabrocini 1988). Densities from their Table VI were fitted to a quartic polynomial in $p^{1/3}$. The quartic polynomial for neutron densities was of the form $\rho = \Sigma_n a_n (p^{1/3})^n$. The coefficients are: a_0= -0.9167367, a_1=6.960282, a_2=-1.462936, a_3=0.267646, a_4=-0.0112676. Fitting errors were below one percent over the range of densities used in these calculations.

After selection of an initial central density and corresponding pressure, the solution proceeds in steps outward with corresponding decrements of pressure until reaching P=0. As a check on the validity of using the polynomial, the field equations of the Schwarzschild metric were solved numerically in the same stepwise fashion described above. Stellar radii and masses agreed to better than one percent with those shown in Table VI of Wiringa, Fiks & Fabrocini. Since their values were calculated via the Tolman, Oppenheimer, Volkoff equation which was derived for the Schwarzschild metric, this should be expected; however, a failure to agree would have indicated a problem with the numerical methods used here.

For the limited purpose of comparing results for different theories, only a few results have been selected. It is of obvious interest to examine the properties of the canonical 1.4 $M_\odot$ neutron star. A second case of interest occurs for a core pressure of $p(0) = \rho(0)c^2/3$, which is a strongly relativistic case, though it does not reach the "causality limit" for which $\partial P/\partial\rho \to c^2$. Results of these calculations are shown in Table 2.

The tabled values for 1.4 $M_\odot$ show that there is good agreement on surface red shift for all of the metrics used. Radii for the isotropic metrics of Yilmaz and General Relativity are also in good agreement, but quite different from the one obtained for the Schwarschild metric. This merely reflects the fact that the radii are coordinate quantities. The proper length radius is

$$R_p = \int_0^R \sqrt{-g_{rr}}\; dr \tag{64}$$

For the 1.4 $M_\odot$ stars, the proper radii are $R_p = 11.4$ km for all of these metrics. All things considered, the Yilmaz theory calculations produced very realistic

[14]Although λ , ν and ν' are continuous at $r = R$, λ' is not. Continuity of ν' is necessary in order to obtain the correct initial pressure gradient just inside the outer surface of the star.

neutron star properties and, as expected considering the small metric differences, very little change from the isotropic metric of General Relativity for a realistic neutron equation of state. Thus the Yilmaz theory seems quite capable of handling the physics of compact objects.

Eqs. 57 - 60 have been solved analytically (Buchdahl 1982)[15] for an object that would have $\rho = constant$. While this is wildly physically unrealistic for anything but very small planets, it is of some interest because it shows that no event horizon conditions are generated in the interior solutions when matched to an exterior Yilmaz exponential metric. Another reason for interest is that the results are not very different from those of more rigorous calculations if a nuclear density is used for ρ. Buchdahl's solution modified by use of Eq. 11 as a boundary condition is shown in Appendix C.

8. Summary and Conclusions

In Eq. 7 we found a result that is required by special relativity, but incompatible with the known solutions of the Einstein field equations for static fixed mass gravitational sources. By considering gravitational waves, it could be easily seen that the issue is a conceptional problem: Einstein's wish to exclude gravitational field energy densities as sources in the right members of the Einstein field equation appears to be both unwarranted and unfulfilled. Gravitational waves of arbitrary amplitude can occur if field energy terms of second order are permitted.

It was also shown that the addition of the energy density of the gravitational field of a central mass as a source term in the field equations permitted a Newtonian potential to survive as a solution of the field equations. This restored compatibility of the solution with Eq. 7 and had the effect of eliminating the event horizon from the solution for a point mass. This still leaves a point mass singularity as $r \to 0$, for which $g_{00} \to 0$, but this is just a problem with the concept of a point mass and not a pathological feature of the metric as there is no curvature singularity there. In contrast, in the well-known solution of the unmodified Einstein field equations, g_{00} vanishes at the event horizon before we reach $r = 0$.

In this regard, we can see that the added gravitational field energy density had a negative curvature effect on the spacetime metric. If sources of positive mass-energy densities produce positive curvature of spacetime, gravitational field stress-energy densities inherently produce the opposite effect on curvature. Without this negative effect (Lo 1995), spacetime becomes too warped too soon as we approach the point particle and an event horizon forms for $r > 0$.

A similar effect occurs in the case of a cosmological metric. The presently accepted cosmology uses a two parameter combination of the mass-energy of the cosmos and "dark energy"; Einstein's exclusion notwithstanding, to account for the redshift - luminosity distance relation of distant type 1a supernovae. Dark

[15] Buchdahl's field equations require minor corrections

energy represented by a cosmological constant has the effect of partially countering the gravitational attraction of the "cosmic dust" particles and allowing the model cosmos to expand at an accelerating rate at late times.[16] It is clear from the results presented here that the supernovae data can be accommodated very well by the inclusion of the gravitational field stress-energy tensor source t_i^j and only one free parameter, the present mass-energy density of "cosmic dust" in our vicinity. Event horizons and dark energy are simply not necessary for a correct accounting for any astrophysical observations.

Removing event horizons from the astrophysical menagerie does, however, leave a need for a new understanding of the nature of the gravitationally collapsed and compact objects that are presently thought by many to be black holes. Models that require the use of unmodified Schwarzschild or Kerr metrics must make way for models that have no event horizons. Since the objects presently considered to be black holes are too massive and compact to be supported by neutron degeneracy pressure, they most likely would collapse to a size that can be supported by internal radiation pressure (Mitra 2006). They might well become quark-gluon plasmas. At the same time, their surface emissions would occur with such extreme redshifts that their distantly observed luminosity would be quite low. In this regard, the ECO (e.g., Mitra 2000-2006) or MECO (e.g., Robertson & Leiter 2002-2006) models, which only need large gravitational redshifts and/or intrinsic magnetic fields to function can likely be encompassed within the Yilmaz theory (Robertson 2016). This remains to be worked out for spacetimes dominated by electromagnetic radiation fields.

The criticism of the Yilmaz theory by Misner has been shown here to be incorrect, but it raised a subtle point: Symbols representing mass or energy densities in the right members of the Einstein field equations will also represent the gravitational field energy densities unless these are separately specified in explicit expressions for t_i^j. This had the effect of leaving the field equations for neutron stars unchanged from those of the Einstein theory. Nevertheless, the nature of the solutions is affected by new boundary conditions. The metric must match the static exterior Yilmaz metric at the star surface.

The clear implication of these results is that permitting gravitational field energy to serve as a source in the Yilmaz field equations allows the solutions to be valid to more than first order in the potentials. The new theory still passes all of the observational tests that have been devised for relativistic gravity theories. Whether it will remain correct when higher order tests can be devised remains to be seen. In the meantime, the clear message that needs to be understood here is that gravity is a field in its own right. It is NOT merely and entirely and only an effect of the geometry of spacetime. Alley (1995) has opined that the inclusion of t_i^j (as given by Eqs. B.4-B.6) as an additional source term in the Einstein field equations is as important for our understanding of gravitational fields as Maxwell's addition of displacement currents was for the understanding

[16]It is hard to believe that the noisy data of supernovae luminosity and redshift provides a compelling reason to think that "dark energy" makes up $\sim 72\%$ of the stuff of the cosmos.

of the electromagnetic field.

If gravity must be regarded as something more than spacetime geometry, then what is it? That remains to be determined by theorists who take up the task in the future. The Yilmaz theory is a good underpinning for the descriptive and geometric aspects that have been discussed here, but it remains to be seen if it can be generally extended to a matter continuum or to other places where a "harmonic coordinate condition" might fail. This would seem to be a promising research area.

As for the physical reality that undergirds gravity, it is probably some sort of quantum field. It could involve quarks and gluons, Higgs bosons or even photons. For example, it has been proposed that gravity might be electromagnetic in origin (e.g., see Puthoff 1989, 1999 and references therein) and gravity somewhat similar to a van der Waals force. But whatever one may think, a correct theory of gravity will require elements of reality that are more than just spacetime geometry.

Appendix A. Einstein's elevators

Consider two elevators, one at rest on a planet where the local gravitational free-fall acceleration would be g. Let the other be out in free space away from gravitational fields. Let it be equipped with a rocket engine and accelerating at the same rate, g, in the z direction as determined by its on-board accelerometer. At the time the second elevator begins to accelerate, let a photon be emitted from a source at its floor and let it be absorbed later in a detector in its ceiling, a distance L away in the frame of the elevator. While the photon is in transit, the detector acquires some speed, v, relative to inertial frames. From the position of the detector, it is the same as if the source were receding from it at speed v. Thus if the frequency of the photon emitted at the floor is ω_0, the detector will detect the Doppler shifted frequency

$$\omega = \frac{\omega_0(1 - v/c)}{\sqrt{1 - v^2/c^2}} \tag{A.1}$$

We can determine the speed, v, of the ceiling photon detector from the special relativistic relation

$$a_z = \frac{dv}{dt} = \frac{a'_z}{(\gamma^3)(1 + u'_z v/c^2)^3} = \frac{g}{\gamma^3} \tag{A.2}$$

where $\gamma = 1/\sqrt{(1 - v^2/c^2)}$ and $u'_z = 0$ is the detector speed relative to an inertial frame that is comoving and coincident at the time the photon reaches the detector. Time increments, dT in the elevator are contracted such that $dT = dt/\gamma$. Substituting into Eq. A.2, integrating and setting T = L/c, we obtain

$$v/c = \tanh(gL/c^2) \tag{A.3}$$

Substituting Eq. A.3 into Eq. A.1, there follows

$$\omega = \omega_0 e^{-gL/c^2} \tag{A.4}$$

By the principle of equivalence the first elevator, which is at rest in a gravitational field, would have to produce the same frequency shift gravitationally. In this elevator, the change of (dimensionless) gravitational potential between floor and ceiling is, of course, $\Delta\phi = gL/c^2$. So the gravitational red shift is given **EXACTLY** by $1 + z = e^{\Delta\phi}$. This photon red shift result was derived by Einstein in a 1907 paper (Schwartz 1977). For a time after 1907, Einstein maintained that the metric coefficients must be strictly exponential functions in order to conform to the requirements of special relativity, but his final development of general relativity failed to satisfy the requirement.

One last situation should be examined as a two elevator experiment. Consider two identical masses constructed in field free space and then placed separately with one in the ceiling of each elevator. If dropped from the ceilings, there would be kinematical equivalence. The local transit times to the floors would be identical, but they might not be dynamically equivalent. If the mere presence of the earth caused a change of the mass within the elevator on earth, dents in the floor might differ. In order to avoid such a possibility it is necessary that the metric depend only upon gravitational potential differences (Yilmaz 1981) rather than upon absolute values of the potential.

Appendix B. Yilmaz Theory

The right member of the Einstein field equations permits the addition of a "Cosmological Constant", denoted Λ. As part of the right member, $\delta_i^j \Lambda c^4/(8\pi G)$ is considered to be a constant "dark energy" density of the cosmological vacuum. This makes it quite clear that it is apparently acceptable to have an energy density source in addition to the "matter tensor", T_i^j, in the right member of the Einstein field equations. Λ represents an energy density in the otherwise empty space of the cosmological vacuum for which $T_i^j = 0$ where no mass is present.

On planetary or galactic scales, the cosmological constant can be ignored. If it represents the ground state oscillations of all of the fields within the cosmos, we might expect its value to be roughly 120 orders of magnitude larger (Carroll 2004, Sec. 4.5) than the $\sim 10^{-8} erg/cm^{-3}$ that is needed to explain the cosmological redshift observations of type 1a supernovae. In addition to this rather glaring discrepancy, the energy density of matter would decrease in an expanding universe, which would allow only one coincidental moment in time in which matter and vacuum energy densities might be of comparable magnitude as they are at present. This coincidence problem and the difficulties associated with the cosmological constant and the dubious concepts of dark energy and non-baryonic dark matter can be removed by replacing the cosmological constant with a variable stress-energy tensor in the right member of the field equations. Contrary to Einstein's fiat, if a gravitational field exists in space, there will also be a field energy density.

Huseyin Yilmaz (Yimaz 1958 ... 1992) proposed that the right member of the Einstein field equations be modified by removing the cosmological constant and replacing it with a variable gravitational field stress-energy tensor. The modified equation can be written as:

$$G_i^j = -(8\pi G/c^4)(T_i^j + t_i^j) \tag{B.1}$$

where $T_i^j = \sigma u_i u^j$ is the matter tensor when no non-gravitational forces contribute. t_i^j is the gravitational stress-energy tensor. Other requirements are[17]

$$(T_i^j + t_i^j); j = 0 \quad \textit{Bianchi requirment} \tag{B.2}$$

$$\bar{\partial}_j(\sqrt{-K}\; T_i^j) = 0 \quad \textit{Freud requirement} \tag{B.3}$$

Here the overbar represents a covariant derivative with respect to local Minkowskian coordinates which share the same origin and orientation as those of the metric. $\sqrt{-K} = \sqrt{-g}/\sqrt{-\eta}$, where $\sqrt{-g}$ is the determinant of the metric and $\sqrt{-\eta}$ is the determinant of the metric of the Minkowskian background. In rectangular coordinates (x,y,z,t), $\sqrt{-\eta} = 1$ and all Christoffel symbols vanish, leaving a normal partial derivative.

This elaborate derivative procedure is necessary to eliminate pseudotensors that might otherwise arise. The pseudotensor problem has been discussed in detail (Yilmaz 1992). This procedure eliminates them. Pseudotensor problems can be avoided in two ways. The first simply consists of the use of rectangular coordinates, in which they never appear. The second is to take derivatives as covariant derivatives in local Minkowskian coordinates.

Einstein's gravitational energy expression, t_i^j, has been shown to be a pseudotensor; however, it can be expressed in terms that eliminate pseudotensors and leave a true tensor quantity. First define terms

$$\mathbf{g}^{ij} = \sqrt{-K} g^{ij} \quad \mathbf{g}_{ij} = g_{ij}/\sqrt{-K} \tag{B.4}$$

and

$$W_i^j = (1/8\sqrt{-K})\mathbf{g}^{jk}[\bar{\partial}_k \mathbf{g}_{ab} \bar{\partial}_i \mathbf{g}^{ab} - 2\bar{\partial}_k \sqrt{-K} \bar{\partial}_i (1/\sqrt{-K}) - 2\bar{\partial}_a \mathbf{g}_{kb} \bar{\partial}_i \mathbf{g}^{ab}] \tag{B.5}$$

then

$$t_i^j = W_i^j - (1/2)W_k^k \tag{B.6}$$

Yilmaz often used expressions that incorporated a harmonic coordinate condition. The expressions for W_i^j and t_i^j were based on Pauli's decomposition of

[17]Einstein's theory, in which T_i^j would expressly exclude gravitational field energy would require $T_i^j; j = 0$. As noted by Landau & Lifshitz, (Landau & Lifshitz 1962), this does not express any conservation law whatever. It would not be a statement of energy-momentum conservation, because it would not include the energy-momentum of the gravitational field. Eq. B.3 above is the energy-momentum conservation law of the Yilmaz theory.

the Einstein tensor with no harmonic coordinate conditions included. Yilmaz (1992) also provided a general expression for T_i^j.

$$(1/4\sqrt{-K})\bar{\partial}_a[\bar{\mathbf{g}}^{ak}\bar{\mathbf{g}}^{jb}(\bar{\partial}_b\mathbf{g}_{ik} - \bar{\partial}_k\mathbf{g}_{ib}) + \delta_i^j\bar{\partial}_b\mathbf{g}^{ba} - \delta_i^a\bar{\partial}_b\mathbf{g}^{bj}] = T_i^j \qquad \text{(B.7)}$$

Of particular interest here is t_i^j for the metric form

$$ds^2 = e^\nu c^2 dt^2 - e^\lambda(dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\Phi^2) \qquad \text{(B.8)}$$

and complete spherical symmetry for which r and t are the only variables. The background Minkowskian metric is $ds^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - (r\ sin\theta)^2 d\Phi^2$ and $\sqrt{-\eta} = r^2 sin\theta$.

After some tedious tensor algebra, the results from Eqs B.4, B.5 and B.6 are (as also stated in the text Eq.s 32 & 33):

$$t_0^0 = -t_1^1 = (c^4/8\pi G)[e^{-\nu}3\dot{\lambda}^2/4 + e^{-\lambda}(\lambda'\nu'/2 + \lambda'^2/4)] \qquad \text{(B.9)}$$

and

$$t_2^2 = t_3^3 = (c^4/8\pi G)(-e^{-\nu}3\dot{\lambda}^2/4 + e^{-\lambda}(\lambda'\nu'/2 + \lambda'^2/4) \qquad \text{(B.10)}$$

Here dots represent partial derivatives with respect to time, and primes represent partial derivatives with respect to the radial coordinate, r. In the special case of the static point particle metric, (Eq. 11 of the text) for which $\nu = -\lambda = 2\phi$,

$$t_0^0 = -(c^4/8\pi G)(e^{-\lambda}\lambda'^2/4) = -(c^4/8\pi G)e^{-2\phi}(\partial_r\phi)^2 \qquad \text{(B.11)}$$

This is the necessary right member for Eq. 27 of the main text.

In addition to the previous equations, it is assumed that the equations of motion of particles moving in the metric spacetime would be geodesic equations if no non-gravitational forces act. The gravitational forces that drive the motion are the stress tensor right members of Eq. B.1. We can examine an interesting aspect of this (in rectangular coordinates) by expanding Eq. B.2

$$(T_i^j + t_i^j)_{;j} = (1/\sqrt{-g})\partial_j(\sqrt{-g}(T_i^j + t_i^j) - 1/2(\partial_i g_{ab})(T^{ab} + t^{ab}) = 0 \qquad \text{(B.12)}$$

but by Eq. B.3, $\partial_j(\sqrt{-g}\,T_i^j) = 0$. Thus $(1/\sqrt{-g})\partial_j(\sqrt{-g}\,t_i^j) = 1/2(\partial_i g_{ab})(T^{ab} + t^{ab})$ and the geodesic equation becomes

$$\sigma du_i/ds = 1/2(\partial_i g_{ab})(T^{ab} + t^{ab}) = (1/\sqrt{-g})\partial_j(\sqrt{-g}\,t_i^j) \qquad \text{(B.13)}$$

This shows that particle motions in the Yilmaz theory are actually driven by the particle interactions with the gravitational field stress-energy tensor. In cases involving multiple particles as sources, they interact via the collectively established t_i^j. These can be most easily constructed in terms of the metric coefficients constructed from the gravitational potentials described below.

Appendix B.1. Time dependence

Thus far it has been shown that it is possible to include a true gravitational field stress-energy tensor with only minor impact on Einstein's grand concept of curved spacetime, yet it would drastically change the black hole-dark energy orthodoxy of current astrophysics. Einstein's theory is more radically altered when gravitational radiation is considered. Einstein's gravitational radiation formula is supported by observations of the decay of binary pulsar's orbits (Hulse & Taylor 1975, Taylor & Weisberg 1984); however, as Einstein discovered, his formula is not compatible with his field equations. Lo (1995) cited Vlasov & Denisov (1982) as showing that the calculated rate of energy emission depends on the choice of coordinates. This is a result of calculations being based on Einstein's gravitational field pseudotensor. Lo (1995) provided a derivation of the formula based on the inclusion of a field energy tensor but failed to recognize that it was actually compatible with the Yilmaz theory. Lo stated that t_i^j must be added as a source term in the right member of Einstein's field equation, yet it could not be a physical cause of the curvature exhibited by the metric, otherwise gravity would be self-generating. In addition to the anti-gravity coupling explicitly demonstrated in this text for the point mass metric, the Yilmaz theory obviates this problem with a stress-energy tensor that depends on potentials rather than directly upon the metric coefficients.

A hint of this was shown in the previous discussion of the metric for cosmology where the metric coefficients were expressed as $\lambda = 2\phi_0 + 2\phi_1$ and $\nu = 6\phi_1 - 2\phi_0$. The functions ϕ_0 and ϕ_1 are potentials that actually form the metric rather than the reverse. As originally presented (Yilmaz 1971), his general theory was formulated in terms of a potential tensor ϕ_μ^ν and the metric coefficients g_{ij} determined as functions or functionals of the ϕ_μ^ν. The theory was later presented with the Einstein tensor in the form used here just to clearly show how it would modify the Einstein field equations. This last correspondence was based on Pauli's decomposition of the Einstein tensor (Yilmaz 1992), which Yilmaz writes as[18]

$$(4\pi G/c^4)T_\mu^\nu = \Box^2\phi_\mu^\nu - (1/\sqrt{-g})\partial_\alpha(\sqrt{-g}\partial^\nu\phi_\mu^\alpha) \tag{B.14}$$

where $\Box^2 = (1/\sqrt{-g})\partial_\alpha(\sqrt{-g}\ \partial^\alpha)$ is the Laplace-Beltrami operator and T_μ^ν satisfies the Freud requirement, Eq. B.3.

Yilmaz has proposed, by analogy with electromagnetism, that one can regard the rightmost terms of Eq. B.14 as gauge terms that can initially be neglected[19]. After the remaining equation has been solved, the gauge terms can be regenerated from the solution to satisfy Eq. B.11. Thus one has the simpler equation

$$\Box^2\phi_\mu^\nu = (4\pi G/c^4)T_\mu^\nu \tag{B.15}$$

[18]As previously noted, in other than rectangular coordinates the derivatives must be taken as covariant derivatives relative to the Minkowskian background and $\sqrt{-g}$ replaced by $\sqrt{-K}$.

[19]In a Lagrangian formulation of the theory, the gauge terms contribute to the Lagrangian density by an ordinary divergence and they do not affect the field equations.

Here $T^\nu_\mu = \sigma u_\nu u^\nu$, where σ is the mass density of particles in a volume element, excluding any gravitational field energy. This is a theory of particles and fields considered separately and not a matter plus field continuum. The solutions of Eq. B.15 are

$$\phi^\nu_\mu = (G/c^4)\int T'^\nu_\mu dV'/r' \tag{B.16}$$

Here primes denote the usual retarded condition $t - r/c$.

In his first presentation of the theory in terms of potentials, Yilmaz (1971) stated the relation between the fields ϕ^ν_μ and the metric as a functional differential equation

$$dg_{\mu\nu} = 2(g_{\mu\nu}d\phi - g_{\mu\alpha}d\phi^\alpha_\nu - g_{\nu\alpha}d\phi^\alpha_\mu) \tag{B.17}$$

where ϕ is the trace of ϕ^ν_μ. Substituting the solutions of Eq. B.15 leads to a formal exponential metric, which is exact for many cases of physical interest

$$g_{\mu\nu} = (\eta e^{[2(\phi\hat{I}-2\hat{\phi})]})_{\mu\nu} \tag{B.18}$$

where η is the metric of the Minkowskian background, $\phi = \phi^k_k$ is the trace of $\hat{\phi} = \phi^\nu_\mu$ and $\hat{I}$ is the identity matrix. The exponential function is defined in terms of its ordered Taylor expansion and in rectangular coordinates, $\eta = (1, -1, -1, -1)$.

As an example, a diagonal exponential metric to describe a spherically symmetric spacetime, could have potentials $\phi^0_0, \phi^1_1, \phi^2_2, \phi^3_3$. Then

$$g_{00} = \eta_{00}e^{2(\phi^0_0+\phi^1_1+\phi^2_2+\phi^3_3-2\phi^0_0)} \tag{B.19}$$

$$g_{11} = \eta_{11}e^{-2(\phi^0_0+\phi^1_1+\phi^2_2+\phi^3_3-2\phi^1_1)} \tag{B.20}$$

etc. For the isotropic metric of Eq. 1 of the text, we need $\phi^1_1 = \phi^2_2 = \phi^3_3$, thus we obtain metric coefficients

$$g_{00} = e^\nu = \eta_{00}e^{6\phi^1_1-2\phi^0_0}, \quad g_{ii} = -e^\lambda = \eta_{ii}e^{2\phi^1_1+2\phi^0_0} \tag{B.21}$$

The harmonic coordinate condition that was applied in the cosmology discussion required that ϕ^0_0 be time independent and that ϕ^1_1 depend only on time. At that point we could have solved Eqs. B.15 for ϕ^0_0 and ϕ^1_1 without bothering with the Einstein tensor. In fact, subject to the harmonic coordinate condition, these yield Eqs. 41 & 42 of the text.

In the case of a mass distribution consisting of discrete particles moving at low speeds, i.e., $u_i \ll c$, $T^j_i = \sigma u_i u^j = \sigma u_0 u^0 = \sigma c^2$.

$$T^0_0 = \Sigma_i m_i c^2 \delta(\mathbf{x} - \mathbf{x_i}) \tag{B.22}$$

$T^\nu_\mu = 0$ in the space outside the particles. The field equations are $G^j_i = -(8\pi G/c^4)t^j_i \neq 0$, as has been shown (Lo (1995). The solution of Eq. B.15, from which the metric (Eq. 6) can be constructed, is

$$\phi = \phi^0_0 = \Sigma_i G m_i/(c^2|\mathbf{x} - \mathbf{x_i}|) + C \tag{B.23}$$

Note that these are not necessarily static conditions, but only circumstances for which the particle speeds are much less than light speed. This means that for any distribution of sufficiently slow moving particles, no matter how complex, we can immediately write down the corresponding *exact* curved spacetime solution. The motions of such distributions can then be studied by Hamilton-Jacobi methods (Yilmaz 1994).

Appendix B.2. Additional Comments

As shown by Yilmaz (1975) the passage from general relativity to Newtonian mechanics requires first passing through the special relativistic limit. The failure to honor this limit exactly for gravitational redshifts may at first glance seem minor, but it shows clearly that second order corrections are needed.

The treatment of free-fall differs between the Yilmaz and Einstein theories. In the Yilmaz theory, the presence of free fall is indicated by a locally Minkowskian metric. Removal of constraining forces that prevent free fall is represented by subtracting constants from the potentials that appear in the metric coefficients. This is not a coordinate transformation. The ability to set the potentials to zero at the location of an observer produces a locally Minkowskian spacetime for any observer, including even those in accelerating systems. While it is true that t_i^j can be compensated at the location of the accelerated observer, one cannot conclude that it must vanish everywhere.

Misner, Thorne & Wheeler (1973) have argued that there can be no localized gravitation field stress-energy because the transition to free-fall requires a coordinate transformation to a local Minkowskian spacetime for which the first derivatives of the metric coefficients vanish. The gravitational field stress-energy tensor is quadratic expression in the first derivatives and vanishes in that transformation. Therefore, it is argued that it must vanish in any coordinates, but this argument does not apply to the Yilmaz theory because it does not rely on a coordinate transformation to achieve free fall. This also permits local energy conservation in the Yilmaz theory.

Appendix C. Analytic solution for a neutron star model

Eqs. 57 -60 for a sphere of constant ρ (Buchdahl 1982) can be matched to the exterior Yilmaz metric Eq. 11. Matching λ, ν and ν' across the exterior boundary is sufficient to determine the integration constants of the solution. Using $u = -\phi(R) = GM/(c^2R)$, the solutions are given in terms of

$$k = \frac{5 + 3\sqrt{1+u^2}}{4 - 3u} \tag{C.1}$$

Results that reduce to $\lambda = -\nu = 2u$ at $r = R$ can then be expressed as:

$$e^{\lambda} = \{\frac{3e^u(k-1)}{2k-1+(k-2)(r/R)^2}\}^2 \tag{C.2}$$

$$e^{\nu} = \{\frac{3e^{-u}[(2k-1)+k(k-2)(r/R)^2]}{(k+1)[2k-1+(k-2)(r/R)^2]}\}^2 \tag{C.3}$$

$$\frac{p}{\rho c^2} = \frac{(2k-1)(k-2)[1-(r/R)^2]}{3[(2k-1+k(k-2)(r/R)^2]} \tag{C.4}$$

$$R = R_0 e^{-u}\sqrt{u} \tag{C.5}$$

where $R_0 = \sqrt{c^2/(4\pi G\rho)}$.

As can be seen from Eqs. C.1 and C.4, infinite central pressure would be generated for $u \to 4/3$ and $k \to \infty$. Although this would also result in $g_{00} \to 0$, infinite central pressure can be rejected as unphysical. The $u = 2$ condition for the event horizon in the isotropic Schwarzschild solution would correspond to an unphysical negative pressure here. For $u \leq 4/3$, the metric coefficients remain well-behaved. A causality limit of $p(0) = \rho c^2$ would be reached first for $u = 3/4,\ k = 5$; again without creating any event horizon conditions. Realistically, the maximum mass of a neutron star should correspond to $p(0) = \rho c^2/3$, for which $k = 3$ and $u = 5/12$. Using a nuclear density $\rho = 2.8 \times 10^{14}\ g\ cm^{-3}$, the mass of such a star would be $2.34 M_{\odot}$. In addition, the solution for a $1.4 M_{\odot}$ star would have $u = 0.268$, $R = 7.75$ km, a central pressure of $p(0) = 0.178\rho c^2$, and a surface redshift of $z = 0.307$. Mass, radius and surface redshift are all very close to the results calculated for a realistic neutron equation of state and shown in Table 2. The simple analytical model is better than might have been expected.

Appendix D. Mach's Principle and the Exponential Metric

Assuming that the (appropriately retarded) gravitational potentials are additive, the potential of a collection of masses, $(M_1,\ M_2,\ M_3,\ \ldots)$ would be

$$\phi = \Sigma_i \phi_i = -(\frac{GM_1}{c^2 r_1} + \frac{GM_2}{c^2 r_2} + \frac{GM_3}{c^2 r_3} + ...) \tag{D.1}$$

and the metric would retain the form

$$ds^2 = e^{2\phi}c^2 dt^2 - e^{-2\phi}(dx^2 + dy^2 + dz^2) \tag{D.2}$$

Now suppose that one is relatively near to one of the masses, say M_1, such that only its potential would vary significantly over a region of interest. Then one can redefine time and distance scales such that

$$dt' = dt e^{-(\frac{GM_2}{c^2 r_2} + \frac{GM_3}{c^2 r_3} + ...)} \tag{D.3}$$

and

$$dx' = dx e^{(\frac{GM_2}{c^2 r_2} + \frac{GM_3}{c^2 r_3} + ...)},\ \ dy' = dy e^{(\frac{GM_2}{c^2 r_2} + \frac{GM_3}{c^2 r_3} + ...)} ....\ r' = r_1 e^{(\frac{GM_2}{c^2 r_2} + \frac{GM_3}{c^2 r_3} + ...)} \tag{D.4}$$

Thus

$$ds^2 = e^{2\phi_1}c^2dt'^2 - e^{-2\phi_1}(dx'^2 + dy'^2 + dz'^2) \tag{D.5}$$

This amounts to letting the collective effects of all distant masses determine the local measures of distance and time. To complete the removal of all vestiges of our former coordinates, we can also redefine

$$M' = M_1 e^{\phi_2+\phi_3+\cdots} \tag{D.6}$$

which is just what we would expect from special relativity in accord with Eq. 2. Substituting from Eq. D.6 for M_1 and from Eq. D.4 for r_1 in ϕ_1, and then dropping subscripts and primes, the local metric near a dominating mass M is

$$ds^2 = e^{2\phi}c^2dt^2 - e^{-2\phi}(dx^2 + dy^2 + dz^2) \tag{D.7}$$

where $\phi = -GM/c^2r$ for the mass M.

In this way, we can regard our local measures of mass, length and time to have been determined by the distant masses of the universe, as was suggested by Mach. This factoring of the metric is unique to the exponential metric and is a powerful argument in its favor.

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