The motion of a liquid with constant and variable volume
Whirlwinds and Tea leaf paradox

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***Annotation**: A set of experiments is presented. The two main types of the movement of incompressible liquid were investigated. In the first case liquid moved with no changes in volume. The elastic deformation made liquid move. In the second case a formation of curl took place during the expansion of the liquid getting an additional kinetic energy. Some formulas modeling curl's evolution were derived.*

***Keywords**: kinetic energy; elastic deformation; curl, experiment; second order differential equation.*

Intro

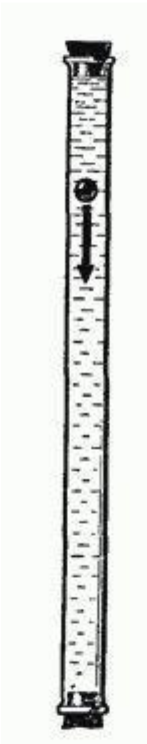
Incompressible liquid is a liquid which density does not change under pressure. A liquid can change its shape with no change in a volume.

Let's show how this property can lead to the movement of parts of liquid in a close volume. We may assume that there is the smallest part of the liquid remaining incompressible and being able to change its shape with the constant volume.

1. **An experiment. The movement is made due to elastic deformation.**

The compressibility factor of the water has a negligibly small value. So the water can be considered as an incompressible liquid.

The widely known experiment, Pic. 1



Pic. 1

Of cause an observer can see the movement of a solid body only. Let’s describe the movement of the liquid taking into account that the volume remain constant.

Let’s take the smallest incompressible part of the liquid as a cube with edges 2x2x1 of some units. Now we can carry out a set of mental experiments.

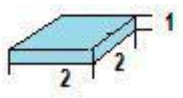


Рис. 2

Mental experiment 1.

There is a cubic container with edges 4x4x1. The container is made of a perfectly solid material. Let’s place 4 particles into the container, three particles of the liquid and one of a solid body. The volume of all particles is 2x2x1. Pic. 3a. The mass of the solid particle is bigger than the mass of the liquid particles. The particle 0 presses the particle 2 due to gravitation. According to Pascal low the pressure applies to all particles of the liquid. And this makes the liquid’s particles move. Pic. 3b and 3c

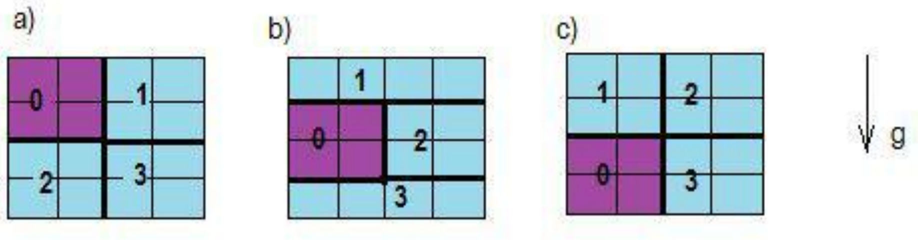


Рис. 3

Mental experiment 2.

Let’s assume that the solid particle has the mass less than the the mass of liquid’s particle. And the solid particle is placed on the bottom of the container. Pic. 4a. In this case the solid particle floats up. Pic. 4b and 4c. And again we can see the movement of the liquid’s particles apart from the movement of the rigid one.

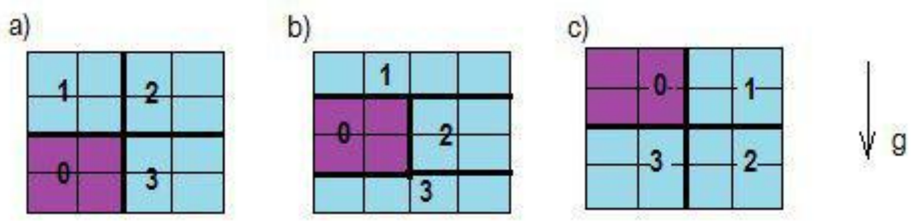


Рис. 4

Mental experiment 3.

Let’s replace the solid particle with the liquid one same as the three other particles but with different momentum. In this case particles will exchange momentum by elastic collisions. In the case when the difference between momentums is big enough the particles will experience an elastic deformation and a topological movement will take place. Of cause the topology of the particle form change will be much more complex than it is presented in the previous experiments. In the real liquid the topological movement tends to zero.

2. **Experiments showing a curls formation in a liquid experiencing an expansion due to getting extra kinetic energy.**

Experiment 4.

Tea leaves are place into a container with clear sides. Let's close the container and rotate it. The tea leaves will move to the side of the container due to centrifugal and friction forces. The influence of air can be neglected.

Experiment 5.

The same container is fully filled with water with some tea leaves inside. Let's shake the container several times and put it on the table. We can observe that the tea leaves move chaotically and go gradually down under the gravity force.

Experiment 6.

The same container fully filled with water and some tea leaves on the bottom is places on a spinning disk. Let's rotate the container for some time and stop the disk. We can observe that the tea leave tend to the center of the bottom.

Experiment 7.

Let's carry out an experiment same as the sixth one but with a cubic container. We can observe almost the same result. The leaves tend to the center but the dispersion is bigger.

The water in the 6th and 7th experiments starts to rotate with the container and behave as a rigid body. All molecules get the same angular velocity while the line speed increases from the center to the side. After the rotting disk has been stopped the viscosity and fluidity come to the fore. The friction between the water and the side of the container reduces the rotation speed. Nothing can cause the movement in vertical direction.

Experiment 8.

A container with rigid sides is ¾ full of water and is sealed. Let's shake the container for some time and then put it on a fixed surface. Currents appear inside the water. Some of them tend to close up. Thus curls appear.

All experiments are recorded and files are avaliable on **http://my-files.ru/v4jb9q**.

3. **A derivation of formula modeling and describing curls**

It is known that the pressure inside a curl decreases from side to the center. It means that the density of the water decreases due to expansion. The faster the water moves inside a big enough volume the less is the influence of forces connecting molecules together, up to the break. Thus a force appears acting in the horizontal direction, similar to Archimedes' power.

(1)

where:

ρ_i - density of water in the layer *i* ;

- volume of layer *i* ;

Fp_i - force pushing out;

a_i - acceleration.

If the acceleration is a constant than the formula (1) will make into (2).

$$Fp_i = \rho_i a V_i \qquad (2)$$

So there are two things which contribute to the movement of molecules of an ideal liquid on a plane, molecule's momentum and a pushing out force acting in horizontal direction. It follows from the Newton low that at least two forces are required to make a body to move along a curve. Thus, in 2D, we get a curl consisting of concentric circle currents, which are congruent to trajectory of molecules.

In the 3D case Archimedes' power has to be added.

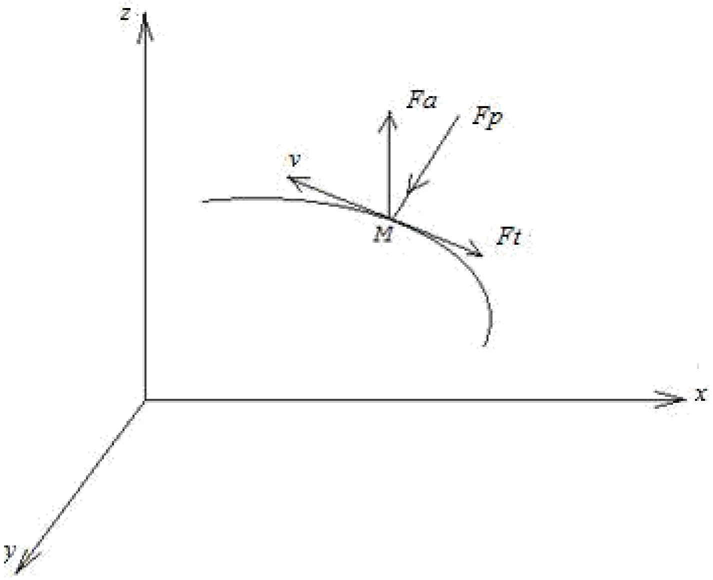
$$Fa_i = \rho_i g V_i \qquad (3)$$

The chaotic movement of molecules tends to take a shape of cones enclosed in each other with tops looking down. As the densities on a side of every cone are equal current's paths can be in form of any curve of second order. It worth mentioning that closed up curves do not exist in vertical sections. Fading of a curl is because of the dispersion of the energy into the environment.

Cole sections are the second order curves. That's why it is possible to use a second order differential equation to calculate the movement across these curves [1]. In the experiment above the container is symmetrical to the vertical axis. As a result the current lines are also symmetric to the vertical axis. It leads us to use the second order differential equation to the center.

$$\ddot{\varphi} = \frac{2*e^2*cos(\varphi(t))*sin(\varphi(t))*(\dot{\varphi})^2}{1-e^2*cos(\varphi(t))^2} \qquad (4)$$

A generalized force Q was used to derive the equation (4). The generalized force is the sum $Q = F_a + F_p + F_t$, Pic. 5, F_a - Archimedes' power, F_p - pushing out force of outer layer, F_t - friction force, v- speed of particle M.



Pic. 5

The experiments above show the way to model and study the movement of a liquid. The system of equations (5) allows the numerical calculation of the movement of a liquid in a curl.

$$\begin{cases}\ddot{\varphi} = \frac{2*\varepsilon^2*cos(\varphi(t))*sin(\varphi(t))*(\dot{\varphi})^2}{1-\varepsilon^2*cos(\varphi(t))^2}\\ Fp_i = \rho_i a V_i \\ Fa_i = \rho_i g V_i\end{cases} \qquad (5)$$

i = 0,1,...,N; *N*– the number of vertical liquid layers in a curl.

Conclusion

1. The first type of the liquid movement is impossible in nature in its pure form because a closed rigid container is required. In the lifeless nature the topological movement has to probably be taken into account. If the side of a living cell is considered to be solid enough then the movement of the inter cell liquid be considered as the movement due to the elastic deformation of its particles.
2. The common differential equation of second order curves [(18), 1] does not put any limits on choosing of the radius.

$$\ddot{\varphi} = 2 * f\left(r(\varphi(t))\right) * \sin(\varphi) * \dot{\varphi}^2$$

It is possible to find a function describing the movement under the gravity. However it is hard to imagine that in the experiments above the molecules have the same speed whenever they move up or down along a closed up vertical curve.

There is similar movement in the nature. “According to the classical theory parts of the water have to move along a circle in the case of deep water wave movement. On the International Oceanography Congress V. Shuleykin pointed at inaccuracy of the classical theory. His observations of movement of suspended particles made in a wave pool revealed that the particles move along an ellipse. “[2]

3. The conditions from the experiments 4-8 can hardly be archived in the nature. First of all it is hard to pick out a part of a liquid with an additional kinetic energy uniformly distributed at the initial stage across the whole volume. Also there is no clear boundary between the curl and the environment. Despite all these limitations the experiments allow to provide investigation of the current line formation and the push out force (1).

Links

1. https://www.academia.edu/36992072/Differential_equations_for_2nd-order_curves
2. International Oceanographic Congress (2; 1966; Moscow)