On Bell's experiment

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Abstract – With the use of tropical algebra operators and a $d = 2$ parameter vectors space, Bell's theorem does not forbid a, physics valid, reproduction of the quantum correlation.

Introduction. – In 1964, John Bell wrote a paper [1] on the possibility of hidden variables [2] causing the entanglement correlation $E(a, b)$ between two particles. In the present paper we continue our study of possible concrete physics theoretical incompleteness. Bell, based his hidden variable description on particle pairs with entangled spin, originally formulated by Bohm [3].

A Bell type experiment is given when two observers, Alice and Bob, are at a (large) distance from each other. Both have a spin measuring instrument. The instruments are denoted with resp. A and B. The instruments have separate and independent setting parameter vectors of unit length. We have a for Alice's parameter vector and b for Bob's. The euclidean length of the parameter vectors a and b is unity. In the middle there is a source S. The source sends to Alice and Bob, particles that belong to entangled pairs cite3, [4]. In the sketchy figure below, wavy lines suggest particles, arrows show the direction of propagation, dots suggests the distance to be traveled and symmetry suggests entanglement. I.e. the source in the wavy symbol moving to the right corresponds to the sink of the wavy symbol moving to the left.

$$[A(a)] \leftarrow\!\sim \ldots \sim\!\leftarrow\!\sim [S] \sim\!\rightarrow\!\sim \ldots \sim\!\rightarrow [B(b)] \qquad (1)$$

In two dimensional parameter space we have on Alice's side (A instrument) of the experiment, $a = (a_1, a_2)$. On Bob's side we have the parameter vector $b = (b_1, b_2)$. The parameter vectors are unitary, $||a|| = ||b|| = 1$.

Bell used hidden variables λ that are elements of a universal set Λ and are distributed with a density $\rho(\lambda) \geq 0$. Suppose, $E(a, b)$ is the correlation between the parameter vectors of the measurement instruments A and B. Then with the use of the λ we can write down the classical probability "correlation" between the two simultaneously measured particles. This is what we will call Bell's correlation formula.

$$E(a, b) = \int_{\lambda \in \Lambda} \rho(\lambda) A(a, \lambda) B(b, \lambda) d\lambda \qquad (2)$$

We have $A = \pm 1$ and $B = \pm 1$ to mimic the spin up and down discrete outcome of measurement.

Bell inequality. From (2) an inequality for four setting combinations, a, b, c and d can be derived as follows

$$\begin{aligned} E(a, b) - E(a, c) = & \quad (3) \\ \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(c, \lambda) A(d, \lambda) B(c, \lambda) & - \\ \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) A(d, \lambda) B(b, \lambda) & + \\ \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) - \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(c, \lambda) & \end{aligned}$$

because, $\{B(c, \lambda)\}^2 = \{B(b, \lambda)\}^2 = 1$. From this it follows

$$\begin{aligned} E(a, b) - E(a, c) = & \quad (4) \\ \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) \{1 - A(d, \lambda) B(b, \lambda)\} & + \\ \int_{\lambda \in \Lambda} d\lambda \rho(\lambda) (-A(a, \lambda) B(c, \lambda)) \{1 - A(d, \lambda) B(c, \lambda)\} & \end{aligned}$$

Hence, because $1 - A(x, \lambda) B(y, \lambda) \geq 0$ for all x, y with $||x|| = ||y|| = 1$ and $A(a, \lambda) B(b, \lambda) \leq 1$ together with $-A(a, \lambda) B(c, \lambda) \leq 1$, it can be derived that

$$E(a, b) - E(a, c) \leq 2 - E(d, b) - E(d, c) \qquad (5)$$

Or,

$$S(a, b, c, d) = E(a, b) + E(d, b) + E(d, c) - E(a, c) \leq 2. \qquad (6)$$

Note, no physics assumptions were employed in the derivation of (5). It is pure mathematics.

Our question here is, is (2) exhausting any possible physics behind the experiment. In other words, is the formula of Bell (2) sufficiently covering for the experiment of Bell in (1)? A *proof* of the inconsistency of Bell's theorem can be found in [11].

Counter proof. –

Tropical algebra operator. Tropical algebra has been used in an attempt to tackle nonlinearity in physical problems [9]. This can be the case in Bell physics as well. If one wants to contest this physics possibility in (1) then the challenge is to come with proof why this can not be the case in entanglement physics. It must be noted that the absence of hidden variables in experiment (1) is solely based on (2) and the inequalties derived thereof. It is based on *mathematical* considerations. There is no explicit physics theory behind the derivation of the inequality from (2). Nobody looked beyond (2) when considering an experiment (1). Hence, when someone contests the physical possibility of the tropical operator, it is legitimate to *insist* on *proof* of the impossibility of the tropic operator in physics reality. This debate is about what we consider reasonable for the description of (1).

Therefore, to the integration of (2) we may add the tropical algebra operation $\oplus$. If there are no physical reasons to disallow it, then it is allowed. The use of tropical operation will provide new insights into the relation Bell formula and Bell experiment.

Tropical sum. Let us define the tropical algebra sum on real, i.e. $\mathbb{R} \cap [-1, 1]$, values for x and y. We define

$$x \oplus y = \begin{cases} x+y, & |x+y| \leq 1 \\ +1, & x+y > 1 \\ -1, & x+y < -1 \end{cases} \tag{7}$$

Interestingly with, $H_{1/2}(x) = 1 \Leftrightarrow x > 0$, with, $H_{1/2}(x) = 0 \Leftrightarrow x < 0$ and $H_{1/2}(0) = 1/2$. This implies, $x \oplus y = (x+y)H_{1/2}(1-|x+y|)) + H_{1/2}(x+y-1) - H_{1/2}(-1-(x+y))$. We note that the summation in (7) is allowed. If readers disagree they have to *prove* that this way of topped summing cannot for sure occur in physics reality. Below we will introduce the other elements of the hidden variables theory and later return to use (7). The tropical semiring is based on the topped sum and normal multiplication. This semi-ring applies to real numbers in the interval $[-1, 1]$.

Density. In the probability density function of (2) there are hidden variables λ. The first hidden variable we introduce here is $n \in \{\epsilon, 1-\epsilon\}$. Here we have the $0 < \epsilon \to 0$. A second spin-like variable is $x \in \{0, 1\}$. An important part of the probability density from Bell's correlation formula is therefore $\rho(n, x) = f(x)g(n, x)$. The function g is defined by

$$g(n, x) = n^x (1-n)^{1-x} \tag{8}$$

with, $n \in \{\epsilon, 1-\epsilon\}_{0<\epsilon\to 0} \equiv n \in E_\epsilon$ and $x \in \{0, 1\}$. The function f is a selection from the set $\mathcal{F}(x) = \{\rho_1(x), \rho_2(x)\}$. Here, $\rho_1(x) = x,\ x \in \{0, 1\}$, while $\rho_2(x) = 1 - x,\ x \in \{0, 1\}$. Hence, obviously,

$$\sum_{x=0}^{1} \rho_1(x) = 1 \tag{9}$$

$$\sum_{x=0}^{1} \rho_2(x) = 1$$

Furthermore, let us introduce an indicator function $\iota(f(x) \in \mathcal{F}(x)) = 1$ when $f(x) \in \mathcal{F}(x)$ and $\iota(f(x) \in \mathcal{F}(x)) = 0$ when $f(x) \notin \mathcal{F}(x)$. Hence, we may look at

$$\sum_{x=0}^{1} f(x)\iota(f(x) \in \mathcal{F}(x)) = \begin{cases} 1, & f(x) \in \mathcal{F}(x) \\ 0, & f(x) \notin \mathcal{F}(x) \end{cases} \tag{10}$$

The outcome 1 in (10), for $f(x) \in \mathcal{F}(x)$, is based on equation (9) and on $\iota(f(x) \in \mathcal{F}(x)) = 1$. The outcome 0 in (10), for $f(x) \notin \mathcal{F}(x)$, is based on $\iota(f(x) \in \mathcal{F}(x)) = 0$. So, given a function $h(x)$ and $x \in \{0, 1\}$, then we have from equation (10)

$$\sum_{x=0}^{1} f(x)\iota(f(x) \in \mathcal{F}(x))h(x) =$$

$$\begin{cases} \sum_{x=0}^{1} \rho_1(x)h(x), & f(x) \in \mathcal{F}(x), \quad f(x) = \rho_1(x) \\ \sum_{x=0}^{1} \rho_2(x)h(x), & f(x) \in \mathcal{F}(x), \quad f(x) = \rho_2(x) \\ 0, & f(x) \notin \mathcal{F}(x) \end{cases} \tag{11}$$

Let us suppose that $h(x) = \sum_{n \in E_\epsilon} g(n, x)$ as defined in (8). Then the first row of equation (11), with $\rho_1(x) = x$, reads, with $0 < \epsilon \to 0$,

$$\sum_{x=0}^{1} \rho_1(x) \sum_{n \in E_\epsilon} g(n, x) = \sum_{x=0}^{1} x \sum_{n \in E_\epsilon} n^x (1-n)^{1-x} = \tag{12}$$

$$\sum_{n \in E_\epsilon} n^1 (1-n)^0 = \sum_{n \in E_\epsilon} n = \epsilon + (1-\epsilon) = 1$$

The second row of equation (11), with $\rho_2(x) = 1 - x$, reads

$$\sum_{x=0}^{1} \rho_2(x) \sum_{n \in E_\epsilon} g(n, x) = \sum_{x=0}^{1} (1-x) \sum_{n \in E_\epsilon} n^x (1-n)^{1-x} = \tag{13}$$

$$\sum_{n \in E_\epsilon} n^0 (1-n)^1 = \sum_{n \in E_\epsilon} (1-n) = (1-\epsilon) + \epsilon = 1$$

Note that equations (12) and (13) remain true when $0 < \epsilon \to 0$. If our hidden variables are $x \in \{0, 1\}$ and $n \in E_\epsilon$, then from equation (11) we can derive

$$\sum_{x=0}^{1} f(x)\iota(f(x) \in \mathcal{F}(x)) \sum_{n \in E_\epsilon} g(n, x)$$

$$= \begin{cases} 1, & f(x) \in \mathcal{F}(x) \\ 0, & f(x) \notin \mathcal{F}(x) \end{cases} \tag{14}$$

If the attention is then directed only to $f(x) \in \mathcal{F}(x)$, the first row of (14) warrants that the probability density function $f(x)g(n, x)$ is correct *and* may be employed in a Bell correlation formula.

Measurement functions. Concerning the defintion of the measurement functions we already defined the two dimensional measurement parameter vectors, $a = (a_1, a_2)$ and $b = (b_1, b_2)$. Let us, subsequently, define two auxiliary function α and β. We have

$$\begin{aligned} \alpha &= a_1\delta_{x,1}\delta_{f,\rho_1} + a_2\delta_{x,0}\delta_{f,\rho_2} \\ \beta &= b_1\delta_{x,1}\delta_{f,\rho_1} + b_2\delta_{x,0}\delta_{f,\rho_2} \end{aligned} \tag{15}$$

Here, the δ for discrete choice is, $\delta_{p,q} = 1$ when, $p = q$ and $\delta_{p,q} = 0$ when $p \neq q$. The δ_{f,ρ_m} means that the function f selects ρ_m, with $m = 1, 2$. Moreover, from $\delta_{x,0}\delta_{x,1} = 0$, it follows that the cross broducts in $\alpha\beta$, defined in (15), that contain a_1b_2 or a_2b_1 terms will not contribute. It also is easy to see that $|\alpha| \leq 1$ and $|\beta| \leq 1$, because $||a|| = 1$ and $||b|| = 1$.

Evaluation I. If we also note that, in effect, $\delta^2_{p,q} = \delta_{p,q}$, then the evaluation of

$$e(a,b) = \sum_{x=0}^{1} f(x) \sum_{n\in E_\epsilon} g(n,x)\alpha(a,x,f)\beta(b,x,f) \tag{16}$$

where $f \in \mathcal{F}$, only will be concerned with two, not-zero-by-definition, terms. Note that we have

$$\begin{aligned} \alpha\beta = a_1b_1\delta^2_{x,1}\delta^2_{f,\rho_1} + \\ (a_1b_2 + a_2b_1)\,\delta_{x,1}\delta_{x,0}\delta_{f,\rho_1}\delta_{f,\rho_2} \\ +a_2b_2\delta^2_{x,0}\delta^2_{f,\rho_2} \end{aligned}$$

and $\delta_{x,1}\delta_{x,0} = 0$. Moreover, $\delta^2_{x,1}\delta^2_{f,\rho_1} = \delta_{x,1}\delta_{f,\rho_1}$ and $\delta^2_{x,0}\delta^2_{f,\rho_2} = \delta_{x,0}\delta_{f,\rho_2}$.

Firstly, because of the δ_{f,ρ_1}, the a_1b_1 containing term in $\alpha\beta$ from (16) is

$$e_1(a,b) = \sum_{x=0}^{1} x \sum_{n\in E_\epsilon} n^x(1-n)^{1-x}a_1b_1\delta_{x,1} \tag{17}$$

This implies that

$$e_1(a,b) = a_1b_1 \sum_{n\in E_\epsilon} n^1(1-n)^0 = a_1b_1 \sum_{n\in E_\epsilon} n = a_1b_1 \tag{18}$$

Secondly, because of the δ_{f,ρ_2}, the a_2b_2 containing term in $\alpha\beta$ gives

$$e_2(a,b) = \sum_{x=0}^{1}(1-x) \sum_{n\in E_\epsilon} n^x(1-n)^{1-x}a_2b_2\delta_{x,0} \tag{19}$$

This, in turn, implies that

$$e_2(a,b) = a_2b_2 \sum_{n\in E_\epsilon} n^0(1-n)^1 = a_2b_2 \sum_{n\in E_\epsilon} (1-n) = a_2b_2 \tag{20}$$

Looking at (16) we can have $e(a,b) = e_1(a,b) + e_2(a,b)$ when the f can be selected from $\mathcal{F}$. It can be compared with the active pumping of f-containg blood through the veins of the formulae. So there must be active ongoing f-selection "above" the right hand terms given in (18) and 20). Hence,

$$e(a,b) = \begin{cases} a_1b_1, f = \rho_1 \\ a_2b_2, f = \rho_2 \end{cases} \tag{21}$$

Suppose, finally, the A and B functions are defined via

$$A = \mathtt{sign}(\alpha - \lambda), \quad B = \mathtt{sign}(\beta - \mu) \tag{22}$$

Here $\mathtt{sign}(y) = 2H_1(y) - 1$, with, $H_1(y) = 1 \Leftrightarrow y \geq 0$ and $H_1(y) = 0 \Leftrightarrow y < 0$, and $y \in \mathbb{R}$. A closed form for $H_1(y)$ is $\lim_{n\to\infty} \exp\left[-e^{-ny}/n\right]$.

If, e.g. in (15) we have $x = 0$, i.e. $\delta_{x,0} = 1$, and $f = \rho_1$, i.e. $\delta_{f,\rho_1} = 1$, then we have $\alpha = 0$ and $\beta = 0$. The definition of H upon which the definition of $\mathtt{sign}$ rests, warrants that there is ±1 for A and B in this case. The λ and μ are both uniform density variables on the interval $[-1, 1]$. We then have that both $A = \mathtt{sign}(0-\lambda)$ and $B = \mathtt{sign}(0-\mu)$ project in $\{-1, 1\}$ and can be meaningfully integrated in a Bell type correlation formula. Hence, they are allowed as measurement functions.

Evaluation II. Let us employ the tropical algebra operator $\oplus$ in relation to f as a part of the integration in (2). The $\oplus_{f\in\mathcal{U}}$ operation is the hart that pumps the f-blood through the veins. Note, $\mathcal{F} \subset \mathcal{U}$, with $\mathcal{U}$ a proper function space. We note here that the integration over f is in fact over the density function space. So this is most likely a proper justification of the use of $\oplus$ related to f. We have for the requirement $\int d\lambda'\rho(\lambda') = 1$

$$\bigoplus_{f\in\mathcal{U}} \iota(f\in\mathcal{F}) \sum_{x=0}^{1} f(x) \sum_{n\in E_\epsilon} g(n,x) \int_{-1}^{1} \frac{d\lambda}{2} \int_{-1}^{1} \frac{d\mu}{2} = 1 \tag{23}$$

Note that because of the $\oplus_{f\in\mathcal{U}}$ operation, the outcome of (23) using (14) and (7) is unity.

The steps to this result can be provided as follows. We know that the μ and λ integrals in (23) are unity. I.e. $\int_{-1}^{1} \frac{d\mu}{2} = 1$. The sum

$$\sum_{x=0}^{1} f(x) \sum_{n\in E_\epsilon} g(n,x) = 1 \tag{24}$$

such as was already demonstrated previously in (14), when we look at it from the perspective $\iota(f \in \mathcal{F}) = 1$. This leaves us with an $\oplus$ operation that looks like

$$\ldots 0 \oplus 1 \oplus 1 \oplus 0 \ldots = 1 \oplus 1 = 1 \tag{25}$$

This evaluation is in accordance with the $\oplus$ definition in (7). Hence, equation (23) is verified. There is a unity outcome but the f are not hidden variables such as in Bell's formula. The f represents probability densities for the variable $x \in \{0, 1\}$. We have two of them $\rho_1 = x$ and $\rho_2 = 1 - x$.

$$E(a,b)=\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}g(n,x)\int_{-1}^{1}\frac{d\lambda}{2}\int_{-1}^{1}\frac{d\mu}{2}\mathtt{sign}(\alpha-\lambda)\mathtt{sign}(\beta-\mu) \tag{26}$$

Correlation. Note,that $|\alpha|\leq 1$ and $|\beta|\leq 1$. Moreover, there is distributivity for $a,b,c\in\{0,1\}$. This is true because as can be verified, $(a\oplus b)c=(ac\oplus bc)$. This is relevant to the computation of the correlation because both ρ_1 and ρ_2 in $\{0,1\}$. Because $(a\oplus b)c$ is a number in $\{0,1\}$, it can be employed in further "normal" mathematics when selection of f, via the iota and Kronecker delta funtions has taken place. Kronecker delta also projects in $\{0,1\}$. The computation of the $E(a,b)$ is rather lengthy but it can be easily followed. Let us begin with looking at (26). We know that

$$\int_{-1}^{1}\frac{d\lambda}{2}\int_{-1}^{1}\frac{d\mu}{2}\mathtt{sign}(\alpha-\lambda)\mathtt{sign}(\beta-\mu)=\alpha\beta \tag{27}$$

This reduces (26) to

$$E(a,b)=\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}g(n,x)\alpha\beta \tag{28}$$

From the definition of α and β in (15) and the discussion, we then arrive at two terms. The first is:

$$\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}g(n,x)a_1b_1\delta_{x,1}\delta_{f,\rho_1}= \tag{29}$$

$$a_1b_1\sum_{x=0}^{1}\sum_{n\in E_\epsilon}g(n,x)\delta_{x,1}\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})f(x)\delta_{f,\rho_1}$$

Note that $a_1b_1\in[-1,1]$ and falls under the spell of the semi-ring defined with the topped sum $\oplus$. This justifies the commutation of a_1b_1 with $\oplus$. Therefore, with $\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})f(x)\delta_{f,\rho_1}=\dots 0\oplus x\oplus 0\oplus 0\dots=x$, with $x\in\{0,1\}$, the first term in the $E(a,b)$ is, looking at (29)

$$\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}g(n,x)a_1b_1\delta_{x,1}\delta_{f,\rho_1}= \tag{30}$$

$$a_1b_1\sum_{x=0}^{1}\sum_{n\in E_\epsilon}g(n,x)\delta_{x,1}x=$$

$$a_1b_1\sum_{n\in E_\epsilon}n=a_1b_1\left(\epsilon+1-\epsilon\right)=a_1b_1$$

The f summation $\oplus$ on the one hand and the x and n summations on the other are independent of each other. That is why $\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})$ and $\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}$ can be interchanged. The second term from the product $\alpha\beta$ is

$$\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}g(n,x)a_2b_2\delta_{x,0}\delta_{f,\rho_2}= \tag{31}$$

$$a_2b_2\sum_{x=0}^{1}\sum_{n\in E_\epsilon}g(n,x)\delta_{x,0}\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})f(x)\delta_{f,\rho_2}$$

$a_2b_2\in[-1,1]$, hence under the spell of the semi-ring algebra of $\oplus$. We know, $\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})f(x)\delta_{f,\rho_2}=\dots 0\oplus 0\oplus(1-x)\oplus 0\dots=1-x$, with $x\in\{0,1\}$, therefore $1-x\in\{0,1\}$, the second term in the $E(a,b)$ evaluation is

$$\bigoplus_{f\in\mathcal{U}}\iota(f\in\mathcal{F})\sum_{x=0}^{1}f(x)\sum_{n\in E_\epsilon}g(n,x)a_2b_2\delta_{x,0}\delta_{f,\rho_2}= \tag{32}$$

$$a_2b_2\sum_{x=0}^{1}\sum_{n\in E_\epsilon}g(n,x)\delta_{x,0}(1-x)=$$

$$a_2b_2\sum_{n\in E_\epsilon}(1-n)=a_2b_2(1-\epsilon+(1-(1-\epsilon))=a_2b_2$$

Because $\alpha\beta$ in (28) is given as $a_1b_1\delta_{x,1}^2\delta_{f,\rho_1}^2+a_2b_2\delta_{x,0}^2\delta_{f,\rho_2}^2$ and squared Kronecker deltas are Kronecker deltas, we find $E(a,b)=a_1b_1+a_2b_2$.

Conclusion & discussion. – The presented local model shows that in $d=2$ euclidean unity parameter vector space, Bell's inequality can be violated. The local model reproduces the $d=2$ quantum correlation and in a similar way like [11], it is a conflicting branch of the physics behind Bell's theorem.

A sceptical reader may want to hit the brakes here and claim that this is not Bell's formula. Agreed, but can the sceptical reader give reasons why this refers *not* to the Bell experiment? If the counting methodology of a Bell experiment is used, that is, if in experiment

$$E(a,b)=\frac{N_{=}(a,b)-N_{\neq}(a,b)}{N_{=}(a,b)+N_{\neq}(a,b)}$$

is used, with $N_{=}(a,b)$ the number of equal spin measurements under settings pair (a,b) and $N_{\neq}(a,b)$ the number of unequal spin measurements under setting pair (a,b), then is there any real tested idea beyond theoretical assumptions, about how $N_{=}(a,b)$ or $N_{\neq}(a,b)$ are generated?

The model has the advantage that the model is relatively simple. The question, "show us where Bell is wrong", the reader is refered to [10], [11] and [12] for more mathematical details. That question is not relevant here because we are looking at Bell's experiment and not Bell's formula per se. For a computational violation of the CHSH the reader is referred to [5] which connects to [6] in its method.

Of course one can ask questions about the Bell - validity of a selection of functions $f\in\mathcal{F}$. Note first that the total probability density is written down as

$$\rho_{Bell}=\frac{1}{4}H(1+\lambda)H(1-\lambda)H(1+\mu)H(1-\mu)f(x)g(n,x)$$

Here, $f \in \mathcal{F} \equiv \{\rho_1, \rho_2\}$ with the functional forms, $\rho_1 = x$ and $\rho_2 = 1 - x$ and the variable $x \in \{0, 1\}$. So, $\rho_{Bell} \geq 0$ as required. Then, secondly, the integral of ρ_{Bell} is unity for $\oplus_{f \in \mathcal{U}} \iota(f \in \mathcal{F})$.

The only thing one can hold against this presented claim of Bell completeness rejection, is that f expressed as $\rho_1 = x$ is associated to the first slot of the measuring instrument parameter vector while the second slot has a different f with $\rho_2 = 1 - x$ and $x \in \{0, 1\}$ associated to it. Nobody knows if the first slot of a measuring system, in an actual physical instrument, is associated to another probability density form, via δ_{f,ρ_1}, than the second slot, via δ_{f,ρ_2}.

So, our claim represents a possible *physics* of a Bell experiment (1). In addition, the slot probability density variation is *not* a form of contextuality [7], [8]. This is so because, for instance, the density does not change when a and/or b changes. The slots (i.e. dimensions) of the parameter vector in the measurement machine are fixed but the values attached to the slots, the a_k and b_k $(k = 1, 2)$ can differ although the parameter vectors are of unit length. From the definitions of α and β we see that slot-1 (dimension 1) of both a and b parameter vector is associated to ρ_1. Slot-2 (dimension 2) is for both measrement instruments associated to ρ_2.

Therefore, if one wants to reject slot dependent density, one first has to *proof*, that this *physics* possibility is for sure ruled out in (1). One has to show that both slots are under the spell of a single density function. The second point is the use of tropical algebra operators as a valid representation of possible physics. Perhaps reasons are to be found such that tropical algebra is ruled out in physics.

The $\oplus$ operator is distributive to common multiplication in the domain we are looking at. For $a, b, c \in \{0, 1\}$, we have $(a \oplus b)c = (ac \oplus bc)$. This is relevant in our case because for $x \in \{0, 1\}$ both ρ functions project in $\{0, 1\}$.

The use of $\oplus_{f \in \mathcal{U}} \iota(f \in \mathcal{F})$ is an operation that is perhaps alien to Bell's formalism. However, we ask if it is alien to the *physics* of an experiment such as represented in (1). We then note that f is not a random *variable*. The ρ_{Bell} function also is not a *variable* subjected to the laws of classical probability. It is a *probability density* function and therefore plays a different role than the variables it governs.

In the present paper we tried to argue that the conclusion is not justified that in actual *experiment* (1) the system does not entangle along the lines of hidden variables physics. This could increase our insight into the physics behind the theorem [13].

Of course the sceptical reader will respond that this is all sheer speculation. However, that is a character trait of theory. The bias is that the speculative aspect of Bell's formula is overlooked. We conclude that the descriptioon of the Bell experiment is *not* fully covered by Bell's formula. The use of per-slot density cannot be ruled out beforehand. The use of topped summation cannot be ruled out beforehand. The use of tropical algebra tackling the possible deep nonlinearity of the physics behind the experiment cannot be ruled out beforehand.

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