The Quantised Angular Momenta of Astronomical Bodies

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The spin angular momentum of the sun precisely equals the quantum ħ multiplied by 2 raised to the power 250. Between them the next five nearest G or K-type stars take the exponents 247, 248, 249, 250 and 251, the binary star Alpha Centauri AB taking exponents (249 and 251) either side of 250 and Epsilon Eridani taking the exponent 250. The red dwarfs Proxima Centauri and Barnard's Star both take the exponent 240. The spin angular momentum quantisation scheme also applies to the planets of the solar system. Again, exponents that are multiples of 5 are preferred. Jupiter and Saturn behave as a binary planet, with exponents centred on 240. Uranus and Neptune behave as a binary planet, with exponents centred on 233. Earth and Mars take the exponents 225 and 220, respectively. The geometric mean orbital angular momentum of the eight planets equals the spin angular momentum of the sun. With spin parameters of intermediate value, the supermassive black holes located in the Milky Way and M31 galaxies take the exponents 295 and 305, respectively.

Introduction

The spin angular momentum of the sun is calculated in natural units (c = G = ħ = 1) as a power of 2 and shown to have an arresting value. The same procedure is applied to the nearest stars for which rotational periods have been measured and to the planets of the solar system. The orbital angular momenta of the planets and the Galilean moons of Jupiter are also calculated. Some observations are made before calculating the spin angular momenta of black holes.

CODATA values of the Planck scales are used in the analysis [1]. Unless stated, data on the sun and planets are taken from the NASA Space Science Data Coordinated Archive fact sheets [2].

Stars

The moment of inertia $I_\odot$ of the sun has been calculated as the product of the moment of inertia factor (0.070) and $M_\odot R_\odot{}^2$, where the mass $M_\odot$ = 1.98850 × 10^{30} kg and the volumetric mean radius $R_\odot$ = 6.957 × 10^8 m. In natural units, $I_\odot$ = 1.185 × 10^{124} or $\pi^{249.6}$, and so one may write

$$I_\odot \approx \pi^{250} \tag{1}$$

The Carrington sidereal rotation period $P_\odot$ = 25.38 days is used in the spin angular momentum calculation. In natural units, $P_\odot$ = 4.07 × 10^{49}, or $\pi^{99.8}$. One may write

$$P_\odot \approx \pi^{100} \tag{2}$$

Powers of π, π/2 and e where the exponent is a multiple of 5 and particularly 25 feature widely in the phenomenological Planck Model of Quantum Gravity [3]. For example, the Bohr radius equals $(\pi/2)^{125.00}$ in natural units.

The spin angular momentum of the sun, $S_\odot = I_\odot \omega_\odot$, or

$$S_\odot = 2\pi I_\odot / P_\odot \tag{3}$$

Using the above values of $I_\odot$ and $P_\odot$, $S_\odot$ equals $2^{250.02}$ in natural units, and so one may write

$$S_\odot = 2^{250}\ \hbar \tag{4}$$

Cang et al have calculated the spin angular momentum of the sun to be 1.8838×10^{41} kg.m^2.s^{-1} [4], which equals $2^{249.98}$ ħ.

The spin angular momenta S_* of Alpha Centauri A (G-type), Alpha Centauri B (K-type) and Epsilon Eridani (K-type) have been calculated from the values of mass, radius and rotation period in Table 1, adopting a moment of inertia factor equal to that (0.070) of the sun. The resulting values of S_* are shown in natural units (ħ) as powers of 2.

Star	Mass ($M_\odot$)	Radius ($R_\odot$)	Rotation Period (days)	Spin Angular Momentum S_* (ħ)
Sun	1	1	25.38*	$2^{250.02}$
Alpha Centauri A	1.1055(39) [5]	1.2234(53) [6]	22.5(5.9) [7]	$2^{250.92\pm0.34}$
Alpha Centauri B	0.9373(33) [5]	0.8632(37) [6]	36.2(1.4) [8]	$2^{248.99\pm0.06}$
Epsilon Eridani	0.82(2) [9]	0.735(5) [10]	11.2 [11]	$2^{250.02\pm0.04}$

* Carrington sidereal rotation period

Table 1: Parameters of the nearest four stars of type G or K, and the calculated values of S_*

While N = 250 for both the sun and Epsilon Eridani, the values of N (251 and 249, respectively) found for Alpha Centauri A and Alpha Centauri B are arranged symmetrically about N = 250. The arrangement of the binary Alpha Centauri AB resembles that of an isospin doublet on the mass levels of the Planck Model.

With $S_* = 2^N$ ħ, the values of N found for the fifth and sixth nearest G or K-type stars, the binary Cygni AB (both stars are of K-type), are 247 and 248, as shown in Table 2. The doublet arrangement of the binary star is centred on N = 247.5, a half-integer multiple of 5.

Star	Mass ($M_\odot$)	Radius ($R_\odot$)	Rotation Period (days)	Spin Angular Momentum S_* (ħ)
61 Cygni A	0.79 [12]	0.665 [13]	35.37 [14]	$2^{248.02\pm0.04}$
61 Cygni B	0.52 [12]	0.595 [13]	37.84 [14]	$2^{247.00\pm0.04}$

Table 2: Parameters of 61 Cygni AB and the calculated values of S_*

The values of N calculated for the six nearest G or K-type stars are plotted on a ladder in Figure 1.

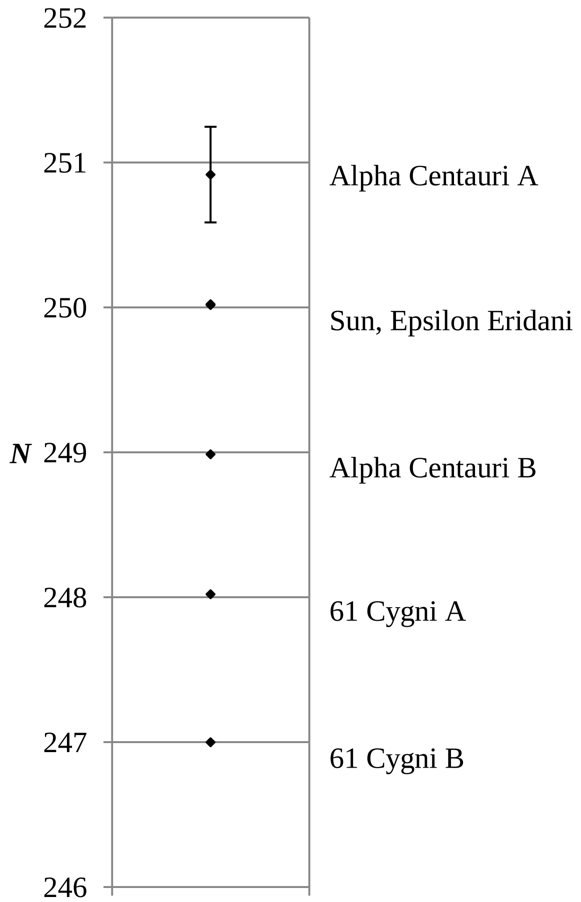

Figure 1: Values of N in $S_* = 2^N \hbar$ for the six nearest G or K-type stars. The error bar on the Alpha Centauri A marker reflects the large uncertainty in the measurement of rotation period for this star.

The spin angular momentum S_* has also been calculated for the two nearest red dwarfs: Proxima Centauri and Barnard's Star. The relevant stellar parameters and the calculated values of S_* are presented in Table 3.

Star	Mass ($M_\odot$)	Radius ($R_\odot$)	Rotation Period (days)	Spin Angular Momentum S_* ($\hbar$)
Proxima Centauri	0.1221(22) [15]	0.1542(45) [15]	82.6(1) [16]	$2^{239.89\pm0.11}$
Barnard's Star	0.144 [17]	0.196(8) [10]	130.4 [18]	$2^{240.16\pm0.12}$

Table 3: Parameters of Proxima Centauri and Barnard's Star, and the calculated values of S_*

Once again adopting a moment of inertia factor of 0.070, one finds that N = 239.89 ± 0.11 for Proxima Centauri and N = 240.16 ± 0.12 for Barnard's Star, both values being consistent with N = 240.

Planets

Moment of inertia is considered first. The sun's moment of inertia was found to be ~π^{250} in natural units. The moments of inertia of the planets are shown to equal powers of either π or e, both being characteristic bases of the Planck Model. The moment of inertia factors used in calculations on the largely gaseous planets of the solar system are based on the work of Podolak and Helled [19] and are shown in Table 4.

Planet	Moment of inertia factor
Jupiter	0.266
Saturn	0.226
Uranus	0.22
Neptune	0.25

Table 4: Moment of inertia factors used in calculations on the gaseous planets

Consider the outer six planets to comprise three sets of partners: Jupiter and Saturn; Uranus and Neptune; and Earth and Venus, with regard to moment of inertia. A characteristic moment of inertia I_{P} is calculated in natural units for each partnership as the geometric mean of the values found for the two planets of the partnership. Values of n_1 and n_3 are then calculated where $I_{\mathrm{P}} = \pi^{n_1} = \mathrm{e}^{n_3}$. Points (n_1, n_3) are plotted in Figure 2. For each partnership either n_1 or n_3 equals a multiple of 5. Mercury and Mars take fractional values of n_1.

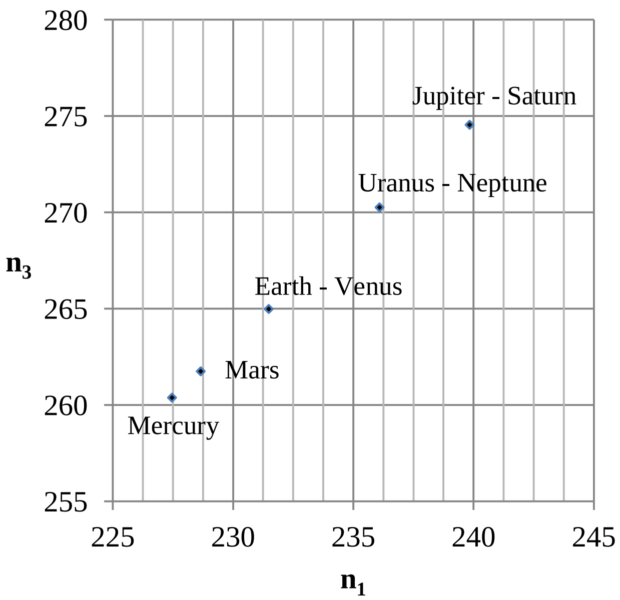

Figure 2: Values of n_1 and n_3 for a planet or planetary partnership where the moment of inertia $I_{\mathrm{P}} = \pi^{n_1} = \mathrm{e}^{n_3}$. The points (n_1, n_3) lie on a straight line since n_1 and n_3 are in constant ratio.

As for the rotation periods P_{P} of the planets, it is those of the rocky planets that are most clearly seen, in Figures 3 and 4, to equal powers of π and e.

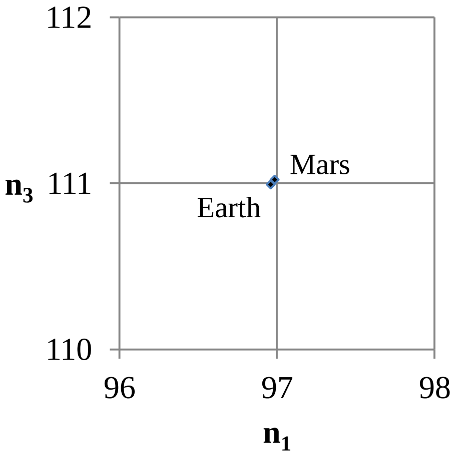

Figure 3: Values of n_1 and n_3 for Earth or Mars where the rotation period $P_{\mathrm{P}} = \pi^{n_1} = \mathrm{e}^{n_3}$

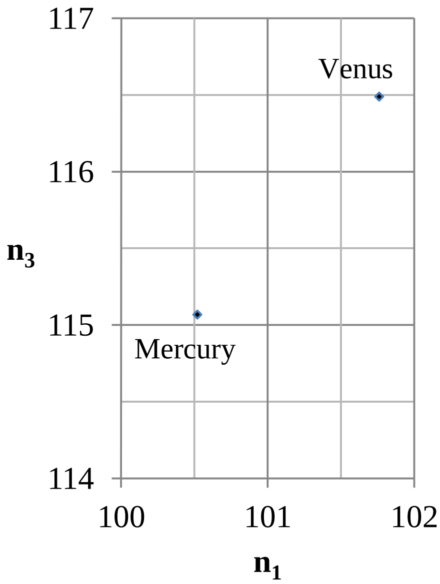

Figure 4: Values of n_1 and n_3 for Mercury and Venus where the rotation period $P_{\mathrm{P}} = \pi^{n_1} = \mathrm{e}^{n_3}$

The values of spin angular momentum $S_{\mathrm{P}} = 2^N$ ħ calculated for the planets are shown in Table 5. The values of N for the outer six planets are plotted on a ladder in Figure 5. Earth (N = 225) and Mars (N = 220) take integer values of N that are multiples of 5. The gaseous planets, as partnerships, take integer values of N. The Jupiter – Saturn partnership is centred on N = 240, a multiple of 5, and the Uranus-Neptune partnership is centred on N = 233. The situation regarding the inner planets Mercury and Venus is unclear.

While the moment of inertia and spin angular momentum of the sun are of value $\sim\pi^{250}\, I_{\mathrm{Planck}}$ and 2^{250} ħ, respectively, the moment of inertia and spin angular momentum characteristic of Jupiter and

Saturn, the predominantly massive planets, are of value $\sim\pi^{240}$ I_{Planck} and 2^{240} ħ, respectively. Interestingly, while the spin angular momentum characteristic of Alpha Centauri AB equals 2^{250} ħ, the spin angular momentum of Proxima Centauri, which orbits Alpha Centauri AB [15], equals 2^{240} ħ.

Planet	Spin Angular Momentum S_P (ħ)
Mercury	$2^{212.30}$
Venus	$2^{216.66}$
Earth	$2^{225.04}$
Mars	$2^{220.11}$
Jupiter	$2^{241.24}$
Saturn	$2^{238.64}$
Uranus	$2^{232.77}$
Neptune	$2^{233.21}$

Table 5: Values of spin angular momentum calculated for the planets of the solar system

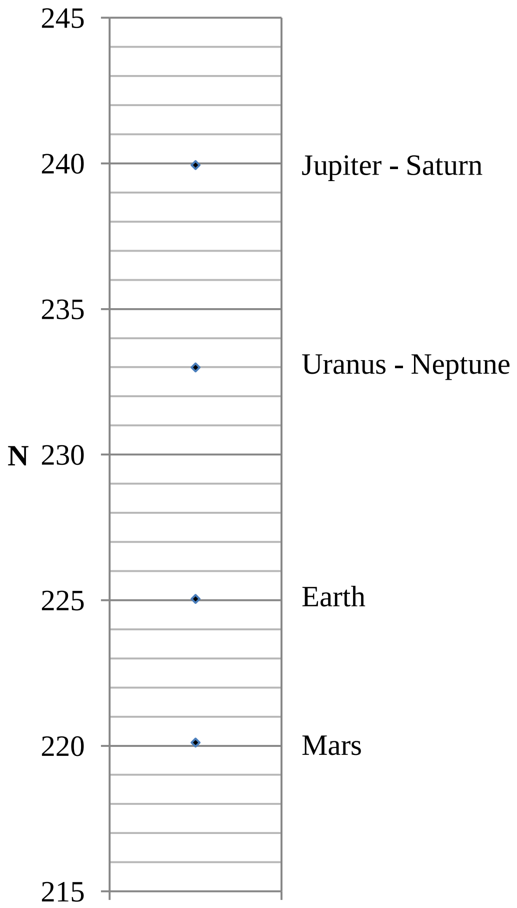

Figure 5: Values of N in $S_P = 2^N$ ħ for the outer six planets

The orbital angular momentum L_P of each of the eight planets of the solar system has been calculated using the equation

$$L_{\mathrm{P}} = M_{\mathrm{P}} R_{\mathrm{P}}{}^{2} \omega_{\mathrm{P}} \tag{5}$$

where M_{P} is the mass of a planet, R_{P} is its mean distance from the sun and ω_{P} (= 2π/orbital period) is its angular velocity. The results are shown in Figure 6 as values of N where $L_{\mathrm{P}} = 2^{N}$ ħ. The geometric mean orbital angular momentum $L_{\mathrm{P,mean}}$ is equal to $2^{249.90}$ in natural units, i.e.

$$L_{\mathrm{P,mean}} \approx 2^{250}\ \hbar \tag{6}$$

and therefore, from (6) and (4)

$$L_{\mathrm{P,mean}} \approx S_{\odot} \tag{7}$$

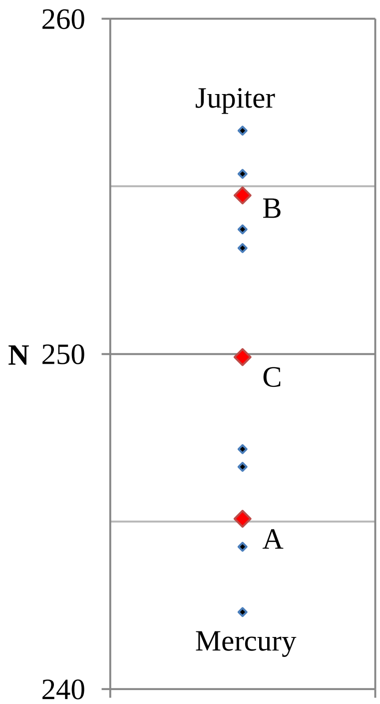

Figure 6: Values of N for the planets in the equation $L_{\mathrm{P}} = 2^{N}$ ħ. The eight planets lie in order of mass. The diamonds represent the mean values of N calculated for A: the rocky planets (245.08), B: the gaseous planets (254.73) and C: all eight planets (249.90).

The Galilean moons of Jupiter

To further investigate the relationship between the geometric mean orbital angular momentum of the major orbiting bodies of a system and the spin angular momentum of the central body the orbital angular momenta L_{G} of the synchronously rotating Galilean moons of Jupiter have been calculated and compared with the spin angular momentum calculated for Jupiter. The values of N for the moons, where $L_{\mathrm{G}} = 2^N$, are shown in Figure 7. The arrangement is characteristic of the model, being centred on a half-integer multiple of 5, and of 25. The geometric mean orbital angular momentum of the moons, $L_{\mathrm{G,mean}} = 2^{212.46}$ ħ, and the spin angular momentum of Jupiter, $S_{\mathrm{Jupiter}} = 2^{241.24}$ ħ, are related by a factor $e^{19.95}$. One may write

$$S_{\mathrm{Jupiter}}/L_{\mathrm{G,mean}} \approx e^{20} \qquad (8)$$

which is characteristic of the Planck Model.

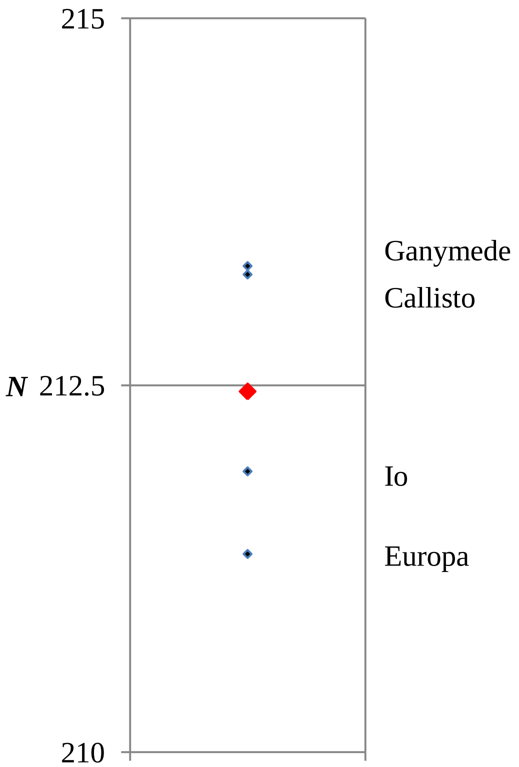

Figure 7: Values of N for the four Galilean moons of Jupiter in the equation $L_{\mathrm{G}} = 2^N$ ħ. The four moons lie in order of mass, Ganymede being the most massive. The diamond represents the mean value of N.

Observations on the above findings

Before moving on to consider the angular momenta of black holes, it is useful to consider the findings described above.

The spin angular momentum of each of the four nearest G or K-type stars to earth is either equal to 2^{250} ħ (Sun and Epsilon Eridani) or is characterised by the value 2^{250} ħ (Alpha Centauri AB). In this respect the sun appears to be an ordinary star, its distinctive value of spin reflecting its location in the middle of the Main Sequence. The value, 2^{250} ħ, of the geometric mean orbital angular momentum of the planets, which is equal to the spin angular momentum of the sun, suggests the sun and planets are arranged together in an optimum configuration.

Certain values of angular momentum seem to be preferred. For stars of G and K-type, $S_* = 2^{250}$ ħ is preferred. For the stars and planets generally, exponents of 2 that are integer multiples of 5 are preferred. Binary stars and planetary partnerships adopt a doublet configuration in which the geometric mean of the two values of spin angular momentum often takes a preferred value.

Supermassive black holes

Using natural units, the spin angular momentum S_{BH} of a Kerr black hole is given by

$$S_{\mathrm{BH}} = a{M_{\mathrm{BH}}}^2 \tag{9}$$

where a is the dimensionless spin parameter ($0 < a < 1$). The quantum number N is then found from

$$2^N = a{M_{\mathrm{BH}}}^2 \tag{10}$$

Values of N are calculated for the supermassive black holes (SMBH) in the Milky Way and M31, the two galaxies that dominate the Local Group. The SMBH at the centre of the Milky Way is of mass $4.15 \times 10^6\, M_\odot$ [20] and the SMBH at the centre of M31 is of mass $1.4 \times 10^8\, M_\odot$ [21]. As the spin parameters are not well established, N is calculated for three values of a: 0.99, a near-maximum value; 0.44 after Kato et al [22]; and 0.2, a low value, conjecturing that the two SMBHs have similar spin parameters. The results are shown in Figure 8. With $a = 0.44$, the values of N calculated from (10) for the SMBHs in the Milky Way and M31 are 294.99 and 305.14, respectively. With intermediate values of spin parameter a (~ 0.44), not only do the SMBHs take values of N that are multiples of 5 (295 and 305) but they also adopt the doublet configuration of the model, about a prominent value of N.

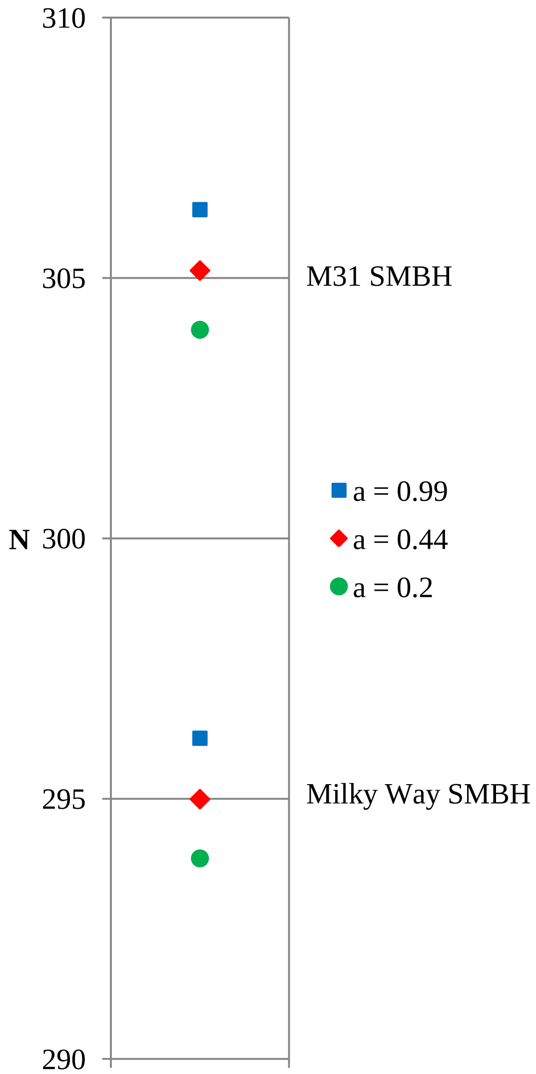

Figure 8: Values of the quantum number N in $2^N = aM_{\mathrm{BH}}{}^2$ for the supermassive black holes in the Milky Way and M31 galaxies. N has been calculated for different values of spin parameter a.

Gravitational wave events

From observations of gravitational wave events the LIGO Scientific Collaboration, and latterly also the Virgo Collaboration, has measured the masses of merging and remnant black holes. The spin parameters are well-constrained for the remnants, allowing calculation of the quantum number N using (10). Five confirmed black hole mergers have been observed to date. The values of mass and spin parameter of the remnants of the five gravitational wave events are shown in Table 6 together with the corresponding values of N, which are also shown in Figure 9.

Gravitational wave event	Remnant mass ($M_{\odot}$)	Remnant spin parameter *a*	*N*
GW150914*	$62.2^{+3.7}_{-3.4}$	$0.68^{+0.05}_{-0.06}$	$263.57^{+0.27}_{-0.30}$
GW151226*	$20.8^{+6.1}_{-1.7}$	$0.74^{+0.06}_{-0.06}$	$260.53^{+0.85}_{-0.37}$
GW170104*	$48.7^{+5.7}_{-4.6}$	$0.64^{+0.09}_{-0.20}$	$262.77^{+0.51}_{-0.83}$
GW170608*	$18.0^{+4.8}_{-0.9}$	$0.69^{+0.04}_{-0.05}$	$260.01^{+0.76}_{-0.26}$
GW170814**	$53.2^{+3.2}_{-2.5}$	$0.70^{+0.07}_{-0.05}$	$263.16^{+0.31}_{-0.28}$

*   LIGO
** LIGO/Virgo

Table 6: Values of mass, spin parameter *a* and quantum number *N* for the remnants of the five confirmed black hole mergers observed by LIGO and LIGO/Virgo

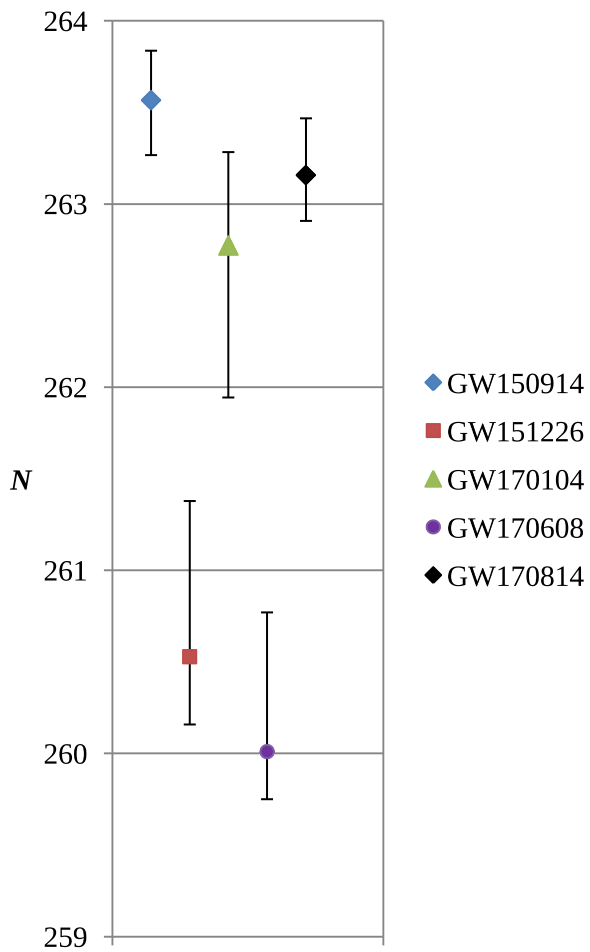

Figure 9: Values of the quantum number N in $2^N = aM_{\mathrm{BH}}{}^2$ for the remnants of the five confirmed black hole mergers observed by LIGO and LIGO/Virgo

The degree of uncertainty shown in Figure 9 is too large for integer numbers to be assigned to N for the remnant black holes. A clearer picture may emerge with more data. The third gravitational wave observing run is planned to commence around February 2019.

Conclusions

The spin angular momentum quantisation scheme for astronomical bodies is $S_{\mathrm{AB}} = 2^N$ ħ, where N is an integer. Values of N that are multiples of 5 are preferred.

Values of N for the nearby G and K-type Main Sequence stars cluster around $N = 250$. N = 240 for the nearby red dwarfs Proxima Centauri and Barnard's Star. Binary stars adopt the symmetrical doublet configuration of the Planck Model with the integer values of N centred on a preferred value.

Earth and Mars take preferred values of N. The gaseous planets do not take integer values of N but form two binary partnerships, each of which is centred on an integer value of N. The Jupiter-Saturn partnership is centred on a preferred value of N. The situation regarding Mercury and Venus is unclear.

The orbital angular momentum quantisation scheme is $L_{\mathrm{P}} = 2^N$ ħ. While the individual planets do not take integer values of N, the mean value of N for: the rocky planets; the gaseous planets; and all eight planets takes preferred integer values. The mean orbital angular momentum of the planets is of equal value to the spin angular momentum of the sun. The mean orbital angular momentum of the Galilean moons of Jupiter is related to the spin angular momentum of Jupiter.

With spin parameters of intermediate value (~ 0.44) the SMBHs in the Milky Way and M31 galaxies take preferred integer values of N and together adopt the symmetrical doublet configuration of the Planck Model.

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