Force on an electric current flowing through the inner area of a long uniformly cylindrical shell current

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Abstract

It is generally considered that an electric current does not experience any force when it flows through any area where the magnetic field is zero and any places having zero magnetic fields are identical in magnetism. However, if Newton's third law works, an electric current must feel a pressure or expansibility when it flowing through the inside area of a uniformly cylindrical shell current, where the magnetic field is zero. Concerning the pressure or expansibility on an electric current, the zero magnetic field area inside a long uniformly cylindrical shell current is not identical in magnetism with the absolute zero magnetic field which is far away from any magnetic sources.

Introduction

It is well known that the magnetic field is zero in the area inside a long uniformly cylindrical shell current [1]. As classical electromagnetic theory and experimental fact, an electric current flowing through a magnetic field is exerted a force by the field, called Ampere force [2, 3]. On the micro level, the Ampere force is originated from the Lorentz force [2, 4]. As far as we think, a current flowing through an area with zero magnetic strength does not experience any force. However, by detailed analysis to the space inside a long uniformly cylindrical shell current, a current flowing through this area will experience a pressure or expanding force although the magnetic field is zero there. The pressure or expanding force comes from the reaction force of cylindrical shell current.

Analysis and discussion

In figure1, there is a long cylindrical conductor shell and a uniformly direct current flowing along the shell. According to Ampere's circuit law, the magnetic field inside the shell is zero. A cylinder conductor is placed inside the shell and a current is flowing through the cylinder conductor. Now, a circular magnetic field around the cylinder conductor is produced and the outside shell current is located in the circuital magnetic field produced by the inner current. As Ampere's force law or Lorentz force law, the shell will experience a force from the magnetic field of inner current. The force is along the radial direction. When the two current are in same direction, the force pulls the shell to the center; when in opposite direct, the force pushes the shell away from the center.

According to Newton's third law, a reaction force from the outside shell will exert on the inner cylinder conduct. The reaction force is also along the radial direction towards to the center or away from the center, and in opposite direction with the force on the outside shell. Exactly, the reaction force should exert on the charge carriers of the inner current, so the force will push the charge carriers of inner current towards to the center of the cylinder conductor when the two currents are opposite direction, and will pull them to the edge of the cylinder conductor when the two currents in same direction. The reaction force on the inner current appears like a pressure or expansion force depending on the relative direction of the two currents.

For the cylindrical shell, suppose that the radial is $\boldsymbol{r}_s$, current $\boldsymbol{I}_s$; for the inner cylinder conductor, the radial $\boldsymbol{r}_i$, current $\boldsymbol{I}_i$. According to Ampere's force law, the magnitude of the force on the outside shell from the inner current is,

$$\boldsymbol{F}_s = \frac{\mu_0 \boldsymbol{I}_s \boldsymbol{I}_i}{\boldsymbol{r}_s} \qquad (1)$$

According to Newton's third law, a force with the same magnitude and opposite direction is exerted on the inner current from the outside shell current. So, the magnitude of the pressure on the inner current is proportional to the product of the two currents and reversely proportional to the radial of the outside shell.

We know that the force on the outside shell current is originated from Lorentz force. However, for the pressure on the inner current, we cannot say it is originated from Lorentz force because the inner current is flowing through a space where the magnetic field is zero. However, if Newton's third law works in this system, there must be this force. We have no reason to think that Newton's third law does not work in this case. The question is where does the force come from? It is only possible that the force is originated from the interaction of the two currents' magnetic fields, because the two currents' magnetic fields are overlapped in the area outside the cylindrical shell.

As an analogy, we may think that this force is also originated from a Lorentz like force, and imagine there is a circular magnetic field alike inside the uniformly cylindrical shell current. We call it 'pseudo-magnetic field.' Similar to the Ampere's force law, the force is proportional to the current and the pseudo-magnetic field. From the equation (1), the magnitude of the pseudo-magnetic field inside a uniformly cylindrical shell current is,

$$\boldsymbol{B} = \frac{\mu_0 \boldsymbol{I}_s}{\boldsymbol{r}_s} \qquad (2)$$

From equation (2), we see that the magnitude of this pseudo-magnetic field is same everywhere inside the shell current. It is proportional to the shell current and reversely proportional to the radial of the shell.

Not like normal magnetic field, a charged particle moving across this pseudo-magnetic field does not experience a net force, but an electric current flowing through will feel pressure or expansibility from the outside shell current, which driving the charge carriers of the current moving along radial direction to center or edge. If the direction of the force followed the similar rule as Lorentz force law, this pseudo-magnetic field will have opposite direction to that outside the shell, where it abbeys Ampere's circuital law, as shown in figure2.

As what we analyzed above, the pressure from the outside shell current drives the charge carriers of inner current to move along radial direction. This will cause the charge carriers more in center or in edge of the cylinder conductor, so an electric field will establish between the center and the edge of the cylinder conductor. This electric field drives the charge carriers of the cylinder conductor to resist the reaction force from the outside shell current until the charge carriers stop moving along the radial direction.

In the system of figure1, supposing the two electric currents are in opposite direction, let's estimate the magnitude of the established electric field in the inner cylinder conductor. In steady state, the force on a charge carrier by the established electric field is equal to force from the reaction interaction of the outside shell current, so,

$$\boldsymbol{E}ne = \frac{\mu 0 \boldsymbol{I}_{s} \boldsymbol{I}_{\mathrm{i}}}{\boldsymbol{r}_{s}} \qquad (3)$$

$$\boldsymbol{E} = \frac{\mu 0 \boldsymbol{I}_{s} \boldsymbol{I}_{\mathrm{i}}}{ne\, \boldsymbol{r}_{s}} \qquad (4)$$

Where, *E* is the established electric field inside the body of cylinder conductor, *e* electronic charge, *n* free electron density of the inner conductor.

As classical electromagnetism, the magnetic fields are both zero for a space far away from any magnetic sources and a space inside a uniformly cylindrical shell current, and the two zero fields are identical in magnetism. However, according to what we analyzed above, an electric current does not experience any force when flowing through the former zero magnetic field, but that does feel pressure or expansibility when flowing through the later zero magnetic field. So, at least concerning the interaction with an electric current, the two zero magnetic fields are not identical in magnetism. Based on the fact, we may say that it has absolute zero magnetic field for a space far away from any magnetic sources, but it has relative zero magnetic field for the space inside the uniformly cylindrical shell current. For the former one the zero magnetic fields are intrinsic zero, but for the later one the zero is due to canceling each other of non-zero magnetic fields.

Conclusions

For the system that an electric current flowing through the area inside a long uniformly cylindrical shell current, if Newton's Third Law works in this case, we will have following conclusions:

1. An electric current, flowing through this area, will experience a pressure or expansibility, although the magnetic field is zero there. The pressure or expansibility comes from the reaction force of the cylindrical shell current.
2. When the two currents are in opposite direction, the pressure pushes the charge carriers of inner current towards to the center, and when in same direction, pulls the charge carriers to the edge. So, an electric field will establish between the center and the edge of the inner cylinder conductor that an electric current flowing through.
3. The magnitude of the established electric field is proportional to the product of the two currents, reversely proportional to the radial of the outside shell and charge carrier density of the inner conductor.
4. For the space inside a long uniformly cylindrical shell current, though the magnetic field is zero, it is not identical with the zero magnetic field space which is far away from any magnetic or electric current sources.
5. Based on the analysis of this paper, it is more proper to say that there two kinds of zero magnetic fields, absolute zero and relative zero magnetic fields.

References

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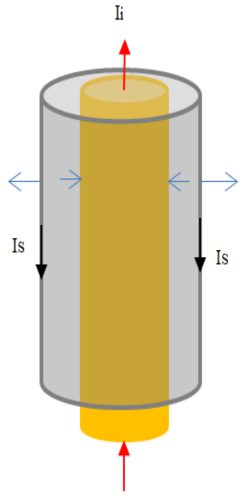

Fig.1. a cylinder conductor with current $\boldsymbol{I}_{\mathrm{i}}$ is inside a long uniformly shell current of $\boldsymbol{I}_{\mathrm{s}}$**.** The two currents are in opposite direction. The blue arrows indicate the direction of the forces on the cylinder conductor and the outside shell.

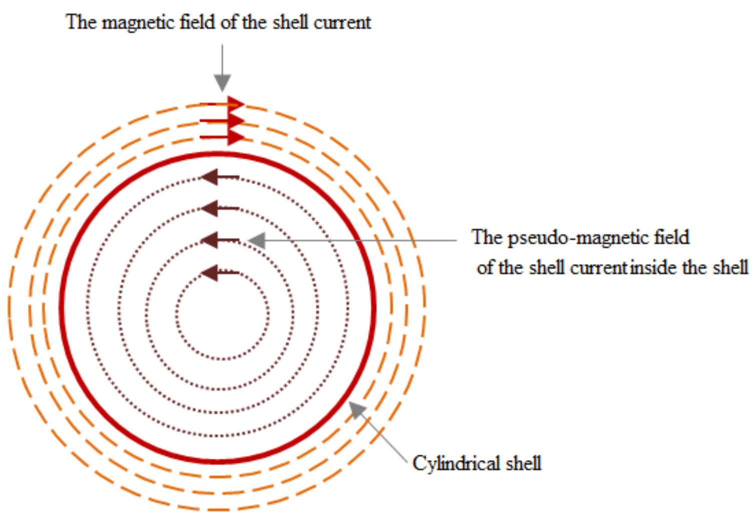

Fig.2. The cross section of a uniformly cylindrical shell current.
The current flows into the paper.