Black Hole Mass Decreasing, The Power and The Time of Two Black Holes in Coalescence, in The Quintessence Field

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Abstract

In this paper, we investigate some consequences of the black hole stabilization of Schwarzschild in the presence of quintessence type of dark energy which leads the way to the black hole mass decreasing other than the Hawking radiation process. The results show that in the quintessence field, the black hole shows a second-order phase transition, implying the existence of a stable phase. However, this stabilization implies some paradoxical effects on the black hole, which gives us a new regard about black holes, precisely we obtain the negative absolute temperature and we propose a process permitting us to appreciate well the likely cause of this phenomenon. These results allow us to give a new definition of the surface gravity for the Schwarzschild black hole in the field of quintessence, which depends on the flux of dilatation produced by the quintessence type of dark energy. Afterward, we analyze the impact of dark energy on the power and the time of two black holes needed to coalesce.

Keywords: Quintessence, black hole, second-order phase transition, negative absolute temperature, gravitational waves.

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1 Introduction

The discovery in the early 1970s that black holes radiate as black bodies has radically affect our understanding of general relativity, and offer us some early hints about the nature of quantum gravity. Since the seminal works of Hawking[1] and Bekenstein[2], it is understood that black holes behave as thermodynamic objects, with characteristics temperature and entropy.

Accelerated expansion of the universe is the most recent fascinating result of observational cosmology. To explain the accelerated expansion of the universe, it is proposed that the universe is regarded as being dominated by an exotic scalar field with a large negative pressure called "dark energy", which constitutes about 70% of the total energy of the universe[3]. There are several candidates for dark energy. "Quintessence" is one among them. It is characterized by a parameter ϵ, the ratio of the pressure to energy density of the dark energy, and the value of ϵ falls in the range $-1 \leq \epsilon \leq -\frac{1}{3}$[3]. Dark energy is the major component of the universe, so we can not talk about the thermodynamics of a black hole without knowing the influence of dark energy on it.

Phase transitions are ubiquitous in Nature. We are all familiar with the different phases of water (vapour, liquid and ice), and with the change from one to the other. Changes of phase are called phase transitions. Phase transition is a vast subject, and a lot of research effort is still being dedicated, both from experimental and theoretical points of view[4]. The phase transition of several systems, such as water, can be investigated precisely in laboratory, no laboratory is yet equipped with the tools necessary to probe the formation of a black hole in a phase transition of thermal space-time.

Several works have been done on the black hole phase transition[5–8]. Already in 1983, the classic paper by Hawking and Page[9] on black hole phase transitions appeared. The black hole phase transition can be studied theoretically in the light of the expression of its heat capacity[3]. Husain and Mann [8] suggested that the specific heat of a black hole becomes positive after a phase transition near the Planck scale. A lot of scientist have studied phase transitions caused by quintessence type of dark energy; Kamiko et *al.* [10] and Tharanath et *al.* [11] investigated second-order thermodynamic phase transition for the Schwarzschild black hole, Mahamat et *al.* [12, 13] investigated the second-order thermodynamic phase transition of the Reissner-Nordström and Regular Bardeen black hole. Dunkel and Hilbert[14] argue that negative absolute temperatures, a well-established thermodynamic concept, are inconsistent with thermodynamics. negative absolute temperatures is a well-known idea proposed and expounded by Ramsey [15] and Landau et *al.* [16]. Also, it is contradiction with usual meaning of temperature and with some basic concepts of physics and mathematics.

The theory of black hole mass decreasing was first introduced by Babichev et *al*[17]. In their paper, due to hydrodynamical analysis, they have shown that accretion of phantom energy is accompanied by the gradual decrease of the black hole mass. Since this, many investigations have been done about the possibility of black hole to loose its mass, due to phantom dark energy [18–22]. Thus, it would be important to study the behaviour of the black hole surrounded by quintessence type of dark energy, by a thermodynamics analysis.

Of the four fundamental interactions known to exist in nature, gravitation has the distinction of being the one with the longest history of observation. Yet, at the present time, the one about which we have the least experimental information[23]. Gravitational waves are propagating fluctuations of gravitational fields that is ripples in space-time, generated mainly by moving bodies. These distortions of space-time travel at the speed of light [24]. Several works have already been done on gravitational waves, since the work of Einstein, from the theoretical point of view as practical[25–29]. In particular, PC Peters et *al.*[30] have founded the power of gravitational waves emitted by a binary system, in a Minskowski space-time, without the influence of dark energy during the collision.

Our aim in this paper is to study by what process the black hole could lose its mass, otherwise than the Hawking radiation, and thus to present which role dark energy could have about black holes; we also analyze the modified formulas of the power of gravitational radiation and the time required for the coalescence of two black holes, taking into account dark energy.

The paper is organized as following: In Section 2, we briefly review the Schwarzschild black hole surrounded by quintessence and we discuss the thermodynamic quantities of this black hole by analyzing the phase transition. In section 3, we analyze some consequences of the second-order phase transition, including the decrease of the black hole mass and the modified surface gravity. In Section 4, we precisely study the the power and the time of two black holes needed to coalesce completely in the presence of dark energy. The last Section is devoted to a conclusion.

2 Thermodynamics phase transition of the Schwarzschild black hole

Kiselev[31] has derived a static spherically symmetric exact solution of Einstein equations for a black hole surrounded by quintessence, with the energy momentum tensor, which satisfies the conditions of additivity and linearity. The geometry of this black hole can be expressed using spherical coordinates $(r,\ \theta,\ \phi)$ as

$$ds^2 = g(r)dt^2 - \frac{1}{g(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where

$$g(r) = 1 - \frac{2M}{r} - \frac{a}{r^{3\epsilon+1}} \tag{2}$$

Here, M is the black hole mass and a is the normalization factor, which is positive, depending on the energy density of quintessence[11], like

$$\rho_q = -\frac{3a\epsilon}{2r^{3(\epsilon+1)}}. \tag{3}$$

The event horizon of the black hole can be found by using the following equation[32]

$$g(r_h) = 0, \tag{4}$$

and we can establish the relation between the mass of black hole and its horizon radius directly from Eq.(3) as

$$M = \frac{r_h}{2} - \frac{a}{2r_h^{3\epsilon}}. \tag{5}$$

We know that entropy can be written as [33, 34]

$$S = \frac{A}{4} = \pi r_h^2, \tag{6}$$

so that r_h can be written in terms of S as

$$r_h = \sqrt{\frac{S}{\pi}}. \tag{7}$$

Let us rewrite Eq.(5) using Eq.(7) as

$$M = \frac{1}{2}\left[\sqrt{\frac{S}{\pi}} - a\left(\frac{\pi}{S}\right)^{\frac{3\epsilon}{2}}\right]. \tag{8}$$

Now, we can deduce the thermodynamical quantities from the above expression of mass in terms of entropy[35]

$$T = \left(\frac{\partial M}{\partial S}\right), \tag{9}$$

$$C = T\left(\frac{\partial S}{\partial T}\right). \tag{10}$$

From Eq.(8) and Eq.(9), we will get the black hole temperature as

$$T = \frac{1}{4\sqrt{\pi}S^{\frac{1}{2}}} + \frac{3a\epsilon\pi^{\frac{3\epsilon}{2}}}{4S^{\frac{3\epsilon}{2}+1}}. \tag{11}$$

The same results can be found in ref.[36, 37].

Let us recall that Kuriakose et *al.*[3] also have computed the black hole temperature. Comparison with our results (see Eq.(11)), we can notice a difference in the denominator coefficient of the second term of the two temperatures T. But this difference does not affect the overall behavior of the temperature T.

By differentiating Eq.(11), we have

$$\left(\frac{\partial S}{\partial T}\right) = \left(\frac{\partial T}{\partial S}\right)^{-1} = -\left[\frac{64\pi^{\frac{1}{2}}S^{\frac{3\epsilon+7}{2}}}{8S^{\frac{3\epsilon+4}{2}} + 24a\epsilon\pi^{\frac{3\epsilon+1}{2}}(3\epsilon+2)S^{\frac{3}{2}}}\right].$$

Now, we can deduce the heat capacity, which is given in terms of entropy and quintessence parameters by:

$$C = -\left[\frac{16S^{3\epsilon+5} + 48a\epsilon\pi^{\frac{3\epsilon+1}{2}}S^{\frac{3\epsilon+9}{2}}}{8S^{3\epsilon+4} + \left(72a\epsilon^2\pi^{\frac{3\epsilon+1}{2}} + 48a\epsilon\pi^{\frac{3\epsilon+1}{2}}\right)S^{\frac{3\epsilon+7}{2}}}\right] \tag{12}$$

Tharanath et *al.*[3, 11] have founded the expression of the heat capacity of the Schwarzschild black hole in the quintessence field and by comparing our expressions, we can also notice a difference between our formulas which might have been caused by the difference in the expressions of temperature T.

To find out if a phase transition occurs, we first have to remember the relation between the heat capacity and the free enthalpy, and then plot it. For that, the best thermodynamic function adapted to the study of these transformations is then the free enthalpy G defined by

$$dG = -SdT + Vdp \qquad S = -\left.\frac{\partial G}{\partial T}\right|_V . \tag{13}$$

Therefore, we have

$$C = -T\frac{\partial^2 G}{\partial T^2} \tag{14}$$

Since C is the second derivative of the free enthalpy, we have to study the curve of heat capacity and to detect if we have any presence of a second-order phase transition. That is because any discontinuity of C expresses the presence of a second-order phase transition. As it's known, the Schwarzschild black hole in the model without dark energy, has a negative heat capacity, and does not present any phase transition; which means that the Schwarzschild black hole is unstable[38].

Let us remark that heat capacity(see Eq.(12)) could have this form after simplifications:

$$C = -2S\left[\frac{1+3a\left(\frac{S}{\pi}\right)^{-\frac{3\epsilon+1}{2}}}{1+3a(3\epsilon+2)\left(\frac{S}{\pi}\right)^{-\frac{3\epsilon+1}{2}}}\right], \tag{15}$$

and the same result has been found in ref.[37].

Now, we can plot the heat capacity of Schwarzschild black hole surrounded by quintessence.

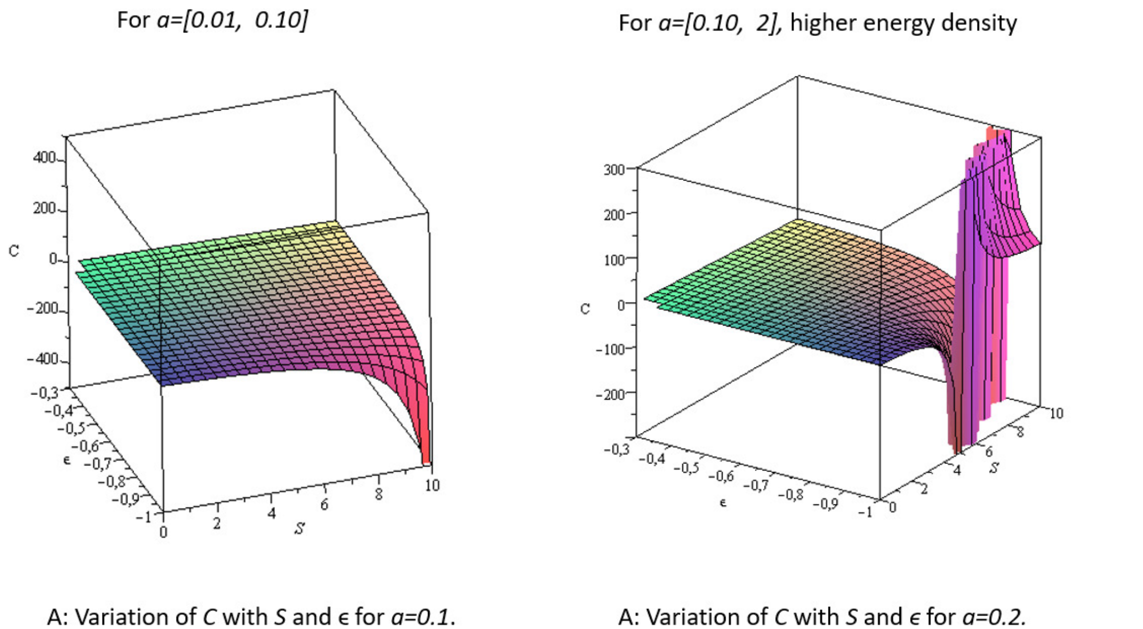

Figure 1: Comparison of heat capacities $C(S, \epsilon)$ for $S =]0, 10]$ and $\epsilon = [-1, -0.3]$
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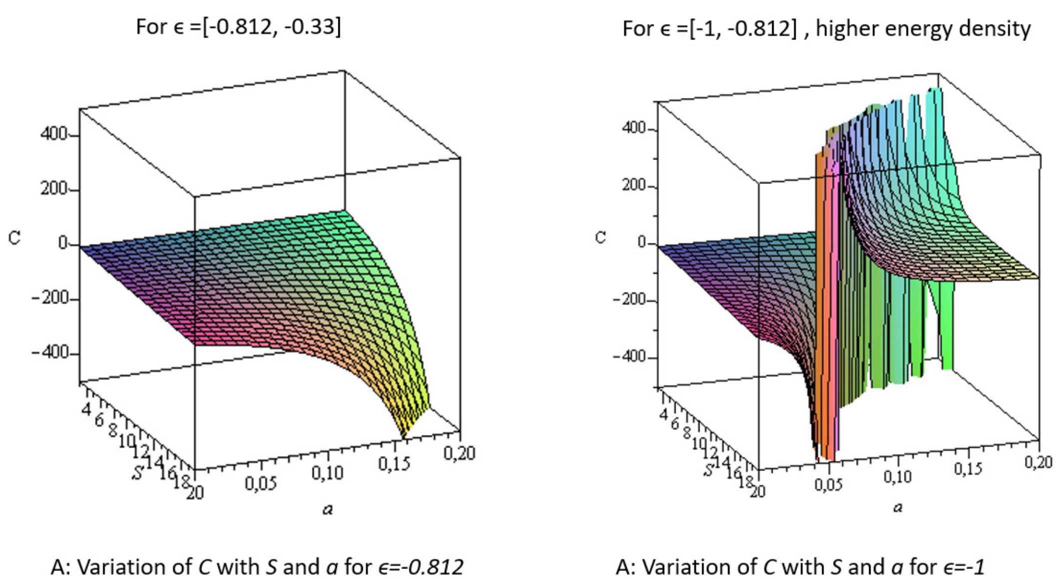


Figure 2: Comparison of heat capacities $C(S, a)$ for $S]0, 20]$ and $a[0, 0.20]$
.

First of all, if we analyze the Figures(1-A) and (1-B), we could see that the black hole becomes stable through the second-order phase transition when $a \geq 0.1$, corresponding to higher energy density(see Eq.(3)). Now, if we analyze the Figures(2-A) and (2-B), we could also see that the black hole becomes stable through the second-order-phase transition when $\epsilon \leq -0.812$. corresponding to higher energy density(see Eq.(3)).

Consequently, one note that for a certain variation of entropy and quintessence parameters, dark energy has a great influence on the thermodynamics stability of the Schwarzschild black hole.Thus, allowing it to be stable by a second-order phase transition. Furthermore, the stabilization of the black hole appears easily for higher values of dark energy density(see Eq.(3)).

At the moment, we know dark energy permits the black hole becoming stable by a second-order phase transition. Now, we are going to determine what consequences it might have on the black hole dynamics. This will be the aim of the next section.

3 Consequences of the second-order phase transition (Black hole stabilization)

Actually, we know that phase transition undergone by the black holes are caused by the presence of quintessence type of dark energy and we have some of certain thermodynamics functions which could permit us to apprehend modifications about the behavior of the black hole.

Before doing this exercise, let us remember Hawking and Bekeinstein[2] results about thermodynamics of the Schwarzschild black hole. Without dark energy($a = 0$), i.e in the vacuum, the solution of the Einstein equation and the thermodynamics parameters get the form

$$ds^2 = g(r)dt^2 - \frac{1}{g(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{16}$$

where

$$g(r) = 1 - \frac{2M}{r}, \tag{17}$$

and then we have

$$M = \frac{1}{2}\sqrt{\frac{S}{\pi}}, \quad T = \frac{1}{4\sqrt{\pi}S^{\frac{1}{2}}}, \quad \text{and} \quad C = -2S < 0.$$

Because $C < 0$, the black hole would be unstable and the consequences are:

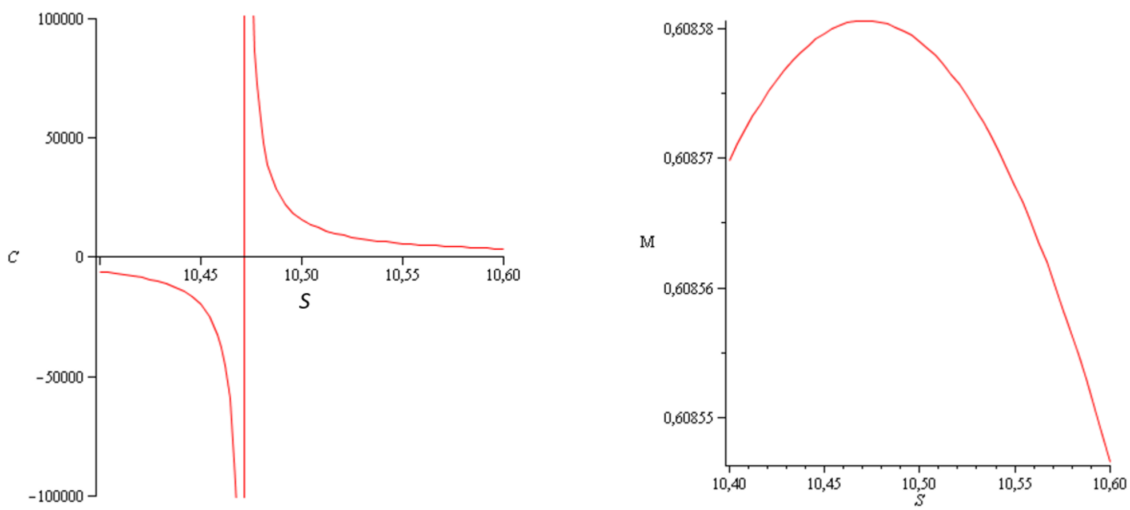
- In term of entropy, the black hole temperature should decrease without being negative;

- In term of entropy, the black hole mass should increase forever.

But in mechanics, if we suppose that it is not possible for any system to increase its mass(internal energy) infinitely, it should reach a critical state of instability which would enable him to be stabilized by explosion, like a Inner tube charged beyond its maximum capacity. However, it is known that the black hole is not an ordinary system. Indeed, there is one process which has been proposed by Soltan[39] that manages to slow the growth mass of black hole preventing it from becoming too massive. It is called the Soltan argument, namely that: The amount of light radiated into X-rays from accretion disk around black holes during the history of the universe permits to slow the growth mass of black holes.

Now, let us choose one case of stabilization and analyze the variation of mass M(see Eq.(8)), temperature T(see Eq.(11)) and the heat capacity C(see Eq.(15)).

For *a=0.1, ϵ=-1*



A: Heat capacity *C*

B: Mass *M*

Figure 3: Plots of heat capacity C and mass M as a function of S, for $a = 0.1,$ and $\epsilon = -1$

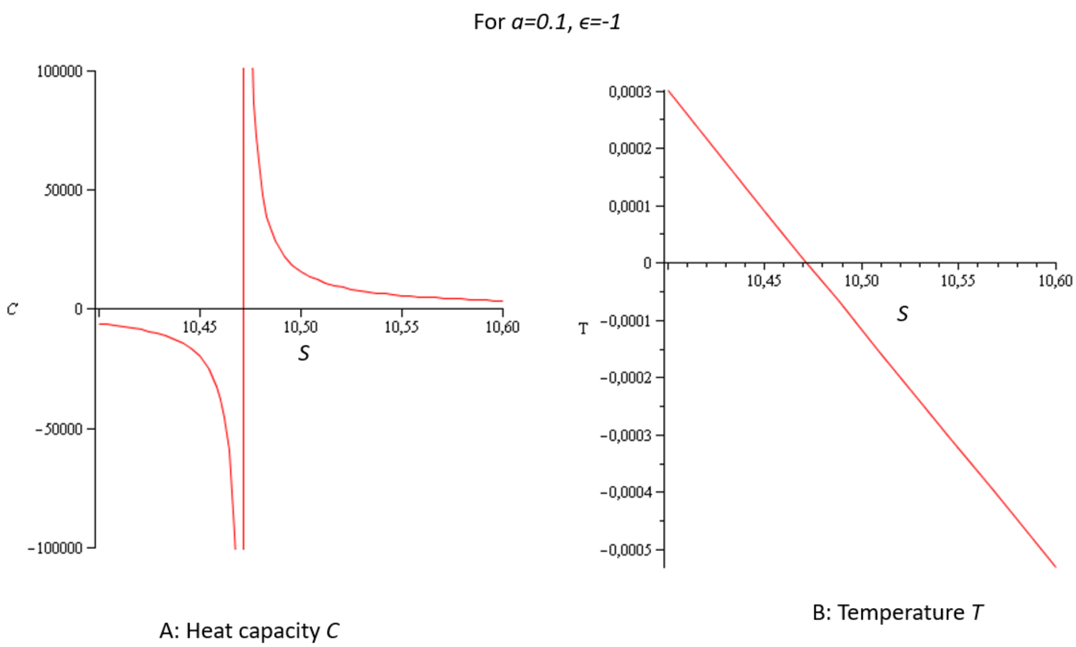


Figure 4: Plots of heat capacity C and temperature T as a function of S, for $a = 0.1$, and $\epsilon = -1$

Here we can determine that when the second-order phase transition occurs(presence of a discontinuity in the plot of Heat capacity) at the critical entropy, into the Schwarzschid black hole, it becomes stable, permitting it stop its mass increasing and now starts to decrease(see Fig.(3-B)). At the same time, its temperature(see Fig.(4-B)) becomes to be absolute negative. Now, how could we explain such a result?

According to the thermodynamics laws of black holes demonstrated by Carter, Hawking and Bardeen[34], it is mentioned that the temperature T is analogous with the surface gravity κ. The surface gravity, κ, of an astronomical or other object is the gravitational acceleration experienced at its surface. Also, we know that black hole is the result of the gravitational collapse of massive stars and then, it severely curves the space-time continuum, because of its high gravitational attraction effect. However, we know that the cause of the accelerated expansion of universe would be dark energy and it pushes away galaxies with its negative pressure. In some theories[40], it is asserted that dark energy would not be above the space-time continuum but inside; and then, causes its dilatation(pushing effect).

Explanation of the result

- There would be two antagonistic effects around the black hole: The attraction effect, caused by the great gravitational attraction(Φ_{GA}) on the space-time continuum; and the pushing effect, caused by the quintessence type of dark energy(Φ_{DE}) also in the same space-time continuum. Thus, the surface gravity κ would be the result of the superposition of these flux near the black hole event horizon. Then, we could write:

$$\kappa \propto [\Phi_{GA} - \Phi_{DE}]. \tag{18}$$

- Before the phase transition(when the black hole is unstable $C < 0$), the black hole has born, the gravitational attraction is stronger than the space-time dilatation caused by dark energy; then the black hole would increase its mass(see Fig.(3-B)) and that means $\kappa \approx T > 0$(see Fig.(4-B)). Therefore, we have

$$\Phi_{GA} > \Phi_{DE}. \tag{19}$$

- After the phase transition(when the black hole becomes stable $C > 0$), the gravitational attraction is instead weaker than the space-time dilatation caused by dark energy; which would give the possibility to the black hole to decrease its mass(Fig.(3-B)) and that means $\kappa \approx T < 0$(Fig.(4-B)). Therefore, we have

$$\Phi_{DE} > \Phi_{GA}. \tag{20}$$

Then, we can determine that dark energy effectively perturbs the black hole since it allows its to be stable.

Indeed, after the black holes formation, they are firstly unstable(Negative capacity $C < 0$), which allow them to increase their mass by attracting everything around them. At this epoch, the dark energy density was high, allowing thus easily their stabilization, by a second-order phase transition. Through this stabilization they could loose their mass.

After having studied the consequences of the Schwarzschild black hole stabilization, we will analyze now the impact of dark energy on the mechanics of two black holes in coalescence.

4 Power P of the gravitational radiation and time t_{spiral} of two black holes in coalescence in the presence of dark energy

Gravitational waves are propagating fluctuations of gravitational fields that is ripples in space-time, generated mainly by moving bodies. These distortions of space-time travel at the speed of light [24].

The weakness of the gravitational field is once again expressed as our ability to decompose the metric into the flat Minkowski metric plus a small perturbation:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \text{ where } |h_{\mu\nu}| \ll 1. \tag{21}$$

We will restrict ourselves to coordinates in which $\eta_{\mu\nu}$ takes its canonical form, $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$.

4.1 Power P of the gravitational radiation

Let us consider two black holes of mass M in a circular orbit in the $x^1 - x^2$ plane, at distance r from their common centre of mass. We will treat theirs motions in the Newtonian approximation.

Basing on the cosmological principle in the $x^1 - x^2$ plane, namely, that apart from local irregularities, the universe presents the same general aspect at every point[41] and in relation to the expansion of universe, meaning that everywhere in the universe, the expansion takes place in such a way to produce a dilatation at the same rate in all directions. In term of the angular frequency orbit $\Omega = (\frac{GM}{4r^3})^{1/2}$, we can write down the explicit path of black holes as

$$\begin{cases} x_a^1 &= r\cos(\Omega t) + \alpha,\ x_a^2 = r\sin(\Omega t) + \alpha, \\ \\ x_b^1 &= -r\cos(\Omega t) - \alpha,\ x_b^2 = -r\sin(\Omega t) - \alpha. \end{cases} \tag{22}$$

Then, the term responsible of the dark energy action would be α.

For reasons of homogeneity for our known about quintessence type of dark energy, we will suppose by (Eq.(2)) that

$$\alpha \propto a^{\frac{1}{3\epsilon+1}}, \tag{23}$$

where ϵ is the quintessence parameter and a the normalization factor.

The corresponding energy-momentum density [42] is

$$\begin{aligned} T^{00}(t,x)=&M\delta(x^3)\left[\delta(x^1-r\cos(\Omega t)-\alpha)\delta(x^2-r\sin(\Omega t)-\alpha)\right.\\ &\left.+\delta(x^1+r\cos(\Omega t)+\alpha)\delta(x^1+r\sin(\Omega t)+\alpha)\right]. \end{aligned} \tag{24}$$

Before computing the power P, we have first to find the expression of Q_{ij}, the traceless part of the quadrupole moment q_{ij}, which can be computed by the relation:

$$Q_{ij}=q_{ij}-\frac{1}{2}\delta_{ij}\delta^{kl}q_{kl}.$$

The quadrupole moment tensor is given by [30, 42]

$$q_{ij}(t)=3\int y^iy^jT^{00}(t,\vec{y})\ d^3y. \tag{25}$$

Then, the non-zero components of the quadrupole moment tensor are

$$\left\{\begin{array}{lll} q_{11} &=& 6M(r\cos(\Omega t)+\alpha)^2,\\ q_{22} &=& 6M(r\sin(\Omega t)+\alpha)^2,\\ q_{12} &=& 6M(r\cos(\Omega t)+\alpha)(r\sin(\Omega t)+\alpha)=q_{21}. \end{array}\right. \tag{26}$$

Thus, we can compute the components of Q_{ij}

$$\left\{\begin{array}{lll} Q_{11} &=& 6M\left[r\cos(\Omega t)+\alpha)^2-(r\sin(\Omega t)+\alpha)^2\right]\\ Q_{22} &=& 6M\left[r\sin(\Omega t)+\alpha)^2-(r\cos(\Omega t)+\alpha)^2\right]\\ Q_{12} &=& 6M(r\cos(\Omega t)+\alpha)(r\sin(\Omega t)+\alpha). \end{array}\right. \tag{27}$$

Now, the power P of the radiation is given by the formula[42]

$$P=\frac{G}{45}\left[\frac{d^3Q^{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3}\right]_{tr}. \tag{28}$$

Substituting Eq.(27) into Eq.(28), we obtain

$$P=\frac{75}{45}M^2\Omega^6\left[16r^4+2\alpha^2r^2+8\alpha r^3(\cos(\Omega t)+\sin(\Omega t))\right] \tag{29}$$

Setting $\alpha=0$, we get the following results which agrees with those of Ref [42].

In Figure (5), we have plotted the power variation P in terms of r and we can see that when the two black holes are going to coalesce (decrease of r), the power P naturally increases, but makes it slightly in the presence of dark energy ($\alpha \neq 0$). This implies that dark energy could decrease the power of gravitational radiation when the two black holes coalesce. Moreover, we determine a gradual increase of the radiation power level when we increase the value α.

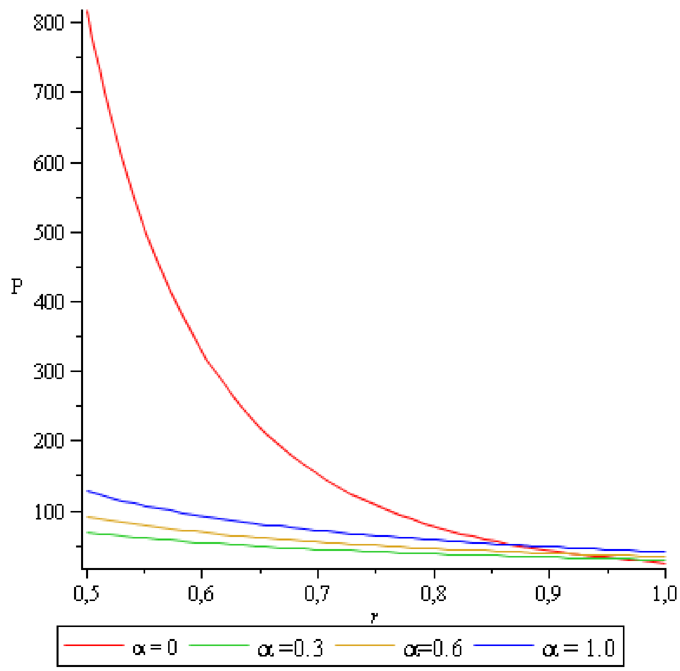


Figure 5: Evolution of the power P.

4.2 Time t_{spiral} of two black holes in coalescence

The lost energy can be described with its relation between the power of gravitational radiation as [42, 43]

$$P = \frac{dE}{dt} = -\frac{GM^2}{2r^2}\frac{dr}{dt}. \tag{30}$$

With the power computed above, we get the differential nonlinear equation which traduces the evolution of the distance between black holes

$$\frac{dr}{dt} = -B\frac{[16r^2 + \alpha + 8\alpha r(\cos\Omega t + \sin\Omega t)]}{r^5}, \quad \text{with} \quad B = \frac{G^2M^3}{20}, \tag{31}$$

which solutions are plotted in Fig.(6)

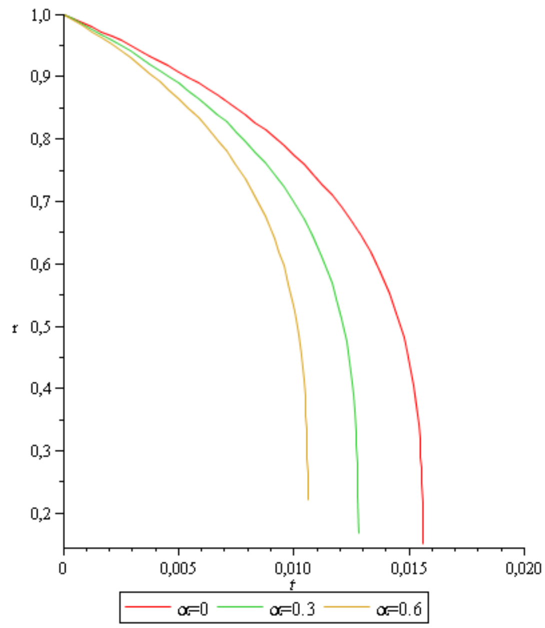


Figure 6: Evolution of the distances r between black holes with the time $t(s)$.

Analysis of this figure shows us that when the black holes coalesce, it is natural to think that the distance between black holes decreases in term of time. Now, if we introduce the term responsible of the presence of dark energy(α), we can determine that the time needed to coalesce completely t_{spiral} becomes smaller than what we have in the vacuum space-time. This time needed to coalesce t_{spiral} is determined when the distance takes the value $r(t) = 0$.

Summary and Conclusion

In summary, we have studied the influence of dark energy on two cases. The first is the Schwarzschild black hole and the second is the mechanics of two black holes in coalescence. In the first system, we have studied some thermodynamics properties of the Schwarzschild surrounded by quintessence type of dark energy. For that, we have first determined the temperature and the heat capacity and plotted them. The plot has shown the presence of a second-order phase transition, which happens more easily for high dark energy density and leads us to think that the black hole should have a stable thermodynamic phase.

Moreover, we have analyze some consequences of the black hole stabilization and By an analysis of the plot of temperature and mass, we have shown a strange behavior of the black hole. Precisely, the stabilization could lead it to loose its mass and to have a Negative absolute temperature and we putted out a new definition of surface gravity. Then, regardless of its mass, any black hole can stabilize itself in the presence of dark energy and thus, could lose some of its mass. Moreover, this phenomenon occurs easily in the higher energy density of quintessence.

For the second case, we calculated the power of the gravitational radiation that could be emitted by two black holes in coalescence and the time necessary for it. Our results showed that, in the presence of dark energy, the power would be less and the time required for the fusion would also be lower than when we ignore it. Therefore, dark energy would have the ability to decrease the level of collision power of black holes and increase the speed of their collision.

Our conclusion is that the dark energy, responsible for the accelerated expansion of the universe, would have a great goal in our universe as well for black holes as gravitational waves, observed by the black hole stabilization, the decreasing of the level of collision power and the time during a coalescence of two black holes.

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