Theorem for $w^n$and Fermat's Last theorem

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Abstract

We give an expression of w^n and the possible to apply for solving Fermat's Last theorem

Theorem 1. *$w^n = (u \pm v)^n$ can be always expressed as $(u \pm v)^n = u.F^2 \pm v.G^2$ when n is odd natural number, and can be always expressed as $(u \pm v)^n = F^2 \pm u.v.G^2$ when n is even natural number.(u,v are integers then F,G are also integers)*

Proof. n is odd, $n = 2m + 1$
Write : $u - v = (\sqrt{u} + \sqrt{v})(\sqrt{u} - \sqrt{v})$, then :
$(u - v)^{2m+1} = (\sqrt{u} + \sqrt{v})^{2m+1}(\sqrt{u} - \sqrt{v})^{2m+1}$
$= [\dfrac{(\sqrt{u} + \sqrt{v})^{2m+1} + (\sqrt{u} - \sqrt{v})^{2m+1}}{2}]^2 - [\dfrac{(\sqrt{u} + \sqrt{v})^{2m+1} - (\sqrt{u} - \sqrt{v})^{2m+1}}{2}]^2$
$= u.F^2 - v.G^2$
Write:$u + v = (\sqrt{u} + i\sqrt{v})(\sqrt{u} - i\sqrt{v})$, then :
$(u + v)^{2m+1} = (\sqrt{u} + i\sqrt{v})^{2m+1}(\sqrt{u} - i\sqrt{v})^{2m+1}$
$= [\dfrac{(\sqrt{u} + i\sqrt{v})^{2m+1} + (\sqrt{u} - i\sqrt{v})^{2m+1}}{2}]^2 - [\dfrac{(\sqrt{u} + i\sqrt{v})^{2m+1} - (\sqrt{u} - i\sqrt{v})^{2m+1}}{2}]^2$
$= u.F^2 + v.G^2$
n is even,$n = 2m$ then:
$(u - v)^{2m} = (\sqrt{u} + \sqrt{v})^{2m}(\sqrt{u} - \sqrt{v})^{2m}$
$= [\dfrac{(\sqrt{u} + \sqrt{v})^{2m} + (\sqrt{u} - \sqrt{v})^{2m}}{2}]^2 - [\dfrac{(\sqrt{u} + \sqrt{v})^{2m} - (\sqrt{u} - \sqrt{v})^{2m}}{2}]^2$
$= F^2 - u.v.G^2$
And:
$(u + v)^{2m} = (\sqrt{u} + i\sqrt{v})^{2m}(\sqrt{u} - i\sqrt{v})^{2m}$
$= [\dfrac{(\sqrt{u} + i\sqrt{v})^{2m} + (\sqrt{u} - i\sqrt{v})^{2m}}{2}]^2 - [\dfrac{(\sqrt{u} + i\sqrt{v})^{2m} - (\sqrt{u} - i\sqrt{v})^{2m}}{2}]^2$
$= F^2 + u.v.G^2$
Here:
i : imaginary unit $i^2 = -1; i^{4k} = 1; i^{4k+2} = -1$,$F = f(u, v), G = g(u, v)$ will not contain i (Since $i^{4k+1} = i; i^{4k+3} = -i$ is lost).

Special cases:
$u = u_0^2, v = v_0^2$:
$(u \pm v)^n = (u_0^2 \pm v_0^2)^n = u_0^2.F^2 \pm v_0^2.G^2 = (u_0F)^2 \pm (v_0G)^2$ for n is odd
$(u \pm v)^n = (u_0^2 \pm v_0^2)^n = F^2 \pm u_0^2v_0^2.G^2 = F^2 \pm (u_0v_0G)^2$ for n is even
Consequently,

Theorem 2. *The equation $x^2 \pm y^2 = z^n$ always has many infinite solutions in integer for any positive integer n*

Note:
Above expression is the only way or not, it depends on w (even or odd), u and v (square e^2or not square e, fe^2).
So that, be carefully when apply for specific case.
For the case w is odd, u and v are squares, $u = a^2, v = b^2$,a and b different parity, $w = a^2 - b^2$,

Above expression is the only way.
However, the case below:
$3 = 5 - 2$, then $3^3 = (5-2)^3 = [\frac{(\sqrt{5}+\sqrt{2})^3 + (\sqrt{5}-\sqrt{2})^5}{2}]^2 - [\frac{(\sqrt{5}+\sqrt{2})^3 - (\sqrt{5}-\sqrt{2})^3}{2}]^2$
$= 5.11^2 - 2.17^2$
But, there is other way such that:
$3^3 = (5-2)^3 = 5.5^2 - 2.7^2$.

1 Applying for FLt

$$x^n + y^n = z^n \tag{1}$$

n is odd, n = 2m + 1
The left hand side:

$$x^{2m+1} + y^{2m+1} = (x+y)(x^{2m} - x^{2m-1}y + x^{2m-2}y^2 - ... + y^{2m}) \tag{2}$$

we can write :$x^{2m+1} + y^{2m+1} = (x+y)Q$, here : $Q = x^{2m} - x^{2m-1}y + x^{2m-2}y^2 - ... + y^{2m}$
to consider FLt, it is enough to consider n prime, n = p.

Assume x and y are odd, we express Q as one of two formulas below:

$$Q_p = M^2 + pN^2 \tag{3}$$

or ;

$$Q_p = M^2 - pN^2 \tag{4}$$

Here: $M = f(a,b), N = g(a,b), a+b = x, a-b = y$. M and N are coprime.
For p = 3:
$Q_3 = a^2 + 3b^2$
For p = 5:
$Q_5 = (a^2 + 5b^2)^2 - 5(2b^2)^2$
For p = 7:
$Q_7 = a^2(a^2 + 7b^2)^2 + 7b^2(b^2 - a^2)^2$
For p = 11:
$Q_{11} = a^2(a^4 - 22a^2b^2 - 11b^4)^2 + 11b^2(b^4 + 2a^2b^2 - 3a^4)^2$
...
Since $x^p + y^p = z^p$, then $Q_p = w^p$ or $Q_p = pw^p$
w is not divisible by p.

Two equations must be considered :

$$M^2 + pN^2 = w^p (or M^2 - pN^2 = w^p) \tag{5}$$

$$M^2 + pN^2 = pw^p (or M^2 - pN^2 = pw^p) \tag{6}$$

For (6), $M = pM_0$, it yields:$pM_0^2 + N^2 = w^p$ (or $pM_0^2 - N^2 = w^p$)

2 The algorithm

Express w^p as:

$$w^p = M'^2 + pN'^2 \qquad (7)$$

or

$$w^p = M'^2 - pN'^2 \qquad (8)$$

Apply theorem above, let $w = c^2 + pd^2$ or $w = c^2 - pd^2$

For p = 3 :
$w^3 = (c^2 + 3d^2)^3 = (c + i\sqrt{3}d)^3(c - i\sqrt{3}d)^3$
$= [\frac{(c + i\sqrt{3}d)^3 + (c - i\sqrt{3}d)^3}{2}]^2 - [\frac{(c + i\sqrt{3}d)^3 - (c - i\sqrt{3}d)^3}{2}]^2$
$=c^2(c^2 - 9d^2)^2 + 3.3^2d^2(c^2 - d^2)^2$
$a = c(c^2 - 9d^2)$ and $b = 3d(c^2 - d^2)$;(in Euler' proof-1770 year)

For p = 5:

$w^5 = (c^2 - 5d^2)^5 = (c + \sqrt{5}d)^5(c - \sqrt{5}d)^5$
$= [\frac{(c + \sqrt{5}d)^5 + (c - \sqrt{5}d)^5}{2}]^2 - [\frac{(c + \sqrt{5}d)^5 - (c - \sqrt{5}d)^5}{2}]^2$
$c^2(c^4 + 50c^2d^2 + 125d^4)^2 - 5.5^2d^2(c^4 + 10c^2d^2 + 5d^4)^2$
$a^2 + 5b^2 = c(c^4 + 50c^2d^2 + 125d^4)$ and $2b^2 = 5d(c^4 + 10c^2d^2 + 5d^4)$;(in Dirichlet's proof-1825 year)
and

For p = 7:
$w^7 = (c^2 + 7d^2)^7 = (c + i\sqrt{7}d)^7(c - i\sqrt{7}d)^7$
$= [\frac{(c + i\sqrt{7}d)^7 + (c - i\sqrt{7}d)^7}{2}]^2 - [\frac{(c + i\sqrt{7}d)^7 - (c - i\sqrt{7}d)^7}{2}]^2$
$c^2(c^6 - 3.7^2c^4d^2 + 5.7^3c^2d^4 - 7^4d^6)^2 + 7.7^2d^2(c^6 - 5.7c^4d^2 + 3.7^2c^2d^4 - 7^2d^6)^2$
...

If it is the only way for the specific case, then there is only one choice, and not more. We obtain the two equations below:

$M = M'$

$N = N'$

If they have no solution in integer, FLt is solved for that case, if they have a solution in integer, then continue consider if it satisfy to condition $2a = w'^p$ $(w'w = z)$ when $p \nmid z$(or $2a = p^{p-1}w'^p$, when $p \mid z$) or not.

If the only way is not shown, then the proof of FLt by algorithm above is not completed (flawed)!

3 About Fermat's margin-notes

Around 1637, Fermat wrote his Last Theorem in the margin of his copy of the Arithmetica next to Diophantu's sum - of- squares problem:
It is impossible to separate a cube in two cubes, or a fourth power into two fourth powers, or in general, any power higher than second,into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.
It is not known whether Fermat had actually a valid proof for all exponents n.

I am Quang, Math independent researcher. In the letter was sent to The Annals of Math in 2015 year, I supposed that the short proof of Flt will appear , and Fermat could have a proof of FLt as he wrote (margin - notes). Indeed the short proof of FLt was found.

In my opinion, Fermat is famous enough , if he had a proof of Flt, publishing a proof of Flt or not, no problem for him, but for us . The short proof could be kept in mind without writing for memory.

4 Acknowledgement

I published a proof of the four color theorem in 2016 year, I think that professional and none -professional mathematicians could understand and verify it . I am very happy if my proof of the four color theorem by induction is properly verified and recognized before I publish the short proof of Flt.Thanh you!

□

References

Quang N V, A proof of the four color theorem by induction Vixra: 1601.0247 (CO)

APPENDIS

About proof of the FLt for n = 5

Dirichlet have proved FLt for n = 5 by infinite descent, his proof is correct if $w^5 = (c^2 - 5d^2)^5 = c^2(c^4 + 50c^2d^2 + 125d^4)^2 - 5.5^2d^2(c^4 + 10c^2d^2 + 5d^4)^2$ is the only way for expression $w^5 = M^2 - 5N^2$.*
If the condition* was shown! I give a very simple poof of FLt for n =5 without using infinite descent below:
Since $a^2 + 5b^2 = c(c^4 + 50c^2d^2 + 125d^4)$, and $2b^2 = 5d(c^4 + 10c^2d^2 + 5d^4)$, then $5 \mid b$, that means $x = a + b$ and $y = a - b$ is not divisible by 5. In other hand, if $x^5 + y^5 = z^5$, then one of x,y and z must divisible by y, it yields $5 \mid z$
It gives:
$5 \mid a^2 + 5b^2$, hence $5 \mid a$, it yields $5 \mid x$; $5 \mid y$ and $5 \mid z$,that means x,y and z have a common factor, a contradiction!

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