Relationship between dark energy with quantum mechanics and Fundamental forces

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Abstract

In this paper we discuss on the geometrical reason that leading to expansion of the universe. This reason tells us that vector fields and tensor fields in the manifolds lead to accelerating the manifolds. If we accept this theory we can describe basis of the quantum mechanics. We mean that if we imagine atoms as manifolds because of that acceleration is so low, all the vector fields and tensor fields that exist in atoms will be curved. So, all the vectors treat like waves, and this is basic reason for quantum mechanics that tell us particle in the atoms like electron treat like wave. In this paper we evaluate 4-vector as vector that treats like wave and finally we will talk about unification of forces with tensor fields. Since tensor fields lead to accelerating the manifolds, fundamental forces lead too. As this approach we could unify our forces in one equation.

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Introduction

After Einstein introduced his field equations for describing gravitation, Edwin Hubble discovered that our universe has been accelerating. everyone thought that this acceleration has been abating because of gravitation but after many years scientists discovered that acceleration have been increasing. Wonder is starting in here when one magical energy overcome to the gravity. The name of this energy is dark energy. For describing this phenomenon, we have many models. The most popular model is cosmological constant. This model adds a constant term to Einstein field equations. With this model we can measure density and pressure of our universe but density is constant. In this paper we evaluate a model that could predicts extending our universe and acceleration of our universe. Besides, in quantum mechanics there is an ambiguous problem that tell us why one particle in the atom treats like a wave. Our model can describe how this happens [4]. Einstein tried to unified the fundamental forces but he could not. In this paper with dark energy approach we unify the fundamental forces. we will understand that origin of all the vectors and tensors that exist in the universe are the fundamental forces so the factors of accelerating of our universe are fundamental forces.

Dark energy equation

For proving our claim that was said in abstract , we mean that "vector fields in the manifolds lead to accelerating the manifolds". We start with differential geometry equations [1]:

$$\nabla.\mathcal{V} = \partial_{\mu}\mathcal{V}^{\mu} + \mathcal{V}^{\upsilon}\Gamma^{\mu}{}_{\mu\upsilon} \quad , \tag{1}$$

$$\nabla.\mathcal{V} = \frac{1}{\sqrt{g}}\partial_{\mu}(\sqrt{g}\mathcal{V}^{\mu}) \quad . \tag{2}$$

In here we see v^{μ} is arbitrary vector field that exists in our manifold and g is determinant of the metric tensor. now we combine (1) and (2)

$$\frac{1}{\sqrt{g}}\partial_{\mu}(\sqrt{g})\mathcal{V}^{\mu}=\Gamma^{\mu}{}_{\mu\upsilon}\mathcal{V}^{\upsilon} \quad . \tag{3}$$

Equation (3) has really deep meaning. This equation describes relationship between arbitrary vector and curved space but this equation is so mere. We should make physical sprit for it.

Now we are coming back to the dark energy. For describing dark energy, we should use (3) for proving our claim.

As we know dark energy does not relate to matter [2] so that we can take out matter term in our Lagrangian. Therefore, we can write Lagrangian of the free particle [3], [4]

$$\mathcal{L}=\sqrt{-g}\mathcal{R}=\sqrt{g_{\alpha\beta}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}} \quad , \tag{4}$$

And now put (4) into the (3)

$$\frac{1}{\sqrt{g}}\partial_{\alpha}(\frac{\mathcal{L}}{i\mathcal{R}})\mathcal{V}^{\alpha}=\mathcal{V}^{\upsilon}\Gamma^{\alpha}{}_{\alpha\upsilon} \quad , \tag{5}$$

And take partial differential from L toward X^{α}

$$\frac{\partial\mathcal{L}}{\partial\mathcal{X}^{\alpha}}=\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt} \quad , \tag{6}$$

Now put (6) into the (5)

$$\frac{1}{\sqrt{g}}(\frac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}}{-\mathcal{R}^{2}})\mathcal{V}^{\alpha}=\mathcal{V}^{\upsilon}\Gamma^{\alpha}{}_{\alpha\upsilon} \quad , \tag{7}$$

$$(\frac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}}{-\mathcal{R}^{2}})\mathcal{V}^{\alpha}=-\sqrt{-g}\mathcal{V}^{\upsilon}\Gamma^{\alpha}{}_{\alpha\upsilon} \quad , \tag{8}$$

$$(\frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial \mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}-\frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}}\mathcal{L}^{2})\mathcal{V}^{\alpha}=\sqrt{-g}\mathcal{V}^{\upsilon}\Gamma^{\alpha}{}_{\alpha\upsilon}\mathcal{L}\mathcal{R}^{2}, \tag{9}$$

$$\mathcal{L}=\sqrt{-g}\mathcal{R}\rightarrow\sqrt{-g}=\frac{\mathcal{L}}{\mathcal{R}} \quad , \tag{10}$$

Now we put (10) into the (9)

$$(\frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial \mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}-\frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}}\mathcal{L}^{2})\mathcal{V}^{\alpha}=\frac{\mathcal{L}^{2}}{\mathcal{R}}\mathcal{V}^{\upsilon}\Gamma^{\alpha}{}_{\alpha\upsilon}\mathcal{R}^{2} \quad . \tag{11}$$

for keeping our calculations, we need christoffel symbol[1] :

$$\Gamma^{\beta}{}_{\alpha\beta}=\frac{1}{2}g^{\alpha\beta}\frac{\partial g_{\alpha\beta}}{\partial x^{\alpha}}\rightarrow 2\Gamma^{\beta}{}_{\alpha\beta}g_{\alpha\beta}=4\frac{\partial g_{\alpha\beta}}{\partial x^{\alpha}}\rightarrow\frac{\partial g_{\alpha\beta}}{\partial x^{\alpha}}=\frac{1}{2}g_{\alpha\beta}\Gamma^{\beta}{}_{\alpha\beta} \quad , \tag{12}$$

Now we put (12) into the (11)

$$(\frac{1}{4}g_{\alpha\beta}\Gamma^{\beta}{}_{\alpha\beta}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}-\frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}}\mathcal{L}^{2})\mathcal{V}^{\alpha}=\mathcal{V}^{\upsilon}\Gamma^{\alpha}{}_{\alpha\upsilon}g_{\alpha\beta}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R} \quad , \tag{13}$$

We should use geodesic equation [3] for make simple our equation and bring acceleration into our equation and make contraction $\upsilon\rightarrow\beta$:

$$\Gamma^{\beta}{}_{\alpha\beta}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}=-\frac{d^{2}\mathcal{X}^{\beta}}{dt^{2}} \quad , \tag{14}$$

Now We put (14) into the (13)

$$(-\frac{1}{4}g_{\alpha\beta}\frac{d^{2}\mathcal{X}^{\beta}}{dt^{2}}\mathcal{R}-\frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}}\mathcal{L}^{2})\mathcal{V}^{\alpha}=-\mathcal{V}^{\beta}g_{\alpha\beta}\frac{d^{2}\mathcal{X}^{\alpha}}{dt^{2}}\mathcal{R} \quad . \tag{15}$$

We know

$$\mathcal{R}=g^{\alpha\beta}\mathcal{R}_{\alpha\beta}, \tag{16}$$

We put (16) into the (15)

$$(-\frac{1}{4}g_{\alpha\beta}g^{\alpha\beta}\mathcal{R}_{\alpha\beta}\frac{d^{2}\mathcal{X}^{\beta}}{dt^{2}}-\frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}}\mathcal{L}^{2})\mathcal{V}^{\alpha}=-\mathcal{V}^{\upsilon}g_{\alpha\beta}g^{\alpha\beta}\mathcal{R}_{\alpha\beta}\frac{d^{2}\mathcal{X}^{\alpha}}{dt^{2}} \quad , \tag{17}$$

$$(-\mathcal{R}_{\alpha\beta}\frac{d^{2}\mathcal{X}^{\beta}}{dt^{2}}-\frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}}\mathcal{L}^{2})\mathcal{V}^{\alpha}=-4\mathcal{V}^{\upsilon}\mathcal{R}_{\alpha\beta}\frac{d^{2}\mathcal{X}^{\alpha}}{dt^{2}} \quad , \tag{18}$$

$$(\frac{d^2\mathcal{X}^\beta}{dt^2}\mathcal{V}^\alpha - 4\mathcal{V}^\beta\frac{d^2\mathcal{X}^\alpha}{dt^2})\mathcal{R}_{\alpha\beta} + \frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2\mathcal{V}^\alpha = 0 \quad . \tag{19}$$

As we see equation (19) show us that manifold is accelerating. Reason of this acceleration is that vector exists in the manifold. We call This equation as dark energy equation. In fact, dark energy is geometrical nature of every manifold that has vector field.

With this equation we understand many things about the universe even atoms because atoms could be manifold too. With this theory that every space has vector field, we can conclude that this space is extending. With this equation we can measure extending of the universe with Friedmann–Lemaître–Robertson–Walker metric.

How does quantum mechanics relate to the dark energy?

As we see in (19), if manifold has vector field, then manifold is accelerating.

Now see your perimeter carefully, our universe makes up by atoms and each atom could be manifold and every atom has vector field, so every atom is stretching right now, but acceleration is really low in the atoms because atom`s space is so small, we can prove it.

Look at equation (19) and limit from it. We mean that limit from our metric to the λ that is very close to the zero:

$$\lim_{g_{\alpha\beta}\to\lambda}\left[(\frac{d^2\mathcal{X}^\beta}{dt^2}\mathcal{V}^\alpha - 4\mathcal{V}^\beta\frac{d^2\mathcal{X}^\alpha}{dt^2})\mathcal{R}_{\alpha\beta} + \frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2\mathcal{V}^\alpha\right] = 0 \quad , \tag{20}$$

$$(\frac{d^2\overline{\mathcal{X}}^\beta}{d\bar{t}^2}\overline{\mathcal{V}}^\alpha - 4\overline{\mathcal{V}}^\beta\frac{d^2\overline{\mathcal{X}}^\alpha}{d\bar{t}^2})\overline{\mathcal{R}}_{\alpha\beta} + \frac{\partial\overline{\mathcal{R}}}{\partial\overline{\mathcal{X}}^\alpha}\overline{\mathcal{L}}^2\overline{\mathcal{V}}^\alpha = 0 \quad . \tag{21}$$

Bar symbol for each term means that our space-time has been limited, now we contract $\alpha \to \beta$

$$-3\overline{\mathcal{V}}^\beta\frac{d^2\overline{\mathcal{X}}^\beta}{d\bar{t}^2}\overline{\mathcal{R}}_{\beta\beta} + \frac{\partial\overline{\mathcal{R}}}{\partial\overline{\mathcal{X}}^\beta}\overline{\mathcal{L}}^2\overline{\mathcal{V}}^\beta = 0 \ . \tag{22}$$

Not with standing Lagrangian that limited and squared, we can consider a small number instead of second term of equation (22).

$$\frac{\partial \overline{\mathcal{R}}}{\partial \overline{\mathcal{X}}^{\beta}} \overline{\mathcal{L}}^{2} \overline{\mathcal{V}}^{\beta} \approx \lambda \quad . \tag{23}$$

So we come back to equation (22)

$$3\overline{\mathcal{V}}^{\beta} \frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}} \overline{\mathcal{R}}_{\beta\beta} \approx \lambda \quad , \tag{24}$$

So, we can conclude that acceleration ($\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}}$) is so low and we can conclude that in the small space, vector fields are curved, because vector field depends on Ricci tensor.

So far, we have understood that in the atoms, acceleration is very low because space is limited. Now we should prove that vector field in the atoms has wave-like behavior. If we prove that, we can conclude that the dark energy leads to that particles in atoms treat like a wave, because:

1- Space is limited
2- Vector fields are curved
3- Dark energy stretches the Manifold with low acceleration

With above conditions we want to prove mathematically treatment of wave-like particle in the atom.

We back to (21) and consider (23) and we select our arbitrary vector field as four-vector ($\overline{\mathcal{X}}^{\alpha} = (\bar{t}, \varphi, \psi, \xi)$)

$$\left(\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}} \overline{\mathcal{X}}^{\alpha} - 4\overline{\mathcal{X}}^{\beta} \frac{d^{2}\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^{2}}\right)\overline{\mathcal{R}}_{\alpha\beta} + \frac{\partial \overline{\mathcal{R}}}{\partial \overline{\mathcal{X}}^{\alpha}} \overline{\mathcal{L}}^{2} \overline{\mathcal{X}}^{\alpha} = 0 \quad , \tag{25}$$

Then we multiply $d\bar{t}^{2}$ into the (25)

$$(d^{2}\overline{\mathcal{X}}^{\beta}\overline{\mathcal{X}}^{\alpha}-4\overline{\mathcal{X}}^{\beta}d^{2}\overline{\mathcal{X}}^{\alpha})\overline{\mathcal{R}}_{\alpha\beta}+\lambda d\bar{t}^{2}=0 \quad , \tag{26}$$

$$(d^{2}\overline{\mathcal{X}}^{\beta}\overline{g^{\alpha\beta}}\overline{\mathcal{X}}_{\beta}-4\overline{g^{\alpha\beta}}\overline{\mathcal{X}}_{\alpha}d^{2}\overline{\mathcal{X}}^{\alpha})\overline{\mathcal{R}}_{\alpha\beta}+\lambda d\bar{t}^{2}=0 \, , \tag{27}$$

$$(d^{2}\overline{\mathcal{X}}^{\beta}\overline{\mathcal{X}}_{\beta}-4\overline{\mathcal{X}}_{\alpha}d^{2}\overline{\mathcal{X}}^{\alpha})\overline{g^{\alpha\beta}}\overline{\mathcal{R}}_{\alpha\beta}+\lambda d\bar{t}^{2}=0 \quad , \tag{28}$$

$$\overline{g^{\alpha\beta}}\overline{\mathcal{R}}_{\alpha\beta}=\overline{\mathcal{R}} \quad , \tag{29}$$

$$(d^{2}\overline{\mathcal{X}}^{\beta}\overline{\mathcal{X}}_{\beta}-4\overline{\mathcal{X}}_{\alpha}d^{2}\overline{\mathcal{X}}^{\alpha})\overline{\mathcal{R}}+\lambda d\bar{t}^{2}=0 \quad , \tag{30}$$

Then we take integral from (30) and consider that $\overline{\mathcal{R}}\overline{\mathcal{X}}_{\beta}$ is very low

$$\frac{1}{2}\overline{\mathcal{R}}(\overline{\mathcal{X}}^{\beta})^{2}\overline{\mathcal{X}}_{\beta}-4\frac{1}{2}\overline{\mathcal{R}}(\overline{\mathcal{X}}^{\alpha})^{2}\overline{\mathcal{X}}_{\alpha}+\frac{1}{2}\lambda\bar{t}^{2}=0 \, , \tag{31}$$

$$\overline{\mathcal{X}}^{\beta}\overline{\mathcal{X}}_{\beta}=\mathcal{X}^{2} \quad , \tag{32}$$

$$(\overline{\mathcal{X}}^{\beta}-4\overline{\mathcal{X}}^{\alpha})\overline{\mathcal{R}}\mathcal{X}^{2}+\lambda\bar{t}^{2}=0 \quad , \tag{33}$$

Now we should find λ from (21) and (23)

$$\left(\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}}\overline{\mathcal{X}}^{\alpha}-4\overline{\mathcal{X}}^{\beta}\frac{d^{2}\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^{2}}\right)\overline{\mathcal{R}}_{\alpha\beta}+\lambda=0 \, , \tag{34}$$

$$\lambda=\left(-\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}}\overline{\mathcal{X}}^{\alpha}+4\overline{\mathcal{X}}^{\beta}\frac{d^{2}\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^{2}}\right)\overline{\mathcal{R}}_{\alpha\beta} \, , \tag{35}$$

Then we put (35) into the (33)

$$(\overline{\mathcal{X}}^{\beta}-4\overline{\mathcal{X}}^{\alpha})\overline{\mathcal{R}}\mathcal{X}^{2}+\left(-\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}}\overline{\mathcal{X}}^{\alpha}+4\overline{\mathcal{X}}^{\beta}\frac{d^{2}\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^{2}}\right)\overline{\mathcal{R}}_{\alpha\beta}\bar{t}^{2}=0 \, , \tag{36}$$

Then we put $\alpha,\beta=0,0$

$$-3\bar{t}\overline{\mathcal{R}}\mathcal{X}^{2}+\left(-\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}}\overline{\mathcal{X}}^{\alpha}+4\overline{\mathcal{X}}^{\beta}\frac{d^{2}\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^{2}}\right)\overline{\mathcal{R}}_{\alpha\beta}\bar{t}^{2}=0 \quad , \tag{37}$$

$$\overline{\mathcal{R}}=\frac{1}{3\bar{t}\mathcal{X}^{2}}\left(\frac{d^{2}\overline{\mathcal{X}}^{\beta}}{d\bar{t}^{2}}\overline{\mathcal{X}}^{\alpha}-4\overline{\mathcal{X}}^{\beta}\frac{d^{2}\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^{2}}\right)\overline{\mathcal{R}}_{\alpha\beta}\bar{t}^{2} \, , \tag{38}$$

$$\overline{g^{\alpha\beta}}\,\overline{\mathcal{R}}_{\alpha\beta} = \overline{\mathcal{R}}\,, \tag{39}$$

$$\overline{g^{\alpha\beta}} = \frac{1}{3\bar{t}\mathcal{X}^2}\left(\frac{d^2\overline{\mathcal{X}}^{\beta}}{d\bar{t}^2}\overline{\mathcal{X}}^{\alpha} - 4\overline{\mathcal{X}}^{\beta}\frac{d^2\overline{\mathcal{X}}^{\alpha}}{d\bar{t}^2}\right)\bar{t}^2\,, \tag{40}$$

Then for first term we change dummy indices $\alpha \Leftrightarrow \beta$

$$\overline{g^{\alpha\beta}} = \frac{-\bar{t}}{\mathcal{X}^2}\left(\frac{d^2\overline{\mathcal{X}}^{\beta}}{d\bar{t}^2}\overline{\mathcal{X}}^{\alpha}\right)\,,$$

$$\overline{g^{\alpha\beta}} = \begin{bmatrix} 0 & \frac{-\overline{t^2}}{\mathcal{X}^2}\frac{d^2\varphi}{d\bar{t}^2} & \frac{-\overline{t^2}}{\mathcal{X}^2}\frac{d^2\psi}{d\bar{t}^2} & \frac{-\overline{t^2}}{\mathcal{X}^2}\frac{d^2\xi}{d\bar{t}^2} \\ 0 & \frac{-\varphi\bar{t}}{\mathcal{X}^2}\frac{d^2\varphi}{d\bar{t}^2} & \frac{-\varphi\bar{t}}{\mathcal{X}^2}\frac{d^2\psi}{d\bar{t}^2} & \frac{-\varphi\bar{t}}{\mathcal{X}^2}\frac{d^2\xi}{d\bar{t}^2} \\ 0 & \frac{-\psi\bar{t}}{\mathcal{X}^2}\frac{d^2\varphi}{d\bar{t}^2} & \frac{-\psi\bar{t}}{\mathcal{X}^2}\frac{d^2\psi}{d\bar{t}^2} & \frac{-\psi\bar{t}}{\mathcal{X}^2}\frac{d^2\xi}{d\bar{t}^2} \\ 0 & \frac{-\xi\bar{t}}{\mathcal{X}^2}\frac{d^2\varphi}{d\bar{t}^2} & \frac{-\xi\bar{t}}{\mathcal{X}^2}\frac{d^2\psi}{d\bar{t}^2} & \frac{-\xi\bar{t}}{\mathcal{X}^2}\frac{d^2\xi}{d\bar{t}^2} \end{bmatrix}. \tag{41}$$

As you see determinant of this matrix is zero, so we can conclude that manifold in the small space or early universe is Pseudo Riemannian.[9]

As you know, metric represents distance. If you plot this metric you can see our plot is like a wave. As you see, metric depends on acceleration, so this is the reason that particles in the small manifolds like atoms, treat like a wave. So dark energy is fundamental reason that particles in atoms treat like waves. Indeed, space-time is similar to wave in the small manifolds, because acceleration is very low. You can check that in (41), we mean that

$$\overline{g^{\alpha\beta}} = \underline{\underline{\frac{-\bar{t}}{\mathcal{X}^2}}}\,\frac{d^2\overline{\mathcal{X}}^{\beta}}{d\bar{t}^2}\overline{\mathcal{X}}^{\alpha}\quad. \tag{42}$$

Expression $\underline{\underline{\frac{-\bar{t}}{\mathcal{X}^2}}}$ is like to wave and acceleration is very low so our space is like to wave.

In here you can see figure of $\frac{-\bar{t}}{\chi^2}$ (43)

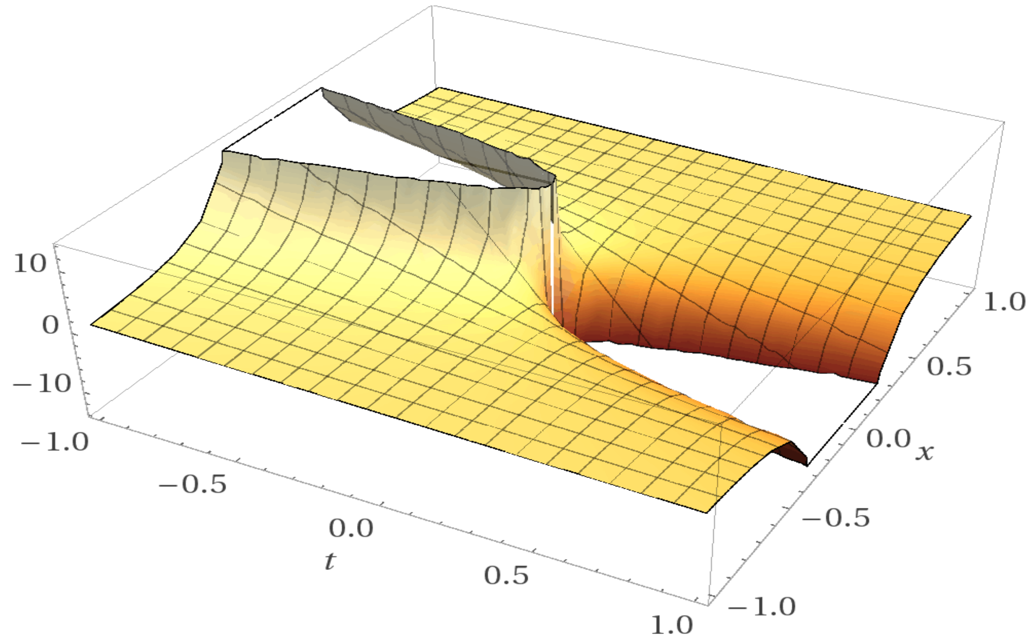

Figure (1). Wavelike metric for just one element of metric

And amazing thing that we can see is relationship between space and time is like what we see in the relativity, as you see in figure (2) light cone is created.

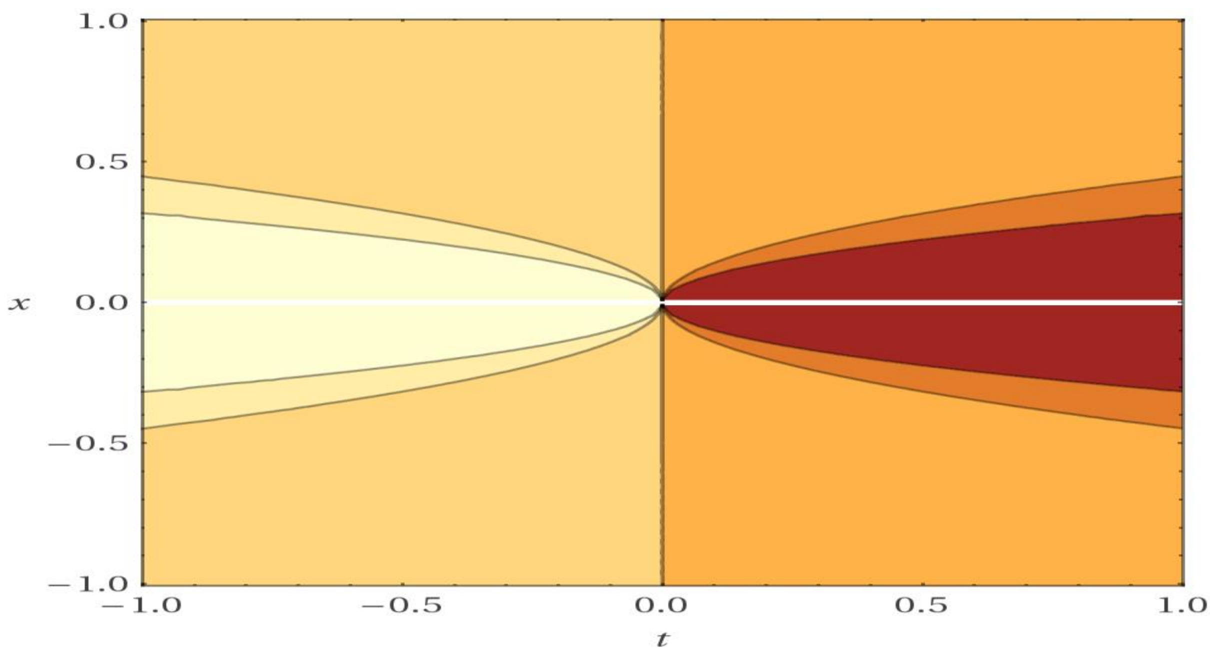

Figure 2 . light cone in our model is created

Tensor Fields

Up to here we discussed about vector fields or tensor rank 1, now we want to talk about tensor fields. As you see equation (3) it is for vector filed now we want to create this equation for tensor fields. So we start the process

υ^{kj} is arbitrary tensor

$$\nabla_k \upsilon^{kj} = \frac{\partial \upsilon^{kj}}{\partial x^k} + \Gamma^k{}_{k\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell} \ , \tag{44}$$

$$\nabla_k \upsilon^{kj} = \frac{\partial \upsilon^{kj}}{\partial x^k} + \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell} \ , \tag{45}$$

$$\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,\upsilon^{kj})}{\partial x^k} - \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^k}\upsilon^{kj} = \frac{\partial \upsilon^{kj}}{\partial x^k} \quad , \tag{46}$$

Now we put (46) into the (45)

$$\nabla_k \upsilon^{kj} = \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,\upsilon^{kj})}{\partial x^k} - \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^k}\upsilon^{kj} + \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell} \ , \tag{47}$$

Then we compare (47) and (44)

$$\frac{\partial \upsilon^{kj}}{\partial x^k} + \Gamma^k{}_{k\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell} = \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}\,\upsilon^{kj})}{\partial x^k} - \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^k}\upsilon^{kj} + \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell}, \tag{48}$$

$$\frac{\partial \upsilon^{kj}}{\partial x^k} + \Gamma^k{}_{k\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell} = \frac{\partial \upsilon^{kj}}{\partial x^k} + \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^k}\upsilon^{kj} - \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^k}\upsilon^{kj} + \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell}, \tag{49}$$

$$\frac{\partial \upsilon^{kj}}{\partial x^k} + \Gamma^k{}_{k\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell} = \frac{\partial \upsilon^{kj}}{\partial x^k} + \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}}{\partial x^\ell}\upsilon^{\ell j} + \Gamma^j{}_{k\ell}\upsilon^{k\ell}, \tag{50}$$

$$\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}}{\partial x^{\ell}}\upsilon^{\ell j}=\Gamma^{k}{}_{k\ell}\upsilon^{\ell j}\ , \tag{51}$$

$$\frac{1}{\sqrt{g}}\partial_{\ell}(\sqrt{g})\upsilon^{\ell j}=\Gamma^{k}{}_{k\ell}\upsilon^{\ell j}\, , \tag{52}$$

$$\frac{1}{\sqrt{g}}\partial_{\alpha}(\frac{\mathcal{L}}{i\mathcal{R}})\upsilon^{\alpha\beta}=\Gamma^{\gamma}{}_{\gamma\alpha}\upsilon^{\alpha\beta}\ , \tag{53}$$

Equation (53) is similar to (3) so we can create equation similar to (19)

So we start the process

$$\frac{1}{\sqrt{g}}\partial_{\alpha}(\frac{\mathcal{L}}{i\mathcal{R}})\upsilon^{\alpha\beta}=\Gamma^{\gamma}{}_{\gamma\alpha}\upsilon^{\alpha\beta}\ , \tag{54}$$

$$\frac{\partial\mathcal{L}}{\partial\mathcal{X}^{\alpha}}=\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}, \tag{55}$$

$$\frac{1}{\sqrt{g}}(\frac{\frac{1}{2\mathcal{L}}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}i\mathcal{R}-i\frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}}{-\mathcal{R}^{2}})\upsilon^{\alpha\beta}=\Gamma^{\gamma}{}_{\gamma\alpha}\upsilon^{\alpha\beta}\ , \tag{56}$$

$$(\frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}-\frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^{2})\upsilon^{\alpha\beta}=\sqrt{-g}\,\upsilon^{\alpha\beta}\Gamma^{\gamma}{}_{\gamma\alpha}\mathcal{L}\mathcal{R}^{2}, \tag{57}$$

$$\mathcal{L}=\sqrt{-g}\,\mathcal{R}\rightarrow\sqrt{-g}=\frac{\mathcal{L}}{\mathcal{R}}, \tag{58}$$

Now we Put (58) into the (57)

$$(\frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial\mathcal{X}^{\alpha}}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}-\frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^{2})\upsilon^{\alpha\beta}=\frac{\mathcal{L}^{2}}{\mathcal{R}}\upsilon^{\alpha\beta}\Gamma^{\gamma}{}_{\gamma\alpha}\mathcal{R}^{2}, \tag{59}$$

We know from (12)

$$\Gamma^{\beta}{}_{\alpha\beta}=\frac{1}{2}g^{\alpha\beta}\frac{\partial g_{\alpha\beta}}{\partial x^{\alpha}}\rightarrow 2\Gamma^{\beta}{}_{\alpha\beta}g_{\alpha\beta}=4\frac{\partial g_{\alpha\beta}}{\partial x^{\alpha}}\rightarrow\frac{\partial g_{\alpha\beta}}{\partial x^{\alpha}}=\frac{1}{2}g_{\alpha\beta}\Gamma^{\beta}{}_{\alpha\beta}\ , \tag{60}$$

We put (60) into the (59)

$$(\frac{1}{4}g_{\alpha\beta}\Gamma^{\beta}{}_{\alpha\beta}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}-\frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^{2})\upsilon^{\alpha\beta}=\upsilon^{\alpha\beta}\Gamma^{\gamma}{}_{\gamma\alpha}g_{\alpha\beta}\frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt}\mathcal{R}, \tag{61}$$

Then we contract $\gamma \to \beta$ and then we use $\Gamma^{\beta}{}_{\alpha\beta} \frac{d\mathcal{X}^{\alpha}}{dt}\frac{d\mathcal{X}^{\beta}}{dt} = -\frac{d^2\mathcal{X}^{\beta}}{dt^2}$

$$(-\frac{1}{4} g_{\alpha\beta} \frac{d^2\mathcal{X}^{\beta}}{dt^2}\mathcal{R} - \frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^2)\upsilon^{\alpha\beta} = -\upsilon^{\alpha\beta} g_{\alpha\beta}\frac{d^2\mathcal{X}^{\beta}}{dt^2}\mathcal{R}, \tag{62}$$

$$(-\frac{1}{4} g_{\alpha\beta} \frac{d^2\mathcal{X}^{\beta}}{dt^2} g^{\alpha\beta}\mathcal{R}_{\alpha\beta} - \frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^2)\upsilon^{\alpha\beta} = -\upsilon^{\alpha\beta} g_{\alpha\beta}\frac{d^2\mathcal{X}^{\beta}}{dt^2} g^{\alpha\beta}\mathcal{R}_{\alpha\beta}, \tag{63}$$

$$(-\frac{d^2\mathcal{X}^{\beta}}{dt^2}\mathcal{R}_{\alpha\beta} - \frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^2)\upsilon^{\alpha\beta} = -4\upsilon^{\alpha\beta}\frac{d^2\mathcal{X}^{\beta}}{dt^2}\mathcal{R}_{\alpha\beta}, \tag{64}$$

$$3\frac{d^2\mathcal{X}^{\beta}}{dt^2}\mathcal{R}_{\alpha\beta}\upsilon^{\alpha\beta} - \frac{\partial\mathcal{R}}{\partial\mathcal{X}^{\alpha}}\mathcal{L}^2\upsilon^{\alpha\beta} = 0 \ , \tag{65}$$

equation (65) tells us if there are tensor fields in the manifolds, it leads to our manifold is accelerated. We mean that our Manifold is extending with an acceleration. For example, the fundamental forces could change size of our universe.

How does the fundamental forces are unified with dark energy approach?

First, we evaluate unification of forces in the present time that is described with Friedmann–Lemaître–Robertson–Walker metric.

As we know, we have four fundamental forces, if we want to describe interaction between them, we should have equation that all the forces be in it, so we should use dark energy approach. Dark energy approach is equation (65).
So we can write

$$\upsilon^{\alpha\beta} = G^{\alpha\beta} + F^{\alpha\beta} + Z^{\alpha}W_{\beta} + \mathcal{G}^{\alpha\beta} \ . \tag{66}$$

$G^{\alpha\beta}$ is gravitation field

$$G^{\alpha\beta} = \mathcal{R}^{\alpha\beta} - \frac{1}{2} g^{\alpha\beta}\mathcal{R} . \tag{67}$$

$F^{\alpha\beta}$ is electromagnetism field

$$F^{\alpha\beta} = \partial^{\alpha} A^{\beta} - \partial^{\beta} A^{\alpha} \ . \tag{68}$$

A^{α} is photon field

$Z^{\alpha} W_{\beta}$ is weak nuclear force, but this force has massive particles mediating, since our Lagragian is for massless particle, we can withdraw this force from our unification.

$\mathcal{G}^{\alpha\beta}$ is the gluon field strength tensor, it is used instead of strong nuclear force, we emphasize again our space is made for free particles, so only particles that have zero mass could take part in the interaction, so only particles that can take part in the strong interaction and have no mass are gluons [5], [6].

$$\begin{gathered}\mathcal{G}^{\alpha\beta} = \partial^{\alpha} \mathcal{A}^{\beta} - \partial^{\beta} \mathcal{A}^{\alpha} \pm i g_s \left[\mathcal{A}^{\alpha}, \mathcal{A}^{\beta} \right], \\ \mathcal{G}^{\alpha\beta} = \partial^{\alpha} \mathcal{A}^{\beta} - \partial^{\beta} \mathcal{A}^{\alpha}.\end{gathered} \tag{69}$$

$\mathcal{A}^{\beta}$ is fields of gluons

g_s is the coupling constant of the strong force;

i is the imaginary unit;

Now we come back to the equation (65)

$$3 \frac{d^2 \mathcal{X}^{\beta}}{dt^2} \mathcal{R}_{\alpha\beta} (G^{\alpha\beta} + F^{\alpha\beta} + \mathcal{G}^{\alpha\beta}) - \frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}} \mathcal{L}^2 (G^{\alpha\beta} + F^{\alpha\beta} + \mathcal{G}^{\alpha\beta}) = 0 \, . \tag{70}$$

Equation (72) is made for long distance, so we can use Friedmann–Lemaître–Robertson–Walker metric, therefore our metric is diagonal and then Ricci tensor is diagonal too. Based on Einstein summation, we can sum up equation (70).

$\mathcal{R}_{\alpha\beta} G^{\alpha\beta}$ is interaction between manifold and gravitation .

$$\begin{gathered}3 \frac{d^2 \mathcal{X}^{\beta}}{dt^2} (\mathcal{R}_{00} G^{00} + \mathcal{R}_{11} G^{11} + \mathcal{R}_{22} G^{22} + \mathcal{R}_{33} G^{33} + \mathcal{R}_{00} F^{00} + \mathcal{R}_{11} F^{11} + \mathcal{R}_{22} F^{22} + \mathcal{R}_{33} F^{33} + \\ + \mathcal{R}_{00} \mathcal{G}^{00} + \mathcal{R}_{11} \mathcal{G}^{11} + \mathcal{R}_{22} \mathcal{G}^{22} + \mathcal{R}_{33} \mathcal{G}^{33}) - \frac{\partial \mathcal{R}}{\partial \mathcal{X}^{\alpha}} \mathcal{L}^2 (G^{\alpha\beta} + F^{\alpha\beta} + A^{\alpha\beta} + \mathcal{G}^{\alpha\beta}) = 0,\end{gathered} \tag{71}$$

$$3\frac{d^2\mathcal{X}^\beta}{dt^2}(\mathcal{R}_{00}G^{00}+\mathcal{R}_{11}G^{11}+\mathcal{R}_{22}G^{22}+\mathcal{R}_{33}G^{33})-\frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2(G^{\alpha\beta}+F^{\alpha\beta}+\mathcal{G}^{\alpha\beta})=0. \quad (72)$$

Now we should calculate $\mathcal{Q}=\mathcal{R}_{00}G^{00}+\mathcal{R}_{11}G^{11}+\mathcal{R}_{22}G^{22}+\mathcal{R}_{33}G^{33}$

$$\mathcal{Q}=\mathcal{R}_{00}(\mathcal{R}^{00}-\frac{1}{2}g^{00}\mathcal{R})+\mathcal{R}_{11}(\mathcal{R}^{11}-\frac{1}{2}g^{11}\mathcal{R})+\mathcal{R}_{22}(\mathcal{R}^{22}-\frac{1}{2}g^{22}\mathcal{R})+\mathcal{R}_{33}(\mathcal{R}^{33}-\frac{1}{2}g^{33}\mathcal{R}), \quad (73)$$

$$\mathcal{Q}=(\mathcal{R}_{00}\mathcal{R}^{00}-\frac{1}{2}\mathcal{R}_{00}g^{00}\mathcal{R})+(\mathcal{R}_{11}\mathcal{R}^{11}-\frac{1}{2}\mathcal{R}_{11}g^{11}\mathcal{R})+(\mathcal{R}_{22}\mathcal{R}^{22}-\frac{1}{2}\mathcal{R}_{22}g^{22}\mathcal{R})+(\mathcal{R}_{33}\mathcal{R}^{33}-\mathcal{R}_{33}\frac{1}{2}g^{33}\mathcal{R}),$$

$$\mathcal{Q}=4-\frac{1}{2}\mathcal{R}_{00}g^{00}\mathcal{R}+4-\frac{1}{2}\mathcal{R}_{11}g^{11}\mathcal{R}+4-\frac{1}{2}\mathcal{R}_{22}g^{22}\mathcal{R}+4-\mathcal{R}_{33}\frac{1}{2}g^{33}\mathcal{R},$$

$$\mathcal{Q}=16-\frac{1}{2}\mathcal{R}(\mathcal{R}_{00}g^{00}+\mathcal{R}_{11}g^{11}+\mathcal{R}_{22}g^{22}+\mathcal{R}_{33}g^{33}),$$

$$\mathcal{Q}=16-\frac{1}{2}\mathcal{R}^2.$$

(74)

Then we should calculate $D=\frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2(G^{\alpha\beta}+F^{\alpha\beta}+\mathcal{G}^{\alpha\beta})$ (75)

As you know $\mathcal{R}$ is function of time so $\alpha=0$

$$D=(\frac{\partial\mathcal{R}}{\partial\mathcal{X}^0}\mathcal{L}^2G^{0\beta}+\frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2F^{0\beta}+\frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2\mathcal{G}^{0\beta}). \quad (76)$$

The we come back to (70)

$$3\frac{d^2\mathcal{X}^\beta}{dt^2}(16-\frac{1}{2}\mathcal{R}^2)-(\frac{\partial\mathcal{R}}{\partial\mathcal{X}^0}\mathcal{L}^2G^{0\beta}+\frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2F^{0\beta}+\frac{\partial\mathcal{R}}{\partial\mathcal{X}^\alpha}\mathcal{L}^2\mathcal{G}^{0\beta})=0.$$

(77)

If we select $\beta=0$ effect of Gravitation will be remain but other forces will be removed

$$\frac{\partial\mathcal{R}}{\partial\mathcal{X}^0}\mathcal{L}^2G^{00}=0, \quad (78)$$

$$\frac{\partial\mathcal{R}}{\partial t}\mathcal{L}^2\mathcal{R}^{00}-\frac{1}{2}\frac{\partial\mathcal{R}}{\partial t}\mathcal{L}^2g^{00}\mathcal{R}=0, \quad (79)$$

So Gravitation could not unify with other forces because Gravitation is diagonal force but other forces are not.

Now if we select $\beta = i, i = 1,2,3$ effect of Gravitation won`t be removed totally , because Gravitation interact with manifold we mean that $(\mathcal{R}_{\alpha\beta} G^{\alpha\beta})$, but gravitation be united with other forces you can see

$$3\frac{d^2\mathcal{X}^i}{dt^2}(16-\frac{1}{2}\mathcal{R}^2)-(\frac{\partial\mathcal{R}}{\partial\mathcal{X}^0}\mathcal{L}^2 G^{0i}+\frac{\partial\mathcal{R}}{\partial\mathcal{X}^0}\mathcal{L}^2 F^{0i}+\frac{\partial\mathcal{R}}{\partial\mathcal{X}^0}\mathcal{L}^2\mathcal{G}^{0i})=0,$$

(80)

$$3\frac{d^2\mathcal{X}^i}{dt^2}(16-\frac{1}{2}\mathcal{R}^2)-\frac{\partial\mathcal{R}}{\partial t}\mathcal{L}^2(F^{0i}+\mathcal{G}^{0i})=0 \; . \tag{81}$$

Now we evaluate unification of forces in the early universe before Higgs mechanism:

Before Higgs mechanism, particles have no mass, so we can unify all the forces. In (38), we understand that our metric in the early universe is pseudo Riemannian, so we should choose manifold that be pseudo Riemannian like weyl manifold, so we can choose weyl metric and choose weyl tensor, there is transformation between Riemannian manifold and weyl manifold that you can see in [7]:

$$S_{\alpha\beta}=\mathcal{R}_{\alpha\beta}-\frac{1}{2(n-1)}g_{\alpha\beta}\mathcal{R} \; , \tag{82}$$

$g_{\alpha\beta}$ FRLW METRIC

$S_{\alpha\beta}$ is weyl tensor

n is dimensions of the manifold

$$\overline{\mathcal{R}}_{\alpha\beta}=S_{\alpha\beta} \; . \tag{83}$$

We defined $\overline{\mathcal{R}}_{\alpha\beta}$ before, it means that very small Riemannian manifold is equal with pseudo Riemannian manifold or weyl manifold.

Now we come back to (65) and we limit from it .

$$3\frac{d^2\overline{\mathcal{X}^\beta}}{d\bar{t}^2}\overline{\mathcal{R}_{\alpha\beta}}\bar{\upsilon}^{\alpha\beta}-\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\mathcal{X}}^\alpha}\bar{\mathcal{L}}^2\bar{\upsilon}^{\alpha\beta}=0\ , \tag{84}$$

$$3\frac{d^2\overline{\mathcal{X}^\beta}}{d\bar{t}^2}\overline{\mathcal{R}_{\alpha\beta}}(\overline{G}^{\alpha\beta}+F^{\alpha\beta}+\mathcal{G}^{\alpha\beta}+Z^\alpha W_\beta)-\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\mathcal{X}}^\alpha}\bar{\mathcal{L}}^2(\overline{G}^{\alpha\beta}+F^{\alpha\beta}+\mathcal{G}^{\alpha\beta}+Z^\alpha W_\beta)=0\,. \tag{85}$$

As we understood, our metric is pseudo Riemannian, so we can use Minkowski manifold for Electromagnetic tensor and strong nuclear force and weak nuclear force because Minkowski manifold is pseudo Riemannian. So we should calculate weyl tensor and gravitation tensor.

$$\overline{G_{\alpha\beta}}=\overline{\mathcal{R}_{\alpha\beta}}-\frac{1}{2}\overline{g_{\alpha\beta}}\overline{\mathcal{R}}\,, \tag{86}$$

$$S_{\alpha\beta}=\overline{\mathcal{R}_{\alpha\beta}}\quad , \tag{87}$$

$$\overline{G_{\alpha\beta}}=\mathcal{R}_{\alpha\beta}-\frac{1}{2(n-1)}g_{\alpha\beta}\mathcal{R}-\frac{1}{2}\overline{g_{\alpha\beta}}\overline{\mathcal{R}}\,, \tag{88}$$

$$\overline{\mathcal{R}}=\overline{g^{\alpha\beta}}\overline{\mathcal{R}_{\alpha\beta}}\quad , \tag{89}$$

$$\overline{G_{\alpha\beta}}=\mathcal{R}_{\alpha\beta}-\frac{1}{2(n-1)}g_{\alpha\beta}\mathcal{R}-\frac{1}{2}\overline{g_{\alpha\beta}}\overline{g^{\alpha\beta}}\overline{\mathcal{R}_{\alpha\beta}}\,. \tag{90}$$

$\overline{g_{\alpha\beta}}\overline{g^{\alpha\beta}}$ based weyl metric we can calculate it , but we can not calculate by (38), because $\overline{g_{\alpha\beta}}$ does not exist , so we should use Weyl vacuum solutions in spherical coordinates metric ,weyl metric is [8]:

$$ds^2=e^{2\psi(r,\theta)}dt^2-e^{2\gamma(r,\theta)-2\psi(r,\theta)}(dr^2+r^2d\theta^2)-e^{-2\psi(r,\theta)}\rho^2d\varphi^2\,.$$

So $\overline{g_{\alpha\beta}}\overline{g^{\alpha\beta}}$ is equal with 4 , so we come back to (94)

$$\overline{G_{\alpha\beta}}=\mathcal{R}_{\alpha\beta}-\frac{1}{2(n-1)}g_{\alpha\beta}\mathcal{R}-2\overline{\mathcal{R}}_{\alpha\beta}\quad . \tag{91}$$

Our manifold is 4 dimensional , so

$$
\begin{aligned}
&\overline{G_{\alpha\beta}} = \mathcal{R}_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta}\mathcal{R} - 2\overline{\mathcal{R}}_{\alpha\beta}, \\
&\overline{\mathcal{R}}_{\alpha\beta} = S_{\alpha\beta} = \mathcal{R}_{\alpha\beta} - \frac{1}{2(n-1)} g_{\alpha\beta}\mathcal{R}, \\
&\overline{G_{\alpha\beta}} = \mathcal{R}_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta}\mathcal{R} - 2(\mathcal{R}_{\alpha\beta} - \frac{1}{2(n-1)} g_{\alpha\beta}\mathcal{R}), \\
&\overline{G_{\alpha\beta}} = \mathcal{R}_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta}\mathcal{R} - 2(\mathcal{R}_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta}\mathcal{R}), \\
&\overline{G_{\alpha\beta}} = \mathcal{R}_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta}\mathcal{R} - 2\mathcal{R}_{\alpha\beta} + \frac{1}{3} g_{\alpha\beta}\mathcal{R}, \\
&\overline{G_{\alpha\beta}} = \frac{1}{6} g_{\alpha\beta}\mathcal{R} - \mathcal{R}_{\alpha\beta}.
\end{aligned} \tag{92}
$$

So we come back to (85) to unify the forces

$$
3\frac{d^2\overline{\mathcal{X}^\beta}}{d\bar{t}^2}\overline{\mathcal{R}_{\alpha\beta}}(\overline{G}^{\alpha\beta} + F^{\alpha\beta} + \mathcal{G}^{\alpha\beta} + Z^\alpha W_\beta) - \frac{\partial\overline{\mathcal{R}}}{\partial\overline{\mathcal{X}}^\alpha}\overline{\mathcal{L}}^2(\overline{G}^{\alpha\beta} + F^{\alpha\beta} + \mathcal{G}^{\alpha\beta} + Z^\alpha W_\beta) = 0. \tag{93}
$$

In here $\overline{\mathcal{R}}$ is not function of time so $\alpha \neq 0$

$$
\begin{aligned}
&3\frac{d^2\overline{\mathcal{X}^\beta}}{d\bar{t}^2}(\mathcal{R}_{\alpha\beta} - \frac{1}{6} g_{\alpha\beta}\mathcal{R})(\frac{1}{6} g^{\alpha\beta}\mathcal{R} - \mathcal{R}^{\alpha\beta} + F^{\alpha\beta} + \mathcal{G}^{\alpha\beta} + Z^\alpha W_\beta) + \\
&-\frac{\partial\overline{\mathcal{R}}}{\partial\overline{\mathcal{X}}^\alpha}\overline{\mathcal{L}}^2(\frac{1}{6} g^{\alpha\beta}\mathcal{R} - \mathcal{R}^{\alpha\beta} + F^{\alpha\beta} + \mathcal{G}^{\alpha\beta} + Z^\alpha W_\beta) = 0.
\end{aligned} \tag{94}
$$

So our unification is created . if we use Einstein summation all the forces become united .

Why does Dark Energy exist with high acceleration in the present time?

Until now, we understood that tensor fields can make acceleration in our universe, on the other hand, we know that every vector fields and tensor files that exist in our universe depend on fundamental forces so, fundamental reason that make acceleration in our universe are fundamental forces.

As you see, gravitational terms are not very big, since acceleration is multiplied by gravitational terms, so it exists between acceleration and gravitational terms and vice versa, in the present time that gravitation is very low, then acceleration is very big.

$$3\frac{d^2\mathcal{X}^i}{dt^2}(16-\frac{1}{2}\mathcal{R}^2)-\frac{\partial\mathcal{R}}{\partial t}\mathcal{L}^2(F^{0i}+\mathcal{G}^{0i})=0\,. \tag{95}$$

So in (94) gravitation is so high because has a many terms , so acceleration is very low.

Conclusion

Totally, we can conclude that if we have vector fields or tensor fields in the Manifolds, our Manifolds have been stretched, this is mathematical interpretation if we change this interpretation to physically interpretation, we can say everything exists in the universe and has direction that somehow depends on the coordinate, it leads to extending our universe. This extending is not just for our universe, it is also included for every curved figure like atoms and etc. We can say definitely this extending has acceleration and this acceleration determine motion of particle, if it is very low, motion of particle becomes similar to wave and if acceleration is very big, motion of particle becomes similar to P\particle movement. From this approach, we can unify the fundamental forces. At first, universe manifold is a pseudo Riemannian, so we should consider weyl geometry for describing the fundamental forces, when we use it we can unify all the forces. we understood that acceleration of universe reversely corresponding to gravitational terms, so in present time, gravity is weak then acceleration is very high. By passing time, from early universe to the present time gravitation has weakened, as you saw in the early universe, gravitation has many terms but in the present time has little terms, so in both situation gravitation is exist but sometime it is very weak and sometime it is very strong.

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