

# zeta odd-numbers

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## Abstract

I tried to find a new expression for zeta odd-numbers, but it seems to have remained a general expression.

However, it may be a new expression and will be published here.

## key words

zeta odd-numbers, new expression

## 1 Introduction

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} + \sum_{n=1}^{\infty} \frac{1}{(2n)^3} = \frac{1}{2^3} \sum_{n=1}^{\infty} \frac{1}{n^3} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{1}{2^3} \zeta(3) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (1)$$

$$\zeta(3) = \frac{2^3}{2^3-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} = \frac{8}{7} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \quad (2)$$

## 2 Discussion

do the same

$$\zeta(5) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} + \sum_{n=1}^{\infty} \frac{1}{(2n)^5} = \frac{1}{2^5} \sum_{n=1}^{\infty} \frac{1}{n^5} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{1}{2^5} \zeta(5) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (3)$$

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$$\zeta(5) = \frac{2^5}{2^5 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} = \frac{32}{31} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \quad (4)$$

do the same

$$\zeta(7) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} + \sum_{n=1}^{\infty} \frac{1}{(2n)^7} = \frac{1}{2^7} \sum_{n=1}^{\infty} \frac{1}{n^7} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{1}{2^7} \zeta(7) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (5)$$

$$\zeta(7) = \frac{2^7}{2^7 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} = \frac{128}{127} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^7} \quad (6)$$

do the same

$$\zeta(9) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} + \sum_{n=1}^{\infty} \frac{1}{(2n)^9} = \frac{1}{2^9} \sum_{n=1}^{\infty} \frac{1}{n^9} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} = \frac{1}{2^9} \zeta(9) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (7)$$

$$\zeta(9) = \frac{2^9}{2^9 - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} = \frac{512}{511} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^9} \quad (8)$$

do the same

$$\zeta(11) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{11}} = \frac{1}{2^{11}} \sum_{n=1}^{\infty} \frac{1}{n^{11}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} = \frac{1}{2^{11}} \zeta(11) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (9)$$

$$\zeta(11) = \frac{2^{11}}{2^{11} - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} = \frac{2048}{2047} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{11}} \quad (10)$$

do the same

$$\zeta(13) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{13}} = \frac{1}{2^{13}} \sum_{n=1}^{\infty} \frac{1}{n^{13}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} = \frac{1}{2^{13}} \zeta(13) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (11)$$

$$\zeta(13) = \frac{2^{13}}{2^{13} - 1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} = \frac{8192}{8191} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{13}} \quad (12)$$

do the same

$$\zeta(15) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{15}} = \frac{1}{2^{15}} \sum_{n=1}^{\infty} \frac{1}{n^{15}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{1}{2^{15}} \zeta(15) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (13)$$

$$\zeta(15) = \frac{2^{15}}{2^{15}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} = \frac{32768}{32767} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{15}} \quad (14)$$

do the same

$$\zeta(17) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{17}} = \frac{1}{2^{17}} \sum_{n=1}^{\infty} \frac{1}{n^{17}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{1}{2^{17}} \zeta(17) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (15)$$

$$\zeta(17) = \frac{2^{17}}{2^{17}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} = \frac{131072}{131071} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{17}} \quad (16)$$

do the same

$$\zeta(19) = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n)^{19}} = \frac{1}{2^{19}} \sum_{n=1}^{\infty} \frac{1}{n^{19}} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{1}{2^{19}} \zeta(19) + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (17)$$

$$\zeta(19) = \frac{2^{19}}{2^{19}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} = \frac{524288}{524287} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{19}} \quad (18)$$

### 3 Conclusion

make official

$$\zeta(2m-1) = \frac{2^{2m-1}}{2^{2m-1}-1} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^{2m-1}} \quad (19)$$

m is a positive integer greater than or equal to 2

## 4 Postscript

These calculations were performed with WolframAlpha.

## References

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