Special relativity in complex space-time. Part 4.
General problems of physics and complex space-time

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Abstract

The article shows that the paravector orthogonal transformation is invariant for the Klein-Gordon equation and for the Hamilton-Jacobi equation of the free particle. This confirms that the alternative theory of relativity, which we create in the complex space-time, does not conflict with other branches of physics. The complex space-time with its natural properties gives hope for ordering of the physics of the great speeds created by the revolution at the turn of the 19th and 20th century.

Keywords: *Complex space-time, alternative special relativity, paravectors*

Introduction

In the series of articles on special relativity (SR) in complex space-time, we try to show that the creators of classic SR made the mistake of assuming that space-time is real. This led to many absurdities that were hidden under the enigmatic term "paradoxes". By expanding the space-time to a complex structure, we obtain interesting and obvious physical interpretations that are free from the mentioned paradoxes. The formalism, constructed for the mathematical description of this theory, refers directly to the Euclidean geometry which makes it simple and intuitive. Other assumptions of our theory are consistent with the existing SR, that is

- the speed of light is constant and maximal at which energy can move
- no ether medium is needed for the existence of an electromagnetic wave.

In order to easily understand the calculations, it is necessary to master the paravector calculus. Our notation is innovative, which will disturb readers, for whom the concept of a paravector is known. To anyone interested in the topic, we persuade not to start with Wikipedia or with Clifford's algebra, but to read the article [1], where the paravectors are presented in a form as close as possible to vectors and matrices. The paravector, depending on what it represents, can be denoted traditionally as a vector or matrix. We often emphasize their relationship with matrices, because they are matrices reduced to two elements: a complex number (scalar) and a complex vector. In order to get the knowledge of the paravector calculus it is not necessary to know tensor calculus or multivector algebra. It is enough to know the basics of linear algebra, which should be known by every engineer. As usual, only practice is necessary, therefore we recommend to repeat each proof the reader has found.

Below there are assumptions that apply to this and all previous papers.

Assumption 1. The physical formulas are described in natural units, that is

- the value of light velocity is equal to one,
- the linear velocity is a dimensionless quantity and the value of it is a fraction of the speed of light,
- the electric permittivity and magnetic permeability of a vacuum are equal to one.

Although in previous works this assumption has not been made explicit yet, it could be noted that if time was mentioned, there was always a time interval Δt. Now we are making the assumption explicitly, because there is no indication that it would be otherwise:

Assumption 2. Time is a quantum size. This means that if we are talking about time, we mean its interval.

In our theory, an image of the physical object's time interval under the paravector orthogonal transformation (POT) is a space-time interval

$$\frac{1}{\sqrt{\alpha^2-\beta^2}}\begin{bmatrix} \alpha \\ \boldsymbol{\beta} \end{bmatrix}\begin{pmatrix} \Delta t^0 \\ 0 \end{pmatrix}=\begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix} \qquad \text{gdzie} \quad \alpha\in C, \quad \boldsymbol{\beta}\in C^3 \quad \text{i} \quad \alpha^2-\beta^2\in R_+\setminus\{0\}. \tag{1}$$

In other words, the interval phase (at the left side of the equation below)

$$\frac{1}{\sqrt{\alpha^2-\beta^2}}\begin{bmatrix} \alpha \\ -\boldsymbol{\beta} \end{bmatrix}\begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix}=\begin{pmatrix} \Delta t^0 \\ 0 \end{pmatrix} \tag{2}$$

is equal to the interval of proper time.

In article [6], we showed that when we deal with a single object, an orthogonal paravector $\Lambda = \frac{1}{\sqrt{\alpha^2-\beta^2}}\begin{bmatrix} \alpha \\ -\boldsymbol{\beta} \end{bmatrix}$ can be replaced with a paravector that describes velocity explicitly $V=\frac{1}{\sqrt{1-v^2}}\begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix}$. Then the relationship between the coordinates of the same object in two systems moving at a relative speed of $\mathbf{v}$ describes the dependence

$$\frac{1}{\sqrt{1-v^2}}\begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix}\begin{pmatrix} \Delta t' \\ \Delta \mathbf{x}' \end{pmatrix}=\begin{pmatrix} \Delta t \\ \Delta \mathbf{x} \end{pmatrix}, \quad \text{where} \quad \Delta t, \Delta t'\in R_+, \quad \Delta\mathbf{x}\in R^3, \quad \Delta\mathbf{x}'\in C^3 \tag{3}$$

The above formula corresponds to the Lorentz transformation in the classical SR. We dealt with the analysis of this formula in articles [4], [6]. It shows that what is simultaneous in one system does not have to be simultaneous in the other, and how it is simultaneous, then the vector has an additional imaginary component which is associated with a kind of space-time deformation. An analysis of a thought experiment consisting of a spherical explosion in the center of a spherical laboratory ([4] section 3.4), in which particles after reflection from the wall of the laboratory return to its center, showed that the simultaneous reflection of all particles in the laboratory system is stretched in time it the moving system. The point of reflection moves at a speed faster than the speed of light, but the shape of the spherical laboratory is not deformed. At the end of the experiment, all the particles meet simultaneously at one point, which is the center of the shifted laboratory. We note that the dilation factor $1/\sqrt{1-v^2}$ works here in all directions, which is not present in the Lorentz transformation.

1 Non-relativistic approximations of the transformation formulas.

For non-relativistic speed ($v \ll c$) the transformation (3) should become a Galilean transformation. Now, we will check what transformation formulas look like for non-relativistic speed. It is better to change the units so that the value of the speed of light can be written explicitly c where it occurs, ie. we are going to the Haeviside-Lorentz units (H-L). This means that instead of the speed $\mathbf{v}$ we will write $\mathbf{v}/c$, and instead of the time $t \to ct$. The complex transformation formulas have a form

$$\begin{pmatrix} c\Delta t \\ \Delta\mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-(v/c)^2}} \begin{bmatrix} 1 \\ -\mathbf{v}/c \end{bmatrix} \begin{pmatrix} c\Delta t' \\ \Delta\mathbf{x}' \end{pmatrix} = \frac{1}{\sqrt{1-(v/c)^2}} \begin{pmatrix} c\Delta t' - \mathbf{v}\Delta\mathbf{x}'/c \\ \Delta\mathbf{x}' - \mathbf{v}\Delta t' - i\mathbf{v}\times\Delta\mathbf{x}'/c \end{pmatrix} \quad (4)$$

For non-relativistic speed ($v/c \to 0$), we get

$$\begin{pmatrix} c\Delta t \\ \Delta\mathbf{x} \end{pmatrix} = \begin{pmatrix} c\Delta t' \\ \Delta\mathbf{x}' - \mathbf{v}\Delta t' \end{pmatrix},$$

which is a Galilean transformation

$$\Delta t = \Delta t' \qquad \text{and} \qquad \Delta\mathbf{x}' = \Delta\mathbf{x} + \mathbf{v}\Delta t' \quad (5)$$

The composition of velocities is described by the formula ([4] eq:17)

$$\frac{1}{\sqrt{1-(v/c)^2+(w/c)^2}} \begin{bmatrix} 1 \\ \mathbf{v}/c + i\mathbf{w}/c \end{bmatrix} = \frac{1}{\sqrt{1-(v_1/c)^2}} \frac{1}{\sqrt{1-(v_2/c)^2}} \begin{bmatrix} 1 \\ \mathbf{v}_1/c \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{v}_2/c \end{bmatrix}, \quad (6)$$

that is
$$\frac{\sqrt{1-(v_1/c)^2}\sqrt{1-(v_2/c)^2}}{\sqrt{1-(v/c)^2+(w/c)^2}} \begin{bmatrix} 1 \\ \mathbf{v}/c + i\mathbf{w}/c \end{bmatrix} = \begin{bmatrix} 1+\mathbf{v}_1\mathbf{v}_2/c^2 \\ \mathbf{v}_1/c + \mathbf{v}_2/c + i\mathbf{v}_1\times\mathbf{v}_2/c^2 \end{bmatrix} \quad (7)$$

Since for low speeds the compound fraction on the left is equal to 1, we get a system of equations:

$$1 = 1 + \mathbf{v}_1\mathbf{v}_2/c^2 \tag{8}$$

$$\mathbf{v}/c = \mathbf{v}_1/c + \mathbf{v}_2/c \tag{9}$$

$$\mathbf{w}/c = \mathbf{v}_1 \times \mathbf{v}_2/c^2 \tag{10}$$

which for very small velocities (in relation to speed of light) comes down to the second equation, that is, to a simple sum of vectors $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$. It can be clearly seen here that by passing to non-relativistic velocities, the POT transformations change into Galilean transformations, and the paravector formulas of velocities composition change into the sum of vectors.

2 Compatibility of the paravector orthogonal transformations with other physics branches

2.1 Compatibility of POT with electrodynamics.

The problem of compliance of POT with electrodynamics appears throughout all previous publications, because it underlies the new theory of relativity, just as it was in classical theory. When we built a new SR, we had assumed that it must be consistent with the postulates of classical theory, and thus demanding the invariant speed of light, the desired transformation must be invariant for the wave equation

$$\frac{\partial^2\varphi(t,\mathbf{x})}{\partial^2 t} - \nabla^2\varphi(t,\mathbf{x}) = \rho(t,\mathbf{x}). \tag{11}$$

Besides, this transformation cannot belong to the Lorentz group. As a result of the research we found a complex paravector orthogonal transformation (POT). The invariance of POT for the wave equation has been proved in the article [2]. Although we transform the wave equation analogically to the classical SR, our results differ in detail from the Maxwell equations. In complex space-time in the equation corresponding to the Maxwell-Ampere equation, there is no current density. In our theory, Maxwell's equations can be formulated so that we either have current density and have no magnetic field, or vice versa. In the second case, we can get Ampere's law, but as a conclusion from the transformation of the electric field. Then, the vector potential has no sense , because both charge density and potential are the invariant scalar fields. In this case, the new Maxwell equations written in the real form are simpler and closer to symmetry, but most importantly, they do not contradict the laws of electrodynamics obtained experimentally. This explains why magnetic charges have not been detected, and the magnetic field is only an imaginary component of a moving electric field. The problem is discussed in more detail in the article [5].

2.2 Compatibility of POT with theoretical mechanics

We do not penetrate the physical sense of the equations of mathematical and quantum mechanics, because we do not have enough competence for this. However, it is our duty to check whether our model does not contradict these branches of physics. Therefore we will make a few mathematical transformations of selected general formulas to show that the basic equations of these physics branches are also invariant under transformation (3).

In the current relativistic mechanics, the Hamilton's function of the free particle is expressed by the formula

$$H = c\sqrt{p^2 + (m_0 c)^2} \quad , \tag{12}$$

and since

$$H = -\frac{\partial S}{\partial t} \qquad \text{and} \qquad \mathbf{p} = \nabla S \tag{13}$$

then the relativistic Hamilton-Jacobi equation for the free particle with the mass m_0 in the H-L units system has the form

$$-\frac{\partial S}{\partial t} = c\sqrt{(\nabla S)^2 + (m_0 c)^2} \tag{14}$$

which after transformation gives

$$\left(\frac{\partial S}{c\partial t}\right)^2 - (\nabla S)^2 = c^2 m_0^2 \quad , \tag{15}$$

In the natural system of units the above equation is written with the operators of differentiation:

$$\begin{bmatrix} \frac{\partial S}{\partial t} \\ \nabla S \end{bmatrix} \begin{bmatrix} \frac{\partial S}{\partial t} \\ -\nabla S \end{bmatrix} = (\partial S)\partial^{-} S = m_0 \tag{16}$$

where S is an action that is a scalar function. From the identities derived in the article [2] it follows that the above equality is transforming under the boost

$$(\partial S)\partial^{-} S = \left(\partial' \Lambda S'\right)\Lambda^{-}\partial'^{-} S' = \left(\partial' S'\right)\Lambda\Lambda^{-}\partial'^{-} S' = \left(\partial' S'\right)\partial'^{-} S' = m_0$$

It follows from the above that the POT conserves the equation (15), and the action is invariant under this transformation.

2.3 Compatibility of POT with quantum mechanics

The Klein-Gordon equation, also known as relativistic Schroedinger equation

$$-\frac{\partial^2 \Psi(t, \mathbf{x})}{\partial^2 t} + \nabla^2 \Psi(t, \mathbf{x}) = \frac{m^2}{\hbar^2} \Psi(t, \mathbf{x}) \tag{17}$$

is transformed with respect to the boost as well as the electromagnetic wave equation [5], so we do not repeat proof of its invariance, but we recommend that the reader do it.

At this point, we would like to draw the reader's attention to a detail that is not addressed in the literature and which may be of great importance. Regardless of whether it happens in the classic SR using the Lorentz transformation, or with us using the POT, the wave equations (11) and (17) should be transformed in the same way. If the potential and charge densities are transformed into 4-vectors containing vector components of vector potential and current density respectively, then it should be the same with the Klein-Gordon equation - the Ψ wave function should have a vector component. Since this is not the case in classical theories, it proves in favor of the complex space-time, where both the potential and the charge density are invariant scalar fields.

From the above cursory considerations, one can see that POT should not clash with electrodynamics, theoretical mechanics or quantum mechanics, either.

3 Summary

We have written many times about the compatibility of our theory with the postulates of the classic STR, and the calculations show that the model proposed by us differs from the classic theory.

- First postulate is: **The laws of physics are the same in any inertial frame of reference**. In the case of our theory, the postulate is fulfilled, because we started from the wave equation, and the entire reasoning was based on the transformation preserving its invariance. The same is true of the relativistic equations of theoretical and quantum mechanics.
- Second postulate: **The speed of light (c) is the same for all observers no matter what their relative speeds**. It is clear that it is the absolute value of the speed of light, not the vector. The compliance of our model with this postulate has also been checked. It cannot be otherwise because this postulate follows from postulate 1, because the speed of light is contained in the wave equation that is invariant under the POT.

So, what is the difference if the results of our theory differ from the classic ones? The creators of the classic SR defaultly put one more assumption: **spatial dilation works only in the direction of the relativistic movement**. In the classic SR, the description of phenomena is divided into components parallel to the direction of motion and perpendicular to it, ie. we deal with spatially one-dimensional relativistic phenomena. This assumption results from the seemingly obvious fact that space-time is real. While studying the mathematically uncomplicated, complex linear transformation preserving the invariance of the wave equation, we looked for consequences for the theory of the electric field and built the framework of the geometry of the space in which this field could reasonably exist. The results turned out to be surprising - the theory developed best when we did not try to limit it to the real space-time. From the mathematical perspective, the adopted definition of a scalar product[1] expands the definition known from the Euclidean geometry and carries the invariance of the sphere shape distinguished in nature. In consequence, this means invariance of shapes, which is more intuitive and simplifies many problems.

Reviewing various studies on special relativity, it can be noted that the authors stress that it is based on Einstein's two postulates and they equate it with these postulates. At the same time, they do not pay any attention to other assumptions that are no less important, and which they use as if these were the "obvious obviousness". These assumptions are:

- Space-time is a four-dimensional real space
- Space-time is an affine space

From the perspective of our results, it seems that what was obvious 100 years ago is not obvious today. At high speeds, the known laws of classical physics do not seem to fit into real space-time, and by treating time as a real continuum consisting of points, the basic property of time, which is time lapse, is erased. We can say that the change proposed by us in the special theory of relativity essentially consists in changing these two assumptions.

- Spacetime is a complex structure,
- Due to the quantum of time a space-time does not have to be an affine space (in the dimension of time).

For now, the above assumptions should be treated as suggestions helpful in organizing the results presented so far. At this point, the situation has already matured to present our hypothesis about the mathematical structure of space-time and the physical objects in it. The next article will be devoted to this issue.

References

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[1]Warning! The scalar product of paravectors does not have a very important feature that is positive definiteness, because $\langle X, X\rangle \in C$.

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