

# Question 480 : An integral for pi

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Abstract. This note presents a definite integral for pi.

- The number pi is defined by:

$$\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 4 \left( 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots \right) = 3.1415926535\dots \quad (1)$$

This short note presents a definite integrals for pi.

## Integral

- Let

$$f(x) = -\frac{10}{27} + \frac{7}{9x} + \frac{1}{18x^2} + \frac{1}{27x^3} \quad (2)$$

$$g(x) = \frac{1}{108} \left( 16 - \frac{64}{x} + \frac{60}{x^2} + \frac{8}{x^3} + \frac{7}{x^4} \right) \quad (3)$$

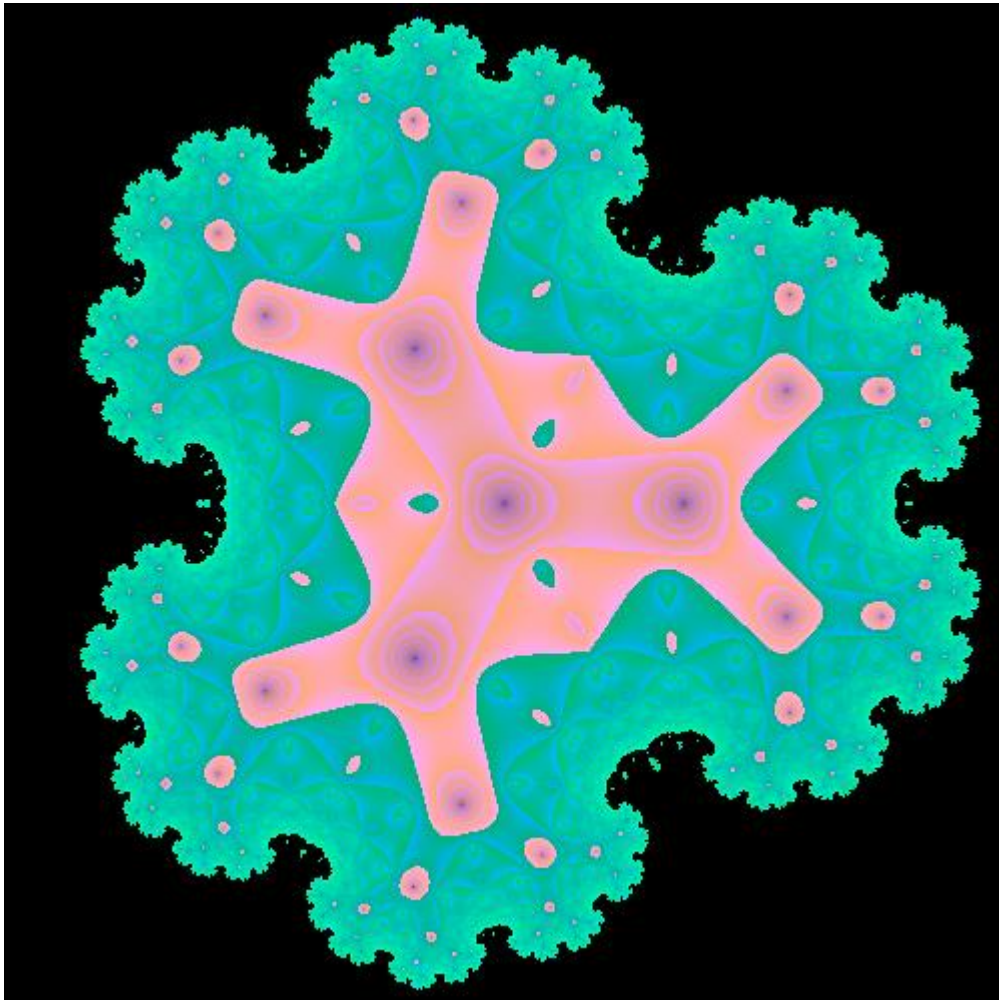
then

$$\frac{\pi}{2} = \int_0^2 \tan^{-1} \left( \frac{1-x}{3x} + \sqrt[3]{f(x) + \sqrt{g(x)}} + \sqrt[3]{f(x) - \sqrt{g(x)}} \right) dx \quad (4)$$

Remark:  $\sqrt[3]{x}$  is the real-valued cube root of  $x$ .

- Change of variable in (4):

$$\pi = \int_0^4 \tan^{-1} \left( \frac{2-x}{3x} + \sqrt[3]{f\left(\frac{x}{2}\right) + \sqrt{g\left(\frac{x}{2}\right)}} + \sqrt[3]{f\left(\frac{x}{2}\right) - \sqrt{g\left(\frac{x}{2}\right)}} \right) dx \quad (5)$$



A fractal Image

## References

1. GRADSHTEYN, I.S., and RYZHIK, I.M.: TABLE of INTEGRALS, SERIES and PRODUCTS. 7th ed. Edited by Alan Jeffrey and Daniel Zwillinger. Academic Press, 2007.