Title: General Relativity, Non-Abelian Gauge Theories, and Quantum Gravity

Author: F. Winterberg

Institution: Carl Friedrich Gauss Academy of Science, Potsdam, Germany and Reno, Nevada, USA.[1]

"Imagination is more important than knowledge."

-Albert Einstein

[1] P.O. Box 18265, Reno, Nevada 89511, USA

Abstract

We adopt here an attempt first pioneered by Dehnen to reformulate Einstein's general theory of relativity as a kind of non-Abelian gauge theory, as is quantum chromodynamics. This approach permits the formulation of Einstein's gravitational field equation by the same Lagrangian of the Einstein-Hilbert action, albeit with two event horizons: an "outer" one in ordinary space for Einstein's theory with a cut-off at the Planck energy ($\sim 10^{19}$ GeV), and an "inner" one in three-dimensional color space for quantum chromodynamics with a cut-of at $\sim 10^{2}$ GeV. This conclusion is reached by the analogy between general relativity and quantum chromodynamics at one hand, and Newtonian point particle classical mechanics and Helmholtz line vortex dynamics at the other hand.

1. Introduction

To this day, we do not yet have a satisfactory theory of quantum gravity that unifies Einstein's general theory of relativity with quantum mechanics despite more than 30 years of effort by a large number of the best theoretical physicists. The most popular of these efforts is string theory, whose solution relies on more than 10^{500} possible theories. Einstein once said, "God is subtle but not malicious." A solution relying on 10^{500} possible theories could certainly be considered "malicious."

To advance a solution to this problem, the author proposes the idea that even though Einstein's gravitational field equation may correctly describe a universe with gravitational field producing positive mass point singularities, the universe we observe hardly satisfies this ideal condition. Rather, the small value of the cosmological constant suggests that the vacuum of space is a kind of plasma made up from positive and negative Planck mass particles in equal numbers [1]. But because the principle of equivalence excludes the existence of negative masses, which would have to move on "anti-geodesics," this plasma would have to be made up of "pole-dipole" particles obtaining their positive mass from the positive gravitational field energy mass of such a pole-dipole positive-negative mass two-body configuration.

We follow the pioneering work of H. Dehnen and his research group [2] to reformulate Einstein's gravitational field equation as a non-Abelian gauge theory like the non-Abelian Yang-Mills theory of quantum chromodynamics, unknown at the time Einstein tried to unify his equations with Maxwell's equations, to correctly describe the standard model of elementary particle physics.

2. Analogies between General Relativity, Non-Abelian Gauge Theories and Superfluid Vortex Dynamics

In Einstein's gravitational field theory the force on a particle is expressed by the Christoffel symbols obtained from first order derivatives of the ten potentials of the gravitational field represented by the ten components of the metric tensor. From the Christoffel symbols the Riemann curvature tensor is structured by the symbolic equation

$$\mathbf{R} = \mathrm{curl}\,\mathbf{\Gamma} + \mathbf{\Gamma} \times \mathbf{\Gamma} \tag{1}$$

where Γ is the Christoffel pseudo-tensor. The expression for the field strength, and hence force, is in Yang-Mills field theories is symbolically given by

$$\mathbf{W} = \mathrm{curl}\,\boldsymbol{A} - g^{-1}\boldsymbol{A} \times \boldsymbol{A} \tag{2}$$

where g is the coupling constant with the dimension of electric charge and **A** the gauge potential. It too has the form of a curvature tensor, albeit not in space-time, but in QCD for example, in color space. It was Riemann who wondered if on small scales there might be a departure from Euclidean geometry. The Yang-Mills field theories have answered this question in a quite unexpected way, not as a non-Euclidean structure in space-time but rather one in charge-space, making itself felt only in the small.

Comparing (1) with (2) one can from a gauge field theoretic point of view consider the Christoffel symbols Γ^i_{kl} as gauge fields. If the curvature tensor vanishes, they can be globally eliminated by a transformation to a pseudoeuclidean Minkowski space-time metric. One may for this reason call the Γ^i_{kl} pure gauge fields, which for a vanishing curvature tensor can always be transformed away by a gauge transformation. Likewise, if the curvature tensor in (2) vanishes, one can globally transform away the gauge potentials.

Newton point-particle dynamics and Einstein's gravitational field theory		Kinematic Quantities	Helmholtz line-vortex dynamics and Yang-Mills field theories	
		$\boldsymbol{r}$	ψ f	velocity potential, gauge functions
Newtonian potential, metric tensor	ϕ g_{ik}	$\dot{\boldsymbol{r}}$	$-\nabla\psi$ A	Force on line vortex, gauge potentials
Force on point particle, gravitational force field expressed by Christoffel symbols	$-\nabla\phi$ Γ	$\ddot{\boldsymbol{r}}$	$W = \nabla \times A - g^{-1}A \times A$	Yang-Mills force field expressed by charge space curvature tensor
Einstein's field equations expressed by metric-space curvature tensor	$R = \nabla \times \Gamma + \Gamma \times \Gamma$			

Fig. 1

From the Newtonian point of view, contained in Einstein's field equations, the force is the first derivative of a potential. Apart from the nonlinear term in (2), this is also true for a Yang-Mills field theory. But with the inclusion of the nonlinear terms, it also has the structure of a curvature tensor. In Einstein's theory the curvature tensor involves second order derivatives of the potentials. This demonstrates a displacement of the hierarchy for the potentials with regard to the forces. A displacement of the hierarchies also occurs in fluid dynamics by comparing Newton's point particle dynamics with Helmholtz's line vortex dynamics. Whereas in Newton's point particle dynamics the equation of motion is $m\ddot{\boldsymbol{r}} = \boldsymbol{F}$, the corresponding equation in Helmoholtz's vortex dynamics is $\mu\dot{\boldsymbol{r}} = \boldsymbol{F}$, where μ is an effective mass [3]. Therefore, whereas in Newtonian mechanics a body moves with constant velocity in the absence of a force, it remains at rest in vortex dynamics. And whereas what is at rest remains undetermined in Newtonian mechanics, it is fully determined in vortex dynamics, where at rest means at rest with regards to the fluid. The same would have to be true with regard to the Planck mass plasma.

The hydrodynamics of the Planck mass plasma suggests that the hierarchal displacement of the curvature tensor for Einstein and Yang-Mills fields is related to the hierarchical displacement of the vortex equation of motion and the Newtonian point particle equation of motion. The hierarchical displacement and analogies to hydrodynamics is made complete by recognizing that the gauge function f is related to the velocity potential of an irrotational flow. A gauge transformation leaving the forces unchanged corresponds in the hydrodynamic analogy to the addition of an irrotational flow. These analogies and hierarchical displacements of Newtonian point mechanics and Einstein gravity, versus Helmholtz's vortex dynamics and Yang-Mills gauge field theories are shown in Figure 1.

This idea can be explored a little further. To do this we consider the force between two magnetic dipoles separated by the distance r. Their dipole moments m_1 and m_2 have the magnetic vector potentials

$$A_1 = \frac{m_1 \times e_r}{r^2} \qquad A_2 = -\frac{m_2 \times e_r}{r^2} \tag{3}$$

Where e_r is perpendicular to m_1 and m_2. The magnetic force on m_2 by m_1 is given by

$$F = \nabla(m_2 \cdot \mathrm{curl}\, A_1) = 6 A_1 \cdot A_2 e_r \tag{4}$$

Which is quadratic in the vector potentials, typical for Yang-Mills theories.

Because of the analogy between magnetic fields generated by current filaments and velocity fields generated by vortex filaments (first recognized by Helmholtz), one has for a current filament of current density j,

$$A = \frac{1}{c} \int \frac{j}{r} dr \tag{5}$$

And for a vortex filament of vorticity ω,

$$\hat{A} = \frac{1}{4\pi} \int \frac{\omega}{r} dr \tag{6}$$

From (5) one obtains $H = \text{curl}\, A$, and from (6) $v = \text{curl}\, \hat{A}$. With the electric current density j and vorticity ω related to each other by $j = (c/4\pi)\omega$, one has $j = (c/4\pi)\, \text{curl}\, \text{curl}\, A$, and $\omega = \text{curl}\, \text{curl}\, \hat{A}$.

While both theories, the general theory of relativity and the Yang Mills theory, can be understood as non-Abelian gauge theories, there is a fundamental difference; it is the Yang Mills theory describing in QCD the standard model of elementary particle physics can be renormalized and thereby quantized, general relativity is non-renormalizeable. This suggests that quantum gravity has its origin in the inner space $r < r_0$, and transmits the quantum behavior by gravitational waves to the outer space through the common event horizon at $r = r_0$.

3. Outer and Inner Space

To be in agreement with quantum mechanics where time is a parameter and not an operator, we use for the following analysis a time orthogonal reference system and divide 3-dimensional space into an "outer" and "inner" space. The "outer" space is distances larger than $r_0 \simeq 10^{-15}$ cm, equal to the radius of a proton, or for energies smaller than the strong interaction scale at ~100 GeV, the energy of the Higgs boson. The "inner" space is for distances smaller than r_0, or energies larger than ~100 GeV. In addition, for distances smaller than r_0, Newton's constant G is replaced by

$$g = (r_0/r_p)^2 G \simeq 10^{36} G \tag{7}$$

Where $r_p \simeq 10^{-33}$ cm is the Planck length.

4. Einstein Gravity in the Outer and Inner Space

Einstein's gravitational field equation can be derived from Hamilton's principle with the Einstein-Hilbert Lagrangian:

$$\delta \int [\frac{1}{2K} R + \Lambda]\sqrt{-g}\, dr = 0 \tag{8}$$

where R is the scalar curvature of the Riemann tensor in 4-dimensional spacetime, Λ the Lagrangian of matter, $K = 4\pi G/c^4$ and for the 4-dimensional volume element in a time orthogonal system

$$dr = dx_1 \cdot dx_2 \cdot dx_3 \cdot cdt \tag{9}$$

Then a solution of (8) in the outer space is given by Einstein's vacuum field equation

$$R_{ik} = 0, r > r_0 \tag{10}$$

For a proton assumed to have a spherical symmetric mass distribution, this is the Schwarzschild solution for the proton mass m.

To the inner solution we assume that to be consistent with a time integration from $\ddot{r}$ to $\dot{r}$, that is from Newton's law of motion for point masses to the Helmholtz law of motion for line vortices, we make a time integration of (8), which is equivalent to multiplying (8) by the operator $1/cdt$, obtaining

$$R + 2K\Lambda = 0, r > r_0, K = \frac{4\pi g}{c^4} \tag{11}$$

For a radiation dominated space, one obtains as the Schwarzschild interior solution, which has an event horizon at $r = r_0$, and for the mass inside $r = r_0$, $m = \frac{4\pi}{3\rho r_0^3}$, with mass density $\rho = \frac{3c^2}{8\pi g r_0^2}$.

In the absence of charged particles, or particles with a rest mass, the line elements of the outer and inner space have three space dimensions, which means that their space solutions can be glued together at $r = r_0$, where the solutions share the same event horizon.

The proposed alternative theory explains the three colors of QCD (quantum chromodynamics) by the three spatial orientation of ring vortices, in agreement with the rotation group of three-dimensional space.

For elementary particles reaching the event horizon, a Lorentzian frame of reference, with a preferred reference system might provide a better description of physical reality, than the more general theory by Einstein, which does not have such a preferred reference system permitting the existence of closed world lines with travel back in time In the proposed theory, the preferred system is at rest with the conjectured Planck mass plasma, filling all of space.

5. Color Confinement and Event Horizon Regularization

It was the crucial ‘t Hooft-Veltman regularization technique for non-Abelian gauge theories with a broken symmentry which made for QCD testable prediction as they were previously only possible for the renormalizeable QED. But while the regularization of QCD with the computational prescription of ‘t Hooft and Veltman [7], involves rather “unphysical” mathematical manipulations, like the regularization of integrals in unphysical dimension d #4, by taking the limit $d \to 4$, and the renormalization of several coupling constants, in a sum with a non-infinite result.

The proposed alternative theory not only provides a simple explanation of color confinement, by the strong gravitational field inside $r < r_0$, but also a simple explanation by a gravitational coupling constant renormalization, bridging over 36 orders of magnitude, by setting the work W of the large gravitational force F to zero over the vanishing thickness d of the event horizon

$$\lim_{d \to 0} W = F \times d \to 0 \tag{12}$$

thereby conserving energy for a particle crossing the event horizon.

In crossing the event horizon, a particle would disintegrate in a burst of radiation, or for quarks, disintegrate into colorless particles [6].

4. Conclusion

The purpose of this is to bring attention to much simpler assumptions than the 10 dimensions of supersymmetric string theories. The mathematical wrong turn began with the Kaluza-Klein theory, where with the introduction of a fifth dimension, the motion of a charged particle in an electromagnetic field could be described by a geodesic in a space-time with five dimensions. It was originally rejected by Einstein, as it required a 10-dimensional space to accommodate the other forces besides electromagnetism.

The older, very successful “Russian doll” approach, solid matter-atom-nucleus-quark, was abandoned with the failure of pre-quark preon models, not observed by either the Large Hadron Collider or detected in Cosmic radiation. Such a model would have to be able to bridge the “desert” from 100 GeV to 10^{19} GeV that is all the way up to the Planck energy. However, the Planck mass plasma conjecture makes it easy to establish such a bridge, via the intermediate energy of ~10^{13} GeV, only six orders of magnitude below the Planck energy [8].

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