The X-cohomology

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Abstract

With help of a closed form over the manifold, an X-cohomology of the manifold is defined

1 Definition

We consider a closed 1-form X over the manifold M; $dX = 0$. Then we can define:

$$d_X\alpha = d\alpha + X \wedge \alpha$$

It is easy to verify that:

Theorem 1

$$d_X \circ d_X = 0$$

Demonstration 1 *Indeed:*

$$d(d\alpha + X \wedge \alpha) + X \wedge (d\alpha + X \wedge \alpha) = 0$$

So, we can have an X-cohomology of the manifold M:

Definition 1

$$XH^*(M, \mathbf{R}) = Ker(d_X)/Im(d_X)$$

2 The X-cohomology as a module

Theorem 2 *The X-cohomology $XH^*(M, \mathbf{R})$ is a module over the cohomology of De Rham viewed as a real algebra, $H^*(M, \mathbf{R})$.*

Demonstration 2 *If:*

$$d_X\alpha = 0$$

then:

$$d_X(\alpha \wedge \beta) = (-1)^{deg(\alpha)}(\alpha \wedge d\beta)$$

And if:

$$d\beta = 0$$

then:

$$d_X(\alpha \wedge \beta) = (d_X\alpha) \wedge \beta$$

References

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