VIKOR based MAGDM Strategy with Trapezoidal Neutrosophic Numbers

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Abstract. VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is a popular strategy for multi- attribute decision making (MADM). We extend the VIKOR strategy for multi- attribute group decision making (MAGDM) problems in trapezoidal neutrosophic number environment. In decision making situation, the attribute values are expressed in terms of single-valued trapezoidal neutrosophic numbers. Then we develop an extended VIKOR strategy to deal with MAGDM with single-valued trapezoidal neutrosophic numbers. The influence of decision-making mechanism coefficient is presented. To illustrate and validate the proposed VIKOR strategy, we solve an illustrative numerical example of MAGDM problem in trapezoidal fuzzy neutrosophic number environment.

Keywords: Neutrosophic set, Trapezoidal neutrosophic fuzzy number, Multi-attribute decision making, VIKOR strategy..

1. Introduction:

Smarandache [1] poineered the neutrosophic set based on neutrosophy in 1998. In 2010, Wang et al. [2] proposed single valued neutrosophic set (SVNS). SVNS has been successfully employed to solve decision making problems [3-31], image processing [32-35], conflict resolution [36], educational problem [37, 38], social problem [39,40], etc. Broumi et al. [41] presents an overview neutrosophic sets. Recently, Peng and Dai [42] presented a bibliometric analysis of neutrosophic sets for last two decades.

Single valued trapezoidal neutrosophic number (SVTrNN) [43] is an extension of SVNS. Each element of SVTrNN is characterized by trapezoidal number with truth membership degree, indeterminacy membership degree and falsity membership degree. Deli and Subhas [10] developed a ranking strategy of SVTrNN and employed the strategy to solve multi attribute decision making (MADM) problem. Biswas et al. [7] presented value and ambiguity based ranking strategy for SVTrNN and employed the strategy to deal with MADM problem. Biswas et al. [44] developed TOPSIS strategy for MADM with trapezoidal neutrosophic numbers (TrNNs). Biswas et al. [45] presented distance measure based MADM strategy with interval trapezoidal neutrosophic numbers (ITrNNs).

In 1998 Opricovic [46] first introduced the VIKOR strategy for multi criteria group decision making (MAGDM) problem with conflicting criteria [47,48] in crisp environment.

 Pouresmaeil et al.[49] proposed an MAGDM strategy based on TOPSIS and VIKOR in SVNS environment. Bausys and Zavadskas[50] proposed the VIKOR strategy in interval neutrosophic set (INS) environment. Huang et al.[51] proposed VIKOR strategy for interval neutrosophic MAGDM. Liu and Zhang[52] studied VIKOR strategy in neutrosophic hesitant fuzzy set environment. In 2017, Hu et al.[53] proposed an interval neutrosophic projection based VIKOR strategy for doctor selection problem. In 2017, Selvakumari et al. [54] studied VIKOR strategy using octagonal neutrosophic soft matrix. Pramanik et al. [55] developed VIKOR based MAGDM strategy in bipolar neutrosophic set environment. In 2018, Pramanik et al. [56] proposed VIKOR strategy for MAGDM with neutrosophic cubic information. Dalapati and Pramanik [57] further revisited VIKOR based MAGDM strategy to remove the shortcoming of [56].

VIKOR strategy in trapezoidal neutrosophic number (TrNN) environment is not studied in the literature. To fill up this research gap, we propose a VIKOR strategy to deal with MAGDM problems in TrNN environment. Also we solve a MAGDM problem based on VIKOR strategy in trapezoidal neutrosophic number.

The rest of the paper is develpoed as follows. In section 2, we briefly describe some definitions of trapezoidal fuzzy number, single valued trapezoidal neutrosophic number, trapezoidal neutrosophic weighted arithmetic averaging (TrNWAA) operator, Hamming distance between two trapezoidal neutrosophic fuzzy numbers. In section 3, we briefly describe extended VIKOR strategy. Thereafter in section 4, we present VIKOR strategy for trapezoidal neutrosophic number. In section 5, we solve an MAGDM problem using the proposed VIKOR strategy. In section 6, we present the sensitivity analysis. In section 7, we present conclusion and future scopen of research.

2. Preliminaries

In this section, we present some basic definitions of fuzzy sets, neutrosophic sets, single-valued neutrosophic sets, and single-valued trapezoidal neutrosophic number.

Definition 2.1 [58] Let Z be a universal set . Then, a fuzzy set F is defined by, (1)

 $F = \{ \}$

where $V_F(z)$ is the degree of membership which map Z to [0,1] i.e , $V_F : Z \rightarrow [0,1]$.

Definition 2.2[1]Let Z be an universal set . A neutrosophic set N can be presented of the form: $N = \{ \langle z : T_{\nu}(z), I_{\nu}(z), F_{\nu}(z) \rangle \}$ $\leq \in \mathbb{Z} \}$ (2)

where the function $T, I, F: Z \rightarrow]0,1^{\dagger}$ define repectively the degree of truth membership, the degree of indeterminacy, and degree of non-membership or falsity of the element $z \in Z$ and satisfy the condition,

 $0 \leq T_{N}(z) + F_{N}(z) + I_{N}(z) \leq 3^{+}$ (3)

Definition 2.3 [2] Let *Z* be a universal set. A single-valued neutrosophic set N in *Z* is defined by $N = \{ \langle z : T_{N}(z), I_{N}(z), F_{N}(z) \rangle \}$ $z \in Z \}$

where $T_N(z)$: $Z \rightarrow [0,1]$, $I_N(z)$: $Z \rightarrow [0,1]$ and $F_N(z)$: $Z \rightarrow [0,1]$ with the condition $0 \le T_N(z) + F_N(z) + I_N(z) \le 3$ for all $z \in Z$. The functions $T_{N}(z)$, $I_{N}(z)$ and $F_{N}(z)$ are respectively, the truth membership function, the indeterminacy membership function and the falsity membership function of the element z to the set N. and supervator displacement
and substitute a finition 2.1 [58] Let Z be a universal set . Then, a fuzzy set F is defined by,
 $\{ \langle z, v_{F}(z): z \in Z \rangle \}$
ere $V_{F}(z)$ is the degree of membership which map Z to [0,1] i.e , $v_{F}:$

Definition 2.4[59] A generalized trapezoidal fuzzy number N denoted by $N = (b_1, b_2, b_3, b_4; v)$ is described as a fuzzy subset of the real number R with membership function κ which is defined by

$$
\kappa_N(x) = \begin{cases}\n\frac{(x - b_1)v}{(b_2 - b_1)}, b_1 \le x < b_2 \\
v, b_2 \le x \le b_3 \\
\frac{(b_4 - x)v}{(b_4 - b_3)}, b_3 < x \le b_4 \\
0, \text{otherwise}\n\end{cases} \tag{4}
$$

where b_1 , b_2 , b_3 , b_4 are real number satisfying $b_1 \le b_2 \le b_3 \le b_4$ and v is the membership degree. **Definition 2.5** [43, 44] Let x be a TrNN. Then, its truth, indeterminacy and falsity membership functions are presented respectively as:

$$
T_x(z) = \begin{cases} \frac{(z-b_1)t_x}{(b_2 - b_1)} b_1 \le z < b_2, \\ t_x, b_2 \le z \le b_3 \\ \frac{(b_4 - z)t_x}{(b_4 - b_3)}, b_3 \le z \le b_4 \\ 0, otherwise \\ 0, otherwise \end{cases}
$$
\n
$$
I_x(z) = \begin{cases} \frac{(b_2 - z) + (z - b_1)i_x}{(b - a)}, b_1 \le z < b_2 \\ i_x, b_2 \le z \le b_3 \\ \frac{z - b_3 + (b_4 - z)i_x}{b_4 - b_3}, b_3 < z \le b_4 \\ 0, otherwise \end{cases}
$$
\n(6)

$$
F_x(z) = \begin{cases} \frac{b_2 - z + (z - b_1)f_x}{b_2 - b_1}, b_1 \le z < b_2\\ f_x, b_2 \le z \le b_3\\ \frac{z - b_3 + (b_4 - z)f_x}{b_4 - b_3}, b_3 < z \le b_4\\ 0, \text{otherwise} \end{cases} \tag{7}
$$

Here $0 \le T_x(z) \le 1, 0 \le I_x(z) \le 1$ and $0 \le F_x(z) \le 1$ and $0 \le T_x(z) + I_x(z) + F_x(z) \le 3; b_1, b_2, b_3, b_4 \in R$. Then $x = ([b_1, b_2, b_3, b_4] : t_x, i_x, f_x)$ is called a neutrosophic trapezoidal number.

Definition 2.6 [43] Let $m_i = \langle (b_{i_1}, b_{2i}, b_{3i}, b_{4i}) ; T_{m_i}, I_{m_i}, F_{m_i} \rangle$ (i=1,2,....n) be a collection of TrNNs, then a trapezoidal neutrosophic number weighted arithmetic averaging (TrNNWAA) operator is defined as follows:

$$
TrNNWAA(m_1, m_2, \dots, m_n) = \sum_{i=1}^{n} w_i m_i
$$
\n(8)

where, w_i is the weight of m_i (i=1,2,...n) such that $w_i > 0$ and $\sum_{i=1}^{n} w_i =$ n $\sum_{i=1}^{n} w_i = 1$. Specially, when $w_i = 1/n$ for

i=1,2,……n, the TrNNWAA operator reduced to the trapezoidal neutrosophic arithmetic averaging operator. **Definition 2.7[44]** Let $m_1 = (p_1, q_1, r_1, s_1]; t_{m_1}, t_{m_1}, f_{m_1})$ and $m_2 = (p_2, q_2, r_2, s_2]; t_{m_2}, t_{m_2}, f_{m_2})$ be any two TrNNs, then the normalized Hamming distance between m_1 and m_2 is defined as:

$$
d(m_1, m_2) = \frac{1}{12} \left(\begin{vmatrix} p_1(2+t_{m_1} - i_{m_1} - f_{m_1}) - p_2(2+t_{m_2} - i_{m_2} - f_{m_2}) \end{vmatrix} + \left| q_1(2+t_{m_1} - i_{m_1} - f_{m_1}) - q_2(2+t_{m_2} - i_{m_2} - f_{m_2}) \right| + \left| q_1(2+t_{m_1} - i_{m_1} - f_{m_1}) - q_2(2+t_{m_2} - i_{m_2} - f_{m_2}) \right| \right)
$$
(9)

2.8. Standardize the decision matrix[44]

Let $D = (b_{ij})_{p \times n}$ be a neutrosophic matrix, where $b_{ij} = ([b_{ij}^1, b_{ij}^2, b_{ij}^3, b_{ij}^4]; t_{b_{ij}}, t_{b_{ij}}, t_{b_{ij}}, t_{b_{ij}})$ is the rating value of the alternative x_i with respect to attribute y_i . Now we eliminate the effect of different physical dimension into decision making process, we will standardized the decision matrix $(b_{ij})_{p \times n}$ based on benefit type and cost type attribute.

The standardized decision matrix by denoted $D^* = (s_{ij})_{p \times n}$

1. For benefit type attribute

$$
b_{ij}^* = \left(\left[\frac{b_{ij}^1}{v_j^+}, \frac{b_{ij}^2}{v_j^+}, \frac{b_{ij}^3}{v_j^+}, \frac{b_{ij}^4}{v_j^+}\right]; t_{b_{ij}}, i_{b_{ij}}, f_{b_{ij}}\right)
$$
\n(10)

2.For cost type attribute:

$$
b_{ij}^* = \left(\left[\frac{b_{ij}^1}{v_j^+}, \frac{b_{ij}^2}{v_j^+}, \frac{b_{ij}^3}{v_j^+}, \frac{b_{ij}^4}{v_j^+} \right]; t_{b_{ij}}, i_{b_{ij}}, f_{b_{ij}} \right)
$$
\n(11)

Where $v_j^+ = \max\{b_{ij}^4 : i = 1, 2, ..., p\}$ and $v_j^- = \min\{b_{ij}^1 : i = 1, 2, ..., p\}$ for j=1,2,.......n Then we obtain standardized matrix

$$
D^* = (s_{ij})_{m \times n} = \begin{pmatrix} b_1 & b_2 & \dots & b_n \\ a_1 & s_{11} & s_{12} & \dots & s_{1n} \\ a_2 & s_{21} & s_{22} & \dots & s_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ a_p & s_{p1} & s_{p2} & \dots & s_{pn} \end{pmatrix}
$$
 (12)

3.VIKOR Strategy for MADM

Suppoose that every alternative is evaluated subject to every criterion function. The compromise ranking is prepared by comparing the measure of closeness to the ideal alternative. Assume that the B_1, B_2, \ldots, B_s are the is alternatives. For the alternative B_i , assume that the rating of the jth aspect is h_{ij} , i.e. h_{ij} is the value of jth criterion for the alternative B_i ; the number of criteria is assumed to be r. VIKOR strategy [47] is presented as follows:

$$
L_{iq} = \{ \sum_{i=1}^{r} [(h_i^+ - h_{ij})/(h_i^+ - h_i^-)]^q \}^{\frac{1}{q}}, 1 \le q \le \infty; i = 1, 2, \dots, s
$$
\n(13)

To formulate ranking measure, $L_{1,i}$ (as S_i) and $L_{\infty,i}$ (as R_i) are employed. The solution obtained by min S_i reflects a maximum group utility ("majority" rule), and the solution obtained by min R_i reflects a minimum individual regret of the ''opponent".

VIKOR stratgey consists of the following steps:

(a) Determine the best h_j^+ and the worst h_j^- values of all criteria j = 1,2,...,n. If the jth criterion is benefit type, then

$$
h_j^+ = \max_i h_{ij}, \mathbf{h}_j^- = \min_i h_{ij}
$$
 (14)

For cost type criterion,

$$
h_j^+ = \min_i h_{ij}, \quad h_j^- = \max_i h_{ij}
$$
\n⁽¹⁵⁾

(b) Compute the values F_i and G_i ; i = 1,2,...,m, by these relations:

$$
F_i = \sum_{j=1}^{n} \frac{w_j (\mathbf{h}_j^+ - \mathbf{h}_{ij})}{(\mathbf{h}_j^+ - \mathbf{h}_j^-)}
$$
(16)

$$
G_i = \max_j \left(\frac{w_j (\mathbf{h}_j^+ - h_{ij})}{(\mathbf{h}_j^+ - h_j^-)} \right) \tag{17}
$$

where w_j (j=1, 2, ..., r) represent the weights of criteria.

(c) Evaluate the value K_i ; i = 1,2,..., s, using following relation:

$$
K_i = \frac{\nu(F_i - F^+)}{(F - F^+)} + \frac{(1 - \nu)(G_i - G^+)}{(G^+ - G^+)}\tag{18}
$$

where

$$
F^+ = \min_i F_i, F^- = \max_i F_i
$$

 $G^+ = \min_i G_i, G^- = \max_i G_i$

Here, v indicates the weight of the strategy of ''the majority of criteria" (or ''the maximum group utility"). Here we set $v = 0.5$.

- (d) Sorting by the values F, G and K in decreasing order, ranking of the alternatives is done.
- (e) Propose the alternative B^1 as a compromise solution that is ranked the best by the measure K (Minimum) subject to the following two conditions:
- C1. Acceptable advantage:

$K(B²) - K(B¹) \ge DK$

where B^2 is the alternative with second position in the ranking list by K; DK= 1/(s-1); s = the number of alternatives.

C2. Acceptable stability in decision making:

Using F or/and G, alternative B^1 must also be the best ranked. The compromise solution is stable that could be "voting by majority rule" (when $v > 0.5$ is needed), or "by consensus" $v = 0.5$, or "with veto" ($v < 0.5$). The weight of the decision making strategy of "the majority of criteria" (or "the maximum group utility") is denoted by v.

- If only the condition C2 is not satisfied, then the alternatives B^1 and B^2 are compromise solutions. Or
- If the condition C1 is not satisfied, alternatives B^1, B^2, \dots, B^M are compromise solutions.

 B^M is determined by the relation $K(B^2) - K(B^1) \geq DK$ for maximum M (the positions of these alternatives are ''in closeness").

The best alternative is chosen based on the minimum value of K.

4.VIKOR strategy for solving MAGDM problem in trapezoidal neutrosophic number environment:

Let $\alpha = {\alpha_1, \alpha_2, \dots, \alpha_r}$ and $\beta = {\beta_1, \beta_2, \dots, \beta_n}$ be the sets of alternatives and attributes. Let $\tau = {\tau_1, \tau_2, \dots, \tau_n}$ be the set of weights of the attributes , where $\tau_i \ge 0$ and $\sum_{i=1}^{\infty}$ $\sum_{i=1}^{n} \tau_i = 1$ $\sum_{i=1}^{\infty}$ ^t τ $\sum_{i=1}^{n} \tau_i = 1$. Let $B = {B_1, B_2, ... B_K }$ be the set of K decision makers and $\sigma = {\{\sigma_1, \sigma_2, \dots \dots \sigma_K\}}$ be the set of weights of the decision makers, where $\sigma_K \ge 0$ and $\sum_{i=1}^{K} \sigma_i = 1$ 1 $\sum \sigma_i = 1$. i $=$

The MAGDM strategy is described as follows:

Step-1: Let $D = (p_{ij}^M)$ (M=1,2,.........s) be the M-th decision matrix where α_i are alternative with respect to attribute β_i . The M-th decision matrix denoted by D^M is presented as:

$$
D^{M} = \begin{pmatrix} \beta_{1} & \beta_{2} & \dots & \beta_{n} \\ \alpha_{1} & p_{11}^{M} & p_{12}^{M} & \dots & p_{1n}^{M} \\ \alpha_{2} & p_{21}^{M} & p_{22}^{M} & \dots & p_{2n}^{M} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{r} & p_{r1}^{M} & p_{r2}^{M} & \dots & p_{rn}^{M} \end{pmatrix}
$$
 (19)

where $M = 1, 2, \dots, s$; i =1,2,...........,r;j = 1, 2,, n.

Step-2: We standardize the decision matrix to tackle the cost and benefit criteria. To standardize the benefit criteria, we use the equation (10) and for cost criteria, we use (11). After standardizing, the decision matrix reduces to

$$
D^{M} = \begin{pmatrix} \beta_{1} & \beta_{2} & \dots & \beta_{n} \\ \alpha_{1} & p_{11}^{*M} & p_{12}^{*M} & \dots & p_{1n}^{*M} \\ \alpha_{2} & p_{21}^{*M} & p_{22}^{*M} & \dots & p_{2n}^{*M} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{r} & p_{r1}^{*M} & p_{r2}^{*M} & \dots & p_{rn}^{*M} \end{pmatrix}
$$

M = 1, 2, ..., s; i=1, 2, ..., r; j = 1, 2, ..., n.

Step-3:To obtain aggregate decision matrix, we use trapezoidal neutrosophic number weighted arithmetic operator(TrNNWAA) which is presented below:

$$
p_{ij} = TrNNWAA(p_{ij}^1, p_{ij}^2, \dots, p_{ij}^M)
$$

=
$$
\sum_{q=1}^{M} \sigma_q p_{ij}^q
$$
 (20)

Therefore, we obtain the aggregated decision matrix as:

$$
D^{M} = \begin{pmatrix} \beta_{1} & \beta_{2} & \dots & \beta_{n} \\ \alpha_{1} & p_{11} & p_{12} & \dots & p_{1n} \\ \alpha_{2} & p_{21} & p_{22} & \dots & p_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha_{r} & p_{r1} & p_{r2} & \dots & p_{rn} \end{pmatrix}
$$

Step -4: Define the positive ideal solution R^+ and negative ideal solution R^-

For the benefit attribute:

 $([b_1^+, b_2^+, b_3^+, b_4^+]; t_h^+, i_h^-, f_h^-)$ (23) $= ([b_1^+, b_2^+, b_3^+, b_4^+]; \max t_b, \min t_b, \min f_b)$

 $([b_{1}^{-}, b_{2}^{-}, b_{3}^{-}, b_{4}^{-}]t_{h}^{-}, i_{h}^{+}, f_{h}^{+})$ (24) $= ([b_1^-, b_2^-, b_3^-, b_4^-]; \min t_b, \max t_b, \max f_b)$ For the cost attribute :

$$
([b_1^+, b_2^+, b_3^+, b_4^+]; t_b^-, t_b^+, f_b^+)
$$
\n
$$
-([b_1^+, b_1^+, b_1^+, b_1^+]; \min_{b \in \mathcal{A}} t_b^-, \max_{b \in \mathcal{A}} t_b^+)
$$
\n(25)

$$
-(\left[\begin{matrix}b_1^-,b_2^-,b_3^-,b_4^-\end{matrix}\right];\min t_b,\max t_b,\max t_b)
$$
\n
$$
(\left[\begin{matrix}b_1^-,b_2^-,b_3^-,b_4^-\end{matrix}\right]t_b^+,t_b^-,t_b^-)
$$
\n(26)

 $= ([b_1^-, b_2^-, b_3^-, b_4^+]; \max t_b, \min t_b, \min f_b)$ Step 5:Compute

$$
\Gamma_m = \sum_{j=1}^n \frac{\tau_m \times d(((b_1^+, b_2^+, b_3^+, b_4^+]; t_b^+, t_b^-, f_b^-), ((b_{j1}, b_{j2}, b_{j3}, b_{j4}]; t_m, t_m, f_m))}{d(((b_1^+, b_2^+, b_3^+, b_4^+]; t_b^+, t_b^-, f_b^-), ((b_1^-, b_2^-, b_3^-, b_4^+]; t_b^-, t_b^+, f_b^+))}
$$
(27)

$$
Z_m = \max \sum_{j=1}^n \frac{\tau_m \times d(([b_1^+, b_2^+, b_3^+, b_4^+]; t_b^+, i_b^-, f_b^-, (([b_{j1}, b_{j2}, b_{j3}, b_{j4}]; t_{nj}, i_{nj}, f_{nj}))}{d(([b_1^+, b_2^+, b_3^+, b_4^+]; t_b^+, i_b^-, f_b^-, (([b_{j1}, b_{j2}, b_{j3}, b_{j4}]; t_{nj}, i_{nj}, f_{nj}))}
$$
\n
$$
(28)
$$

where
$$
\tau_m
$$
 is the weight of β_m .

Using equation (9), we obtain

$$
d(([b_1^+, b_2^+, b_3^+, b_4^+]; t_b^+, t_b^-, f_b^-), ([b_{j1}, b_{j2}, b_{j3}, b_{j4}]; t_{nj}, t_{nj}, f_{nj})
$$

=
$$
\frac{1}{12} \begin{pmatrix} |b_1^+(2+t_b^+ - i_b^- - f_b^-) - b_{j1}(2+t_{nj} - i_{nj} - f_{nj})| + |b_2^+(2+t_b^+ - i_b^- - f_b^-) - b_{j2}(2+t_{nj} - i_{nj} - f_{nj})| \\ + |b_3^+(2+t_b^+ - i_b^- - f_b^-) - b_{j3}(2+t_{nj} - i_{nj} - f_{nj})| + |b_4^+(2+t_b^+ - i_b^- - f_b^-) - b_{j4}(2+t_{nj} - i_{nj} - f_{nj})| \end{pmatrix}
$$

and

and

$$
d((\left[b_{1}^{+}, b_{2}^{+}, b_{3}^{+}, b_{4}^{+}\right]; t_{b}^{+}, t_{b}^{-}, f_{b}^{-}), (\left[b_{1}^{-}, b_{2}^{-}, b_{3}^{-}, b_{4}^{-}\right]; t_{b}^{-}, t_{b}^{+}, f_{b}^{+}))
$$
\n
$$
=\frac{1}{12}\begin{pmatrix}\n\left|b_{1}^{+}(2+t_{b}^{+}-i_{b}^{-}-f_{b}^{-})-b_{1}^{-}(2+t_{b}^{-}-i_{b}^{+}-f_{b}^{+})\right|\n+ \left|b_{2}^{+}(2+t_{b}^{+}-i_{b}^{-}-f_{b}^{-})-b_{2}^{-}(2+t_{b}^{-}-i_{b}^{+}-f_{b}^{+})\right|\n\end{pmatrix}
$$
\n
$$
=\frac{1}{12}\begin{pmatrix}\n\left|b_{1}^{+}(2+t_{b}^{+}-i_{b}^{-}-f_{b}^{-})-b_{1}^{-}(2+t_{b}^{-}-i_{b}^{+}-f_{b}^{+})\right|\n+ \left|b_{2}^{+}(2+t_{b}^{+}-i_{b}^{-}-f_{b}^{-})-b_{3}^{-}(2+t_{b}^{-}-i_{b}^{+}-f_{b}^{+})\right|\n\end{pmatrix}
$$

Step 6: Compute the Θ by the following formula:

$$
\Theta_m = \Psi \frac{(\Gamma_m - \Gamma_m^*)}{(\Gamma_m - \Gamma_m^*)} + (1 - \Psi) \frac{(Z_m - Z_m^*)}{(Z_m - Z_m^*)} \tag{29}
$$

where $\Gamma_m^* = \min_m \Gamma_m$, $\Gamma_m^- = \max_m \Gamma_m$ (30)

$$
Z_m^* = \min_m Z_m \, , \, Z_m^- = \max_m Z_m \tag{31}
$$

Here, Ψ denotes decision-making mechanism coefficient. If $\Psi \ge 0.5$, then it denotes the maximum group utility ; If $\Psi \le 0.5$ then it is the minimum regret and it is both if $\Psi = 0.5$. Step-7: Rank the alternative by. Γ_m , Z_m , Θ_m .

Step-8: Determine the compromise solution

Obtain alternative Δ as a compromise solution, that is ranked as the best by the measure Θ (Minimum) if the following two conditions are satisfied:

Condition 1. Acceptable stability:

$$
\Theta(\Delta^2) - \Theta(\Delta^1) \ge \frac{1}{r - 1} \tag{32}
$$

where Δ^1 , Δ^2 are the alternatives with first and second position in the ranking list by Θ ; r = the number of alternatives.

Condition 2. Acceptable stability in decision making:

Alternative Δ^1 must also be the best ranked by Γ or/and Z. This compromise solution is stable within whole decision making process.

If one of the conditions is not satisfied, then a set of compromise solutions is as follows:

o Alternatives Δ^1 and Δ^2 are compromise solutions if only condition 2 is not satisfied, or

 $\Delta^1, \Delta^2, ... \Delta^r$ are compromise solutions if condition 1 is not satisfied and Δ^r is decided by constraint $\Theta(\Delta^2) - \Theta(\Delta^1) \le \frac{1}{r-1}$ f $\Theta(\Delta^2) - \Theta(\Delta^1) \le \frac{1}{r-1}$ for maximum r.

Figure 1. VIKOR based MAGDM strategy in trapezoidal neutrosophic number environment

5.Numerical example

To iluustrate the proposed VIKOR strategy, we solve an MAGDM problem adapted from [57]. Assume that an investment company intends to invest a sum of money in the best option. The company constitutes a board of of decision making with three decision makers or experts. The decision makers evaluate the alternatives to invest money.

The alternatives are:

- 1. Car company(α_1)
- 2. Food company(α_2)
- 3. Computer company(α_3)
- 4. Arms company(α_4)

Decision makers take the decision based following three attributes

- 1. Risk factor(β_1)
- 2. Growth factor(β_2)
- 3. Environment impact(β_3)

Suppose, $\tau = (0.30, 0.42, 0.28)$ be the set of weigts of the decision makers and $\sigma = (0.33, 0.39.0.28)$ be the set of weights of the attributes.

Step-1: Construction of decision matrix in TrNNs form

Decision matrix $D¹$

Decision matrix D^2

Decision matrix D^3

Step-2: We do not need to standardize the decision matrix as all the criteria are benefit type.

Step-3: Using TrNNWAA operator defined in equation (20), we obtain aggregate decision matrix as:

Step-5:Using equations (27) and (28), we compute $\Gamma_{\rm m}$ and $Z_{\rm m}$ which are presented as:

$$
\Gamma_{1} = \left(\frac{0.33 \times 0.202}{0.294}\right) + \left(\frac{0.39 \times 0.070}{0.121}\right) + \left(\frac{0.28 \times 0.172}{0.327}\right) = 0.601,
$$
\n
$$
\Gamma_{2} = \left(\frac{0.33 \times 0.238}{0.294}\right) + \left(\frac{0.39 \times 0.119}{0.121}\right) + \left(\frac{0.28 \times 0.181}{0.327}\right) = 0.805,
$$
\n
$$
\Gamma_{3} = \left(\frac{0.33 \times 0.298}{0.294}\right) + \left(\frac{0.39 \times 0}{0.121}\right) + \left(\frac{0.28 \times 0}{0.327}\right) = 0.334,
$$
\n
$$
\Gamma_{4} = \left(\frac{0.33 \times 0}{0.294}\right) + \left(\frac{0.39 \times 0.080}{0.121}\right) + \left(\frac{0.28 \times 0.284}{0.327}\right) = 0.501.
$$

Here, we use Hamming distance to measure the distantance between to TrNNs.

$$
Z_{1} = \max \left\{ \left(\frac{0.33 \times 0.202}{0.294} \right) + \left(\frac{0.39 \times 0.070}{0.121} \right) + \left(\frac{0.28 \times 0.172}{0.327} \right) \right\} = 0.227,
$$

\n
$$
Z_{2} = \max \left\{ \left(\frac{0.33 \times 0.238}{0.294} \right) + \left(\frac{0.39 \times 0.119}{0.121} \right) + \left(\frac{0.28 \times 0.181}{0.327} \right) \right\} = 0.383,
$$

\n
$$
Z_{3} = \max \left\{ \left(\frac{0.33 \times 0.298}{0.294} \right) + \left(\frac{0.39 \times 0}{0.121} \right) + \left(\frac{0.28 \times 0}{0.327} \right) \right\} = 0.334,
$$

\n
$$
Z_{4} = \max \left\{ \left(\frac{0.33 \times 0}{0.294} \right) + \left(\frac{0.39 \times 0.080}{0.121} \right) + \left(\frac{0.28 \times 0.284}{0.327} \right) \right\} = 0.258.
$$

Step-6:Using (29), we calculate Θ_i

 $\Theta_1 = 0.283, \Theta_2 = 1, \Theta_3 = 0.342, \Theta_4 = 0.274$ Step -7: The ranking order of alternatives is obtained as: $\Theta_4 \leq \Theta_1 \leq \Theta_3 \leq \Theta_2$

Table 1. Preference ranking order and compromise solution based on Γ , Z and Θ

Step 8: Determine the compromise solution

If we rank Θ in decreasing order, the best position alternative is Δ_4 with $\Theta(\Delta_4)$ =0.274, and the second best position Δ_1 with Θ (Δ_1)=0.342. Therefore, Θ (Δ_1)- Θ (Δ_4)=0.008<0.33(since r = 4;1/(r-1)=0.33), which does not satisfy the condition $1(\Theta(\Delta^2) - \Theta(\Delta^1))$ 1 1 $\overline{}$ \geq $\frac{1}{r-1}$).

Here Δ_4 is ranked best by Γ and Z and satisfies the condition 2.

So, the compromise solution as follows:

- Θ (Δ_1)- Θ (Δ_4)=0.008<0.33,
- Θ (Δ ₂)- Θ (Δ ₄)=0.726>0.33,

 Θ (Δ_3)- Θ (Δ_4)=0.05<0.33,

Therefore, Δ_2 , Δ_4 are compromise solutions.

6.1 The influence of parameter Ψ

Table 2 reflects how the ranking order of alternatives Θ , varies with the change of the value of Ψ

Table 2. Values of Θ_i (i = 1, 2, 3, 4,5) and ranking of alternatives for different values of Ψ .

7. Conclusions

In this paper, we develop a VIKOR strategy for MAGDM in trapezoidal neutrosophic number environment. TrNNWAA operator and Hamming distance are employed to develop the Vikor strategy for MAGDM. Finally, we solve an MAGDM problem to show the feasibility, applicability and efficiency of the proposed VIKOR strategy. Here we also present a sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives. The proposed VIKOR based MAGDM strategy can be employed to solve MAGDM problems such as brick selection [60, 61], logistics center selection [62], teacher selection [63], weaver selection [64], etc.

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